Lecture – 18 Date: 20.03.2017

- The Coupled Line Coupler
- Vector Network Analyzer (VNA) Introduction

The Coupled Line Coupler

Q: The "Quadrature Hybrid" or "Rat Race" are 3dB couplers. How do we build couplers with **less coupling**, say 10dB, 20dB, or 30 dB?

A: Such directional couplers are typically built using **coupled lines.**

Q: How can we **design** a coupled line couplers so that it is an **ideal** directional coupler with a **specific** coupling value?

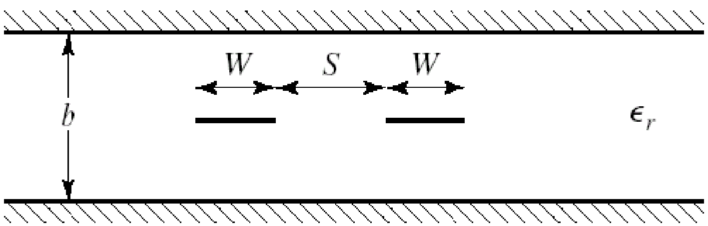
A: This lecture introduces the concept of such a design.

Q: Like all devices with quarter-wavelength sections, a coupled line coupler would seem to be inherently **narrow band**. Is there some way to **increase coupler bandwidth**?

A: Yes! add more coupled-line sections.

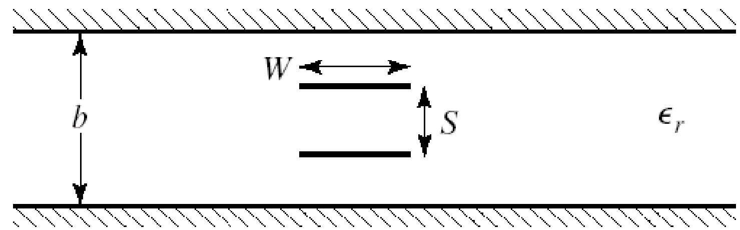
The Coupled Line Coupler (contd.)

- Two transmission lines in **proximity** to each other will **couple** power from one line into another.
- This proximity will **modify** the electromagnetic fields (and thus modify voltages and currents) of the propagating wave, and therefore **alter** the characteristic impedance of the transmission line!



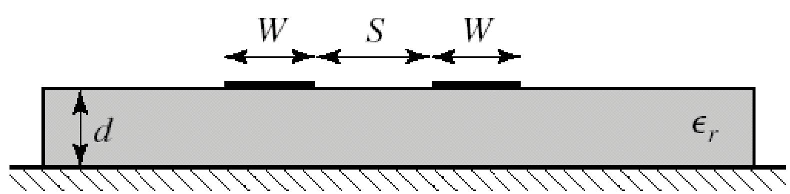
(a)

Coupled Stripline (edge-coupled or planar)



(b)

Coupled Stripline (broadside-coupled or stacked)

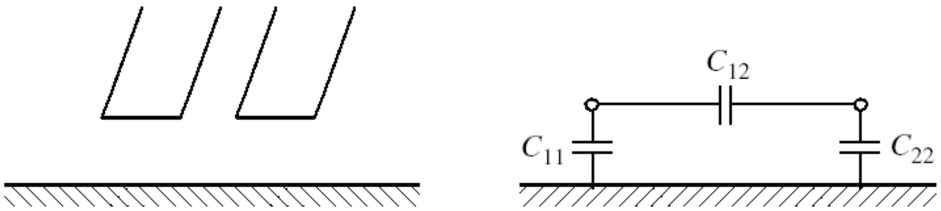


(c)

Coupled Microstrip

The Coupled Line Coupler (contd.)

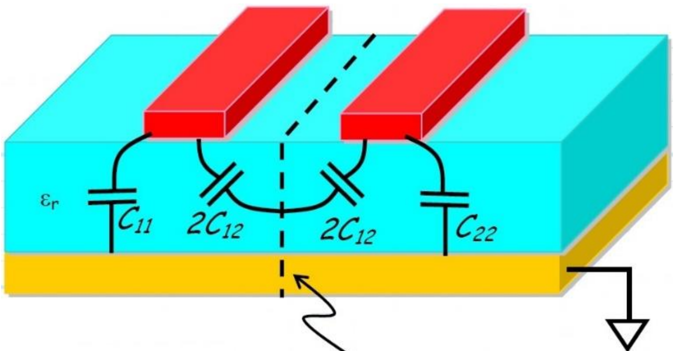
- Generally, speaking, we find that these transmission lines are capacitively coupled (i.e., it appears that they are connected by a capacitor).



A three-wire coupled transmission line and its equivalent capacitance network

If the two transmission lines are **identical** (and they typically are), then $C_{11} = C_{22}$

- Likewise, if the two transmission lines are identical, then a plane of circuit **symmetry** exists. As a result, we can analyze this circuit using **odd/even mode** analysis!



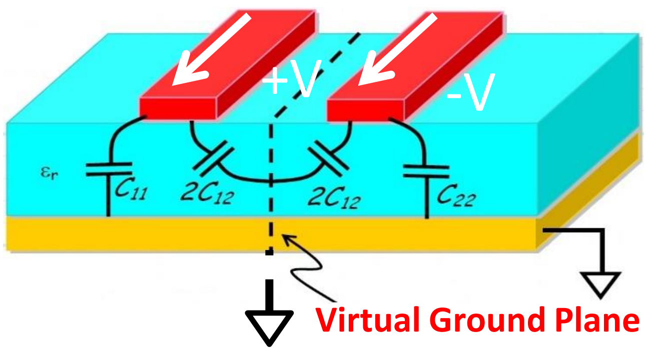
Plane of Coupler Symmetry

Note the capacitor C_{12} has been divided into **two series** capacitors, each with a value of $2C_{12}$

The Coupled Line Coupler (contd.)

Odd Mode

- If the incident wave along the two TLs are **opposite** (i.e., equal magnitude but 180^o out of phase), a virtual **ground plane** is created at the plane of circuit symmetry.



- Thus, the capacitance/ unit length of each TL, in the **odd** mode, is:

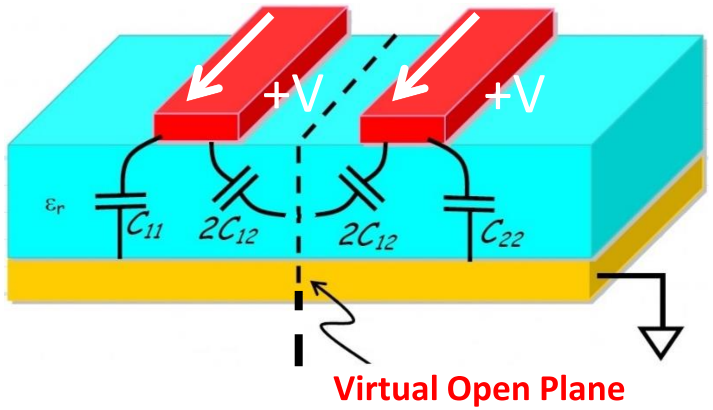
$$C_o = C_{11} + 2C_{12} = C_{22} + 2C_{12}$$

- The corresponding char. impedance is:

$$Z_0^o = \sqrt{\frac{L}{C_o}}$$

Even Mode

- If the incident wave along the two TLs are **equal** (i.e., equal magnitude and phase), a **virtual open** plane is created at the plane of circuit symmetry.



- Note the $2C_{12}$ capacitors have been "**disconnected**", and thus the capacitance/unit length of each TL, in the **even** mode, is:

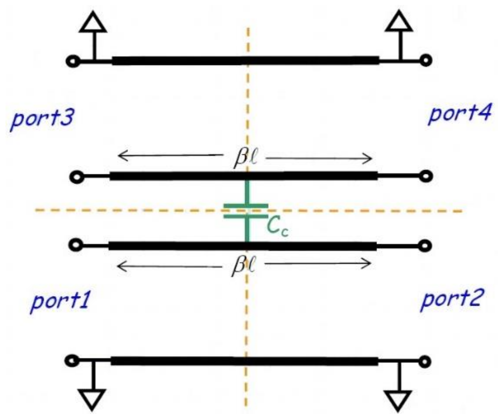
$$C_e = C_{11} = C_{22}$$

- Therefore the corresponding characteristic impedance is:

$$Z_0^e = \sqrt{\frac{L}{C_e}}$$

Analysis and Design

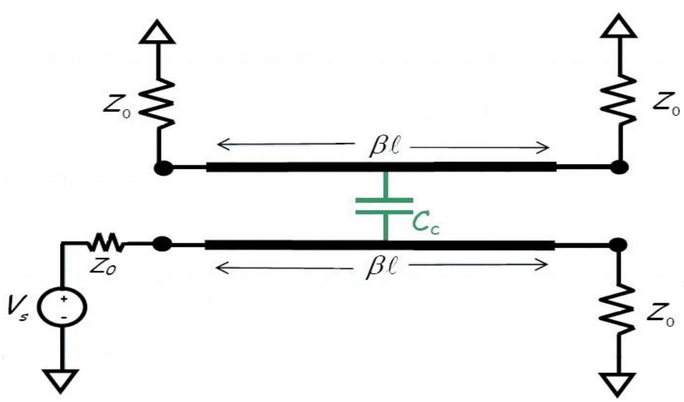
- A pair of coupled lines form a **4-port** device with **two** planes of reflection symmetry.



- As a result, we know that the **scattering matrix** of this four-port device has just **4 independent** elements:

$$\boldsymbol{S} = \begin{bmatrix} S_{11} & S_{21} & S_{31} & S_{41} \\ S_{21} & S_{11} & S_{41} & S_{31} \\ S_{31} & S_{41} & S_{11} & S_{21} \\ S_{41} & S_{31} & S_{21} & S_{11} \end{bmatrix}$$

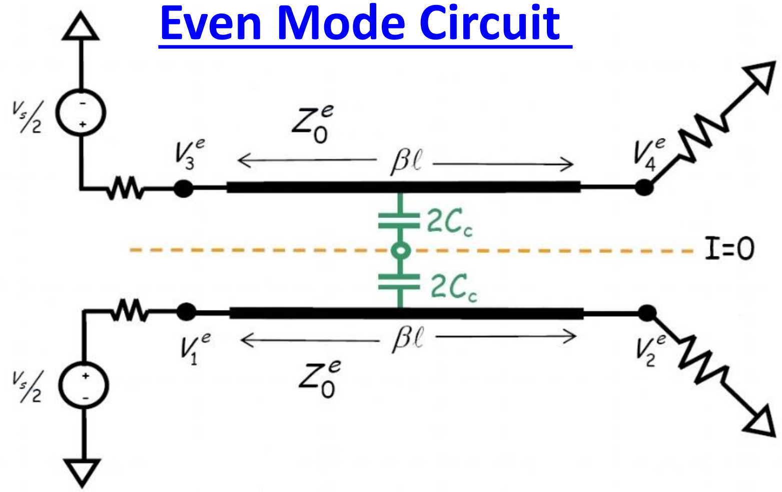
- To determine these four elements, we can apply a **source to port 1** and then **terminate** all other ports:



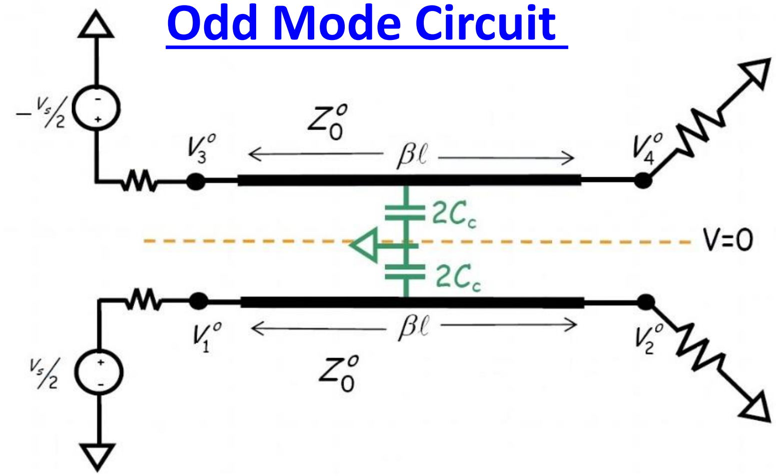
Typically, a coupled-line coupler schematic is drawn **without** explicitly showing the **ground conductors** (i.e., without the ground plane)

Analysis and Design (contd.)

Even Mode Circuit



Odd Mode Circuit



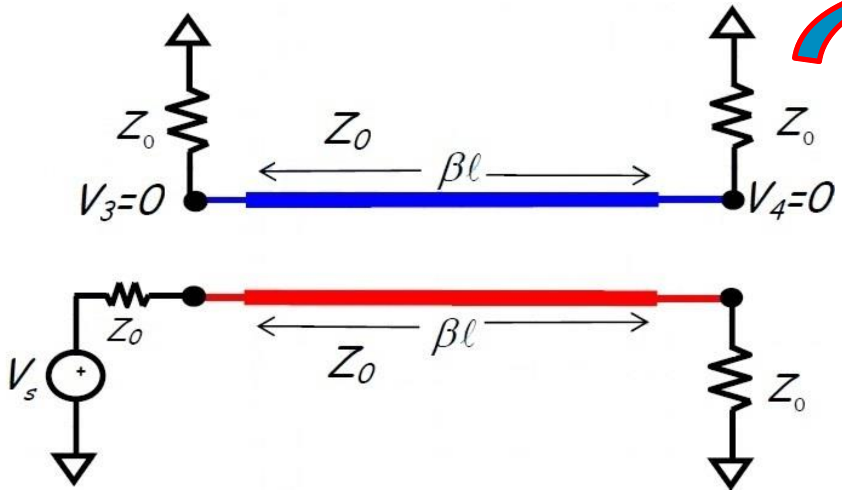
Note that the **capacitive coupling** associated with these modes are different, resulting in a **different** characteristic impedance of the lines for the two cases (i.e., Z_0^e,Z_0^o)

Q: So what?

A: Consider what would happen if the characteristic impedance of each line were **identical** for **each mode**: $Z_0 = Z_0^e = Z_0^o$

- In such a situation we can find that: $V_3^e = -V_3^o$ $V_4^e = -V_4^o$
- Therefore from **superposition**: $V_3 = V_3^e + V_3^o = 0$ $V_4 = V_4^e + V_4^o = 0$
- This indicates that **no power is coupled** from the "**energized**" transmission line onto the "**non-energized**" transmission line.

Analysis and Design (contd.)

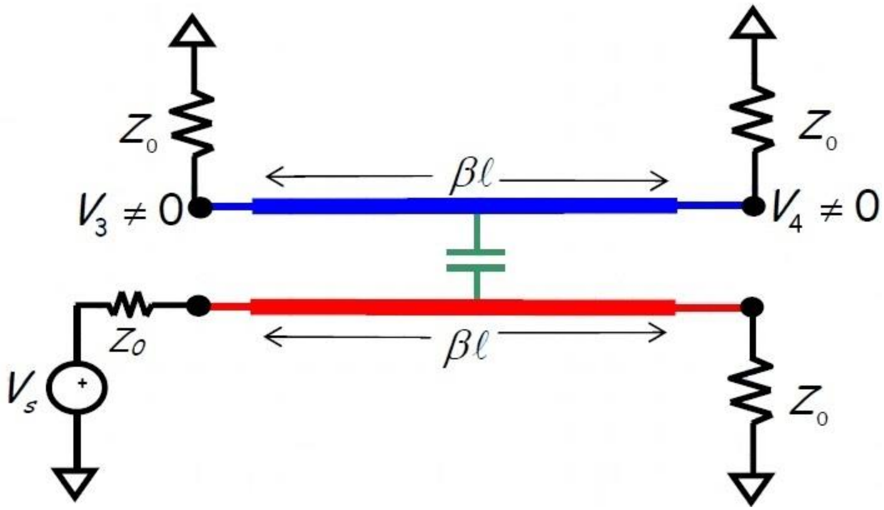


This makes sense! After all, **if** no coupling occurs, then the characteristic impedance of each line is **unaltered** by the presence of the other—their characteristic impedance is Z_0 **regardless** of "mode".

- However, if coupling **does** occur, then $Z_0^e \neq Z_0^o$, meaning in general: $V_3^e \neq -V_3^o$ $V_4^e \neq -V_4^o$
- and thus in general: $V_3 = V_3^e + V_3^o \neq 0$ $V_4 = V_4^e + V_4^o \neq 0$

The odd/even mode analysis can thus reveal the amount of **coupling from** the ***energized*** section **onto** the ***non-energized*** section!

Analysis and Design (contd.)



- Now, our **first step** in performing the odd/even mode analysis will be to determine scattering parameter S_{11}. To accomplish this, we will need to determine voltage V_1:

$$V_1 = V_1^e + V_1^o$$

- The analysis is a bit complicated, so it won't be presented here. However, a pertinent question we might ask is, what value **should S_{11}** be?

A: For the device to be a **matched** device, it must be **zero**!

- From the value of S_{11} derived from our odd/even analysis, it can be shown that S_{11} will be equal to zero **if** the odd and even mode characteristic impedances are related as:

$$\sqrt{Z_0^e Z_0^o} = Z_0$$

- In other words, we should design our coupled line coupler such that the **geometric mean** of the even and odd mode impedances is **equal to Z_0**.

Analysis and Design (contd.)

- Now, assuming the first design rule has been implemented, we also find (from odd/even mode analysis) that the scattering parameter S_{31} is:

$$S_{31} = \frac{j\left(Z_0^e - Z_0^o\right)}{2Z_0\cot(\beta l) + j\left(Z_0^e + Z_0^o\right)}$$

Thus, it can be seen that **unless $Z_0^e = Z_0^o$**, power must be coupled from port 1 to port 3!

Q: But what is the value of line **electrical length** βl ?

A: The **electrical length** of the coupled transmission lines is also a **design parameter**. Assuming that we want to **maximize** the coupling onto port 3, we find from the S_{31} expression that this is accomplished if we set βl such that:

$$\cot(\beta l) = 0 \;\Rightarrow\; \beta l = \frac{\pi}{2} \;\Rightarrow\; l = \frac{\lambda}{4}$$

Once again, our design rule is to set the transmission line length to a value equal to **one-quarter wavelength** (at the design frequency).

- Implementing these **two** design rules, we find that (at the design frequency):

$$S_{31} = \frac{\left(Z_0^e - Z_0^o\right)}{\left(Z_0^e + Z_0^o\right)}$$

Analysis and Design (contd.)

- The value of S_{31} is a **very** important one with respect to coupler performance. Specifically, it is the **coupling coefficient** c !

$$c=\frac{\left(Z_0^e-Z_0^o\right)}{\left(Z_0^e+Z_0^o\right)}$$

- Given this definition, we can **rewrite** the scattering parameter S_{31} as:

$$S_{31}=\frac{jc\tan(\beta l)}{\sqrt{1-c^2}+j\tan(\beta l)}$$

- **Similarly,** the odd/even mode analysis gives (given that $\sqrt{Z^e{}_0\, Z^o{}_0}=Z_0$):

$$S_{21}=\frac{\sqrt{1-c^2}}{\sqrt{1-c^2}\cos(\beta l)+j\sin(\beta l)}$$

- at **design frequency**, $\beta l=\pi/2$,:

$$S_{21}=-j\sqrt{1-c^2}$$

- **Finally**, the odd/even analysis also gives (at design frequency):

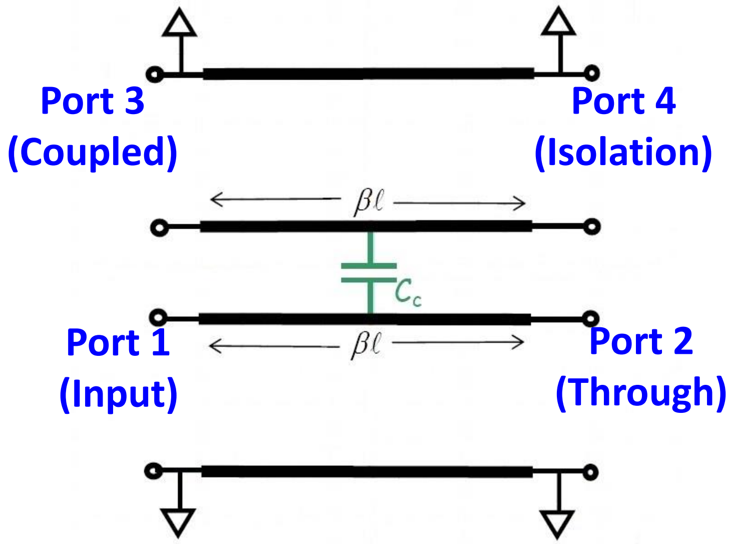
$$S_{41}=\frac{\sqrt{1-c^2}}{\sqrt{1-c^2}\cos(\beta l)+j\sin(\beta l)}$$

- **Combining** these results, at the design frequency, the **scattering matrix** of coupled-line coupler is:

$$S=\begin{bmatrix}0 & -j\sqrt{1-c^2} & c & 0\\ -j\sqrt{1-c^2} & 0 & 0 & c\\ c & 0 & 0 & -j\sqrt{1-c^2}\\ 0 & c & -j\sqrt{1-c^2} & 0\end{bmatrix}$$

The same coupler ! The coupled-line coupler—if our design rules are followed—results in an "ideal" directional coupler.

Analysis and Design (contd.)



- If the **input** is port 1, then the **through** port is port 2, the **coupled** port is port 3, and the **isolation** port is port 4!

Q: But, how do we **design** a coupled-line coupler with a **specific** coupling coefficient c?

A: We know the **two design constraints**:

$$\sqrt{Z_0^e Z_0^o} = Z_0 \qquad c = \frac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}$$

We can **rearrange** these two expressions to find **solutions** for our odd and even mode impedances

$$Z_0^e = Z_0 \sqrt{\frac{1+c}{1-c}} \qquad Z_0^o = Z_0 \sqrt{\frac{1-c}{1+c}}$$

Therefore, **given** the desired values Z_0 and **c**, we can determine the proper values of Z_0^e and Z_0^o for an ideal directional coupler

Analysis and Design (contd.)

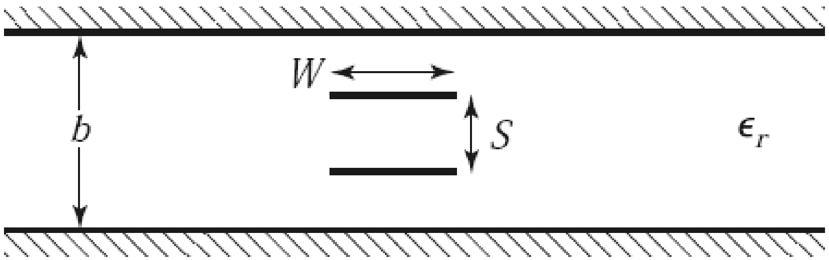
Q: Yes, but the odd and even mode impedance depends on the **physical structure** of the coupled lines, such as substrate dielectric ε_r, substrate thickness, conductor width, and separation distance. How do we determine **these** physical design parameters for desired values of Z_0^e and Z_0^o ?

A: That's a much more difficult question to answer! Recall that there is **no** direct formulation relating microstrip and stripline parameters to **characteristic impedance** (There are numerically derived **approximations**).

- So it's no surprise that there is **no direct formulation** relating odd and even mode characteristic impedances to the specific physical parameters of microstrip and stripline coupled lines.
- Instead, there are again numerically derived **approximations** that allow us to determine (approximately) the required microstrip and stripline parameters, or one can always use a **microwave CAD package** (such as ADS!).

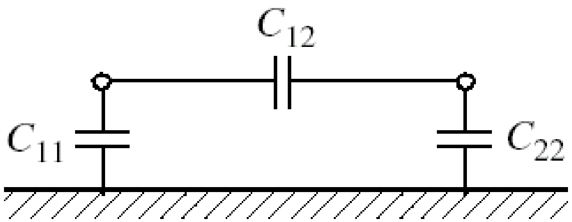
Example – 1

For the broadside coupled stripline geometry shown below, assume $W \gg S$ and $W \gg b$, so that fringing fields can be ignored. Determine the even- and odd-mode characteristic impedances.



Example – 1 (contd.)

Solution: The equivalent circuit is:



- First determine the equivalent network capacitances, C_{11} and C_{12}.

- The capacitance per unit length of broadside parallel lines with width *W* and separation *S* is:

$$C = \frac{\epsilon W}{S} \;\; F/m$$

Ignores the fringing field

- C_{11} and C_{22} are formed by the capacitance of one strip to the ground planes. Thus the capacitance per unit length is:

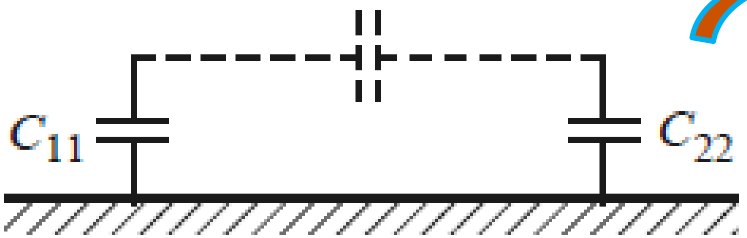
$$C_{11} = C_{22} = \frac{2\epsilon_0 \epsilon_r W}{b - S} \;\; F/m$$

- The capacitance per unit length between the strips is:

$$C_{12} = \frac{\epsilon_0 \epsilon_r W}{S} \;\; F/m$$

- For the **even mode**, the electric field has even symmetry about the center line, and no current flows between the two strip conductors. This leads to the equivalent circuit, where C_{12} is effectively open-circuited.

Example – 1 (contd.)



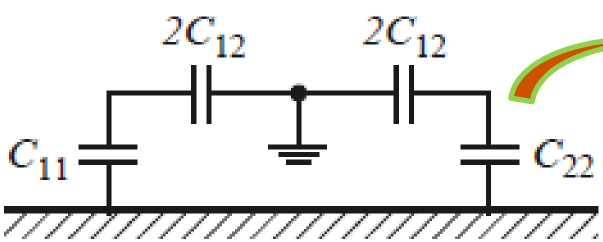
The resulting capacitance of either line to ground for the even mode is:

$$C_e = C_{11} = C_{22} = \frac{2\epsilon_0\epsilon_r W}{b-S} \quad F/m$$

- Therefore: $$Z_{0e} = \frac{1}{v_p C_e} = \eta_0 \frac{b-S}{2W\sqrt{\epsilon_r}}$$

$v_p = c/\sqrt{\epsilon_r}$

- For the **odd mode**, the electric field lines have an odd symmetry about the center line, and a voltage null exists between the two strip conductors. We can imagine this as a ground plane through the middle of C_{12}, which leads to the equivalent



the effective capacitance between either strip conductor and ground is:

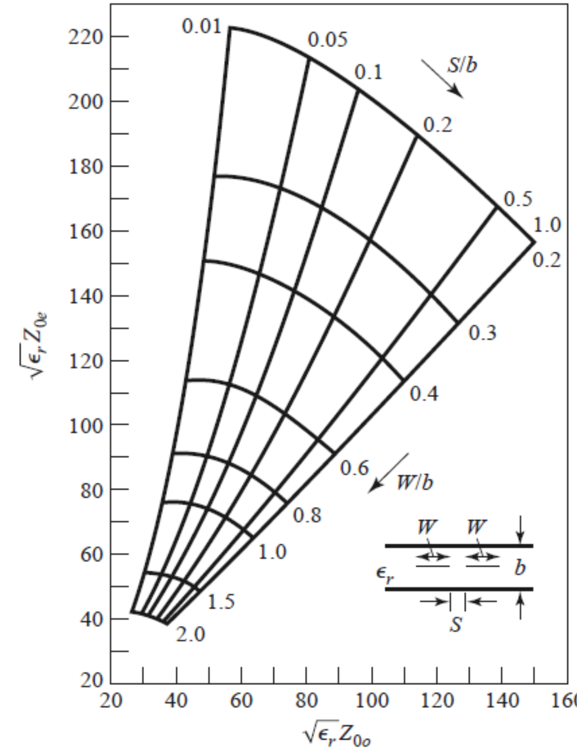
$$C_o = C_{11} + 2C_{12} = C_{22} + 2C_{12}$$

- Therefore: $$Z_{0o} = \frac{1}{v_p C_o} = \eta_0 \frac{1}{2W\sqrt{\epsilon_r}\left[\frac{1}{(b-S)} + \frac{1}{S}\right]}$$

$v_p = c/\sqrt{\epsilon_r}$

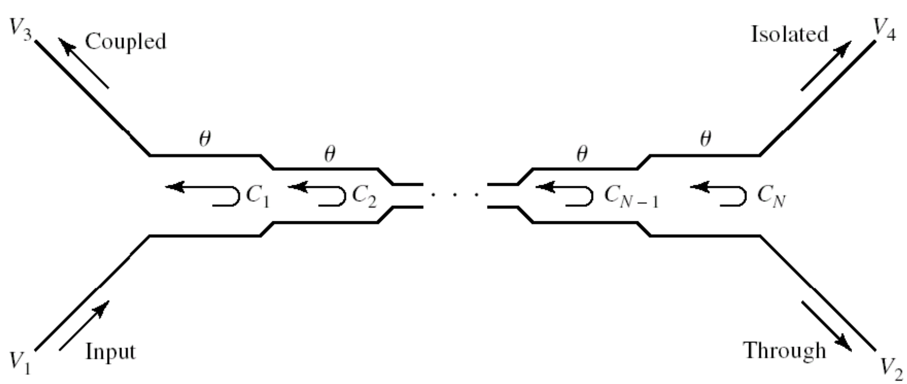
Example – 2

Design a 20 dB single-section coupled line coupler in stripline with a ground plane spacing of 0.32 cm, a dielectric constant of 2.2, a characteristic impedance of 50, and a center frequency of 3 GHz. Plot the coupling and directivity from 1 to 5 GHz. Include the effect of losses by assuming a loss tangent of 0.05 for the dielectric material and copper conductors of 2 mil thickness.



Multi-section Coupled-Line Couplers

- We can add **multiple** coupled lines in series to increase coupler bandwidth.



- The couplers are typically designed such that they are **symmetric**, i.e.:

$C_1 = C_N$ $C_2 = C_{N-1}$ $C_3 = C_{N-2}$ **etc.**

where N is odd.

Because the phase characteristics are usually better

Multi-section Coupled-Line Couplers (contd.)

Q: What is the coupling of this device as a function of **frequency**?

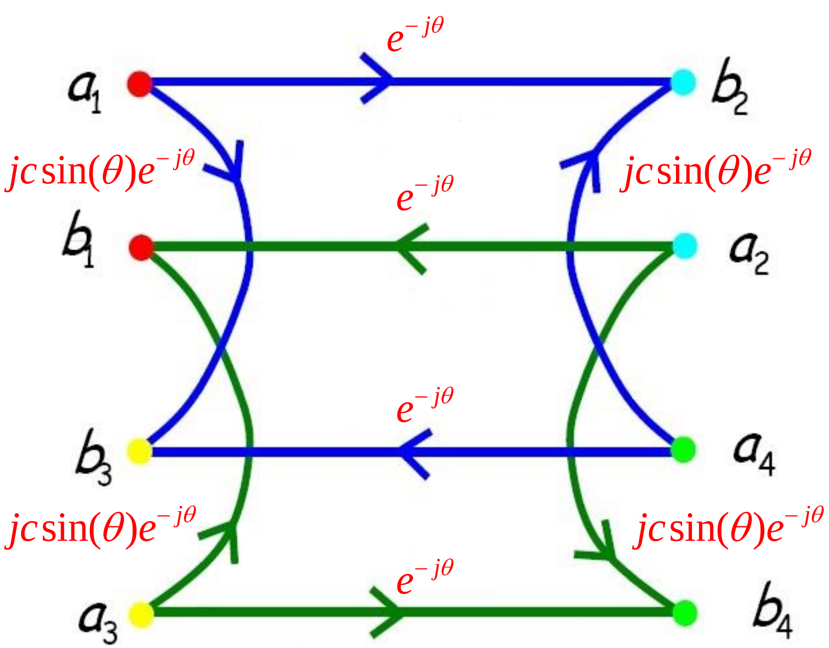
A: To analyze this structure, we make some **approximations:**

$$S_{31}(\theta) = \frac{jc\tan(\theta)}{\sqrt{1-c^2}+\tan(\theta)} \approx \frac{jc\tan(\theta)}{1+j\tan(\theta)} = jc\sin(\theta)e^{-j\theta}$$

$$S_{21}(\theta) = \frac{\sqrt{1-c^2}\,jc\tan(\theta)}{\sqrt{1-c^2}\cos(\theta)+j\sin(\theta)} \approx \frac{1}{\cos(\theta)+j\sin(\theta)} = e^{-j\theta}$$

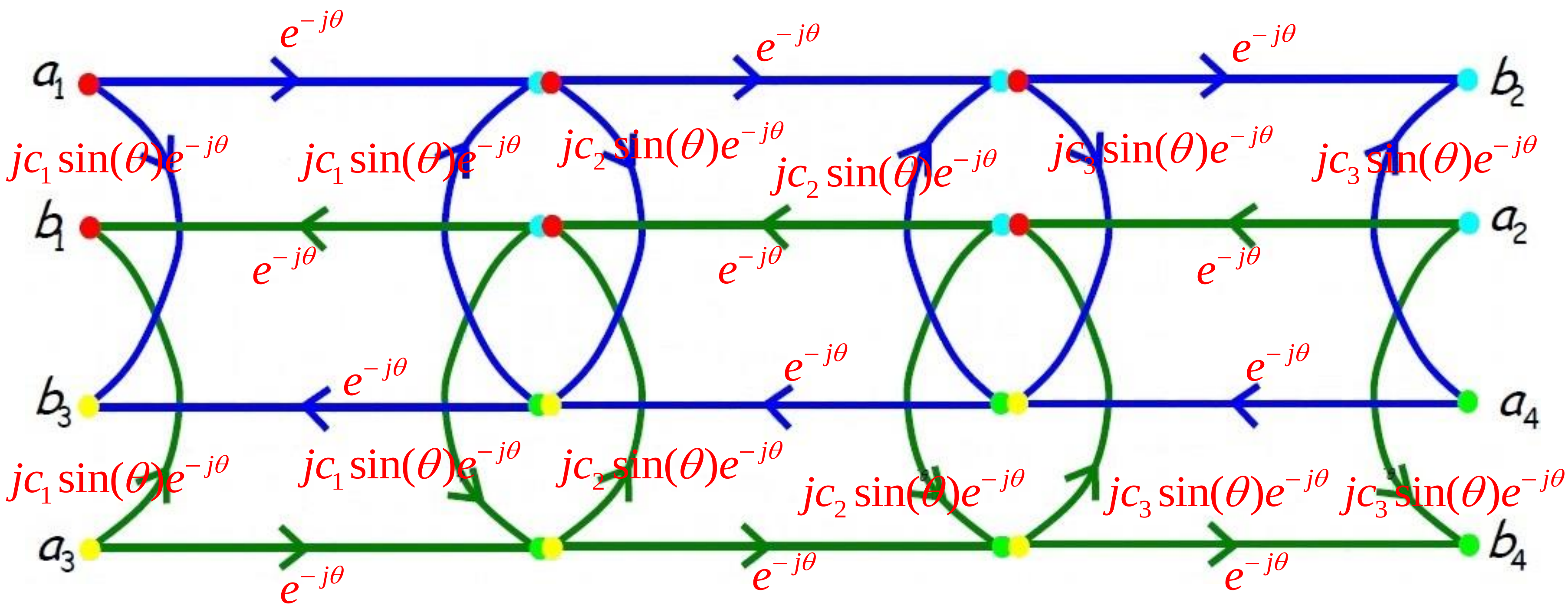
where obviously, $\theta = \beta l = \omega T$, and $T = l/v_p$

- We can use these approximations to construct a **signal flow graph** of a **single**-section coupler:



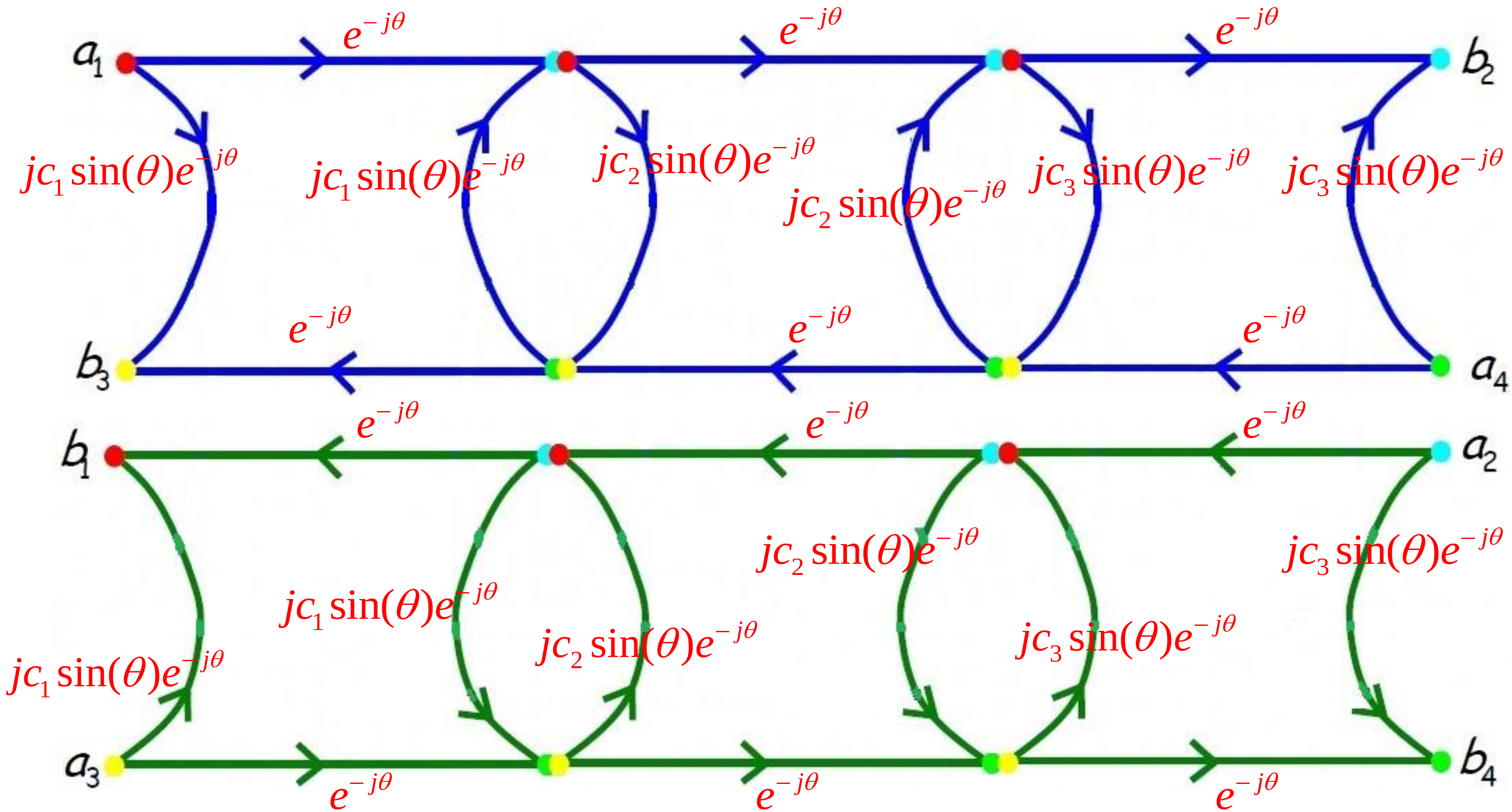
Multi-section Coupled-Line Couplers (contd.)

- Now, say we cascade **three** coupled line pairs, to form a **three section** coupled line coupler. The signal flow graph would thus be:



Multi-section Coupled-Line Couplers (contd.)

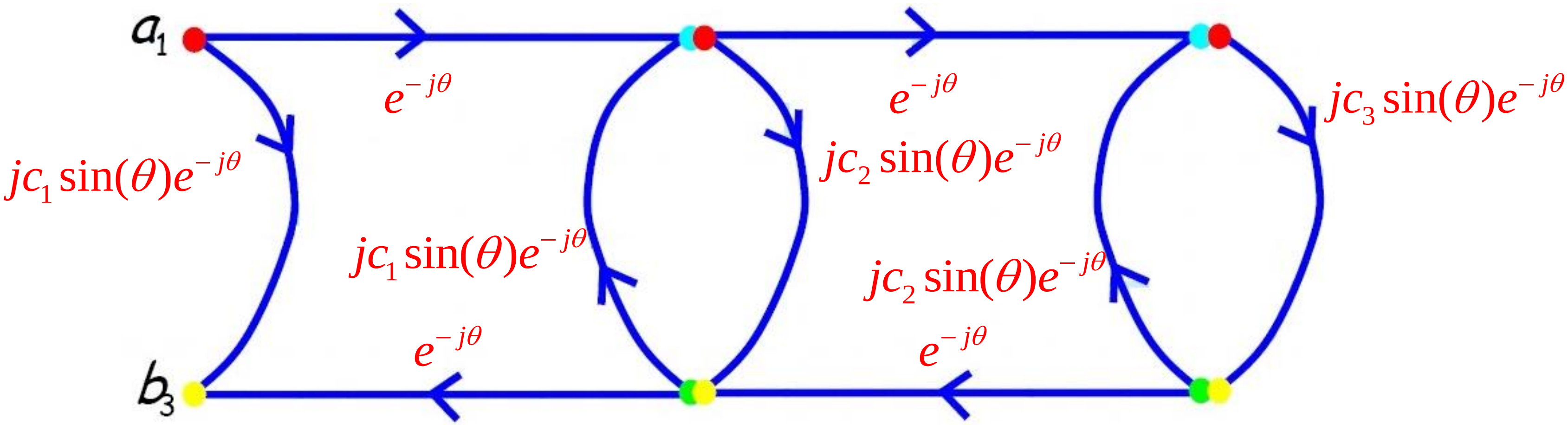
- Note that this signal flow graph **decouples** into two separate graphs (i.e., the blue graph and the green graph).



Note that these two graphs are essentially identical, and emphasize the symmetric structure of the coupled-line coupler.

Multi-section Coupled-Line Couplers (contd.)

- Now, we are interested in describing the **coupled output** (i.e., b_3) in terms of the incident wave (i.e., a_1). For this, assume ports 2, 3 and 4 are **matched** (i.e., $a_4 = 0$), we can reduce the graph to simply:

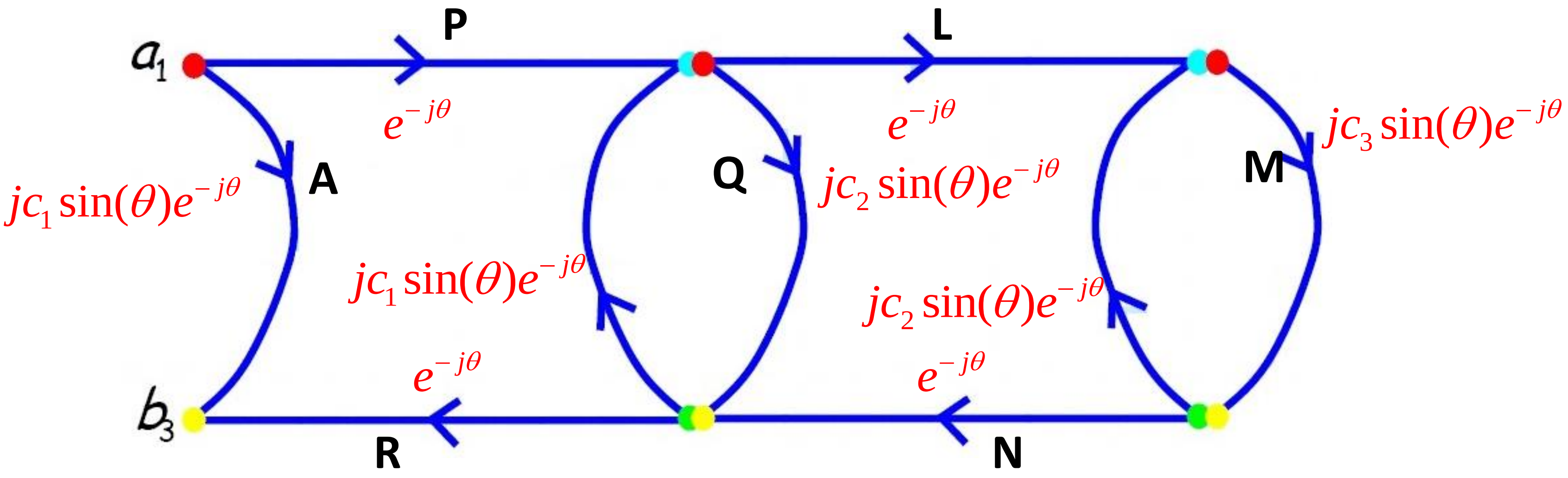


Now, we **could** reduce this signal flow graph even further—**or** we can apply the **multiple reflection viewpoint** explicitly to each propagation term! **(follow Microwave Engineering *by* Collins)**

- As per **theory of multiple reflection small reflections**, one can only consider the propagation paths where **one coupling** is involved—i.e., the signal propagates **across** a coupled-line pair only **once**!

Multi-section Coupled-Line Couplers (contd.)

- In our example, there are **three** propagation paths, corresponding to the coupling across each of the **three** separate coupled line pairs:



Here the propagation paths are: A PQR PLMNR

$$b_3 = \left(jc_1\sin(\theta)e^{-j\theta} + e^{-j\theta} jc_2\sin(\theta)e^{-j\theta}e^{-j\theta} + e^{-j2\theta} jc_3\sin(\theta)e^{-j\theta}e^{-j2\theta} \right) a_1$$

$$b_3 = \left(jc_1\sin(\theta)e^{-j\theta} + jc_2\sin(\theta)e^{-j3\theta} + jc_3\sin(\theta)e^{-j5\theta} \right) a_1$$

Therefore, according to the **approximation**:

$$S_{31}(\theta) = \frac{V_3^-}{V_1^+}(\theta) = \frac{b_3}{a_1}(\theta) = jc_1\sin(\theta)e^{-j\theta} + jc_2\sin(\theta)e^{-j3\theta} + jc_3\sin(\theta)e^{-j5\theta}$$

Multi-section Coupled-Line Couplers (contd.)

Furthermore, for a **multi**-section coupler with N sections:

$$S_{31}(\theta) = \frac{V_3^-}{V_1^+}(\theta) = \frac{b_3}{a_1}(\theta) = jc_1 \sin(\theta) e^{-j\theta} + jc_2 \sin(\theta) e^{-j3\theta} + jc_3 \sin(\theta) e^{-j5\theta} + \ldots.$$
$$\ldots.. + jc_N \sin(\theta) e^{-j(2N-1)\theta}$$

- And for **symmetric** couplers with an **odd** value N , we find:

$$S_{31}(\theta) = j2\sin(\theta)e^{-jN\theta}\left[c_1\cos(N-1)\theta + c_2\cos(N-3)\theta + c_3\cos(N-5)\theta + \ldots. + \frac{1}{2}c_M\right]$$

where M=(N+1) /2. Note M is an **even integer**, as N is an **odd** number

- Thus, we find the coupling **magnitude** as a function of frequency:

$$|c(\theta)| = |S_{31}(\theta)| = c_1 2\sin(\theta)\cos(N-1)\theta + c_2 2\sin(\theta)\cos(N-3)\theta + c_3 2\sin(\theta)\cos(N-5)\theta + \ldots. + c_M 2\sin(\theta)$$

- Therefore, the **coupling in dB** is: $C(\theta) = 10\log_{10}|c(\theta)|^2$

- Now, our design goals are to **select** the coupling values c_1, c_2, c_N such that:
  1. The coupling value $C(\theta)$ is a specific, **desired** value at our design frequency.
  2. The coupling **bandwidth** is as **large** as possible.

Multi-section Coupled-Line Couplers (contd.)

- For the first condition, recall that the at the **design frequency**: $\theta = \beta l = \pi / 2$

i.e., the section lengths are a **quarter-wavelength** at our design frequency

- Thus, we find our **first** design equation:

$$\left|c(\theta)\right|_{\theta=\pi/2} = \left|S_{31}(\theta)\right| = c_1 2\cos\{(N-1)\pi/2\} + c_2 2\cos\{(N-3)\pi/2\} + c_3 2\cos\{(N-5)\pi/2\} + \dots + c_M$$

where we have used the fact that **sin(π/2) = 1**.

- Note the value $|c(\theta)|_{\theta=\pi/2}$ is set to the value necessary to achieve the **desired** coupling value. This equation thus provides **one** design constraint—we have **M-1** degrees of design freedom left to accomplish our **second** goal!

- To **maximize bandwidth**, we typically impose the **maximally flat** condition:

$$\left(\frac{d^m \left|c(\theta)\right|}{d\theta^m}\right)_{\theta=\pi/2} = 0$$

m = 1, 2, M-1

Be careful! Remember to perform the derivative **first**, and **then** evaluate the result at θ = π/2.

Vector Network Analyzer – Introduction

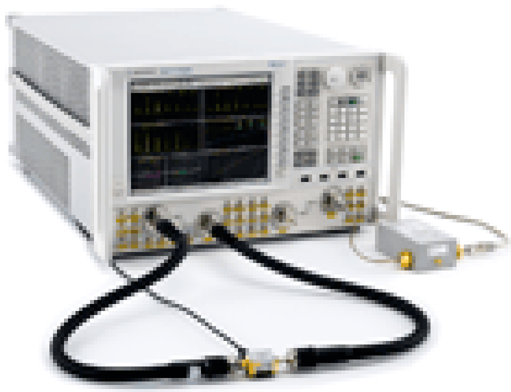
Q: What is VNA? **Q: Why VNA?** **Q: If not VNA, then what?**

Simple answer could be:

- Another instrument (for high frequency measurement)
- Definitely to measure something that a simple low frequency instrument is not able to measure (S-parameter)
- Then a combination of instrument (such as power meter, phase meter, etc.)

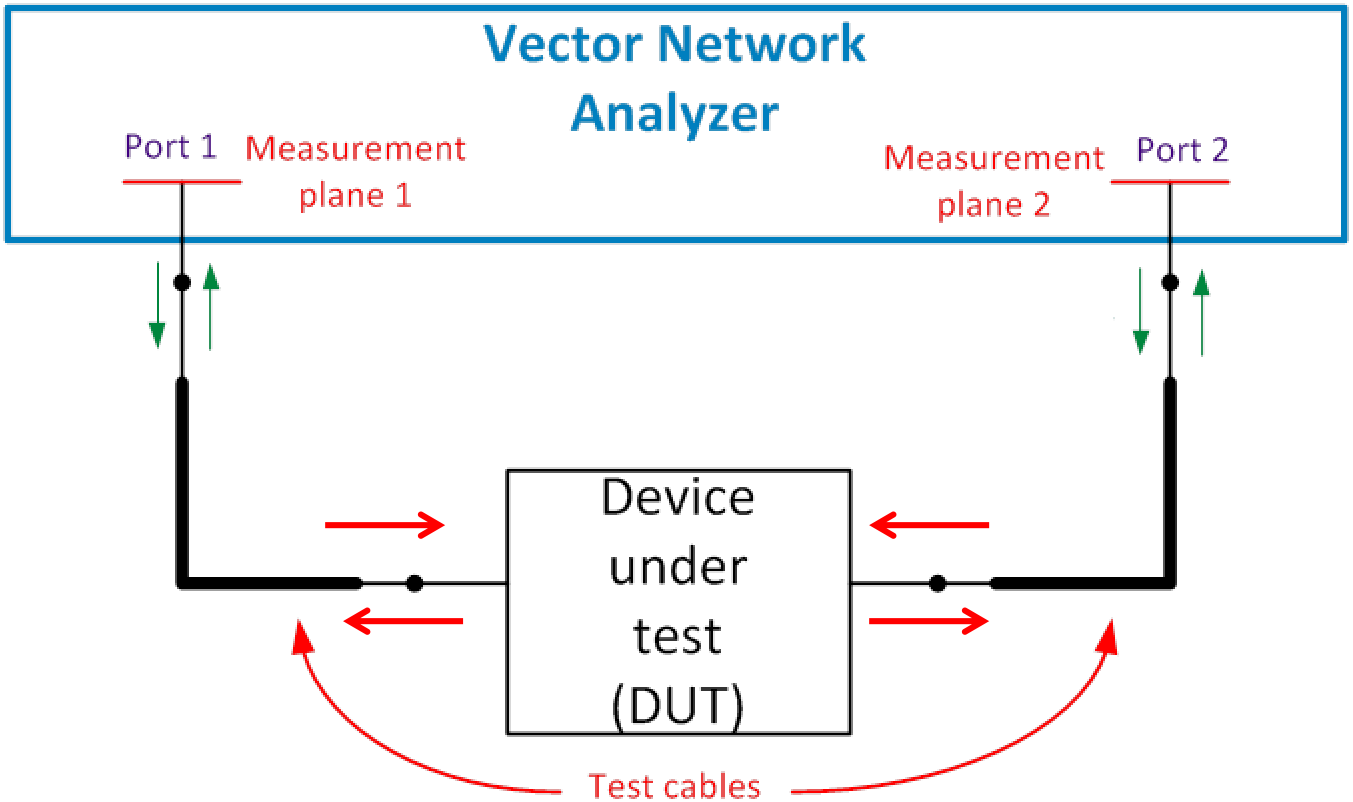
Vector network analyzers are particularly useful items of RF test equipment. When used skilfully, they enable RF devices and networks to be characterised so that an RF design can be undertaken with a complete knowledge of the devices being used. This will provide a better understanding of how the circuit will operate. Vector network analyzers provide a much greater capability than their scalar counterparts, and as a result the vector network analyzers are more widely used, even though they tend to be more expensive.

Introduction (contd.)

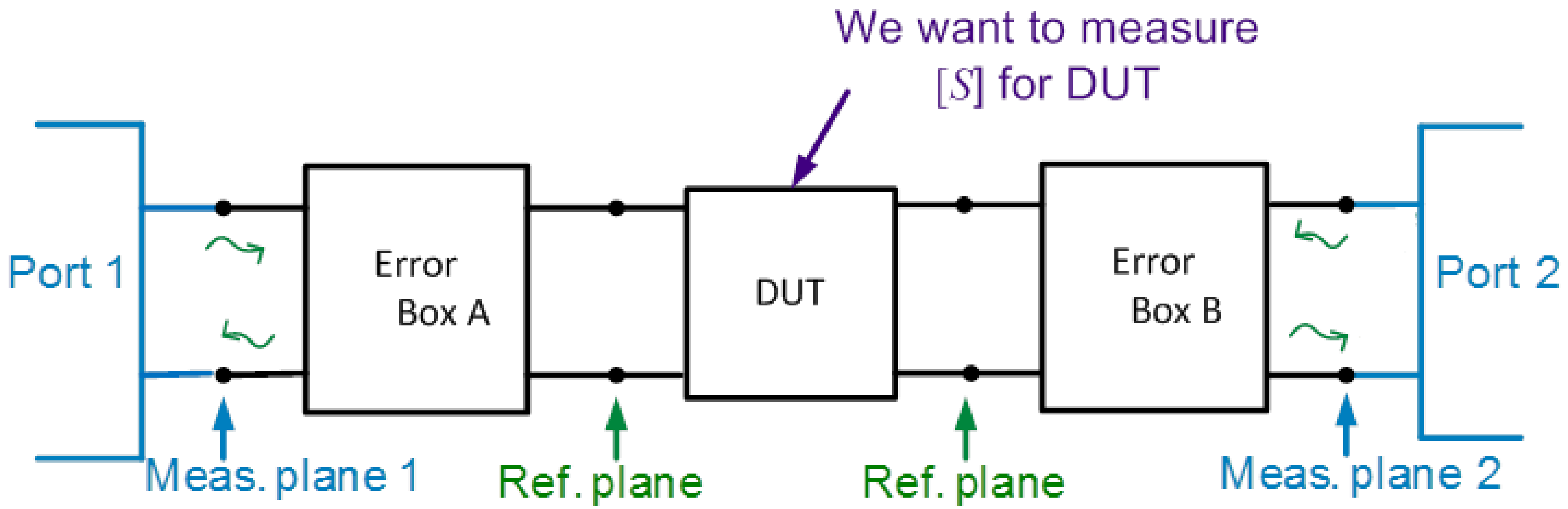


VNA Experimental Setup

Vector Network Analyzer (contd.)



Vector Network Analyzer (contd.)

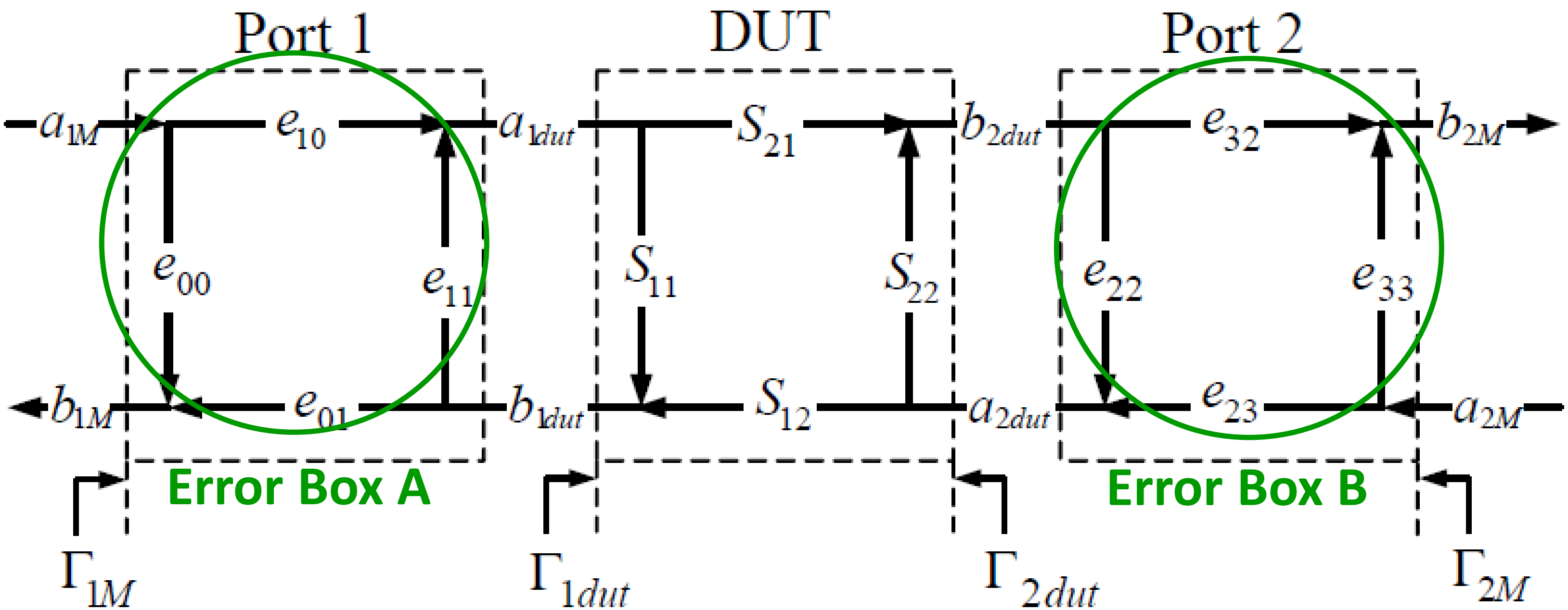


Errors Could be:

- **System Error**
- **Random Error**
- **Drift Error**

→ **Necessitates Calibration**

Vector Network Analyzer – Error Model



- **SFG Simplification:**

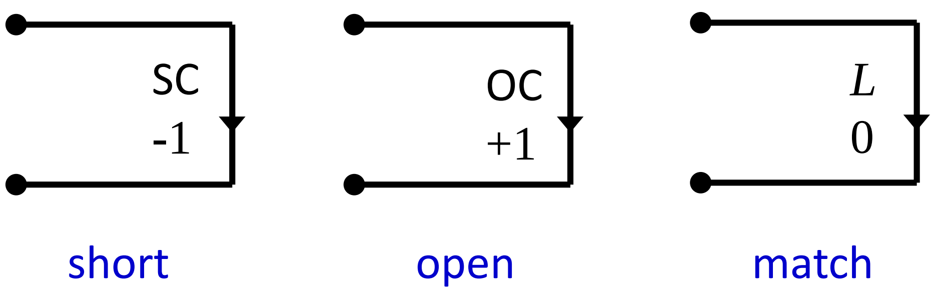
$$a_{1dut} = \left(\frac{e_{01}e_{10} - e_{00}e_{11}}{e_{01}}\right) a_{1M} + \left(\frac{e_{11}}{e_{01}}\right) b_{1M}$$

$$b_{1dut} = \left(\frac{-e_{00}}{e_{01}}\right) a_{1M} + \left(\frac{1}{e_{01}}\right) b_{1M}$$

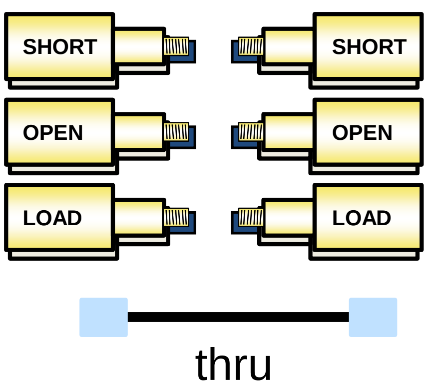
Similarly for 2nd port

Vector Network Analyzer – Calibration

- It is apparent that you need to determine the error terms to get the traveling waves at the DUT ports.
- If you observe carefully, for 1-port error correction at port-1 only three terms ($e_{00}, e_{11},\ e_{01}e_{10}$) need to be determined. Similarly for 1-port correction at port-2.
- For relating these error terms for 2-port measurements → carry out a THRU measurement between the two ports.



Calibration Standards



Vector Network Analyzer – Calibration

- The error terms e_{00}, e_{11}, and $e_{01}e_{10}$ can be determined from the first port measurement by connecting respective calibration standards and then relating the measured reflection coefficient to the reflection coefficient of the respective calibration standards.

$$\Gamma_{1M} = e_{00} + \frac{e_{01}e_{10}\Gamma_{1dut} + e_{00}}{1 - e_{11}\Gamma_{1dut}} = \frac{-\Delta e \Gamma_{1dut} + e_{00}}{-e_{11}\Gamma_{1dut} + 1} \quad \Leftarrow \quad \Delta e = \left(e_{00}e_{11} - e_{01}e_{10}\right)$$

$$\begin{bmatrix} e_{00} \\ e_{11} \\ \Delta e \end{bmatrix} = \begin{bmatrix} 1 & \Gamma_O \Gamma_{MO} & -\Gamma_O \\ 1 & \Gamma_S \Gamma_{MS} & -\Gamma_S \\ 1 & \Gamma_L \Gamma_{ML} & -\Gamma_L \end{bmatrix}^{-1} \times \begin{bmatrix} \Gamma_{MO} \\ \Gamma_{MS} \\ \Gamma_{ML} \end{bmatrix}$$

- Carry out similar measurements at port-2 and determine the error terms e_{22}, e_{33}, and $e_{23}e_{32}$.
- Then perform THRU measurement to ideally determine the tracking errors between port-1 and port-2.