Lecture – 21 Date: 03.04.2017

- RF Transistor Characteristics
- Two-Port Power Gains
- Turning Gain Element into an Amplifier

Introduction

- The most important and useful circuit element ever devised is the transistor
- Among its other applications, transistors can be used to make gain stages for microwave amplifiers and oscillators
- Its application to **digital** devices and machines get all the press, but they are equally invaluable for **analog** applications, including RF and microwave
- Specifically, a transistor allows us to generate **signal gain**—to transfer energy from a DC source and apply it to an RF signal, without otherwise distorting that signal
- Because of this, we can build two crucial items for most microwave systems: a microwave **amplifier** and its **unstable** cousin, the microwave **oscillator**

Transistors as Gain Elements

A **quiz**!

1. To construct a small-signal amplifier, a **BJT** must be **DC biased** to which **mode**:

A. Active
B. Triode
C. Cutoff
D. Saturation

2. To construct a small-signal amplifier, a **FET** must be DC biased to which mode:

A. Active
B. Triode
C. Cutoff
D. Saturation

3. The **BJT** amplifier **configuration** that typically provides the highest open-circuit **voltage gain** is the:

A. common emitter
B. common source
C. common base
D. common collector
E. common drain
F. common gate

Transistors as Gain Elements (contd.)

4. The **FET** amplifier configuration that typically provides the highest open-circuit voltage gain is the:

A. common emitter

B. common source

C. common base

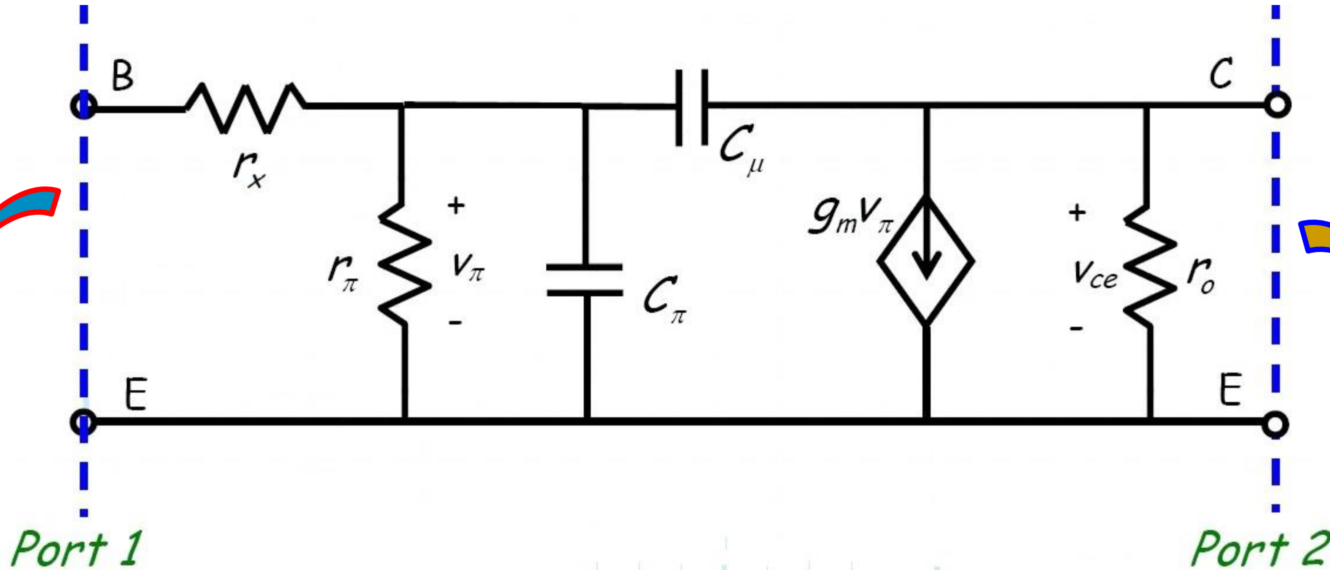
D. common collector

E. common drain

F. common gate

Transistors as Gain Elements (contd.)

- The high-frequency small-signal (hybrid-pi) **model** for a **BJT** in the **common emitter** configuration is:

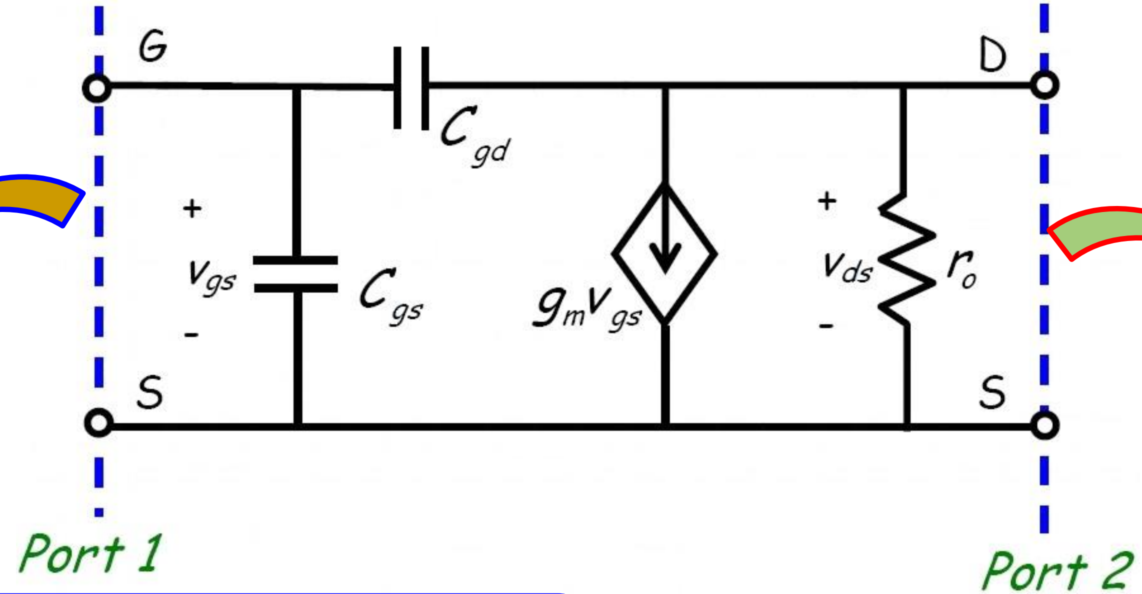


Here the values g_m, r_o, and r_π are all **small-signal parameters**—values determined in part by the **DC bias** of the transistor.

The values r_x, C_π, and C_μ are **parasitic elements**. Generally too small to consider for low-frequency operation, these value make a great difference at microwave frequencies!

Transistors as Gain Elements (contd.)

- Likewise, the high-frequency small-signal **model** for a **MOSFET** device in a **common-source** configuration is:

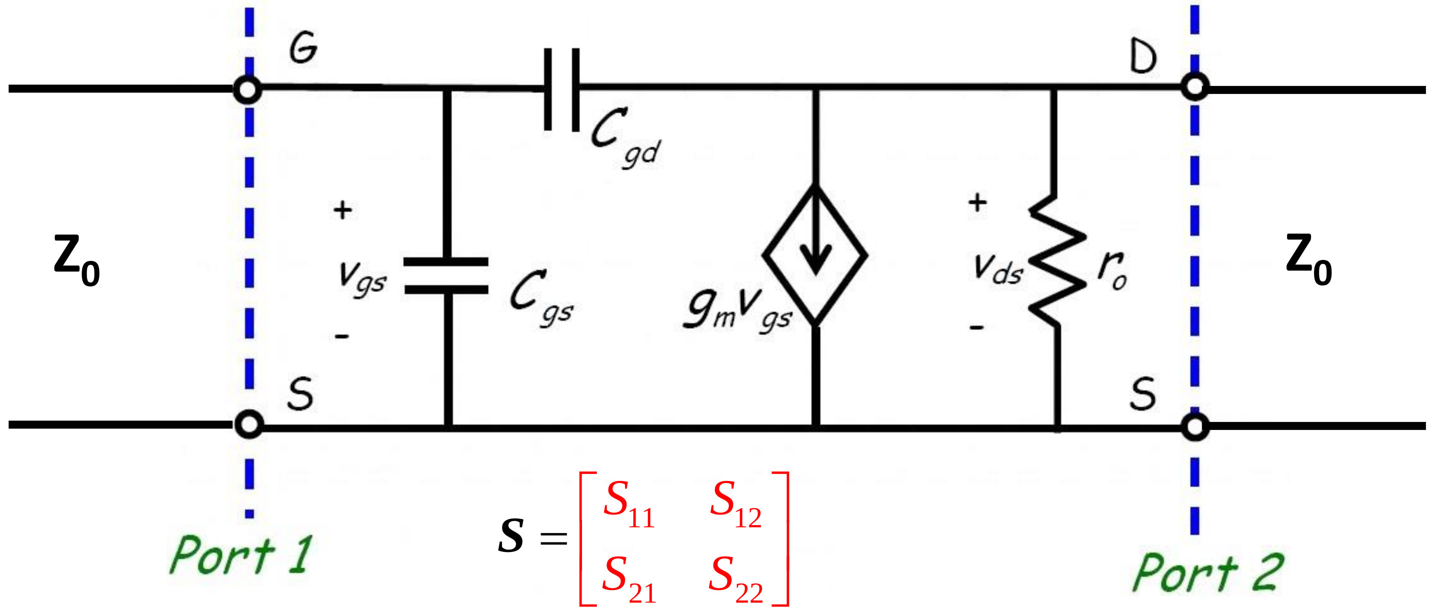


where again g_m, and r_o are small-signal parameters and C_{gs}, and C_{gd} are parasitic elements.

- Note that each of these circuits form a **two-port network!**
- This network we define as a **gain stage**, where port 1 is the **input** port and port 2 the **output** port.

Transistors as Gain Elements (contd.)

- Since they are two-port networks, we can describe them with a **scattering matrix**:



$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

- We can determine this scattering matrix either by direct **measurement** (using a network analyzer) or by **analysis** of the small-signal circuit.

Transistors as Gain Elements (contd.)

- Either way, it can be identified that this two-port network has some **interesting** characteristics!

   1. We will typically find that both $\mathbf{S_{11}}$ and $\mathbf{S_{22}}$ are relatively large (e.g. $0.6 < |S_{11}| < 1.0$).
   2. We will typically find that $\mathbf{S_{12}}$ is relatively small (e.g, $|S_{12}| = 0$)
   3. We will typically find that $\mathbf{S_{21}}$ is much greater than one (e.g, $|S_{21}| = 5.0$)

- As a result, it is evident that this **gain stage** is:

   a. not matched (just look at $|S_{11}|$ and $|S_{22}|$)
   b. not reciprocal (just look at $|S_{12}|$ and $|S_{21}|$)
   c. not lossless—but neither is it lossy ($|S_{11}|^2 + |S_{21}|^2 > 1$)!

This gain stage is an **active** device—the DC bias supplies energy that is converted into RF signal power at the output port. In other words, **more** RF power flows **out** than flows **in**!

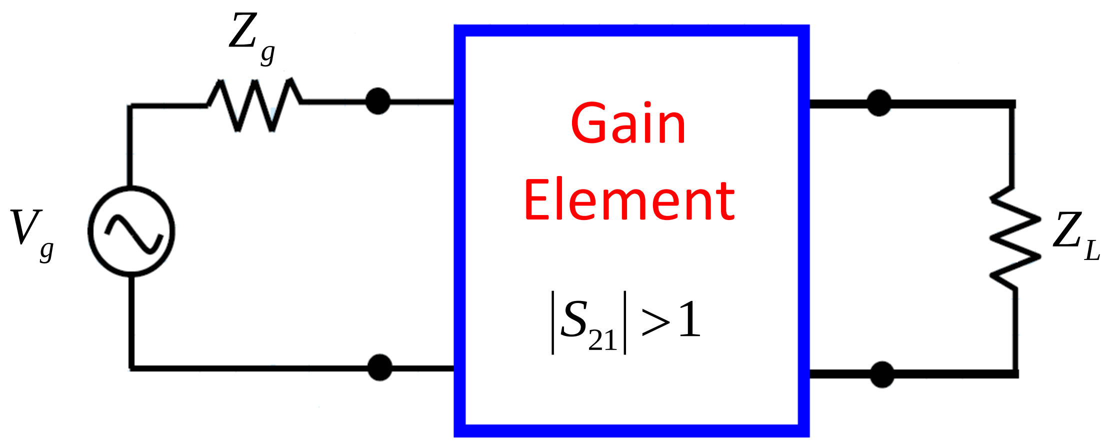
Transistors as Gain Elements (contd.)

Q: So, is this gain stage a high frequency amplifier?

A: It **could** be used as such, but usually we start with this **gain stage** and then carefully design **two** additional networks – one for the **input** and one for the **output**. These three together form a typical high frequency amplifier

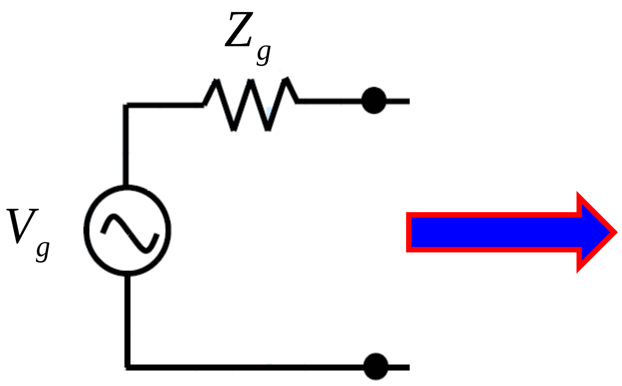
Two-port Power Gains

- Specifying the **gain** of an amplifier is a bit more **ambiguous** than you may think. The problem is that there are so **many** ways to define **power**!
- To begin our discussion of **amplifiers**, we must first define and derive a number of quantities that describe the **rate of energy flow** (i.e., power).
- For this purpose, let us consider a source and a load that are connected together by some gain element



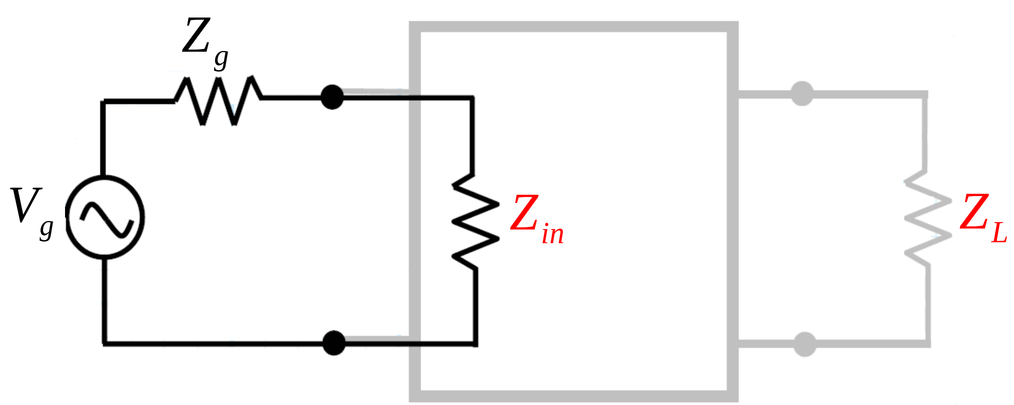
Two-port Power Gains (contd.)

- The first power we consider is the **available power** from **the source**:



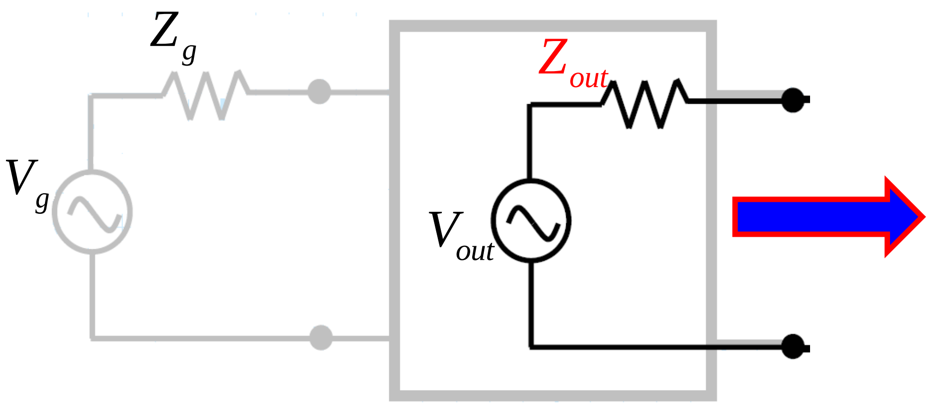
P_{avs} = Power Available from the Source

- We likewise consider the power P_{in} **delivered** by the source; in other words the power **absorbed** by the input impedance of the gain element with a load attached:



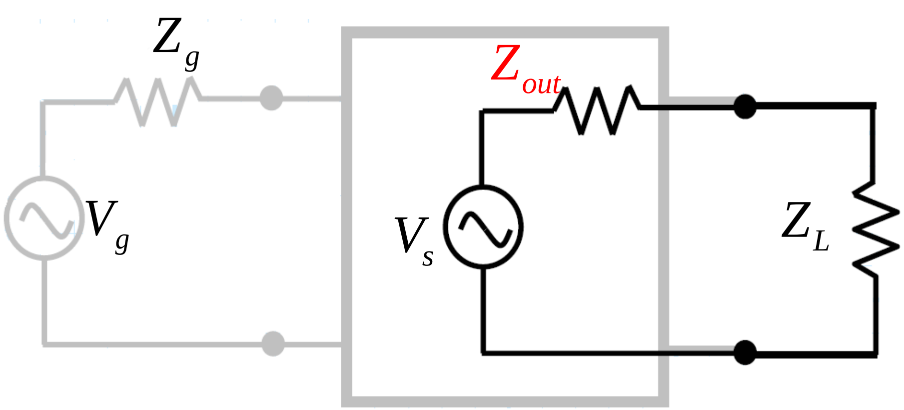
Two-port Power Gains (contd.)

- On the output, we consider the power **available** from the **output** of the gain element:



P_{avn} = Available Power from the Output Port

- And finally, we consider the power P_L **delivered** by the output port—the power absorbed by load Z_L



These four power quantities depend (at least in part) on the **source** parameters V_g and Z_g, **load** Z_L, and the **scattering parameters** S_{11}, S_{21}, S_{22}, S_{12} of the gain element.

Two-port Power Gains (contd.)

Q: Yikes! How can we **determine** the power values in terms of circuit parameters?

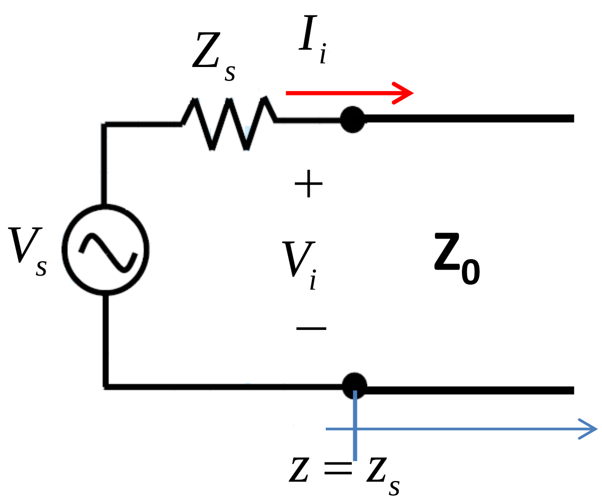
A: Remember, the source, load, and the gain element (i.e. its scattering matrix) are described by a set of **equations**. We simply need to **solve** these simultaneous equations!

This can be algebraically solved (look through Pozar). But lets follow simpler technique using SFG

Two-port Power Gains (contd.)

Q: But there's a **source** in our circuit: How do we handle that in a SFG?

A: For this purpose, let us consider a simple source connected to a transmission line:



- From **KVL** we know that: $V_s = V_i + Z_s I_i$
- Whereas, from the **telegraphers equations** we know that:

$$V_i = V(z = z_s) = V^+ e^{-j\beta z_s} + V^- e^{+j\beta z_s}$$

$$I_i = I(z = z_s) = \frac{V^+}{Z_0} e^{-j\beta z_s} - \frac{V^-}{Z_0} e^{+j\beta z_s}$$

- Let us define following **definitions**:

$a_s \doteq V^- e^{+j\beta z_s}$ (complex amplitude of voltage wave incident on source)

$b_s \doteq V^+ e^{-j\beta z_s}$ (complex amplitude of voltage wave exiting the source)

Two-port Power Gains (contd.)

- Simplification of current and voltage equations give:

$$V_i = V(z = z_s) = b_s + a_s$$

$$I_i = I(z = z_s) = \frac{b_s}{Z_0} - \frac{a_s}{Z_0}$$

- Furthermore, the KVL equation can be written as:

$$V_s = (b_s + a_s) + \frac{Z_s}{Z_0}(b_s - a_s)$$

- Rearrangement results in:

$$b_s = \left(\frac{Z_0}{Z_s + Z_0}\right)V_s + \Gamma_s a_s$$

Where

$$\Gamma_s \doteq \left(\frac{Z_S - Z_0}{Z_s + Z_0}\right)$$

$$\frac{Z_0}{Z_s + Z_0} = \frac{1 - \Gamma_s}{2}$$

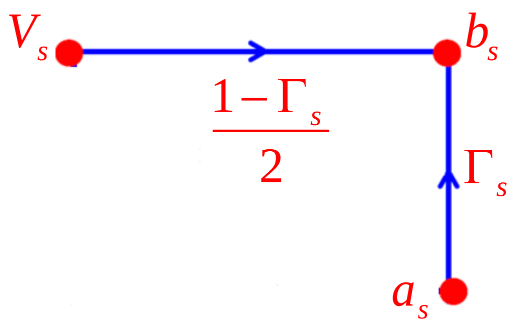
- Therefore:

$$b_s = \left(\frac{1 - \Gamma_s}{2}\right)V_s + \Gamma_s a_s$$

Two-port Power Gains (contd.)

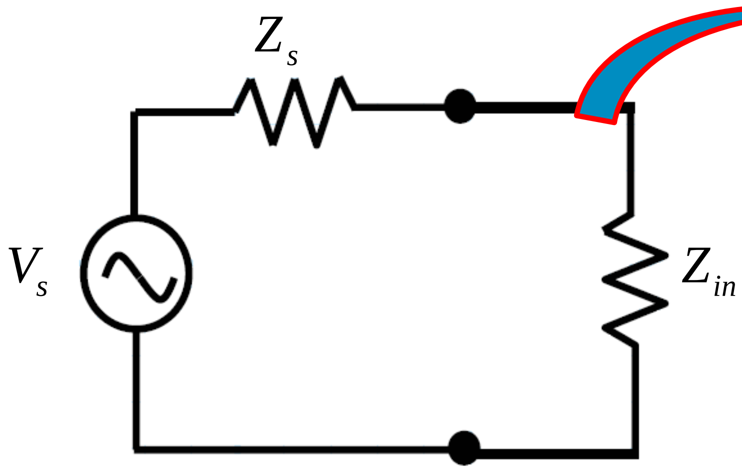
$$b_s = \left(\frac{1-\Gamma_s}{2}\right)V_s + \Gamma_s a_s$$

- This can be expressed using a **SFG** as:



$$b_s = \left(\frac{1-\Gamma_s}{2}\right)V_s + \Gamma_s a_s$$

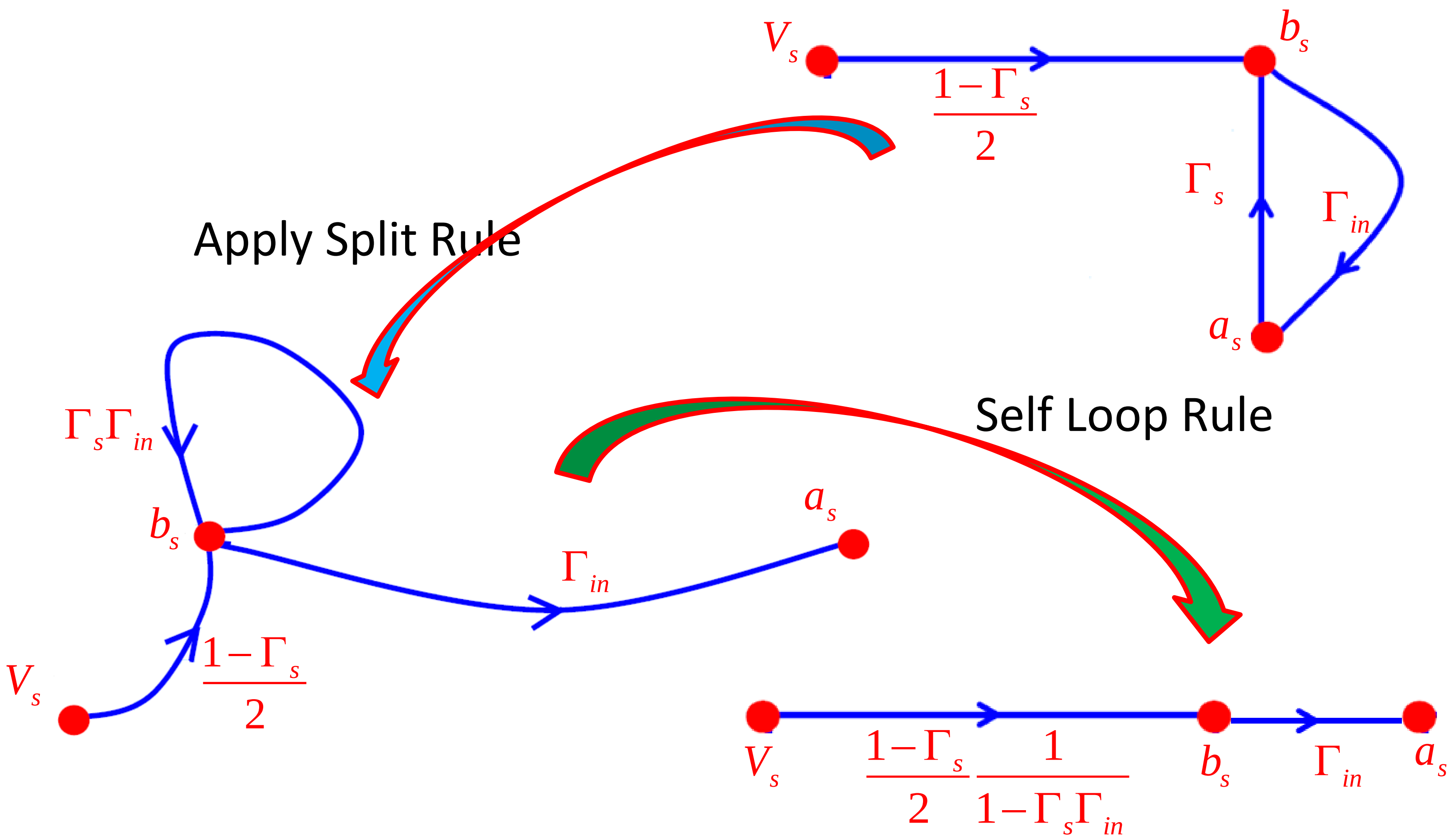
- Now, consider the case where we place an impedance **(e.g., the input** impedance of a two port network) at this source port:



$$\Gamma_{in} \doteq \frac{V^- e^{+j\beta z_s}}{V^+ e^{-j\beta z_s}} = \frac{a_s}{b_s} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

Two-port Power Gains (contd.)

- Thus, the relationship **$a_s = \Gamma_{in} b_s$** can be added to the SFG:



Two-port Power Gains (contd.)

- It can be concluded that:

$$b_s = V_s \frac{1-\Gamma_s}{2} \frac{1}{1-\Gamma_s\Gamma_{in}} \qquad a_s = V_s \frac{1-\Gamma_s}{2} \frac{\Gamma_{in}}{1-\Gamma_s\Gamma_{in}}$$

- Therefore the power incident on the load is:

$$P_{inc} = \frac{|b_s|^2}{2Z_0} = \frac{|V_s|^2}{8Z_0} \frac{|1-\Gamma_s|^2}{|1-\Gamma_s\Gamma_{in}|^2}$$

- And the power reflected from the load is:

$$P_{ref} = \frac{|a_s|^2}{2Z_0} = \frac{|V_s|^2}{8Z_0} \frac{|1-\Gamma_s|^2}{|1-\Gamma_s\Gamma_{in}|^2} |\Gamma_{in}|^2$$

- So that the power absorbed by the load (i.e. the power **delivered by the source) is:**

$$P_{in} = P_{inc} - P_{ref}$$

Two-port Power Gains (contd.)

- Therefore:

$$P_{in} = \frac{|V_s|^2}{2Z_0}\left|\frac{Z_0}{Z_0+Z_s}\right|^2 \frac{1-|\Gamma_{in}|^2}{|1-\Gamma_s\Gamma_{in}|^2}$$

It is evident that the amount of power delivered is **dependent on the value of the input impedance.** To maximize this power, we must find the value Γ_{in} that maximizes the term:

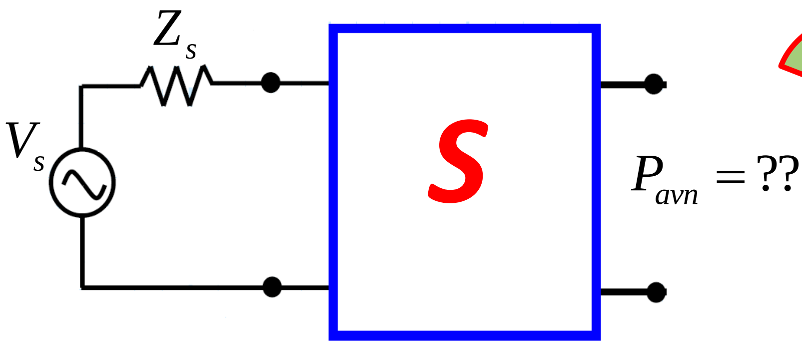
$$\frac{1-|\Gamma_{in}|^2}{|1-\Gamma_s\Gamma_{in}|^2}$$

It can be shown that this term is maximized when $\Gamma_{in} = \Gamma_s^*$. No surprise here; the **conjugate match condition** applies. This maximum value—resulting only when the input impedance is conjugate of the source impedance—is referred to as the **available power of the** source

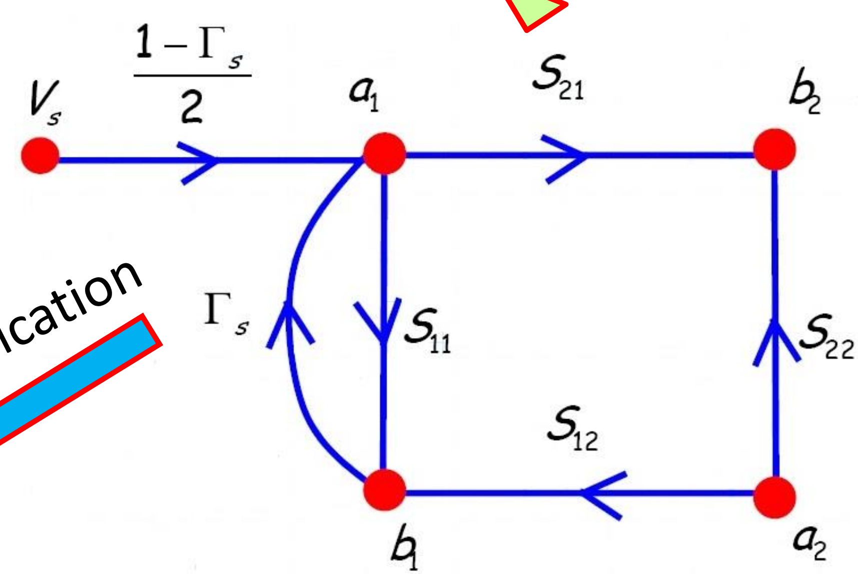
$$P_{avs} = P_{in}\big|_{\Gamma_{in}=\Gamma_s^*} = \frac{|V_s|^2}{8Z_0}\frac{|1-\Gamma_s|^2}{1-|\Gamma_s|^2} = \frac{|V_s|^2}{2}\frac{1}{4\,\mathrm{Re}\{Z_s^*\}}$$

Two-port Power Gains (contd.)

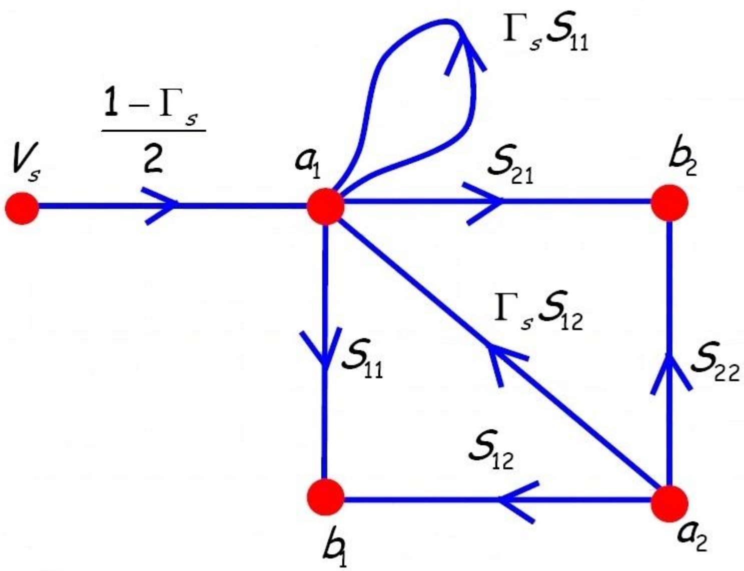
- Now, consider the case where we connect some arbitrary **two port device to the source.** We would like to determine the **available power P_{avn}** from the output port of this two-port device



The SFG for **this network**

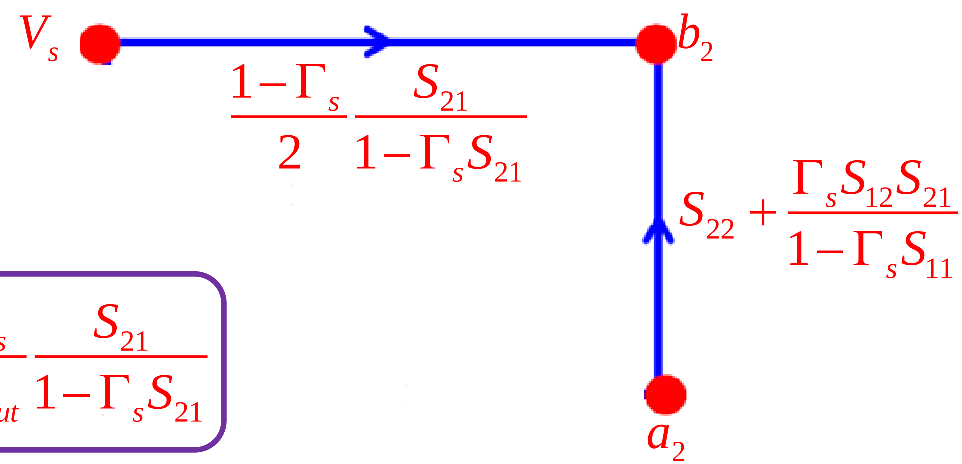


Simplification



Two-port Power Gains (contd.)

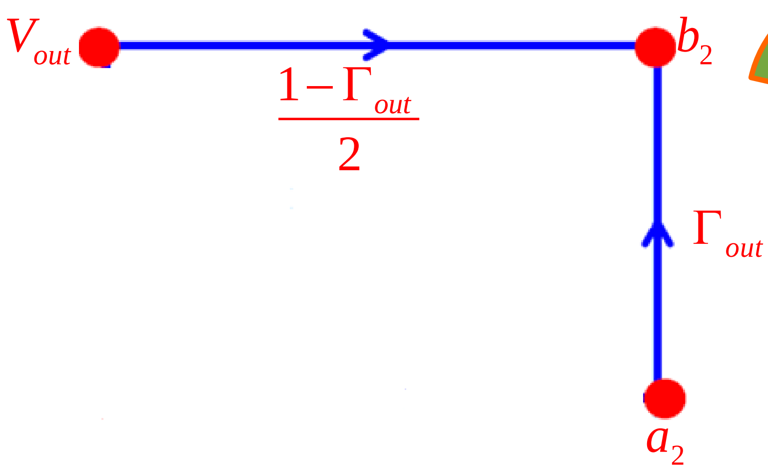
Final simplification results in:



Let us **define following variables**

$$\Gamma_{out} \doteq S_{22} + \frac{\Gamma_s S_{12} S_{21}}{1-\Gamma_s S_{11}}$$

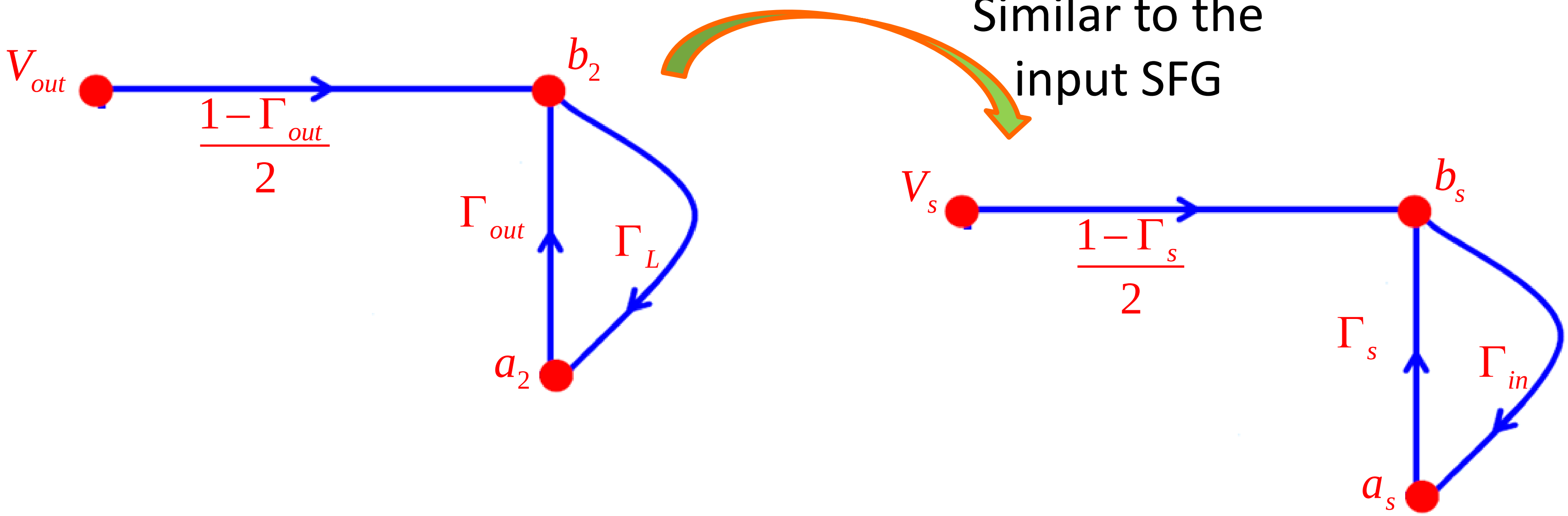
$$V_{out} \doteq V_s \frac{1-\Gamma_s}{1-\Gamma_{out}} \frac{S_{21}}{1-\Gamma_s S_{21}}$$



It is apparent that V_{out} and Γ_{out} define an **equivalent source** created when the original source is connected to a two-port device.

Two-port Power Gains (contd.)

- Thus, when some **load is connected to the output of the two port** device, the SFG will be:



- As a result, the **delivered power is precisely the same as the** original case, with the exception that we use the **equivalent** values defined above:

$$\boxed{P_L = \frac{|V_{out}|^2}{8Z_0}\frac{|1-\Gamma_{out}|^2}{|1-\Gamma_{out}\Gamma_L|^2}\left(1-|\Gamma_L|^2\right)}$$

Two-port Power Gains (contd.)

- This can also be written as:

$$P_L = \frac{|V_s|^2}{8Z_0} \frac{|S_{21}|^2}{|1-\Gamma_s S_{11}|^2} \frac{|1-\Gamma_s|^2}{|1-\Gamma_{out}\Gamma_L|^2}\left(1-|\Gamma_L|^2\right)$$

- The **available power from port 2 is simply the** maximum possible power absorbed by a load Γ_L. This **is** found by maximizing the term:

$$\frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}$$

This **again** occurs when $\Gamma_L = \Gamma_{out}^*$. Thus, maximum power transfer occurs when the load is **conjugate matched** to the equivalent source impedance Z_{out} (Γ_{out}).

As a result the **available power** from port 2 is:

$$P_{avn} = P_L\big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_s|^2}{8Z_0} \frac{|S_{21}|^2}{|1-\Gamma_s S_{11}|^2} \frac{|1-\Gamma_s|^2}{|1-\Gamma_{out}|^2}$$

Two-port Power Gains (contd.)

- There are **three** standard ways of defining amplifier gain:

1. Power Gain

It is defined as: $\boxed{G \doteq \frac{P_L}{P_{in}}}$

It describes the increase in **delivered** (i.e., absorbed) power from input to output

Therefore: $$G \doteq \frac{P_L}{P_{in}} = \frac{|S_{21}|^2}{1-|\Gamma_{in}|^2}\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

2. Available Power Gain

$$G_A \doteq \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2}{1-|\Gamma_{out}|^2}\frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2}$$

it describes the increase in **available** power from input to output at device

Two-port Power Gains (contd.)

3. Transducer Gain

$$G_T \doteq \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2\left(1-|\Gamma_s|^2\right)\left(1-|\Gamma_L|^2\right)}{\left(|1-\Gamma_s\Gamma_{in}|^2\right)\left(|1-\Gamma_L S_{22}|^2\right)}$$

it relates the power available from the source to the power delivered to the load. It in effect describes how **effectual** the amplifier was in extracting the available power from the source, increasing this power, and then delivering the power to the load.

- There are likewise a few **special cases** that we need to be aware of. If both the source and the load impedance are Z_0, then we find $\Gamma_s = \Gamma_L = 0$, and then not surprisingly:

$$G_T = |S_{21}|^2$$

Two-port Power Gains (contd.)

Q: I'm so confused! **Which** gain definitions should I use when specifying an amplifier? **Which** gain definition do amplifier vendors use to specify their performance?

A: We find that for a **well-designed** amplifier, the three gain values generally do **not** provide significantly differing values. Most often then, microwave amplifier vendors do **not** explicitly specify the three values (for an assumed Z_0 source and load impedance). Instead, they provide a somewhat ambiguous value that they simply call **gain***.

* If you are inclined to be mischievous, ask an amplifier vendor if their gain spec. is actually **available** gain or **transducer** gain.

Turning a Gain Element into an Amplifier

- Say the design criteria for our amplifier is to maximize the power delivered to the load (i.e., maximize P_L). This power is maximized when:
    1. The available power from the **source** is entirely delivered to the **input** of the gain element, i.e., $P_{in} = P_{avs}$
    2. The available power from the **output** of the gain element is entirely delivered to the **load** $P_L = P_{avn}$
    3. Recall this happy occurrence results when $\Gamma_{in} = \Gamma_s^*$ and $\Gamma_L = \Gamma_{out}^*$.

Q: But what if this is **not** the case? What if our gain element is not matched to our source, or to our load? Must we simply accept inferior power transfer?

A: Nope! Remember, we can always build **lossless matching networks** to efficiently transfer power between mismatched sources and loads.

Q: I see! We need to **modify** the source impedance Z_s and modify the output impedance Z_{out} such that $Z_{in} = Z_s^*$ and $Z_L = Z_{out}^*$. Right?

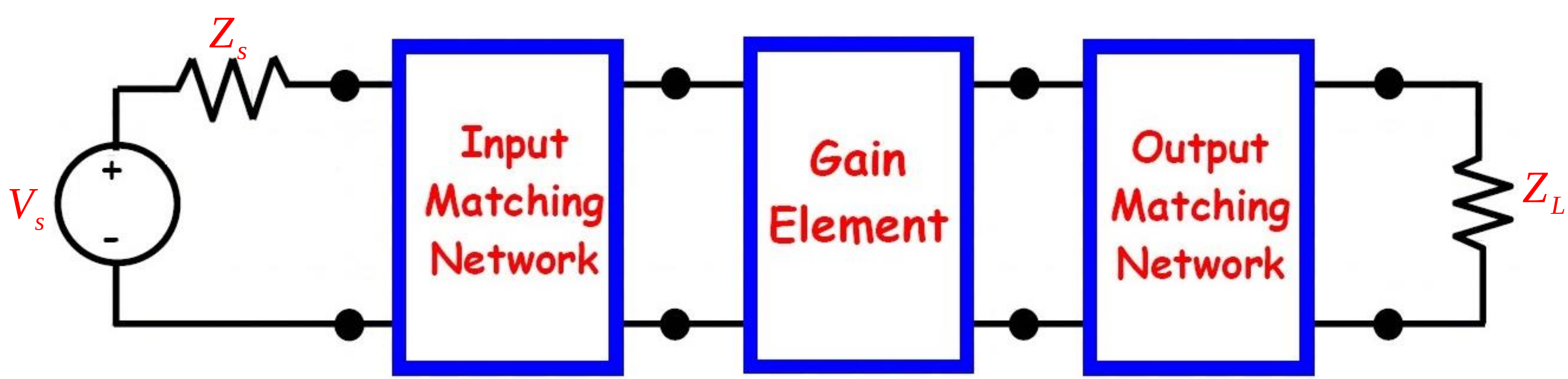
A: Not Exactly!!!

Turning a Gain Element into an Amplifier (contd.)

- Remember, it is true that a lossless matching network can change the source impedance to match a specific load. But the lossless matching network likewise alters the source voltage V_s such that the available power is preserved!

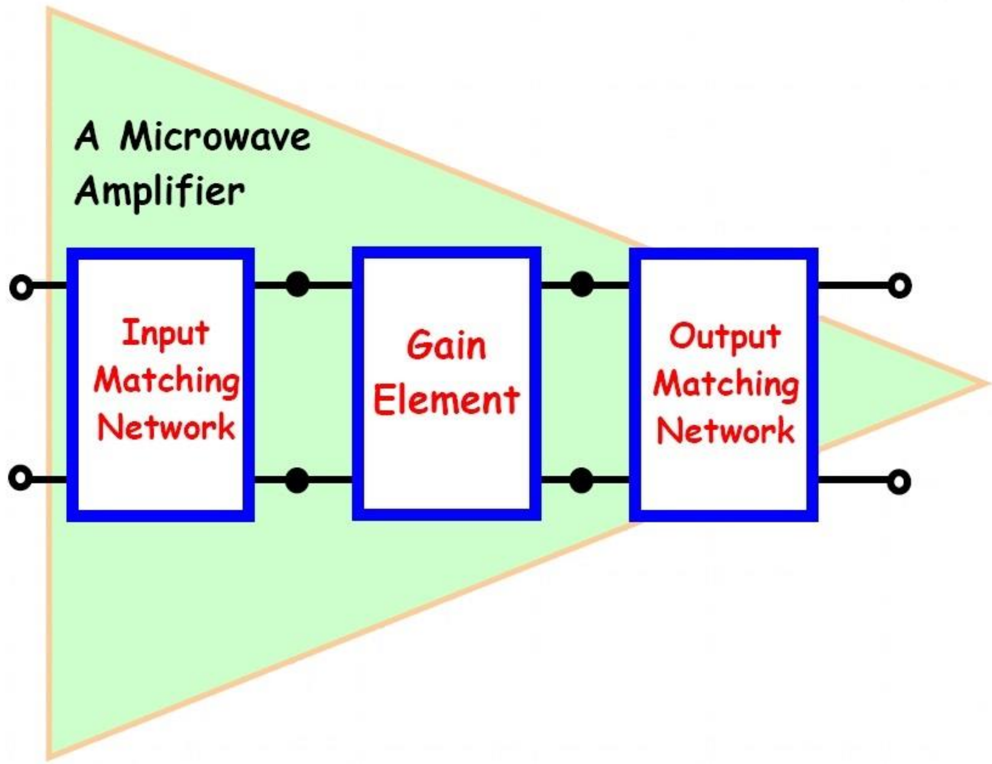
Messing around **directly** with the source impedance will undoubtedly **reduce** the available power of the source (this is bad!).

- So, to maximize the power delivered to a load, we need to insert **lossless matching networks** between the source and gain element, and between the gain element and the load:



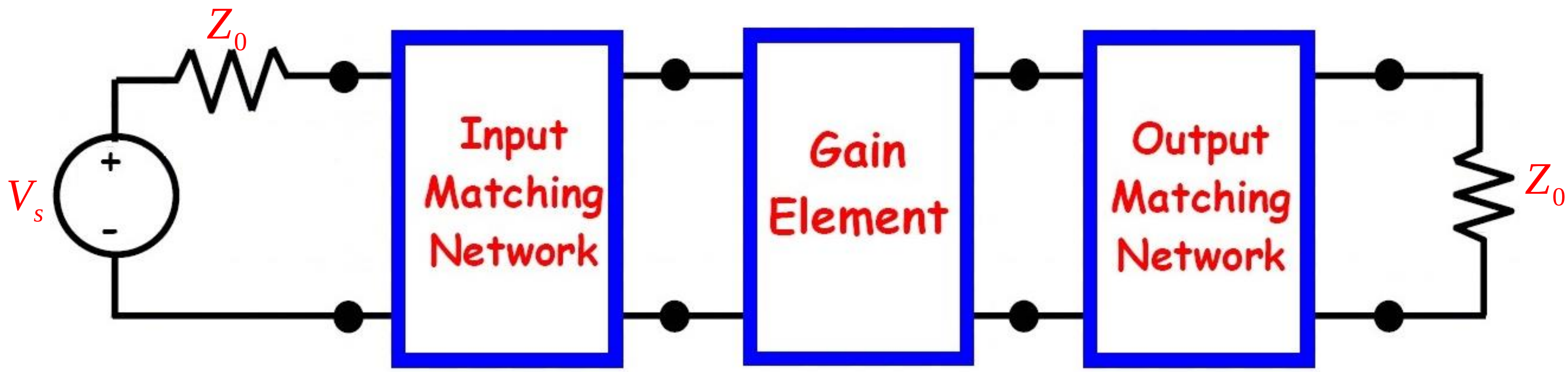
Turning a Gain Element into an Amplifier (contd.)

- The **three stages** together—input matching network, gain element, and output matching network—form a **high frequency amplifier**!



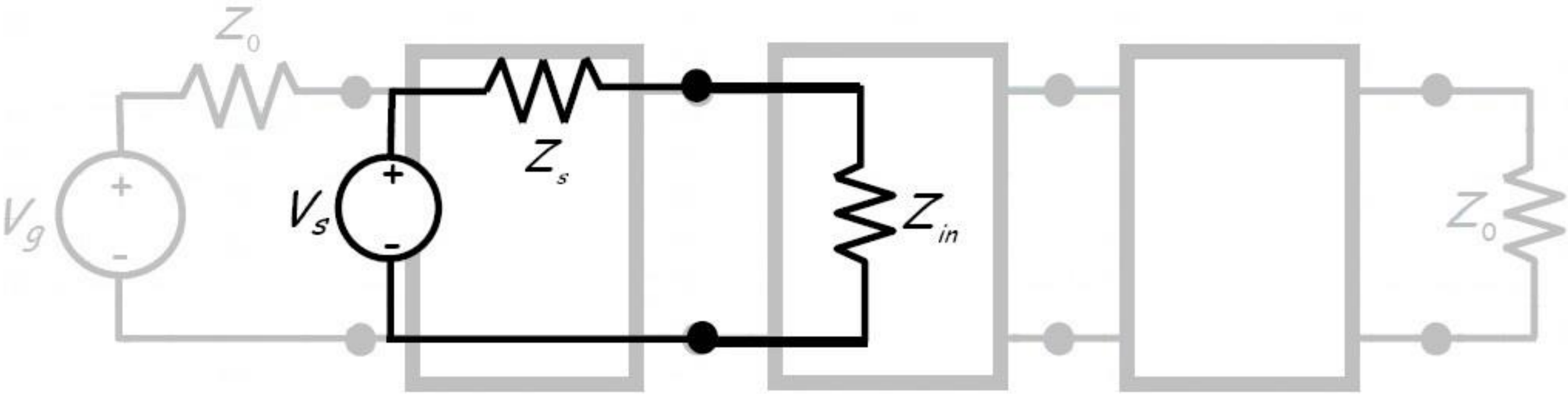
Turning a Gain Element into an Amplifier (contd.)

- Of course, the impedance of both the source and the load connected to this amplifier will most certainly be that of transmission line **characteristic impedance** Z_0. Thus, our amplifier circuit is typically:



Turning a Gain Element into an Amplifier (contd.)

- The **input network** is thus required to match Z_0 to the gain element input impedance Z_{in}. For the purposes of amplifier design, we view the input matching network as one that transforms the source impedance Z_0 into a new source impedance Z_s, one that is conjugate matched to the gain element input impedance Z_{in}:

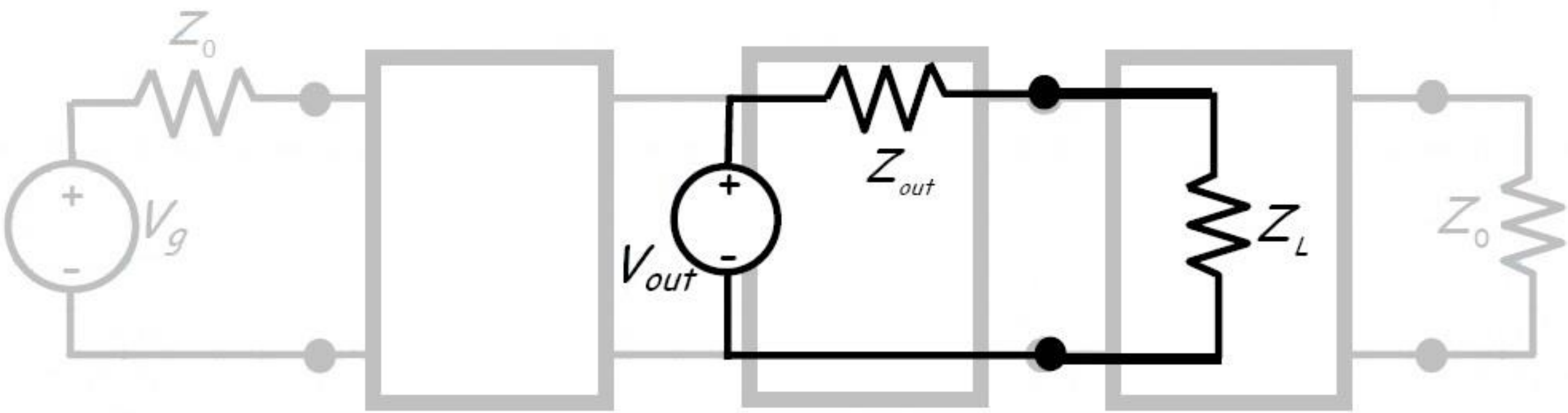


- If our input matching network is properly designed, we then find:

$$Z_{in} = Z_s^* \qquad \Gamma_{in} = \Gamma_s^*$$

Turning a Gain Element into an Amplifier (contd.)

- Likewise, the **output matching network** is used to match Z_0 to the gain element output impedance Z_{out}. For the purposes of amplifier design, we view the output matching network as one that transforms the load impedance Z_0 into a new load impedance Z_L, one that is conjugate matched to the gain element output impedance Z_{out}:

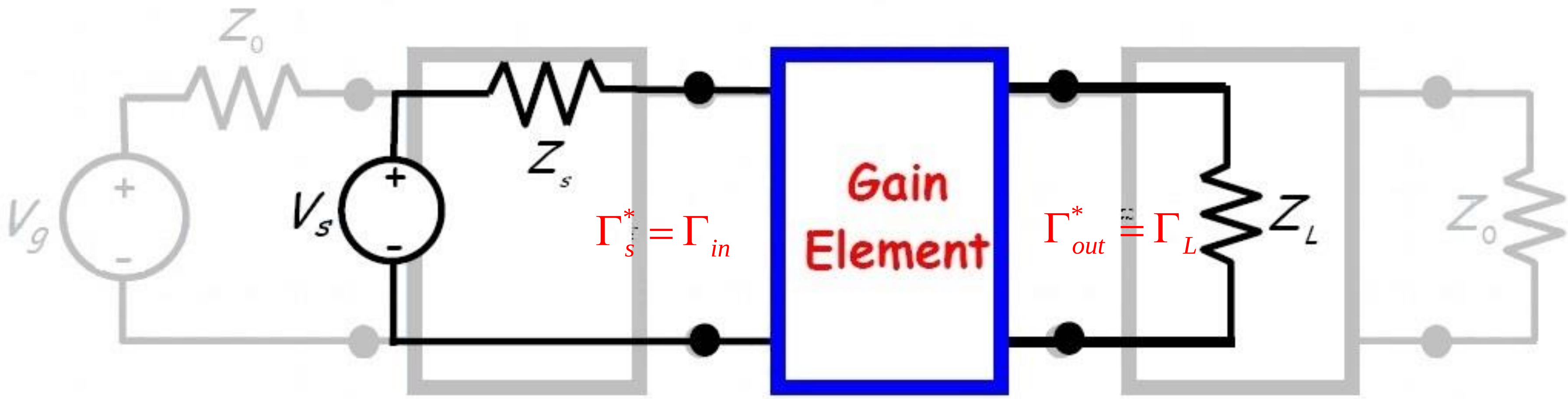


- Thus, if our output matching network is properly designed, we then find:

$$Z_L = Z_{out}^* \qquad \Gamma_L = \Gamma_{out}^*$$

Turning a Gain Element into an Amplifier (contd.)

- Therefore, our amplifier design problem can be described as:



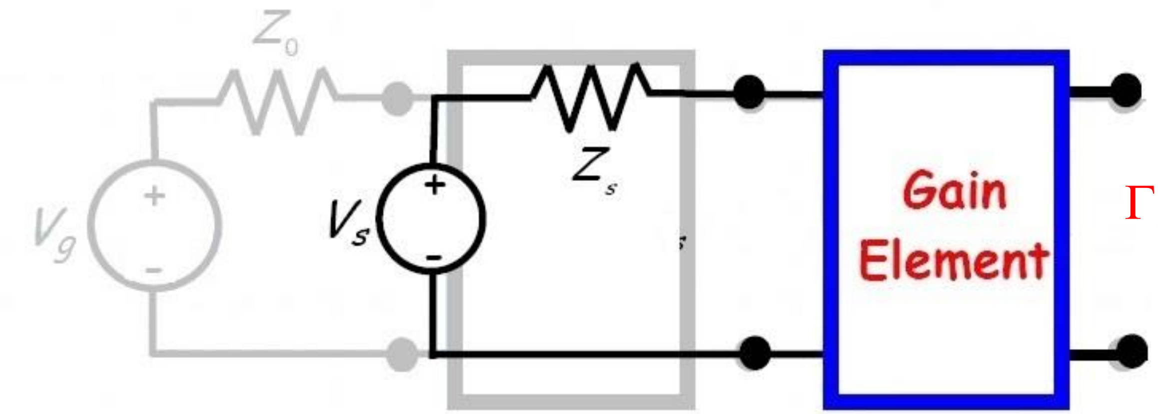
The values of Γ_s and Γ_L depend on the input and output matching networks

Q: Alright, we get it. We **know** how to make matching networks. Can't we move on to something else?

A: Not so fast! There's one little **problem** that makes this procedure more difficult than it otherwise might appear.

Turning a Gain Element into an Amplifier (contd.)

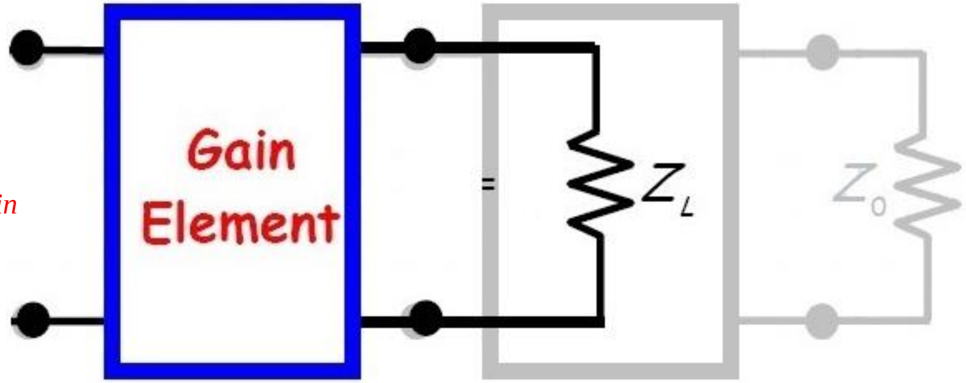
- Note that the value of Γ_{out} depends on the value of Z_s (i.e., depends on Γ_s).



$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$$

- Similarly, value of Γ_{in} depends on the value of Z_L (i.e., depends on Γ_L).

$$S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = \Gamma_{in}$$



Turning a Gain Element into an Amplifier (contd.)

It's a classic **chicken and egg** problem!

1. We can't design the input matching network until we determine Γ_{in} .
2. We can't determine Γ_{in} until we design the output matching network.
3. We can't determine the output matching network until we determine Γ_{out}.
4. We can't determine Γ_{out} until we design the input matching network.
5. But we can't design the input matching network until we determine Γ_{in}!

Our matching network design problems are thus **coupled**.