Lecture – 11 Date: 06.02.2017

- Impedance Matching using Lumped Components
- Forbidden Region, Frequency Response, Quality Factor

Introduction – Impedance Transformation

One of the most important and fundamental two-port networks that microwave engineers design is a **lossless matching network** (otherwise known as an **impedance transformer**).

Q: In high frequency circuits, a source and load are connected by a TL. Can we implement matching networks in transmission line circuits?

Furthermore, these matching networks seem too good to be true—can we **really** design and construct them to provide a **perfect** match?

A: We can **easily** provide a **near** perfect match at **precisely one frequency**

But, since lossless matching and transmission lines are made of entirely r**eactive elements** (not to mention the reactive components of source and load impedance), we find that **changing** the frequency will typically “**unmatch**” our circuit!

Introduction – Impedance Transformation (contd.)

Therefore, a difficult challenge for any RF/microwave design engineer is to design a **wideband** matching network—a matching network that provides an **"adequate"** match over a wide range of frequencies.

- Generally speaking, matching network design requires a **tradeoff** between following parameters for desirable attributes:

1. Bandwidth; **2.** Complexity; **3.** Implementation; **4.** Adjustability

Matching with Lumped Components

- we begin with the **simplest** solution: An **L-network**, consisting of a **single capacitor** and a **single inductor.**

Q: Just **two** elements! That seems simple enough. Do we **always** use these L-networks when constructing lossless matching networks?

A: Nope. L-networks have **two** major drawbacks:

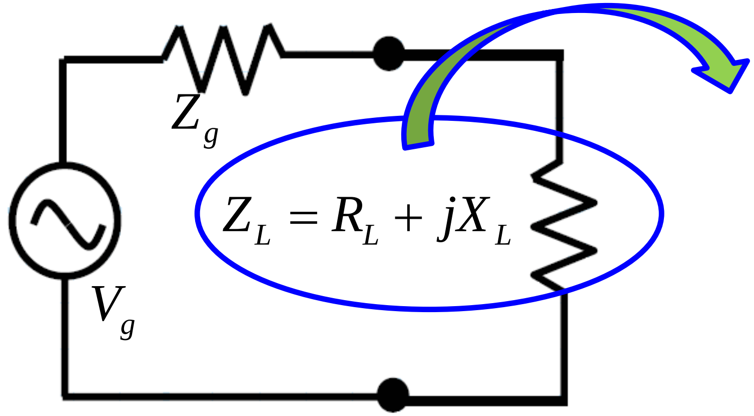
1. They are **narrow-band**.

2. Capacitors and inductors are **difficult to make** at microwave frequencies!

Soon we will see how these L-networks actually **work**

Matching Network Analysis

- Consider following circuit where a **passive load** is attached to an **active source**:



It will **absorb power** — power that is **delivered** to it by the **source** → given by expression

$$P_L = \frac{1}{2}\{V_L I_L^*\}$$

$$\Rightarrow P_L = \frac{1}{2}\mathrm{Re}\left\{\left(V_g \frac{Z_L}{Z_g + Z_L}\right)\left(\frac{V_g}{Z_g + Z_L}\right)^*\right\} \Rightarrow = \frac{1}{2}|V_g|^2 \frac{\mathrm{Re}\{Z_L\}}{|Z_g + Z_L|^2} = \frac{1}{2}|V_g|^2 \frac{R_L}{|Z_g + Z_L|^2}$$

the power delivered to the load will be **maximized** (for a given V_g and Z_g) **if** the load impedance is equal to the **complex conjugate** of the source impedance ($Z_L = Z_g^*$)

- We call the maximized power, the **available power** P_{avl} **from the source** → it is, after all, the **largest** amount of power that the source can **ever** deliver!

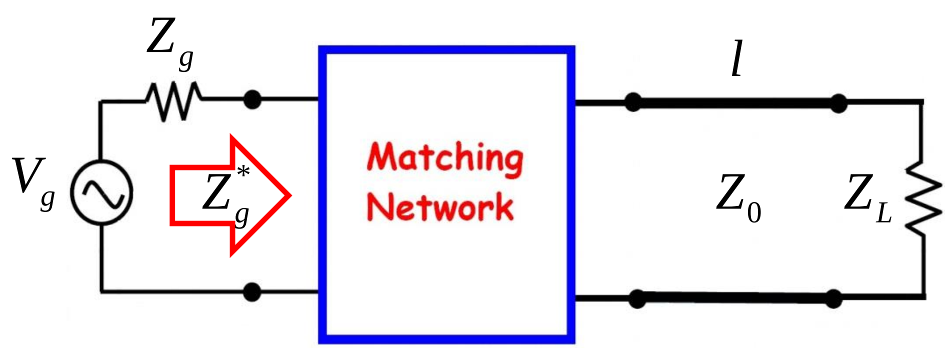
$$P_L^{\max} \doteq P_{avl} = \frac{1}{2}|V_g|^2 \frac{R_g}{|Z_g + Z_g^*|^2} = \frac{1}{2}|V_g|^2 \frac{R_g}{|2R_g|^2} = \frac{|V_g|^2}{8R_g}$$

Matching Network Analysis (contd.)

- Note the available power of the **source** is dependent on **source** parameters **only** (i.e., V_g and R_g). This makes sense! Do **you** see why?
- we must make the load impedance the complex conjugate of the source impedance to "take full advantage" of all the available power of the source,
- Otherwise, the power delivered to the load will be less than power made available by the source! In other "words": $\boxed{P_L \leq P_{avl}}$

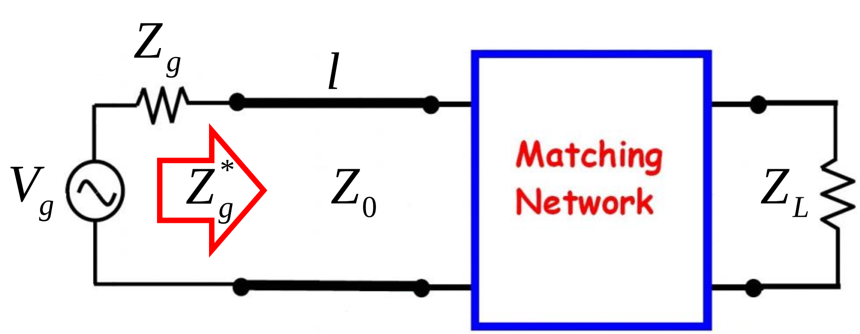
Matching Networks and TLs

Note: we can construct a network to transform the **input impedance** of the transmission line into the complex conjugate of **the source impedance**:



Q: But, do we **have** to place the matching network between the source and the transmission line?

A: Nope! We could **also** place a (different) matching network between the transmission line and the load.

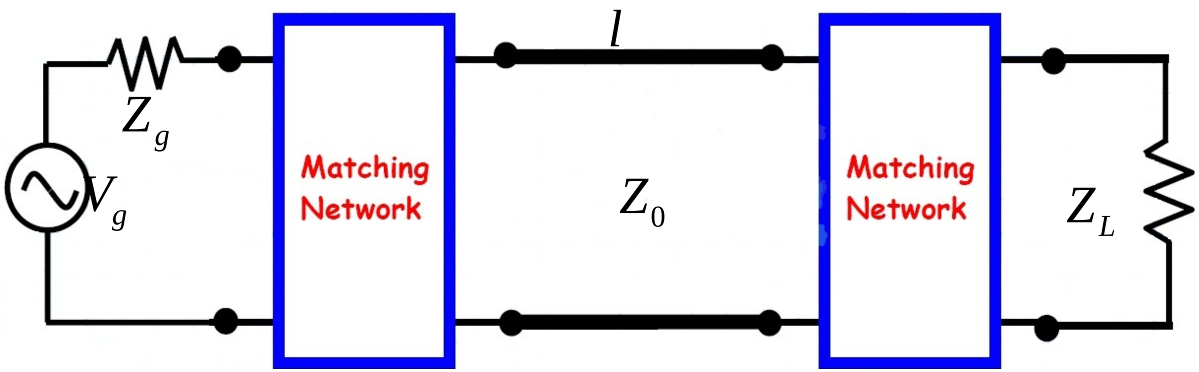


Matching Networks and Transmission Lines (contd.)

Q: So **which** method should we choose? Do engineers typically place the matching network between the source and the transmission line, **or** place it between the transmission line and the load?

A: Actually, the typical solution is to do **both**!

- We find that often there is a matching network between the a source and the transmission line, **and** between the line and the load.



Q: Yikes! Why would we want to build **two** separate matching networks, instead of just **one**?

A: By using two separate matching networks, we can **decouple** the design problem. Recall that the design of a **single** matching network solution would depend on four separate parameters:

1. the source impedance Z_g
2. load impedance Z_L
3. the TL characteristic impedance Z_0
4. the TL length l

Matching Networks and Transmission Lines (contd.)

- Alternatively, the design of the network matching the **source** and **transmission line** depends on **only**:
- In addition, the design of the network matching the **load** and **transmission line** depends on **only**:

1. the source impedance Z_g
2. the transmission line characteristic impedance Z_0

1. the source impedance Z_L
2. the transmission line characteristic impedance Z_0

Note that **neither** design depends on the transmission line **Length *l***!

Q: How is that possible?

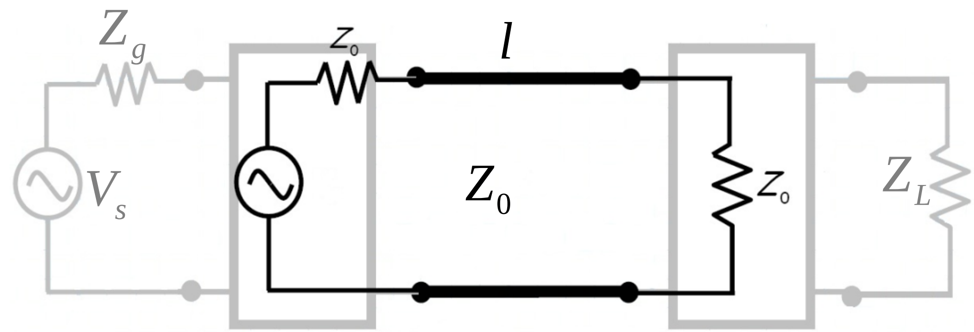
A: Remember the case where $Z_g = Z_0 = Z_L$. For that special case, we found that a conjugate match was the result—regardless of the transmission line length.

Thus, by matching the source to line impedance Z_0 **and** likewise matching the load to the line impedance, a conjugate match is **assured**—but the **length** of the transmission line does **not** matter!

In fact, the typical problem for microwave engineers is to match a load (e.g., device input impedance) to a **standard** transmission line impedance (typically $Z_0 = 50\Omega$); **or** to independently match a source (e.g., device output impedance) to a **standard** line impedance.

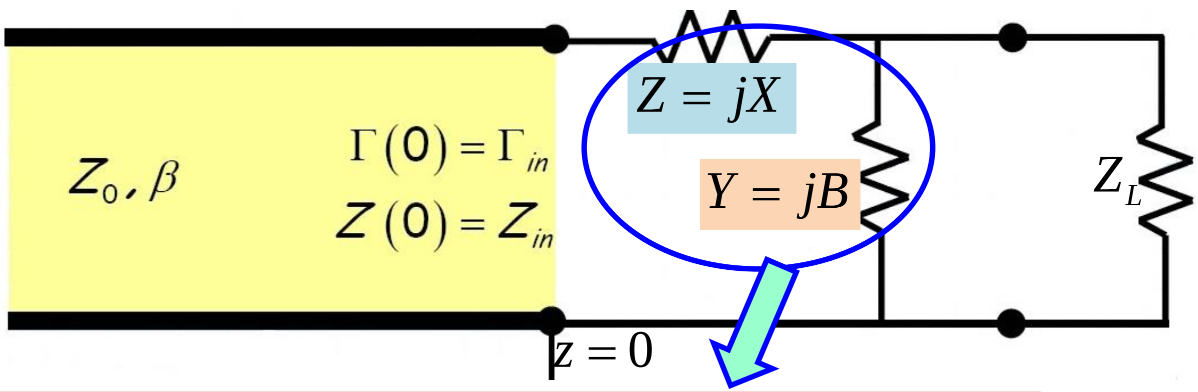
Matching Networks and Transmission Lines (contd.)

A **conjugate match** is thus obtained by connecting the two with a transmission line of **any length**!



L-Network Analysis

- consider the **first** matching L-network, and denote it as matching **network (A)**:



Note that this matching network consists of just **two** lumped elements, which must be **purely reactive** — in other words, a **capacitor** and an **inductor**!

- To make Γ_{in} = 0, the **input impedance** of the network must be:

$$\boxed{Z_{in} = Z_0}$$

- Using **basic** circuit analysis we can find that:

$$Z(z=0) = Z_{in} = jX + \frac{(1/jB)Z_L}{(1/jB)+Z_L} = jX + \frac{Z_L}{1+jBZ_L}$$

L-Network Analysis (contd.)

- a **matched** network, with $Z_{in} = Z_0$, means that: $\text{Re}\{Z_{in}\} = Z_0$ $\quad \text{Im}\{Z_{in}\} = 0$

Gives two equations and aids in the determination of two unknowns (X, and B)

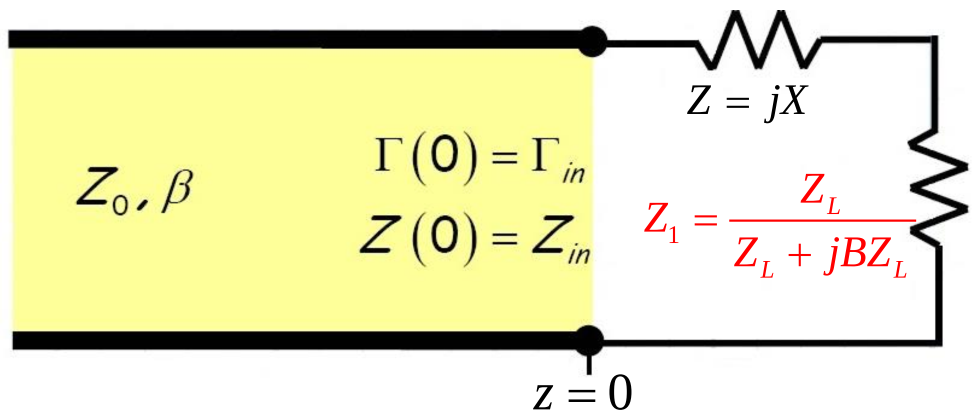
Essentially, the L-network matching network can be viewed as consisting of **two distinct parts**, each attempting to satisfy a specific requirement.

Part 1: Selecting Y = jB: Since the shunt element Y and Z_L are in **parallel**, we can combine them into one element **Y_1**:

$$Y_1 \doteq Y + \frac{1}{Z_L} = jB + Y_L$$

- The impedance of this element is therefore:

$$Z_1 = \frac{1}{Y_1} = \frac{1}{jB + Y_L} = \frac{Z_L}{Z_L + jBZ_L}$$

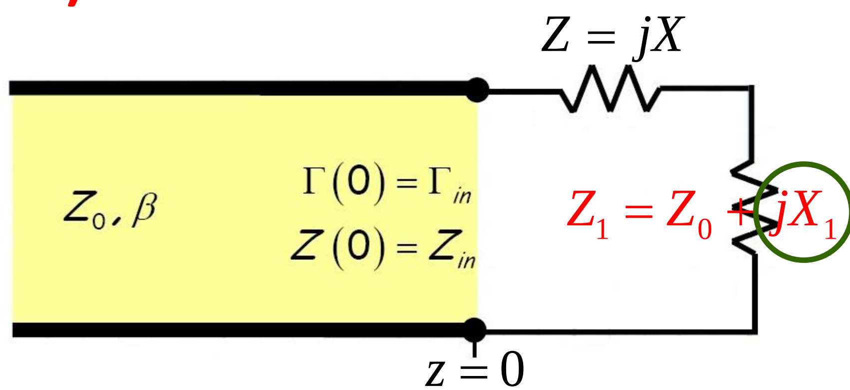


- For perfect match, set the value of susceptance B such that:

$$\text{Re}\{Z_{in}\} = \text{Re}\{Z_1\} \quad \Rightarrow Z_0 = \text{Re}\left\{\frac{Z_L}{Z_L + jBZ_L}\right\}$$

L-Network Analysis (contd.)

- Thus if B is properly selected:



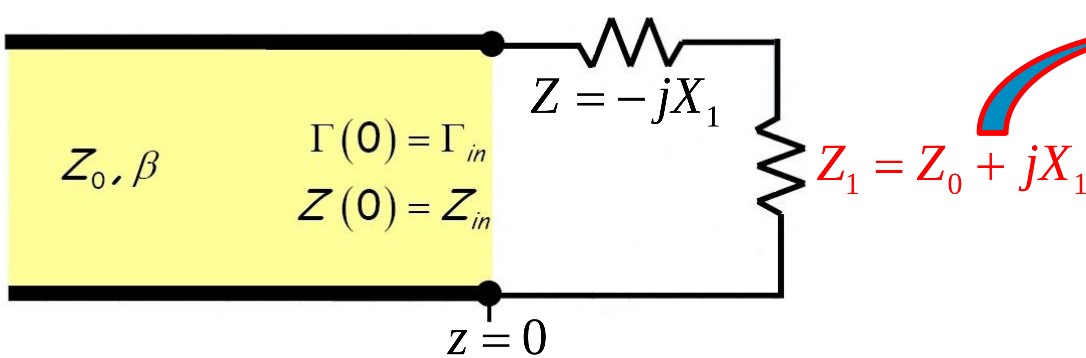
reactive part of matching circuit

Part 2: Selecting Z =jX: Note that the impedance $Z_1 = Z_L || (1/jB)$ has the ideal real value of Z_0. However, it also posses an **annoying** imaginary part of:

$$X_1 = \mathrm{Im}\{Z_1\} = \mathrm{Im}\left\{\frac{Z_L}{Z_L + jBZ_L}\right\}$$

- However, this imaginary component can be easily removed by setting the **series** element Z =jX to its equal but **opposite** value!

$$X = -X_1 = -\mathrm{Im}\left\{\frac{Z_L}{Z_L + jBZ_L}\right\}$$



It gives

$$Z_{in} = Z + Z_1$$

$$\boxed{\Rightarrow Z_{in} = -jX_1 + Z_0 + jX_1 = Z_0}$$

Perfect Match!!!

L-Network Analysis (contd.)

- we can solve the preceding equations for the **required** values X and B to **satisfy** these two equations — to create a **matched** network!

$$B = \frac{X_L \pm \sqrt{R_L / Z_0}\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2}$$

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{BR_L}$$

Where, $Z_L = R_L + jX_L$

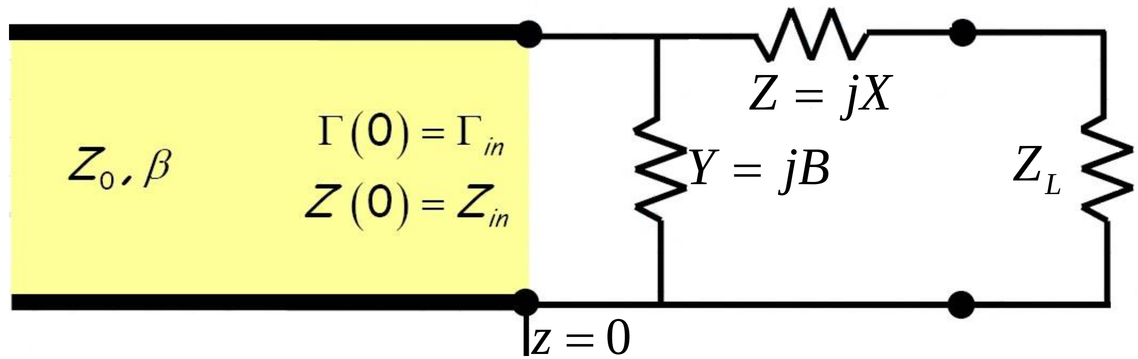
Note: **1)** Because of the ± , there are **two** solutions for B (and thus X)

2) For jB to be purely imaginary (i.e., reactive), B must be **real** → R_L **must** be greater than Z_0 ($R_L > Z_0$) to insure that B and thus X is real.

In other words, this matching network **(type-A)** can only be used when $R_L > Z_0$. Notice that this condition means that the normalized load z_L' lies **inside** the r = 1 circle on the Smith Chart!

L-Network Analysis (contd.)

- Now let's consider the **second** of the two L-networks, which we call **network (B)**. Note it is **also** formed with just two lumped elements.

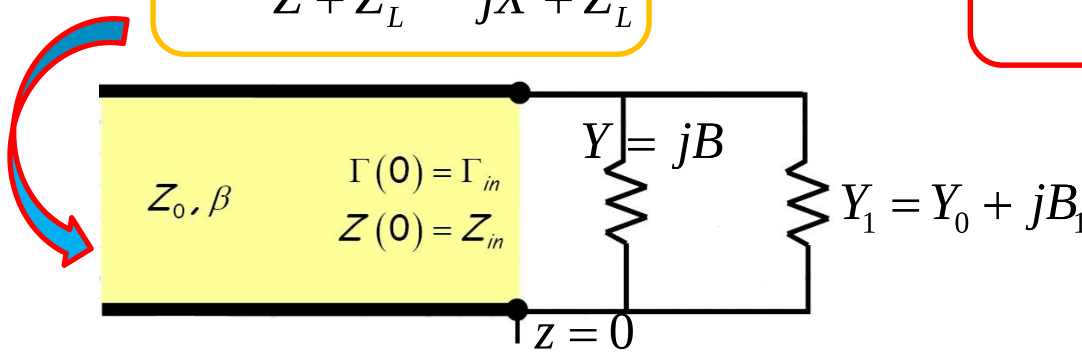


- To make Γ_{in} = 0, the **input admittance** of the network must be: $\boxed{Y_{in} = Y_0}$
- From circuit theory, the input **admittance** for this network is: $\boxed{Y_{in} = jB + \frac{1}{jX + Z_L}}$
- We can describe a **matched** network, with Y_{in} = Y_0 as: $\mathrm{Re}\{Y_{in}\} = Y_0$ $\quad \mathrm{Im}\{Y_{in}\} = 0$
- For this design, we set the value of Z =jX such that:

$$\boxed{Y_1 \doteq \frac{1}{Z + Z_L} = \frac{1}{jX + Z_L}}$$

Gives real part as

$$\boxed{Y_0 = \mathrm{Re}\left\{\frac{1}{jX + Z_L}\right\}}$$



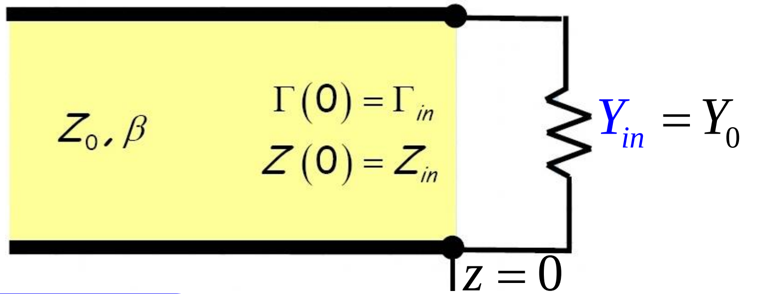
L-Network Analysis (contd.)

- It is apparent that a perfect match will occur if the shunt element Y = jB is set to "cancel" the reactive component of Y_1:

$$B = -\mathrm{Im}\{Y_1\} = -\mathrm{Im}\left\{\frac{1}{jX + Z_L}\right\}$$

So that: $$Y_{in} = Y + Y_1 = -jB_1 + (Y_0 + jB_1) = Y_0$$

Perfect Match!!!



- we can solve the preceding equations for the **required** values X and B to **satisfy** these two equations — to create a **matched** network!

$$B = \pm\frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}$$

$$X = \pm\sqrt{R_L(Z_0 - R_L)} - X_L$$

Where, $Z_L = R_L + jX_L$

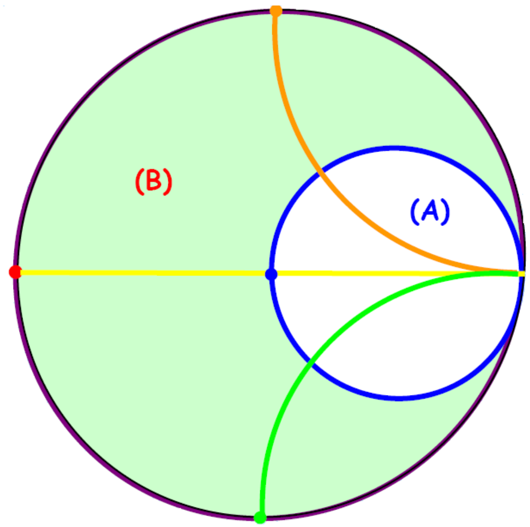
Note: **1)** Because of the ± , there are **two** solutions for B (and thus X)

2) For jB and jX to be purely imaginary (i.e., reactive), B and X must be **real**

→ R_L **must** be less than Z_0 ($R_L < Z_0$) to insure that B and X are real.

L-Network Analysis (contd.)

In other words, this matching network **(type-B)** can only be used when $R_L < Z_0$. Notice that this condition means that the normalized load z_L' lies **outside** the r = 1 circle on the Smith Chart!



Smith Chart Regions Depicting the Validity Regions for Networks A and B

- Once the values of X and B are found, determine the required values of L and/or C, **for the signal frequency ω_0 !**

$$X = \begin{cases} \omega_0 L & \text{If } X > 0 \\ -\dfrac{1}{\omega_0 C} & \text{If } X < 0 \end{cases}$$

$$B = \begin{cases} \omega_0 C & \text{If } B > 0 \\ -\dfrac{1}{\omega_0 L} & \text{If } B < 0 \end{cases}$$

Make sure that you see and know why these equations are true

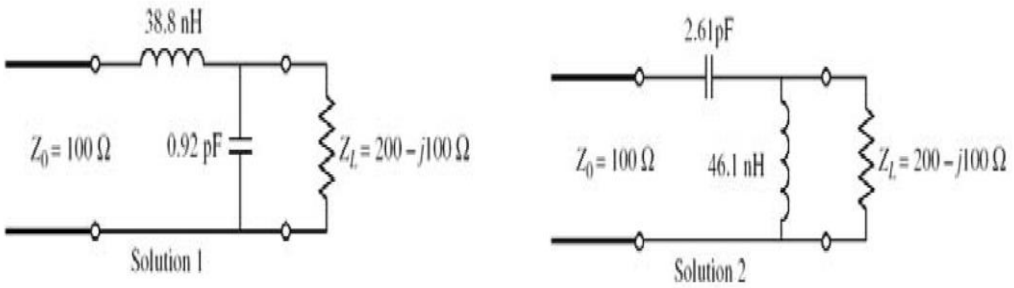
As a result, we see that the reactance or susceptance of the elements of our L-network will have the proper values for matching at precisely **one and only one frequency!**

And this frequency **better be the signal frequency ω_0 !**

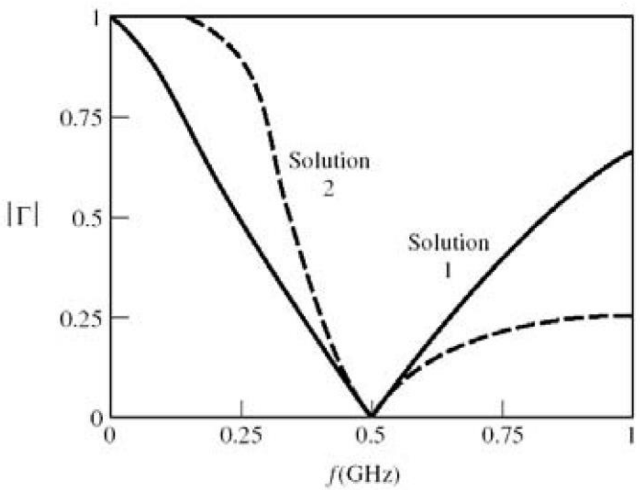
L – Type Matching Network (contd.)

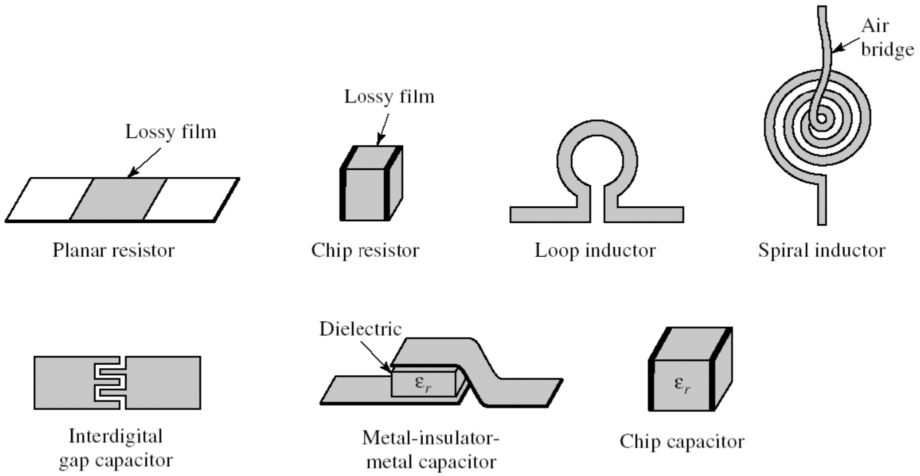
If the signal frequency **changes from the design frequency, the** reactance and susceptance of the matching network inductors and capacitors will change → As a result the circuit will **no longer** be matched

Therefore the L-Type matching network has a **narrow bandwidth!**



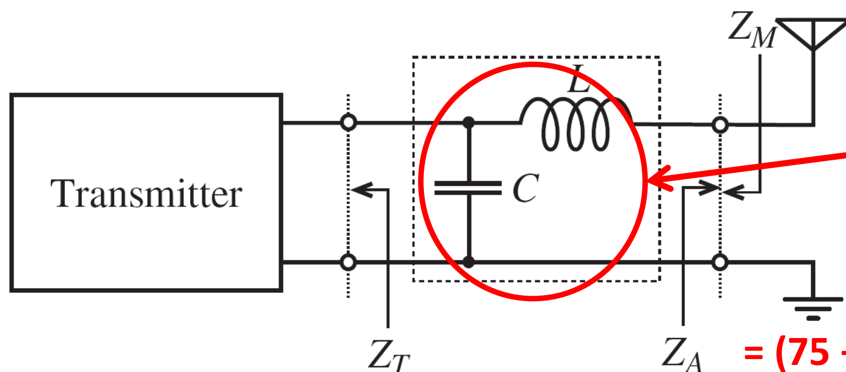
- **In addition; it becomes very difficult to build quality lumped elements with useful values past 1 or 2 GHz. Thus, L-network solutions are generally applicable only in the relatively low RF region (i.e., < 2GHz).**





Example – 1

The output impedance of a transmitter operating at a frequency of **2GHz** is **Z_T = (150 + j75)Ω**. Design an L–Type matching network, as shown below, such that maximum power is delivered to the antenna whose input impedance is **Z_A = (75 + j15) Ω.**

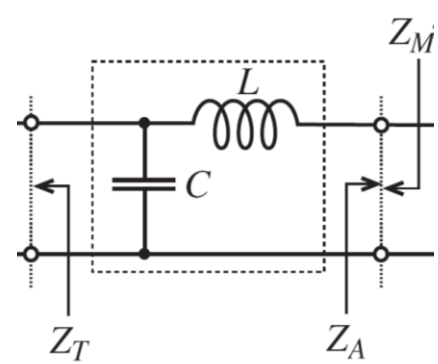


Matching Circuit – determine C and L

Analytical Approach

The condition of the maximum power transfer implies that:

$$Z_M = Z_A^* = (75 - j15)\Omega$$



Series connection of L and an element (of parallel combination of C and Z_T)

$$\Rightarrow Z_M = \frac{1}{Z_T^{-1} + jB_C} + jX_L = Z_A^*$$

$$B_C = \omega C$$

$$X_L = \omega L$$

Let us write:

$$Z_T = R_T + jX_T \qquad Z_A = R_A + jX_A$$

Example – 1 (contd.)

$$\frac{R_T + jX_T}{1 + jB_C\left(R_T + jX_T\right)} + jX_L = R_A - jX_A$$

Simplification gives:

$$R_T = R_A(1 - B_C X_T) + (X_A + X_L)B_C R_T$$

$$X_T = R_T R_A B_C - (1 - B_C X_T)(X_A + X_L)$$

$$B_C = \frac{X_T \pm \sqrt{\frac{R_T}{R_A}\left(R_T^2 + X_T^2\right) - R_T^2}}{R_T^2 + X_T^2}$$

In this example, $R_T > R_A$; therefore the square root is +ve and therefore for positive B_C (capacitor) we must choose the plus sign in this expression.

Therefore:

$$X_L = \frac{1}{B_C} - \frac{R_A(1 - B_C X_T)}{B_C R_T} - X_A$$

- **Insert the given values in the obtained expressions to get:**

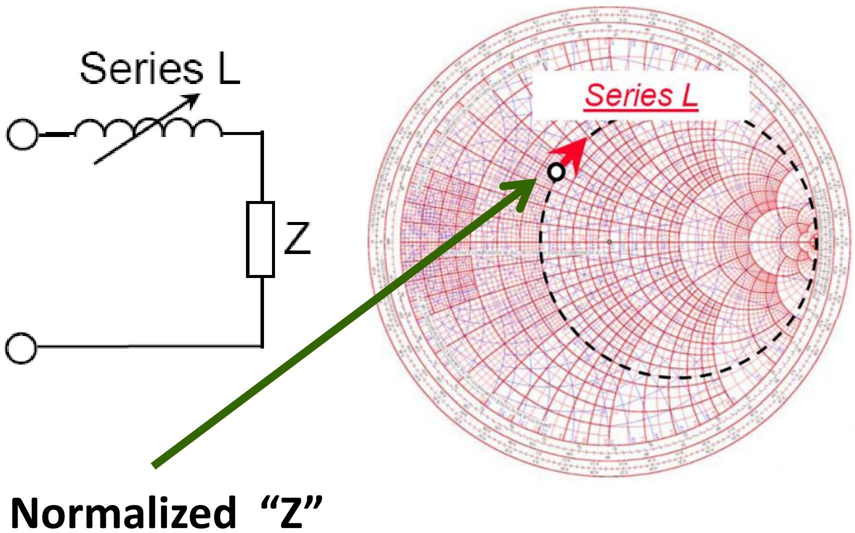
$$B_C = 9.3mS \Rightarrow C = \frac{B_C}{\omega} = 0.73pF \qquad X_L = 76.9\Omega \Rightarrow L = \frac{X_L}{\omega} = 6.1nH$$

Example – 1 (contd.)

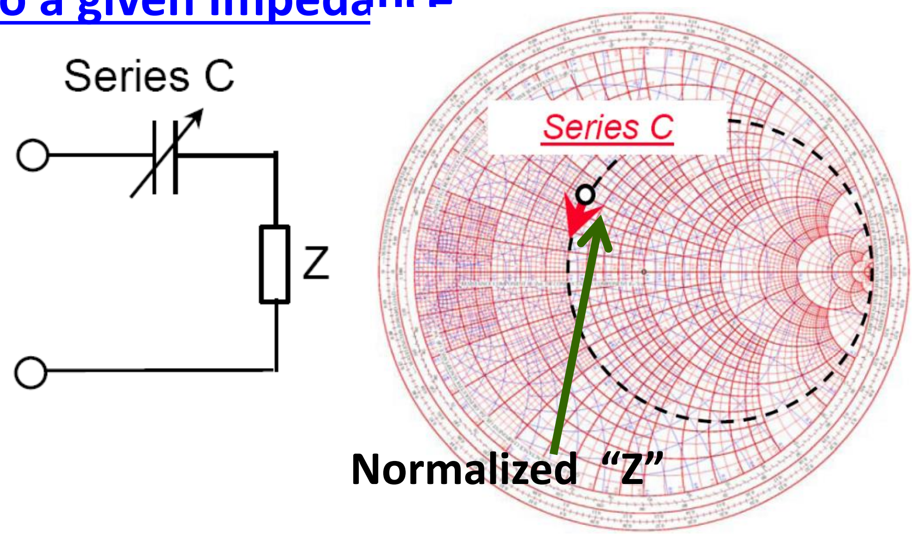
Smith Chart Based Approach

- Before we tread this path, let us have a look on Smith chart navigation when series/shunt reactances are added to any impedance (Z)

Series Connection of Inductance to a given Impedance



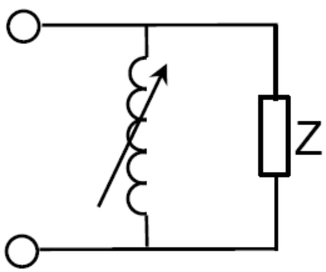
Series Connection of Capacitance to a given Impedance

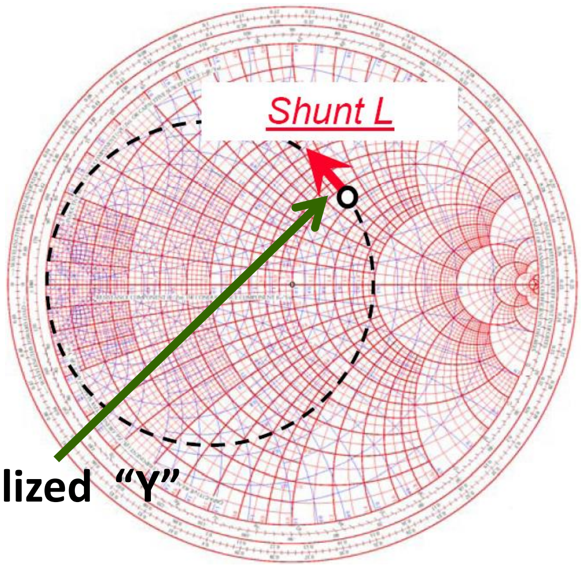


Example – 1 (contd.)

Shunt Connection of Inductance to a given Impedance

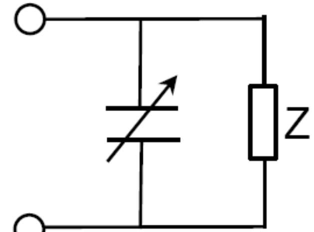
Shunt L

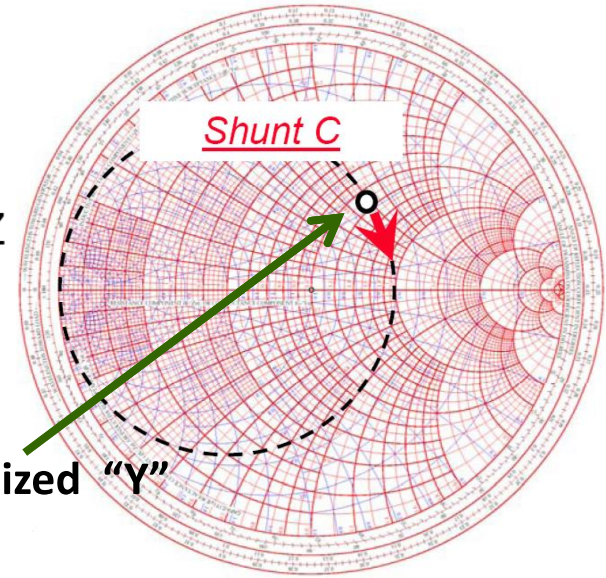




Shunt Connection of Capacitance to a given Impedance

Shunt C





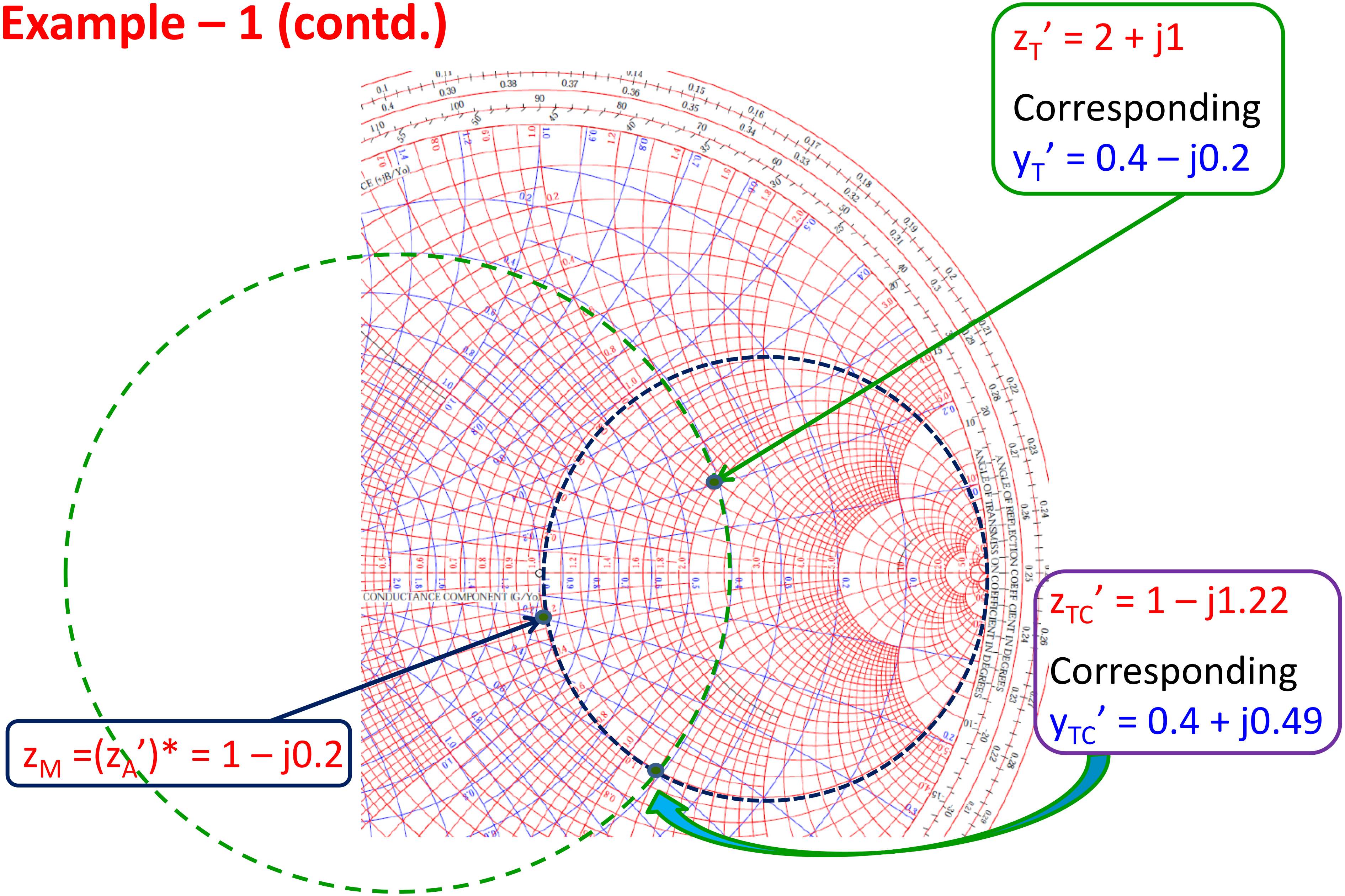
Normalize the transmitter and antenna impedances

- Since no Z_0 is given, one can choose any. In this example, Z_0 = 75Ω makes simplification easier.

$$z_T' = Z_T / Z_0 = 2 + j1 \qquad z_A' = Z_A / Z_0 = 1 + j0.2$$

- C is in shunt with Z_T → the movement will be downward on a constant conductance circle → starting point will be z_T' and the end point will be its intersection with z_M = 1 – j0.2 circle
- Then the movement will be upward on a constant resistance circle (from $z_M = (z_A')^* = 1 – j0.2$) → to account for the series inductance L

Example – 1 (contd.)



Example – 1 (contd.)

Therefore, the normalized susceptance jb_c is:

$$\boxed{jb_c = y'_{TC} - y'_T = 0.4 + j0.49 - (0.4 - j0.2) = j0.69}$$

Similarly, the normalized reactance jx_L is:

$$\boxed{jx_L = z_A^{*'} - z'_{TC} = 1 - j0.2 - (1 - j1.22) = j1.02}$$

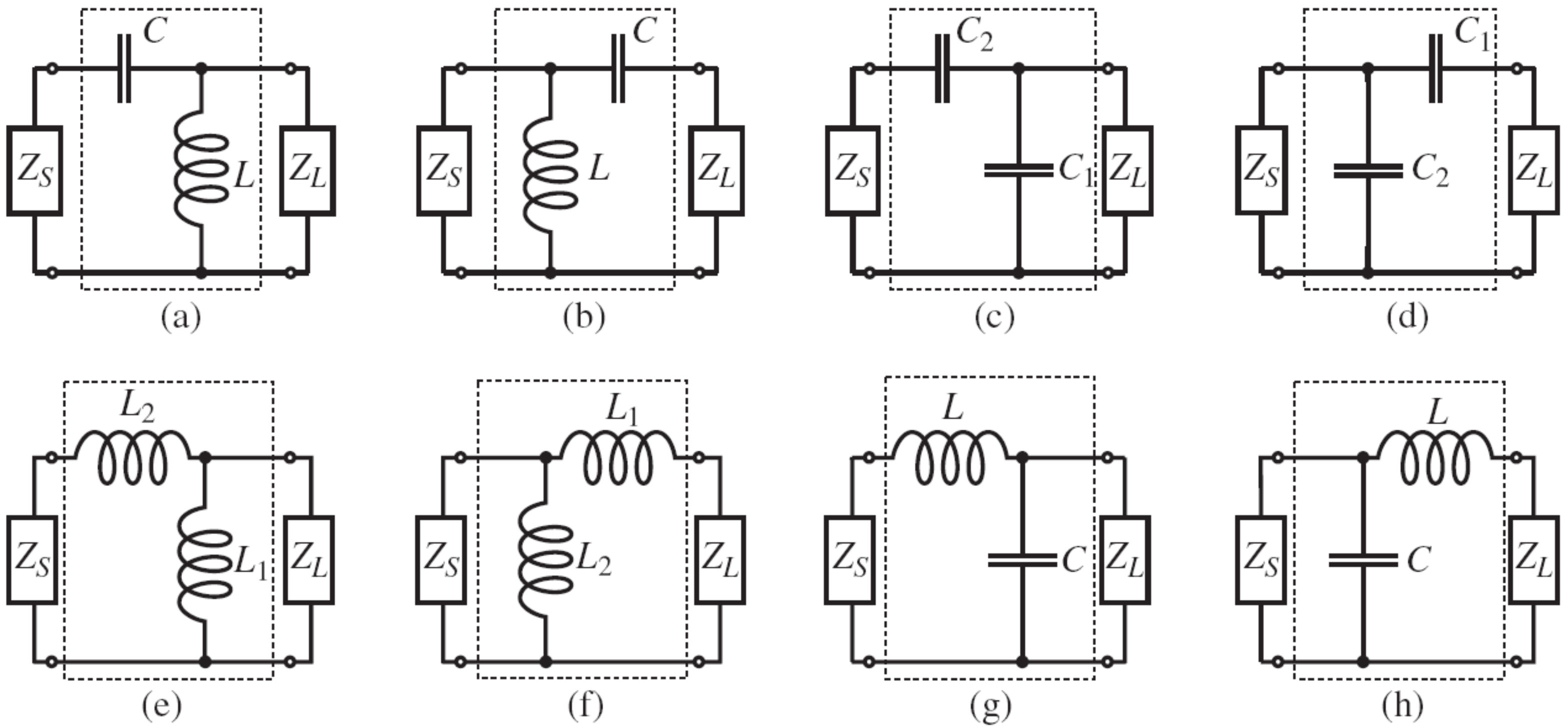
Finally,

$$L = \frac{x_L Z_0}{\omega} = 6.09nH$$

$$C = \frac{b_c}{\omega Z_0} = 0.73pF$$

L – Type Matching Network (contd.)

Various configurations



Their usefulness is regulated by the specified source and load impedances and the associated matching requirements

L – Type Matching Network (contd.)

Design procedure for two element L – Type matching Network

1. Find the normalized source and load impedances.
2. In the Smith chart, plot circles of constant resistance and conductance that pass through the point denoting the source impedance.
3. Plot circles of constant resistance and conductance that pass through the point of the complex conjugate of the load impedance ($z_M = z_L^*$).
4. Identify the intersection points between the circles in steps 2 and 3. The number of intersection points determine the number of possible L-type matching networks.
5. Find the values of normalized reactances and susceptances of the inductors and capacitors by tracing a path along the circles from the source impedance to the intersection point and then to the complex conjugate of the load impedance.
6. Determine the actual values of inductors and capacitors for a given frequency.

Example – 2

Using the Smith chart, design all possible configurations of discrete two-element matching networks that match the source impedance $Z_S = (50+j25)\Omega$ to the load $Z_L = (25-j50)\Omega$. Assume a characteristic impedance of $Z_0 = 50\Omega$ and an operating frequency of f = 2 GHz

Example – 2 (contd.)

1. The normalized source and load impedances are:

$$z_S = Z_S / Z_0 = 1 + j0.5$$

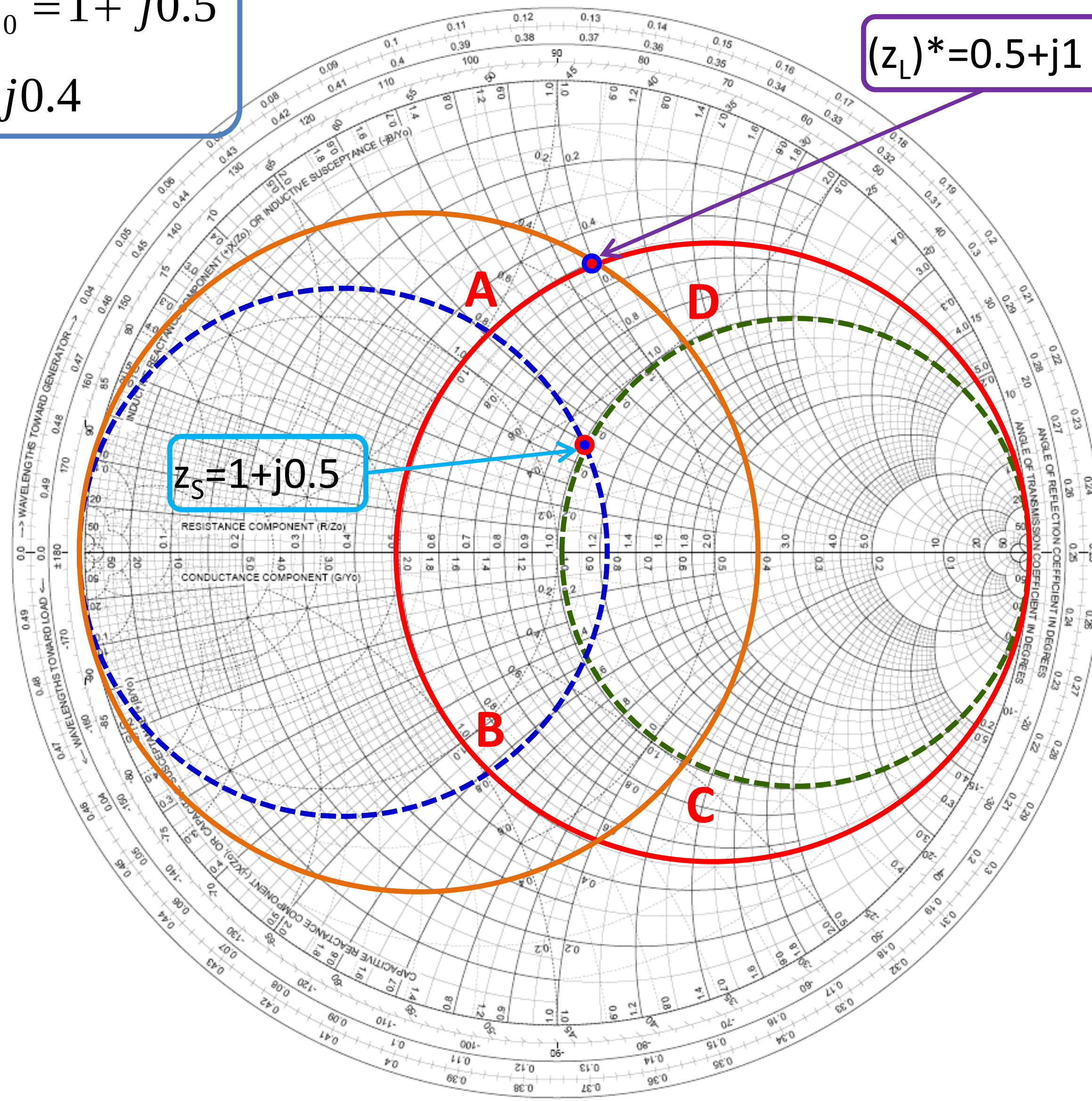
$$y_S = 0.8 - j0.4$$

$$z_L = Z_L / Z_0 = 0.5 - j1$$

$$y_L = 3 + j0.8$$

2. Mark z_S and then plot circles of constant resistance and conductance that passes through z_S.

3. Mark $(z_L)^*$ and then plot circles of constant resistance and conductance that passes through $(z_L)^*$.



Example – 2 (contd.)

4. The intersection points of these circles are A, B, C and D with the normalized impedances and inductances as:

$z_A = 0.5 + j0.6 \qquad y_A = 0.8 - j1$

$z_B = 0.5 - j0.6 \qquad y_B = 0.8 + j1$

$z_C = 1 - j1.2 \qquad y_C = 3 + j0.5$

$z_D = 1 + j1.2 \qquad y_D = 3 - j0.5$

5. There are four intersection points and therefore four L-type matching circuit configurations are possible.

$z_S \rightarrow z_A \rightarrow (z_L)^*$ ⇒ **Shunt L, Series L**

$z_S \rightarrow z_B \rightarrow (z_L)^*$ ⇒ **Shunt C, Series L**

$z_S \rightarrow z_C \rightarrow (z_L)^*$ ⇒ **Series C, Shunt L**

$z_S \rightarrow z_D \rightarrow (z_L)^*$ ⇒ **Series L, Shunt L**

6. Find the actual values of the components

In the first case:

$z_S \rightarrow z_A$: the normalized admittance is changed by

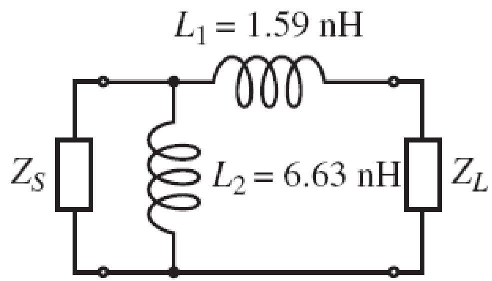
$$jb_{L_2} = y_A - y_S = -j0.6 \qquad \Rightarrow L_2 = -\frac{Z_0}{b_{L_2}\omega} = 6.63nH$$

$z_A \rightarrow z_L$: the normalized impedance is changed by

$$jx_{L_1} = (z_L)^* - z_A = j0.4 \qquad \Rightarrow L_1 = \frac{x_{L_1} Z_0}{\omega} = 1.59nH$$

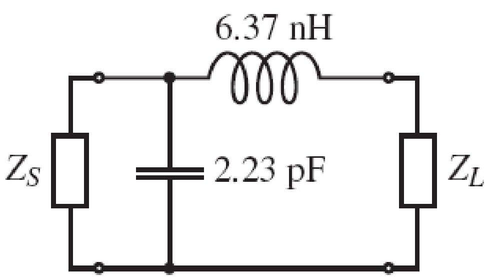
Example – 2 (contd.)

Therefore the circuit is

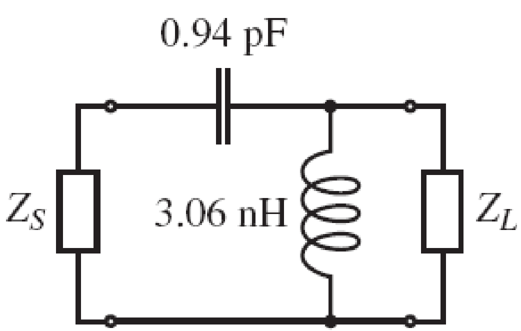


$z_S \rightarrow z_A \rightarrow (z_L)^*$

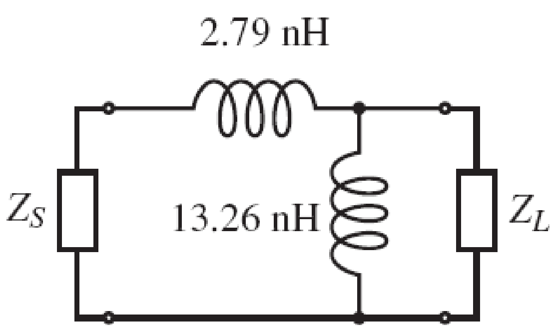
Similarly:



$z_S \rightarrow z_B \rightarrow (z_L)^*$



$z_S \rightarrow z_C \rightarrow (z_L)^*$



$z_S \rightarrow z_D \rightarrow (z_L)^*$

Forbidden Region, Frequency Response, and Quality Factor

Self Study - Section 8.1.2 in the Text Book

- The L-type matching networks can be considered as resonance circuits with f_0 being the resonance frequency.
- These networks can be described by a loaded quality factor, Q_L, given by:

$$Q_L = \frac{f_0}{BW}$$

Similarity to bandpass filter

- However, analysis of matching circuit based on bandpass filter concept is complex → In addition, it only allows approximate estimation of the bandwidth.
- More simpler and accurate method is design and analysis through the use of nodal quality factor, Q_n.
- During L-type matching network analysis it was apparent that at each node the impedance can be expressed in terms of equivalent series impedance $Z_S = R_S + jX_S$ or admittance $Y_P = G_P + jB_P$.
- Therefore, at each node we can define Q_n as the ratio of the absolute value of reactance X_S to the corresponding resistance R_S.

$$Q_n = \frac{|X_S|}{R_S}$$

- Similarly,

$$Q_n = \frac{|B_P|}{G_P}$$

Nodal Quality Factor

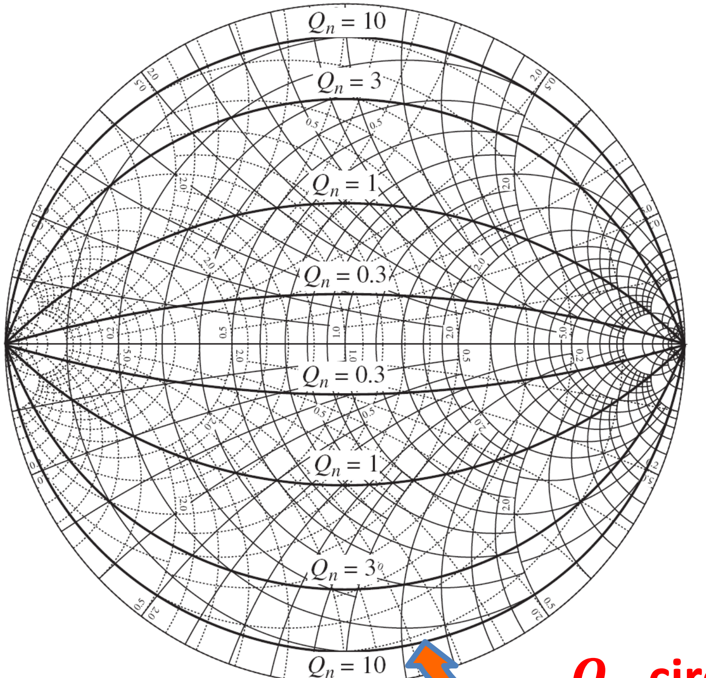
- The "nodal quality factor" and loaded quality factor are related as:

$$Q_L = \frac{Q_n}{2}$$

For any L-matching network

For more complicated networks, $Q_L = Q_n$

- Bandwidth of the matching network can be easily estimated once the "nodal quality factor" is known.
- To simplify the matching network design process even further, we can draw constant Q_n contours in the Smith chart.



We know the Smith chart expression:

$$z = r + jx = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2} + j\frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}$$

- The expression for Q_n:

$$Q_n = \frac{|x|}{r} = \frac{2|\Gamma_i|}{1-\Gamma_r^2-\Gamma_i^2}$$

$$\Gamma_i^2 + \left(\Gamma_r \pm \frac{1}{Q_n}\right)^2 = 1 + \frac{1}{Q_n^2}$$

Q_n **circles!**

Q_n circles in the Smith chart help in direct determination of Q_L.

Nodal Quality Factor (contd.)

- From section 8.1.2, it will be apparent that Q of matching network is extremely important.
- For example, broadband amplifier requires matching circuit with low-Q, while oscillators require high-Q networks to eliminate harmonics in the output signal.
- It will also be apparent that L-type matching networks have no control over the values of Q_n → **Limitation!!!**
- To gain more **freedom in choosing the values of Q or Q_n**, another element in the matching network is incorporated → results in T- or Pi-network

T- and Pi- Matching Networks

- The knowledge of nodal quality factor (Q_n) of a network enables estimation of loaded quality factor → hence the Band Width (BW).
- The addition of third element into the matching network allows control of Q_L by choosing an appropriate intermediate impedance.

Example – 1

- Design a T-type matching network that transforms $Z_L = (60 - j30)\Omega$ into $Z_{in} = (10 + j20)\Omega$ and that has a maximum **Q_n of 3**. Compute the values for the matching network components, assuming that matching is required at $f = 1\text{GHz}$.

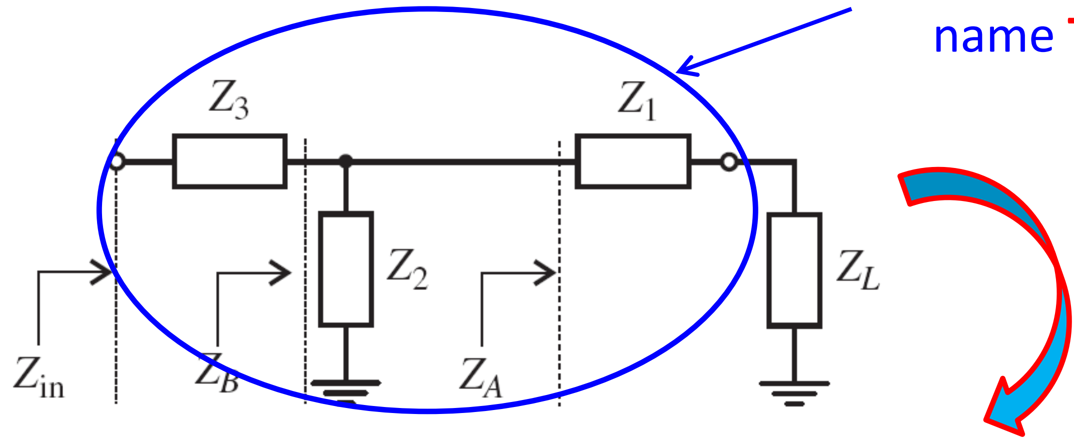
Example – 1 (contd.)

Solution

- Several possible configurations! Let us focus on just one!

Gives the name **T**

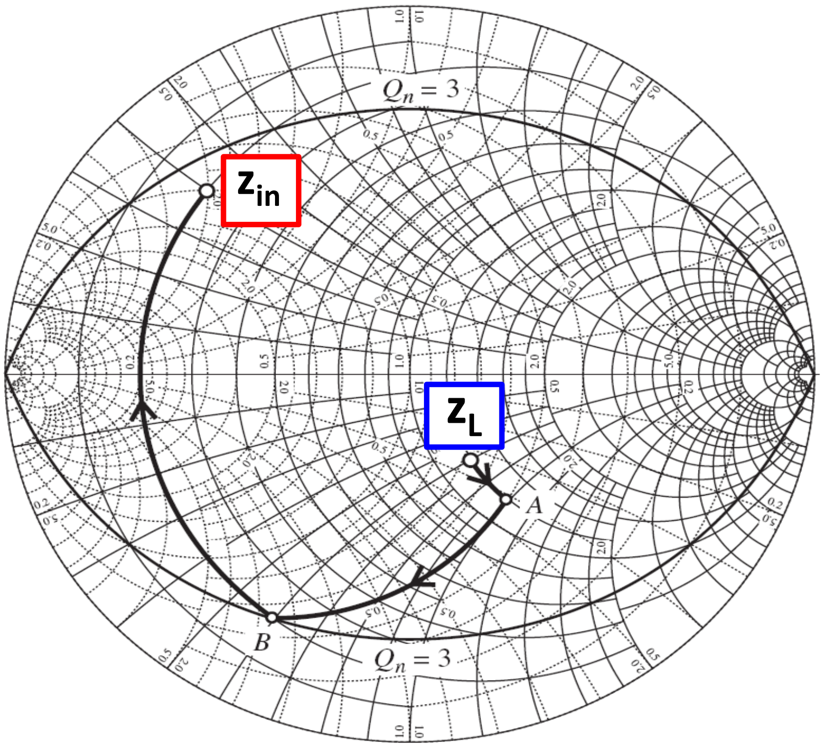
General Topology of a T-matching Network



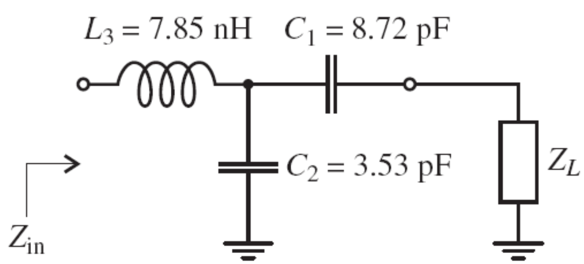
- First element in series (Z_1) is purely reactive, therefore the **combined impedance of (Z_L and Z_1)** will reside on the constant resistance circle of **$r = r_L$**
- Similarly, **Z_3** (that is purely reactive!) **is connected in series** with the input, therefore **the combined impedance Z_B** (consisting of Z_L, Z_1, and Z_2) lies on the constant resistance circle **$r = r_{in}$**
- Network needs to have a Q_n of 3 → we should choose impedance in such a way that **Z_B** is located on the **intersection** of constant resistance circle **$r = r_{in}$** and **Q_n = 3** circle → helps in the determination of **Z_3**

Example – 1 (contd.)

- The constant resistance circle of **z_in** intersects the $Q_n = 3$ circle at point **B**. This gives value of Z_3.
- The constant resistance circle $r = r_L$ and a constant conductance circle that passes through B helps in the determination of Z_2 and Z_1.



Final solution at 1 GHz



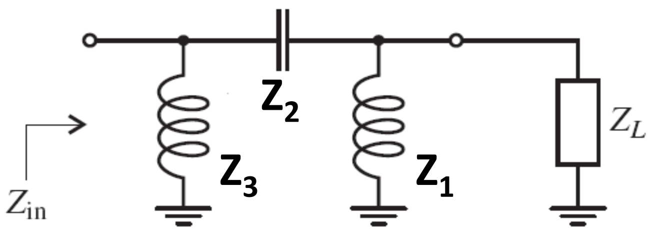
Example – 2

- For a broadband amplifier, it is required to develop a Pi-type matching network that transforms a Z_L = (10 – j10)Ω into Z_{in} = (20 + j40)Ω. The design should involve the lowest possible **Q_n**. Compute the values for the matching network components, assuming that matching is required at f = 2.4GHz.

Example – 2 (contd.)

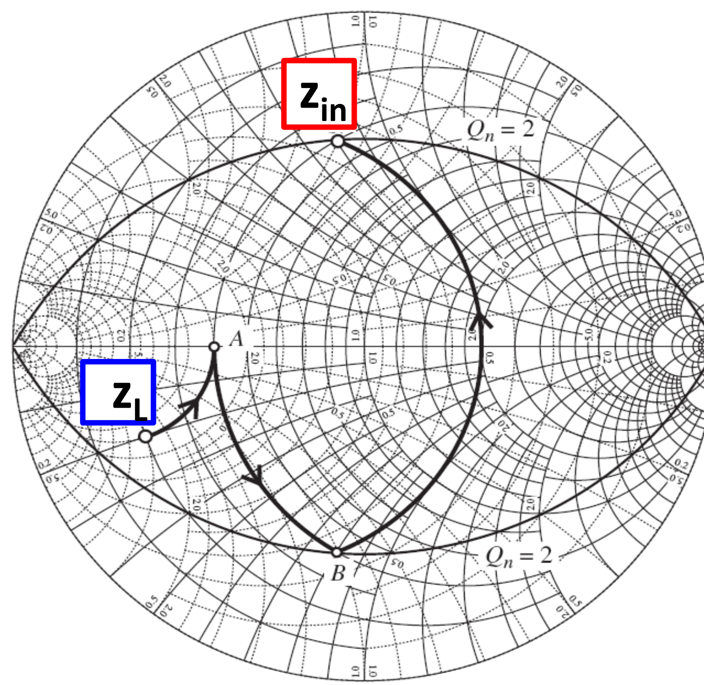
Solution

- Several Configurations possible (including the forbidden!). One such is :



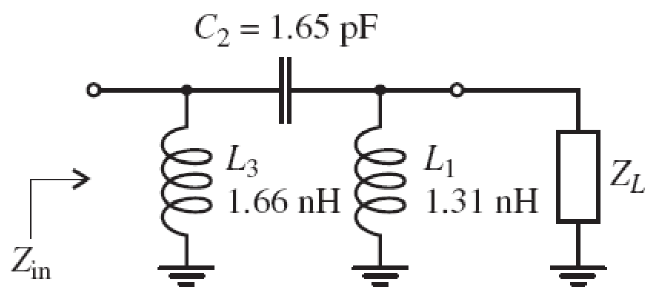
- Since the load and source impedances are fixed, we can't develop a matching network that has Q_n lower than the values at locations Z_L and Z_{in}
- Therefore in this example, the minimum value of Q_n is determined at the input impedance location as $Q_n = |X_{in}|/R_{in} = 40/20 = 2$

- In the design, we first plot constant conductance circle $g = g_{in}$ and find its intersection with Q_n=2 circle (point B) → determines the value of Z_3
- Next find the intersection point (labeled as A) of the $g=g_L$ circle and constant-resistance circle that passes through B → determines value of Z_2 and Z_1



Example – 2 (contd.)

Final solution at 2.4 GHz



- It is important to note that the relative positions of Z_{in} and Z_L **allows only one** optimal Pi-type network for a given specification.
- All other realizations will result in higher Q_n → **essentially smaller BW!**
- Furthermore, **for smaller Z_L** the Pi-matching isn't possible!

It is thus apparent that BW can't be enhanced arbitrarily by reducing the Q_n. The limits are set by the desired complex Z_{in} and Z_L.

With increasing frequency and correspondingly reduced wavelength the influence of parasitics in the discrete elements are noticeable → distributed matching networks overcome most of the limitations (of discrete components) at high frequency