Lecture – 10

Date: 02.02.2017

- The Signal Flow Graph

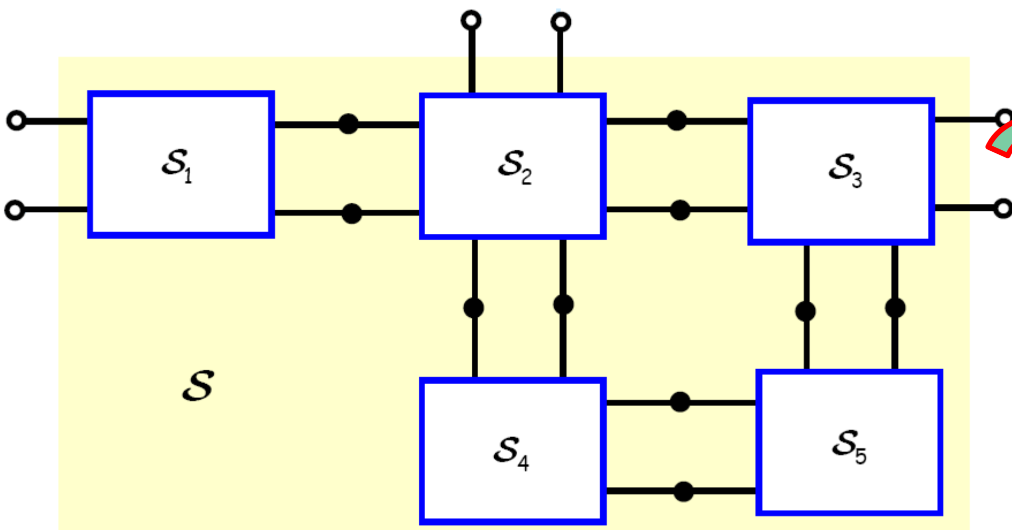
Signal Flow Graph

Q: Using individual device scattering parameters to analyze a complex microwave network results in a lot of **messy** math! Isn't there an **easier** way?

It provides a sort of a **graphical** way to do algebra!

A: Yes! We can represent a microwave network with its **signal flow graph (SFG)** and then decompose this graph using a standard set of rules → results into simpler analysis.

- To understand the significance of SFG, let us consider a complex **3-port** microwave network constructed of **5** simpler microwave devices



S_n is the **scattering matrix** of each device, and S is the **overall** scattering matrix of the **entire** 3-port network

Signal Flow Graph (contd.)

The S-parameter (S) of the whole network can be obtained from the knowledge of S-parameter of individual devices

Tedious Algebra!

Alternative is SFG based solution!

Signal flow graphs are helpful in three ways!

Way 1 – It provide us with a **graphical** means of **solving** large systems of simultaneous equations.

Way 2 – We'll see that it can provide us with a **road map** of the wave **propagation paths** throughout a HF device or network. If we're paying attention, we can glean great **physical insight** as to the inner working of the device represented by the graph.

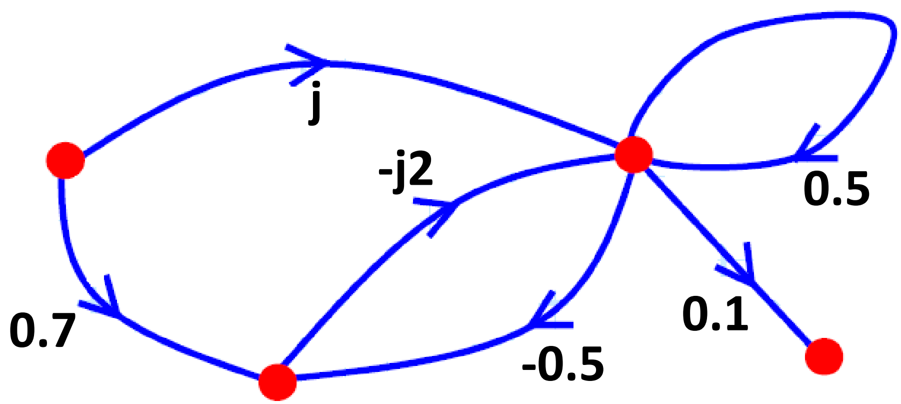
Way 3 – It provide us with a quick and accurate method for **approximating** a network or device. We will find that we can often replace a rather complex graph with a much **simpler** one that is **almost** equivalent.

Signal Flow Graph (contd.)

We find this to be very helpful when **designing** microwave components. From the analysis of these approximate graphs, we can often determine **design rules** or equations that are tractable, and allow us to design components with (near) optimal performance.

Some definitions!

Every SFG consists of a set of **nodes.** These nodes are connected by **branches**, which are simply contours with a specified **direction**. Similarly, each branch has an associated complex **value**.



Q: What could this possibly have to do with **RF/microwave engineering**?

Signal Flow Graph (contd.)

- In RF applications, each **port** of a device is represented by **two nodes**—the "a" node and the "b" node. The "a" node simply represents the value of the **normalized amplitude** of the wave incident on that port, evaluated **at** the plane of that port:

$$\boxed{a_n = \frac{V_n^+(z_n = z_{nP})}{\sqrt{Z_{0n}}}}$$

- Similarly, the "b" node simply represents the **normalized amplitude** of the wave **exiting** that port, evaluated **at** the plane of that port:

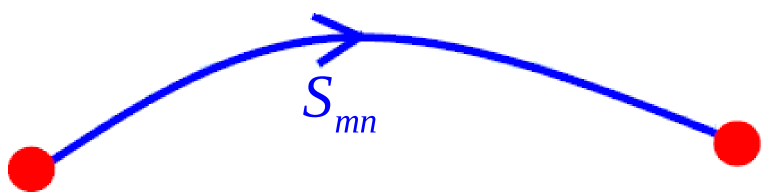
$$\boxed{b_n = \frac{V_n^-(z_n = z_{nP})}{\sqrt{Z_{0n}}}}$$

- Then the **total voltage** at a port is simply:

$$\boxed{V_n(z_n = z_{nP}) = (a_n + b_n)\sqrt{Z_{0n}}}$$

- The value of the **branch** connecting two nodes is simply the value of the **scattering parameter** relating these two voltage values.

$$a_n = \frac{V_n^+(z_n = z_{nP})}{\sqrt{Z_{0n}}}$$

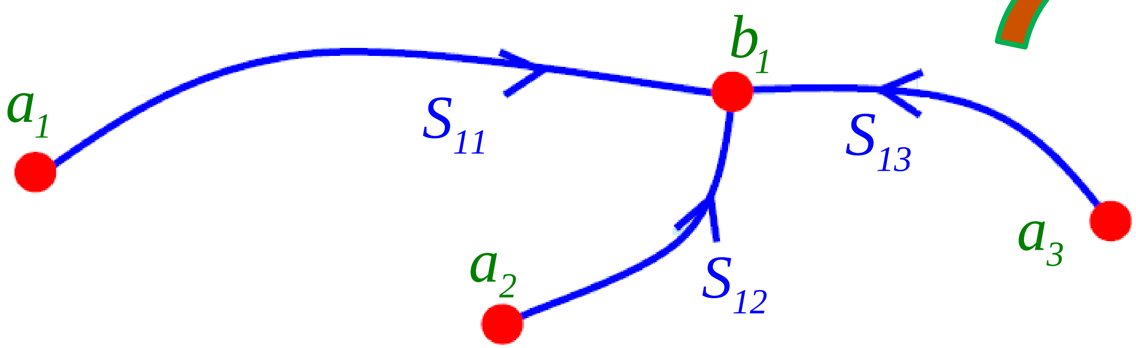


$$b_m = \frac{V_m^-(z_m = z_{mP})}{\sqrt{Z_{0m}}}$$

- The signal flow graph is simply **graphical** representation of: $\boldsymbol{b_m = a_n S_{mn}}$

Signal Flow Graph (contd.)

- Moreover, if **multiple** branches enter a node, then the voltage represented by that node is the **sum** of the values from each branch. For example, following SFG represents:



$$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3$$

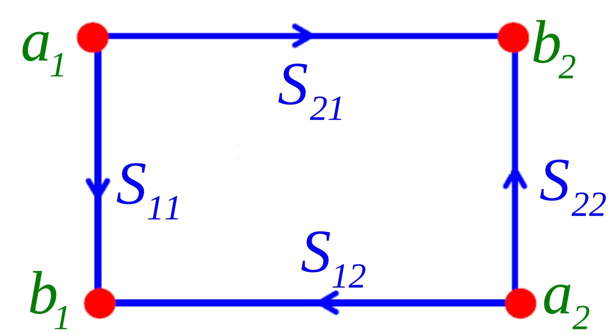
- Now, consider a **two-port device** with a scattering matrix **S:**

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

<u>So that:</u>

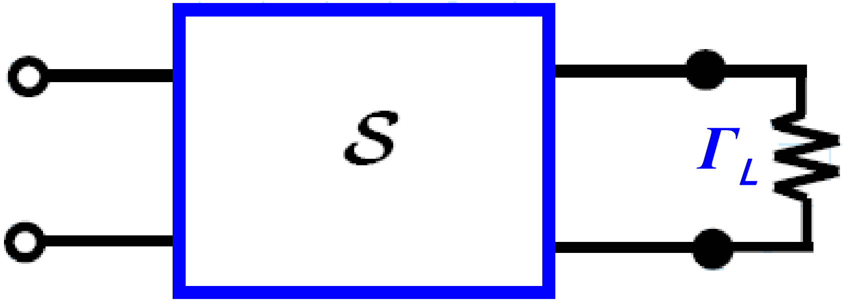
$$b_1 = S_{11}a_1 + S_{12}a_2 \qquad b_2 = S_{21}a_1 + S_{22}a_2$$

- We can then **graphically** represent a **two-port device** as:



Signal Flow Graph (contd.)

- Now, consider a two-port device where the second port is **terminated** by some load Γ_L:

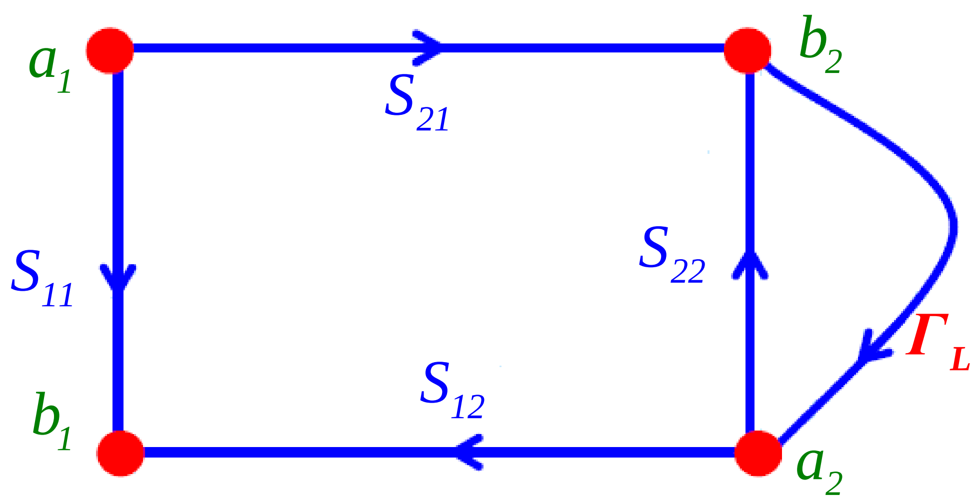


Additional Equation

$$V_2^+(z_2 = z_{2P}) = \Gamma_L V_2^-(z_2 = z_{2P})$$

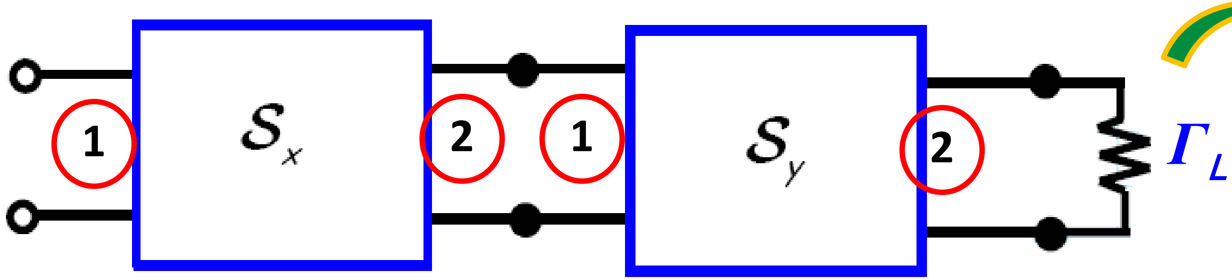
$$\Rightarrow a_2 = \Gamma_L b_2$$

- Therefore, the signal flow graph of this **terminated** network is:



Signal Flow Graph (contd.)

- Now consider cascading of **two different** two-port networks

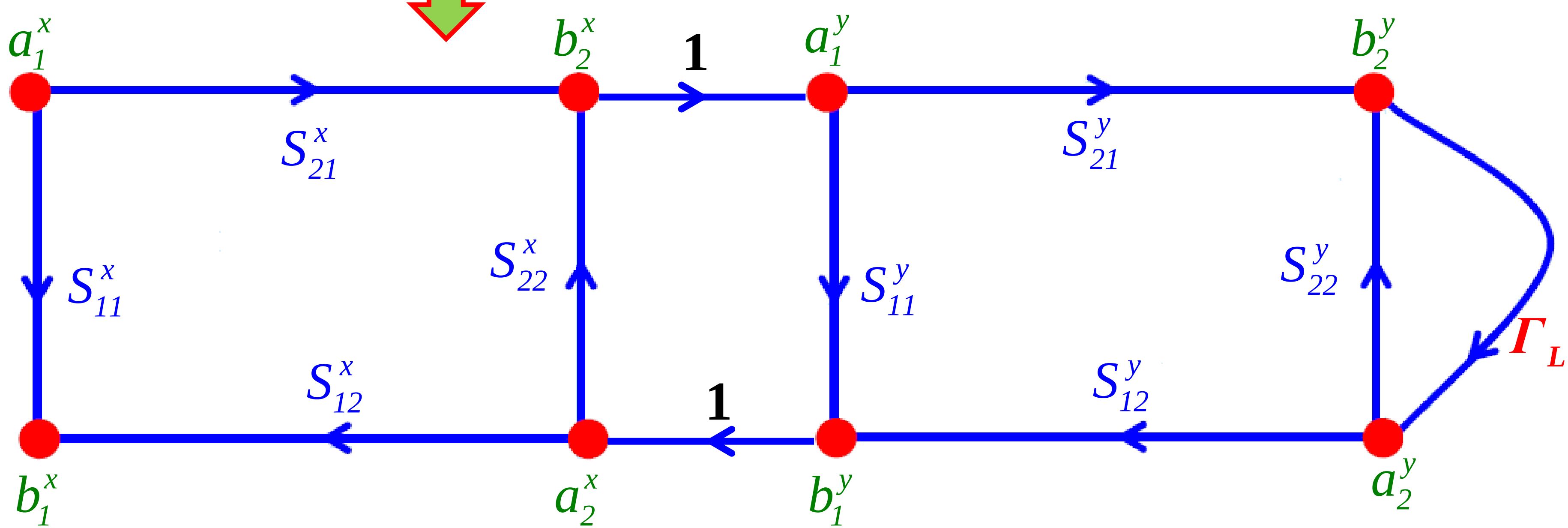


Here:

$a_1^y = b_2^x$

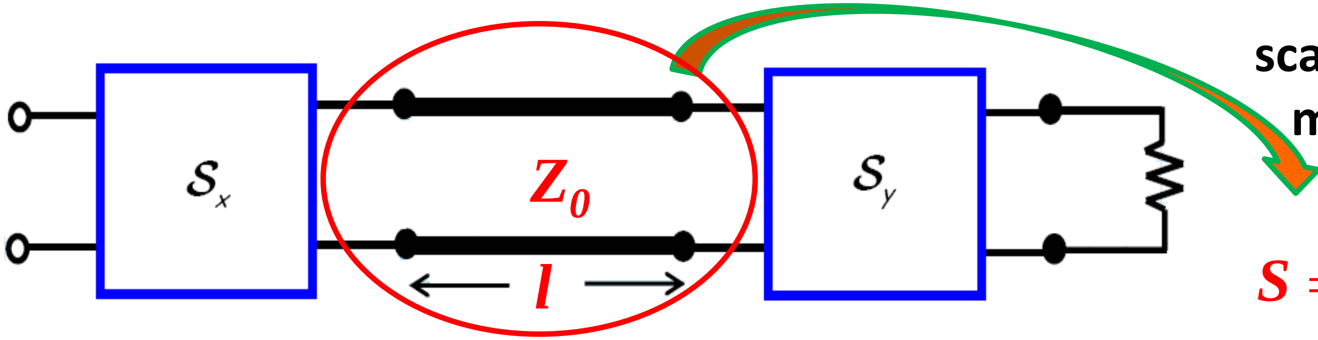
$b_1^y = a_2^x$

SFG

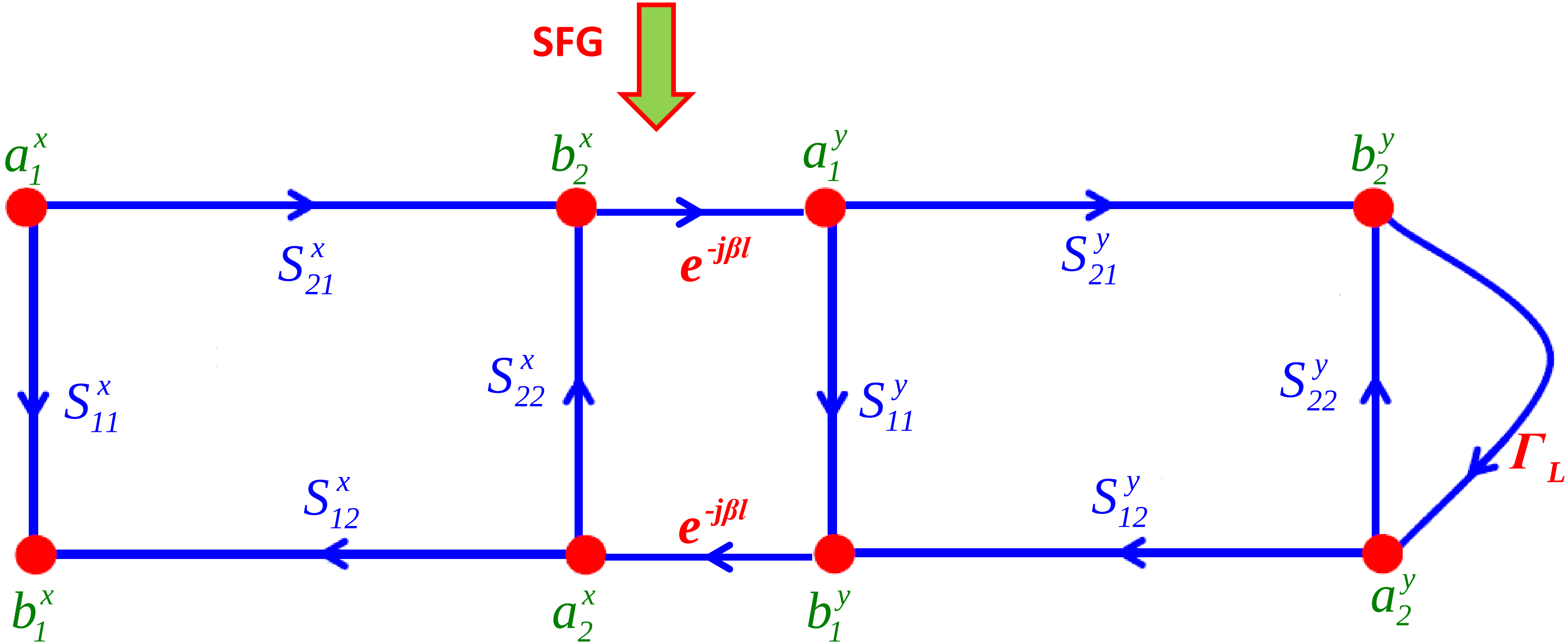


Signal Flow Graph (contd.)

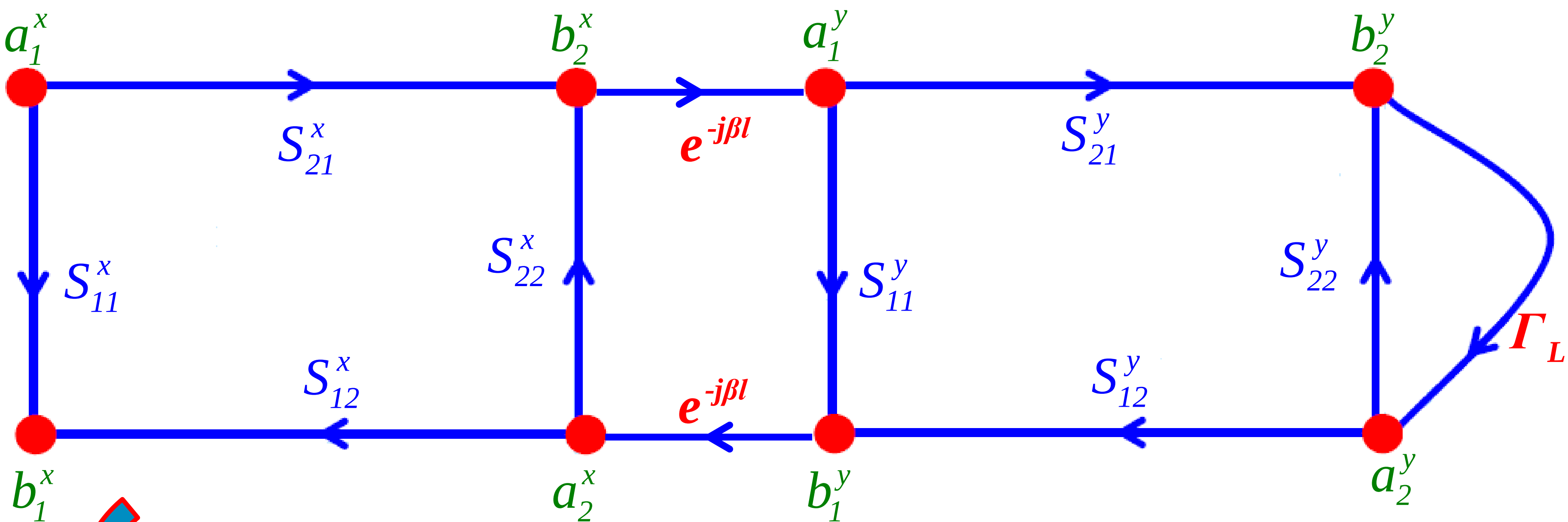
- Now consider networks connected with a **transmission line segment:**



$$S = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix}$$



Signal Flow Graph (contd.)



Note that there is **one** (and only one!) **independent variable** in this graphical representation (i.e., SFG) → $\mathbf{a_1^x}$

This is the only node of the SFG that does **not** have any **incoming** branches. As a result, its value depends on **no other** node values in the SFG

Independent nodes in the SFG are called sources!

Signal Flow Graph (contd.)

Independent nodes in the SFG are called sources!

- This makes sense physically (do **you** see why?)
- The node value $\mathbf{a_1^x}$ represents the complex amplitude of the wave **incident** on the one-port network. If this value is **zero**, then **no power** is incident on the network—the rest of the nodes (i.e., wave amplitudes) will be **zero**!

Now, say we wish to determine, for example:

1. The **reflection coefficient** Γ_{in} of the one-port device
2. The **total current** at port 1 of second network (i.e., network y)
3. The **power absorbed** by the load at port 2 of the second (y) network.

- first case, we need to determine the value of dependent node $b_1{}^x$:

$$\Gamma_{in} = \frac{b_1^x}{a_1^x}$$

- second case, determine the value of wave amplitudes $a_1{}^y$ and $b_1{}^y$:

$$I_1^y = \frac{a_1^y - b_1^y}{\sqrt{Z_0}}$$

- third case, the values of nodes $a_2{}^y$ and $b_2{}^y$ are required:

$$P_{abs} = \frac{\left|b_2^y\right|^2 - \left|a_2^y\right|^2}{2}$$

Signal Flow Graph (contd.)

How do we **determine** the values of these wave amplitude “nodes”?

solve the **simultaneous equations** that describe this network.

Decompose (reduce) the SFG!

- SFG **reduction** is a method for **simplifying** the **complex** paths of that SFG into a more **direct (but equivalent!)** form.
  - Reduction is really just a **graphical** method of **decoupling** the simultaneous equations that are **described** by the SFG.
- SFGs can be reduced by applying one of **four simple rules.**

Q: Can these rules be applied in **any order**?

A: YES! The rules can only be applied when/where the structure of the SFG allows. You must **search** the SFG for structures that allow a rule to be applied, and the SFG will then be (a little bit) reduced. You then search for the **next** valid structure where a rule can be applied. Eventually, the SFG will be **completely reduced**!

It’s a bit like solving a **puzzle**. Every SFG is different, and so each requires a different reduction procedure. It requires a little **thought**, but with a little practice, the reduction procedure can be **easily** mastered → **You may find its kind of a fun! (TRUST ME)**

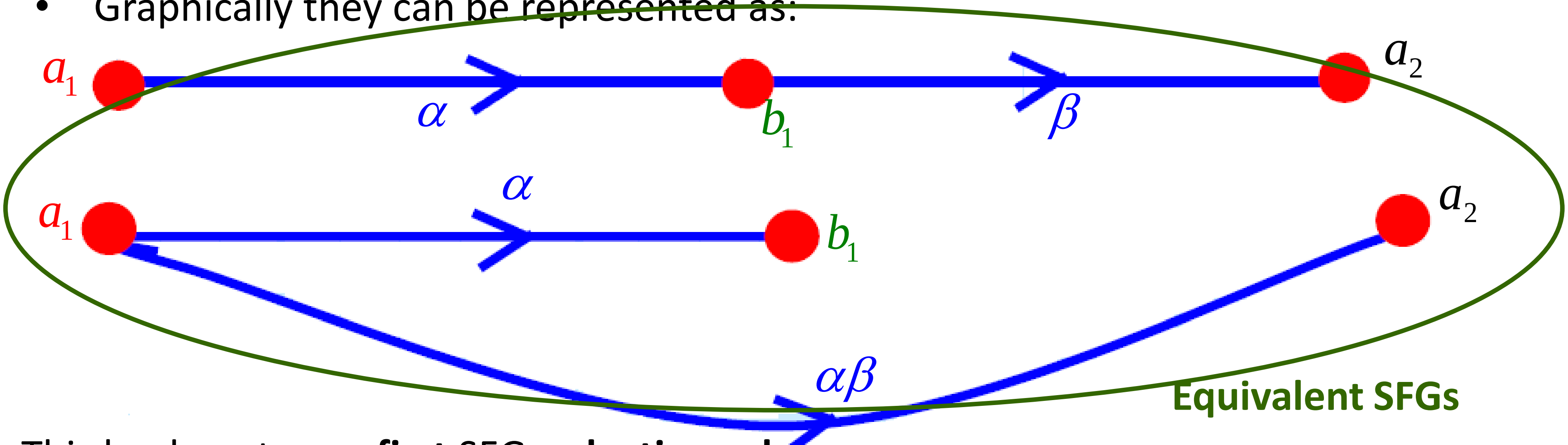
Signal Flow Graph (contd.)

Series Rule

- Consider these two complex equations: $b_1 = \alpha a_1 \qquad a_2 = \beta b_1$
- These two equations can combined to form an **equivalent set** of equations:

$$b_1 = \alpha a_1 \qquad\qquad a_2 = \beta b_1 = \beta(\alpha a_1) = \alpha\beta a_1$$

- Graphically they can be represented as:



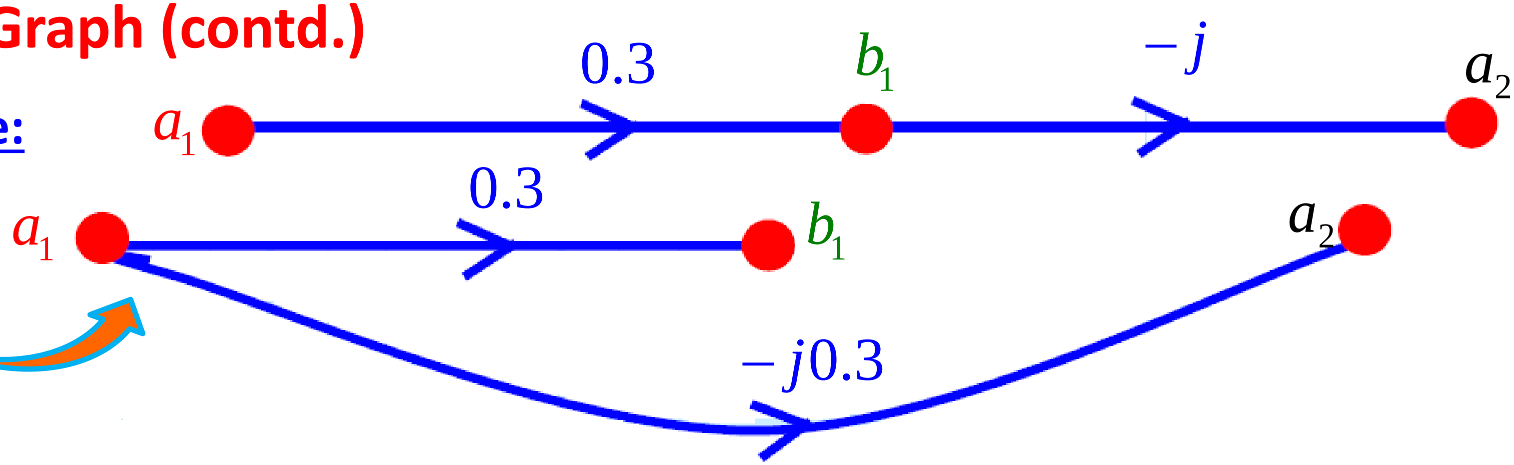
This leads us to our **first** SFG **reduction rule:**

Rule 1 - Series Rule

If a node has **one** (only one!) incoming branch, and **one** (and only one!) outgoing branch, the node can be eliminated and the two branches can be combined, with the new branch having a value equal to the product of the original two.

Signal Flow Graph (contd.)

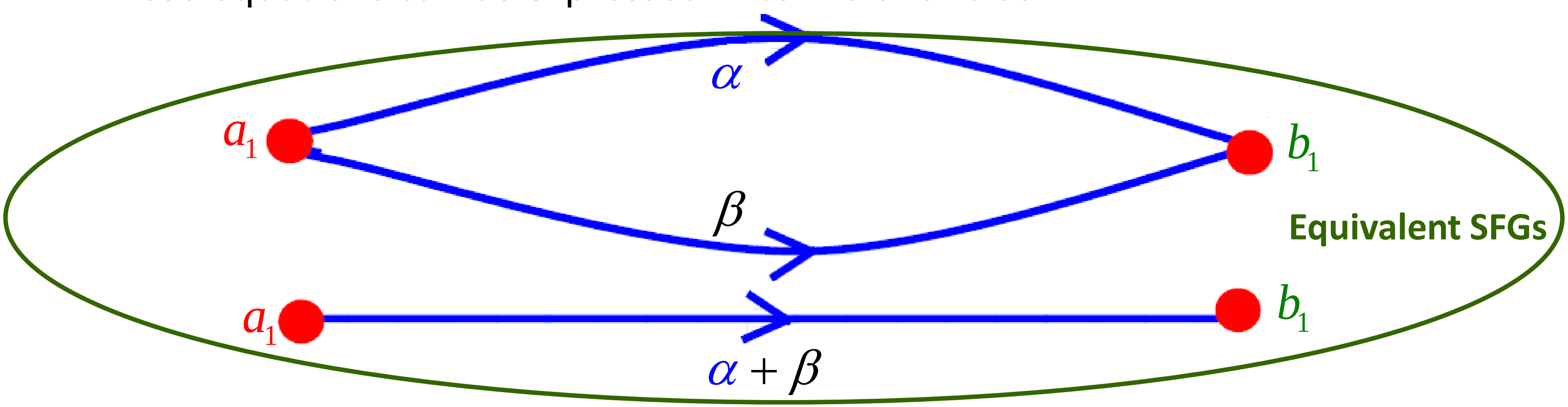
Example:



Can be reduced to

Parallel Rule

- Consider these two complex equations: $b_1 = \alpha a_1 + \beta a_1$
- The equation can also be expressed as: $b_1 = (\alpha + \beta) a_1$
- These equations can be expressed in terms of SFG as:



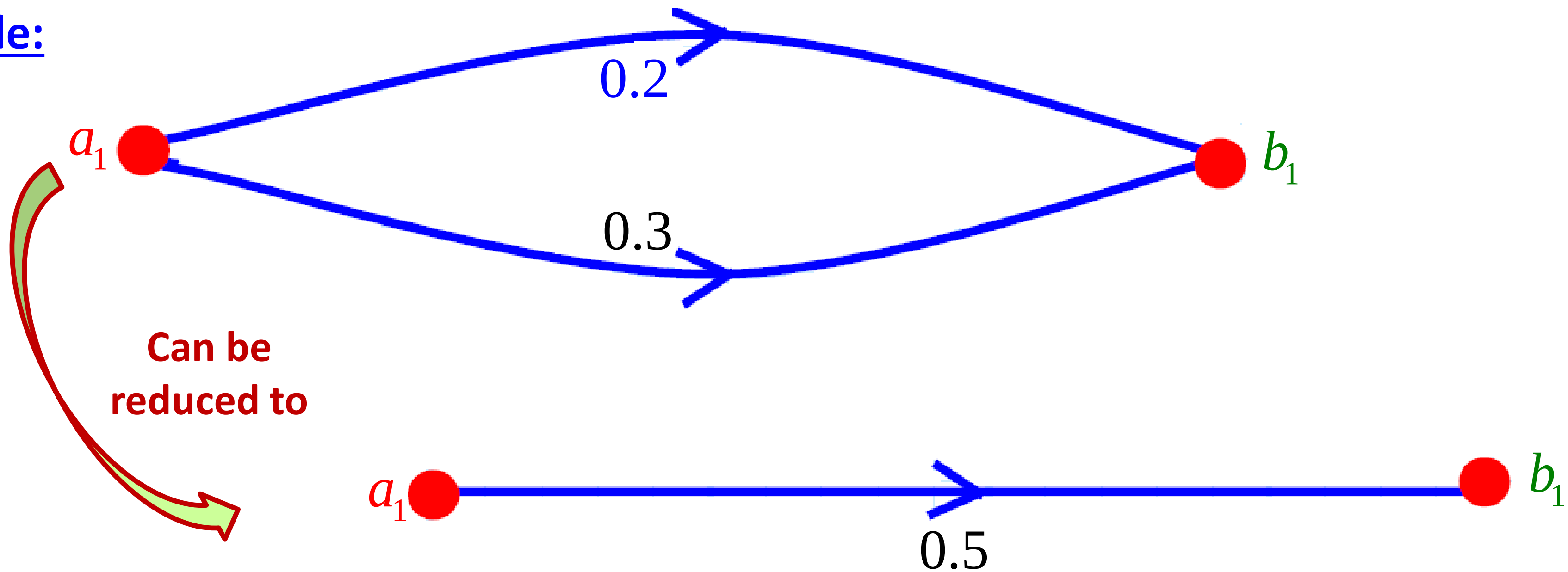
Signal Flow Graph (contd.)

This leads us to our **second** SFG **reduction rule:**

Rule 2 - Parallel Rule

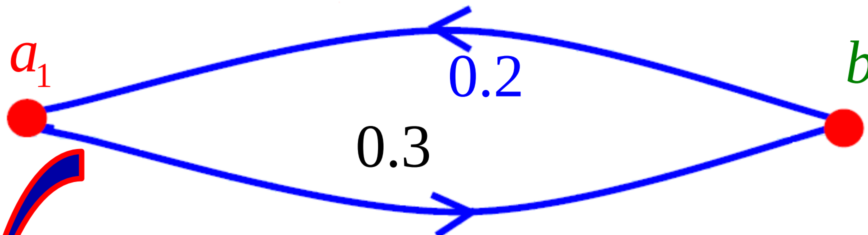
If two nodes are connected by parallel branches—and the branches have the **same direction**—the branches can be combined into a single branch, with a value equal to the **sum** of each two original branches.

Example:

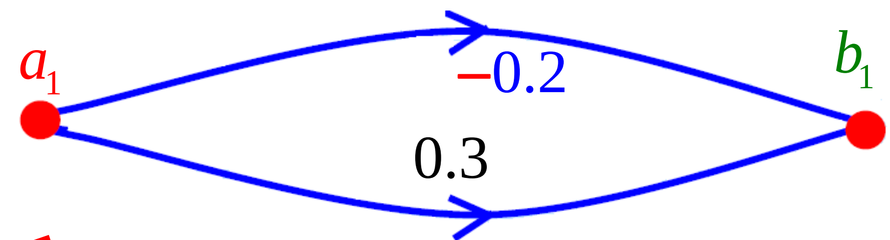


Signal Flow Graph (contd.)

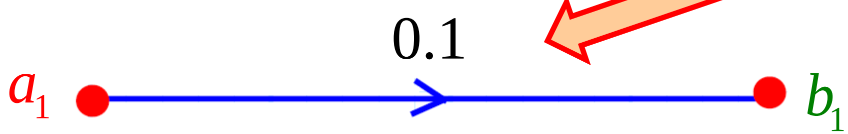
What about **this** signal flow graph?



Can it be transformed into

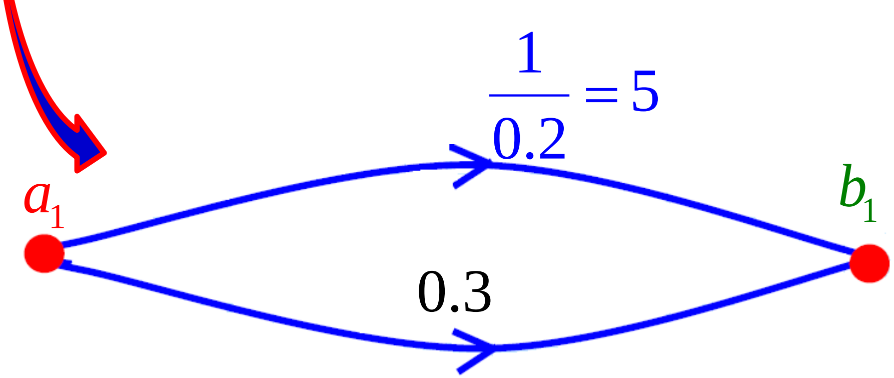


So that



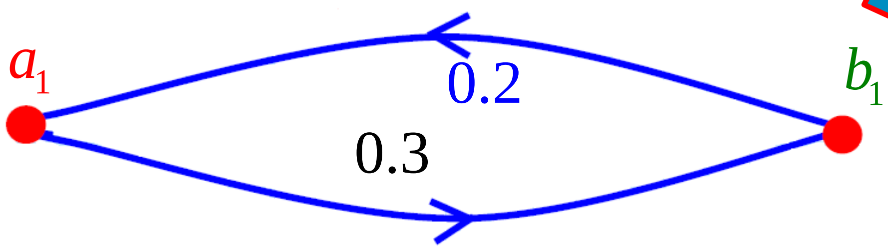
Absolutely not! **NEVER DO THIS!!**

How about:



Absolutely not! **NEVER DO THIS EITHER!!**

Signal Flow Graph (contd.)

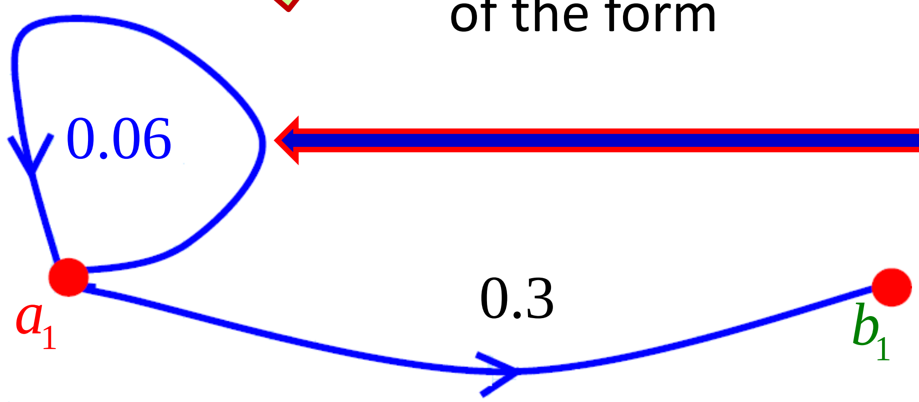


Actually from this SFG we can only conclude that

$b_1 = 0.3a_1$ $\quad$ $a_1 = 0.2b_1$

$a_1 = 0.06a_1$ $\quad$ $b_1 = 0.3a_1$

SFG can be of the form

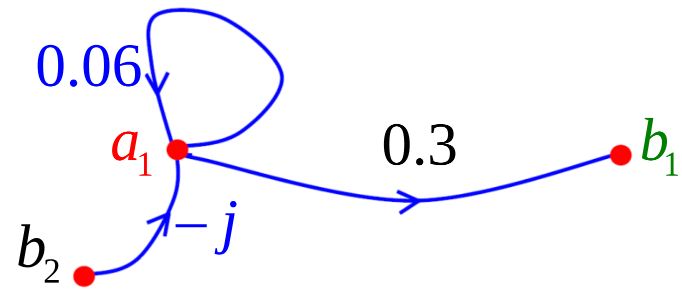


Branches that begin and end at the same node are called self-loops

In practical situations, self-loop node will **always** have at least **one other incoming branch**

Signal Flow Graph (contd.)

Practical example of node with self-loop:



Self-Loop Rule

- Consider the complex equation:

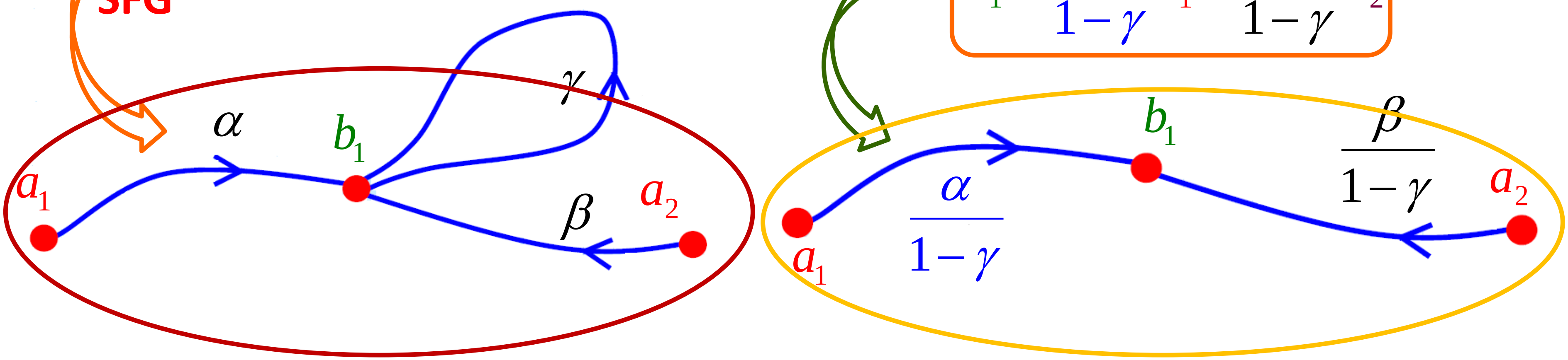
$$b_1 = \alpha a_1 + \beta a_2 + \gamma b_1$$

SFG

- A little bit of **algebra** allows us to determine the value of node b_1:

$$\boxed{b_1 = \frac{\alpha}{1-\gamma} a_1 + \frac{\beta}{1-\gamma} a_2}$$

SFG



Equivalent

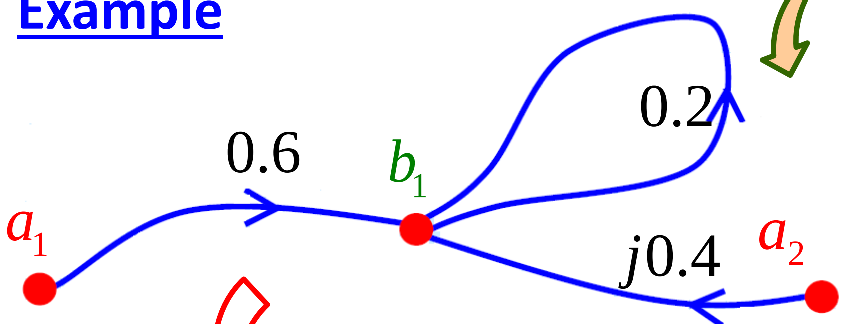
Signal Flow Graph (contd.)

<u>This leads us to our **third** SFG **reduction rule:**</u>

Rule 3 – Self-Loop Rule

A self-loop can be eliminate by multiplying **all** of the branches "**feeding**" the self-loop node by $1/(1-S_{sl})$, where S_{sl} is the value of the self loop branch.

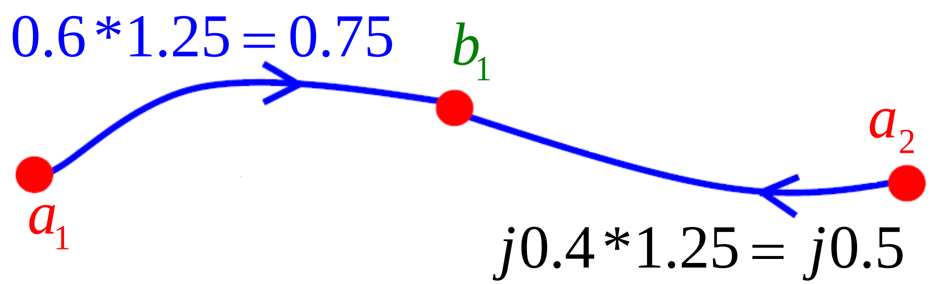
<u>Example</u>



can be simplified by **eliminating the self-loop** → multiply **both** of the two branches **feeding** the self-loop node by:

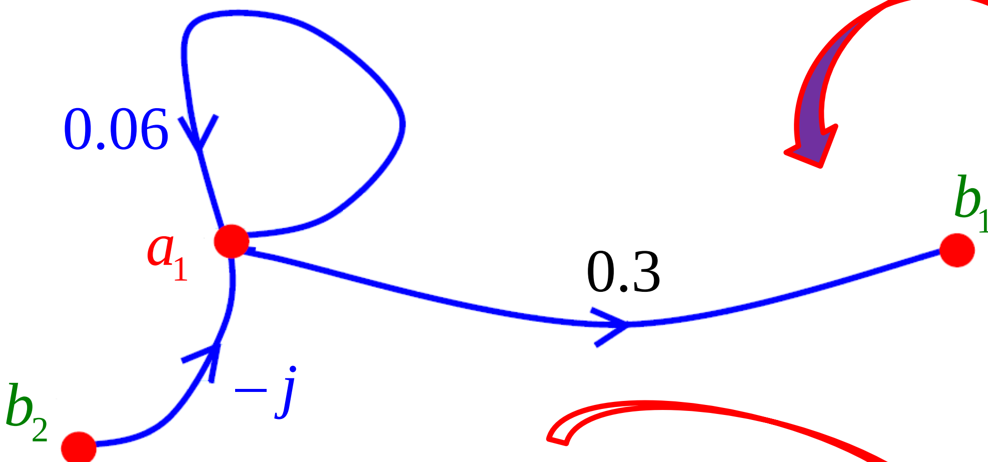
$$\frac{1}{1-S_{sl}} = \frac{1}{1-0.2} = 1.25$$

Simplified and Reduced



Signal Flow Graph (contd.)

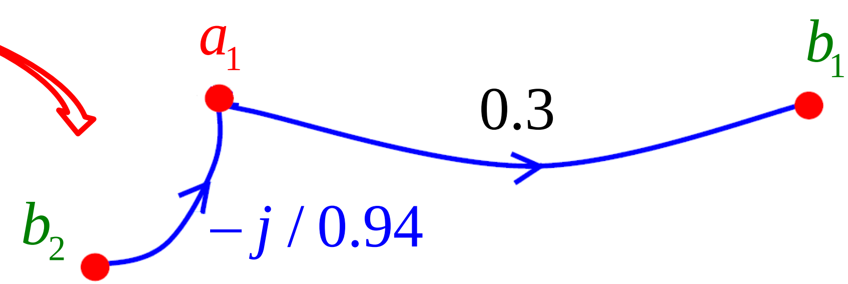
Example



Here

$$\frac{1}{1-S_{sl}} = \frac{1}{1-0.06} = \frac{1}{0.94}$$

Simplified



Only the incoming branches are modified by the self-loop rule! Here, the 0.3 branch is **exiting** the self-loop node a_1 and therefore doesn't get modified. **Only** the –j branch(incoming at node a_1) to the self-loop node are modified by the self-loop rule!

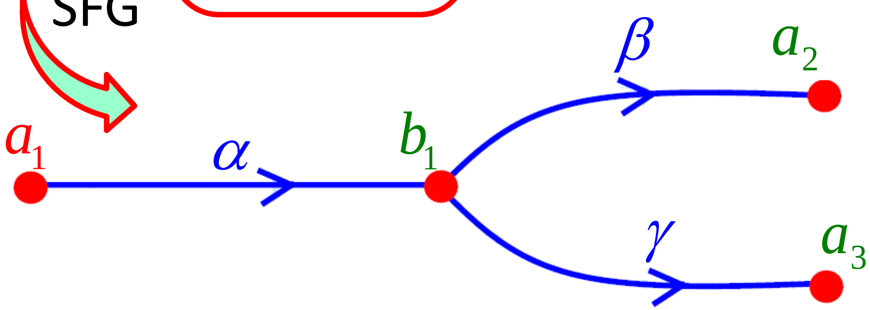
Signal Flow Graph (contd.)

Splitting Rule

- Now consider the three equations

$$
\boxed{\begin{aligned} b_1 &= \alpha a_1 \\ a_2 &= \beta b_1 \\ a_3 &= \gamma b_1 \end{aligned}}
$$

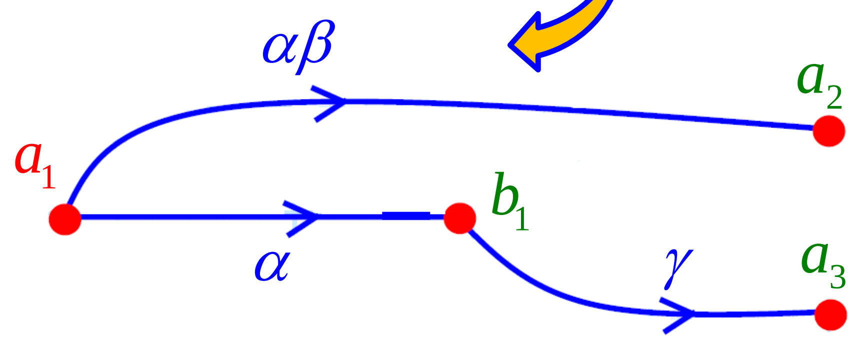
SFG



- These equations can be equivalently written as

$$
\boxed{\begin{aligned} b_1 &= \alpha a_1 \\ a_2 &= \alpha\beta a_1 \\ a_3 &= \gamma b_1 \end{aligned}}
$$

SFG

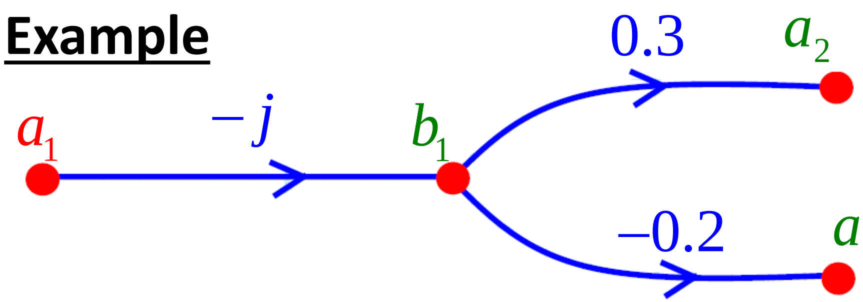


Rule 4 – Splitting Rule

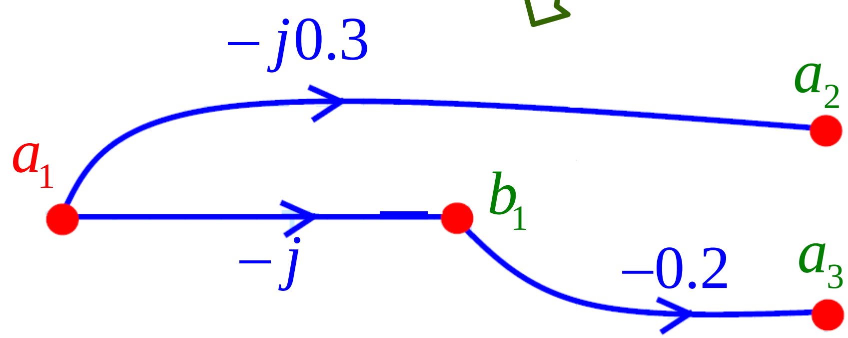
If a node has one (and only one!) incoming branch, and one (or more) exiting branches, the incoming branch can be "split", and directly combined with each of the exiting branches.

Signal Flow Graph (contd.)

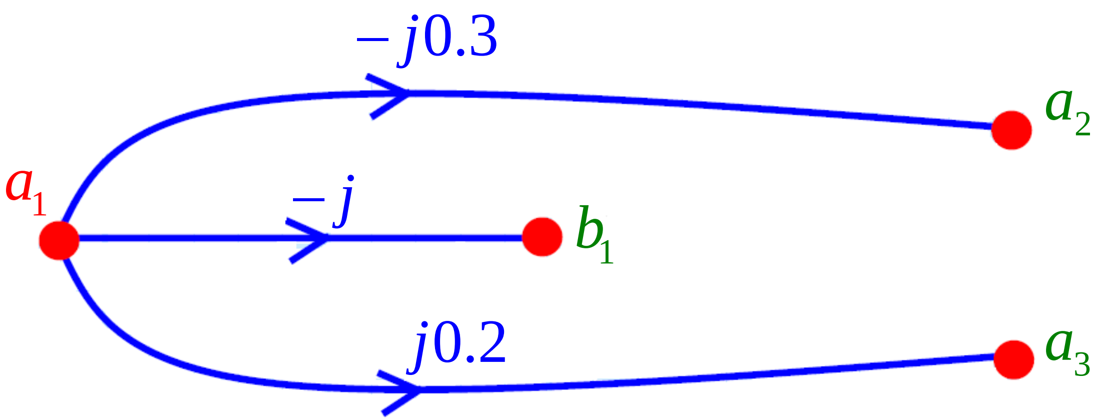
Example



Splitting Rule Gives

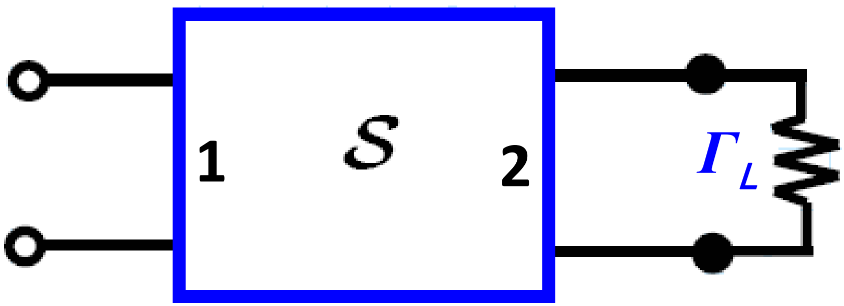


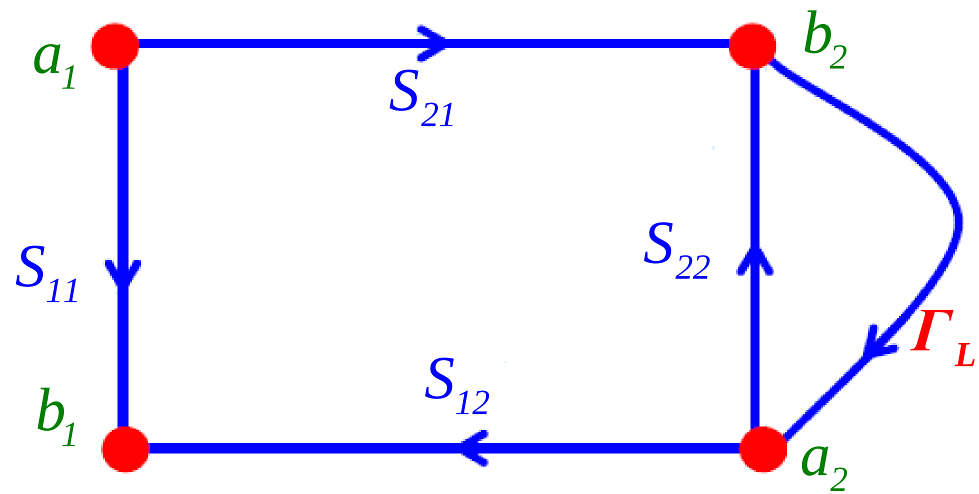
Series Rule Gives



Example-1

Consider the basic 2-port network, terminated with load Γ_L:





determine the value: $\Gamma_1 = \frac{b_1}{a_1}$

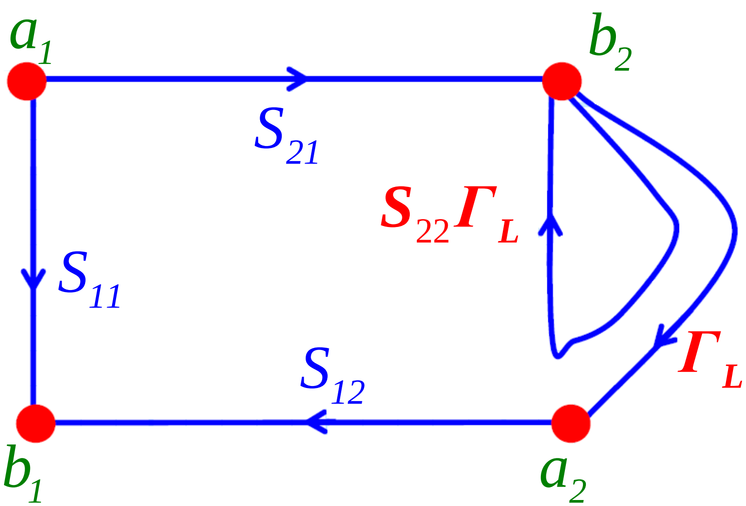
Solution:

- Isn't this simply S_{11} ?
- Only if $\Gamma_L = 0$ (and in this situation it is not!)

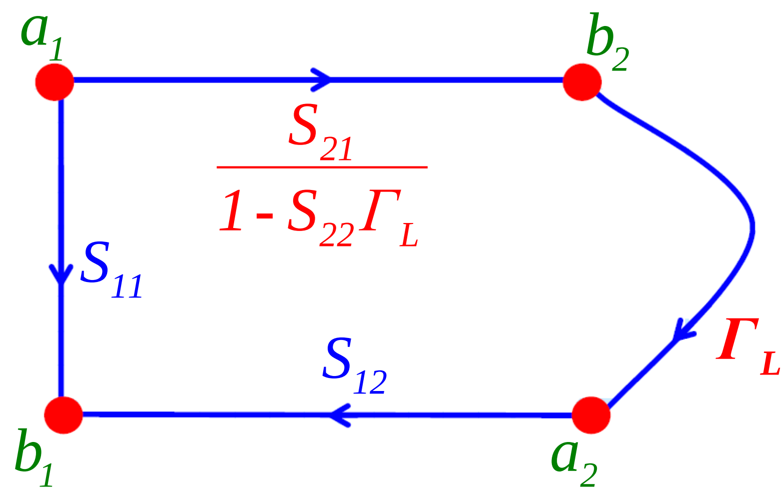
Example-1 (contd.)

- let's decompose (simplify) the signal flow graph and find out!

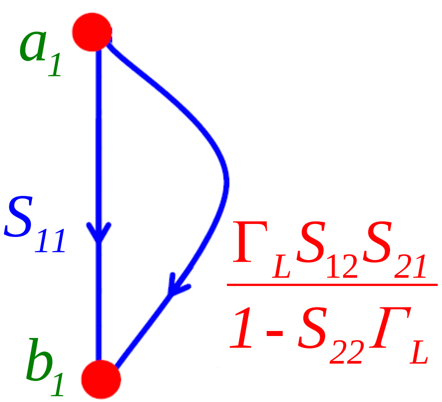
Step-1: **splitting rule** on node a_2



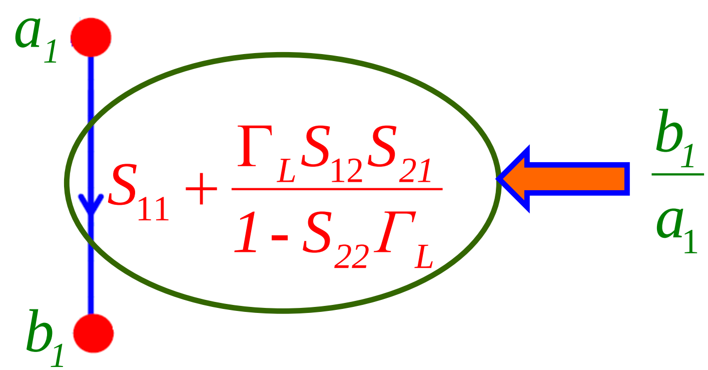
Step-2: **self-loop rule** on node b_2



Step-3: **series rule gives**

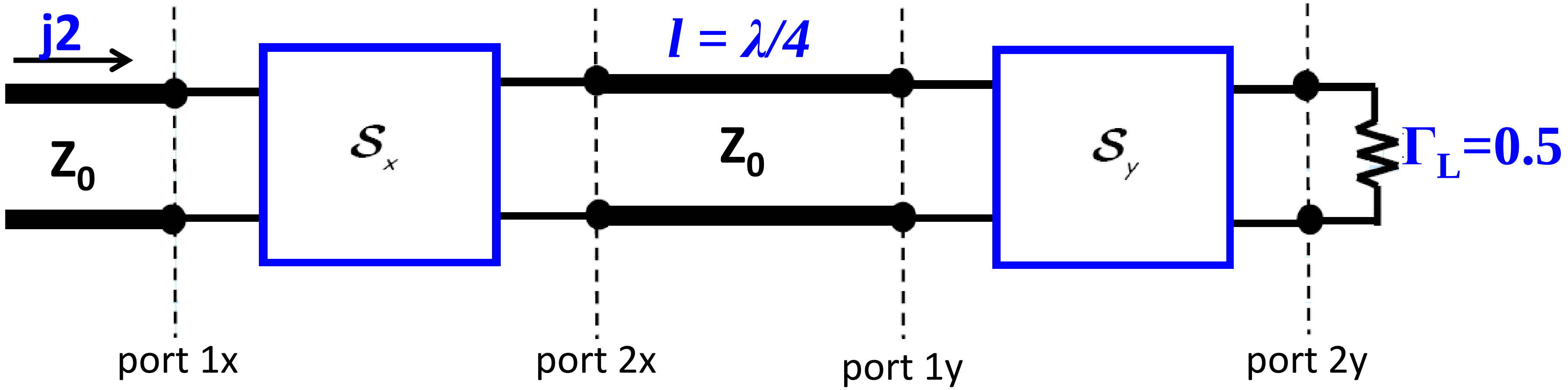


Step-4: **parallel rule gives**



Example – 2

Below is a **single**-port device (with **input** at port 1x) constructed with two two-port devices (**S**$_x$ and **S**$_y$), a quarter wavelength transmission line, and a load impedance.



Given

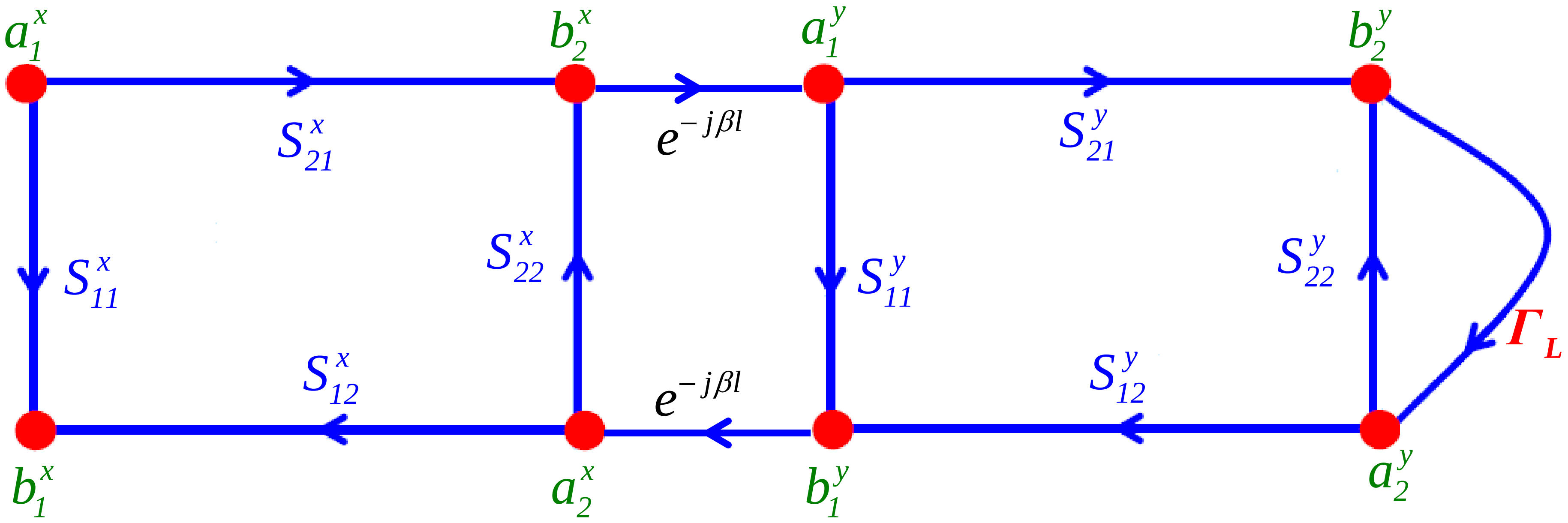
Z_0 = 50Ω

$$S_x = \begin{bmatrix} 0.35 & 0.5 \\ 0.5 & 0 \end{bmatrix} \qquad S_y = \begin{bmatrix} 0 & 0.8 \\ 0.8 & 0.4 \end{bmatrix}$$

Draw the complete **signal flow graph** of this circuit, and then reduce the graph to determine: **a)** The total current through load Γ_L; **b)** The power delivered to (i.e., absorbed by) port 1x.

Example – 2 (contd.)

The signal flow graph describing this network is:

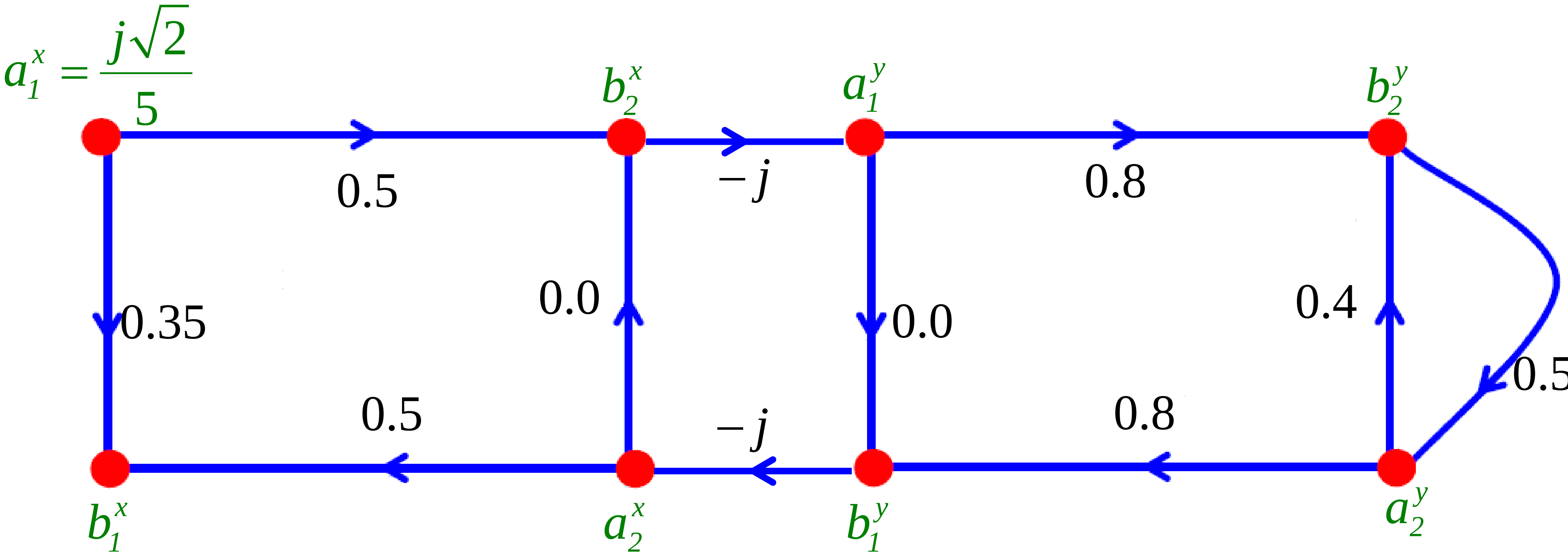


We know that the value of the wave **incident** on port 1 of device $\mathbf{S}_x$ is:

$$a_1^x = \frac{V_{1x}^+ \left(z_{1x} = z_{1xP} \right)}{\sqrt{Z_0}} = \frac{j2}{\sqrt{50}} = \frac{j\sqrt{2}}{5}$$

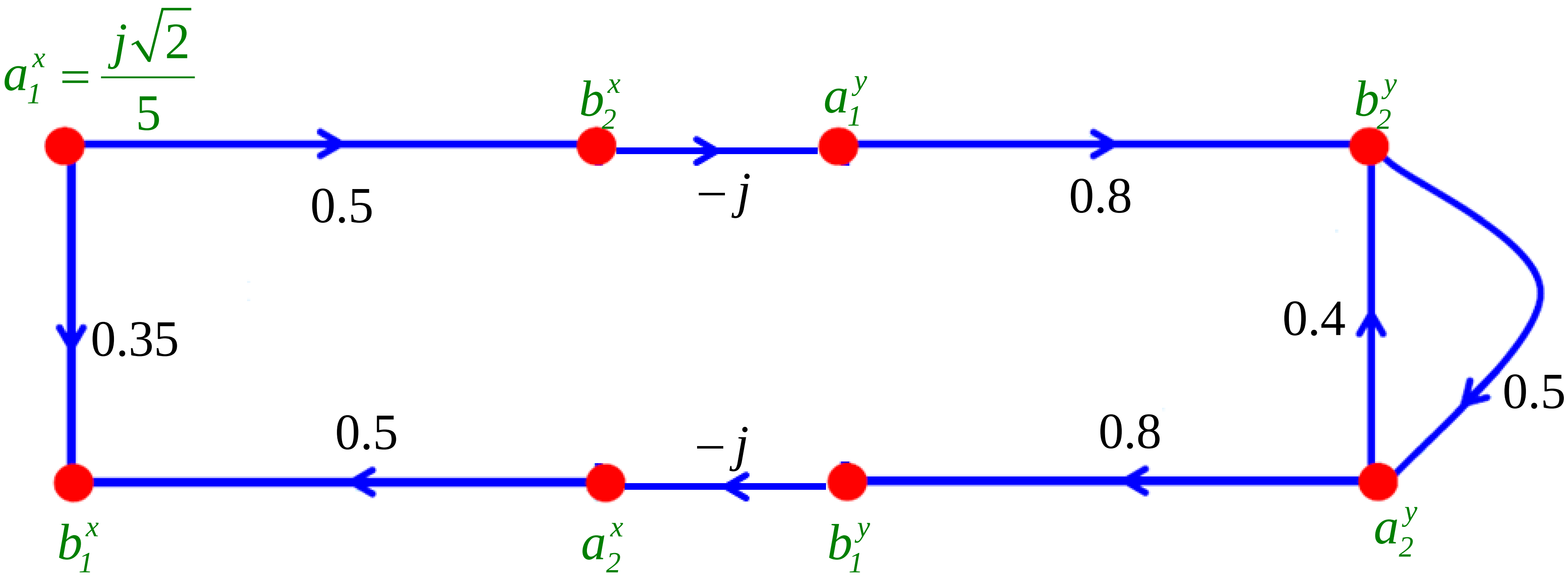
Example – 2 (contd.)

Let us place the given numeric values of branches on this SFG:



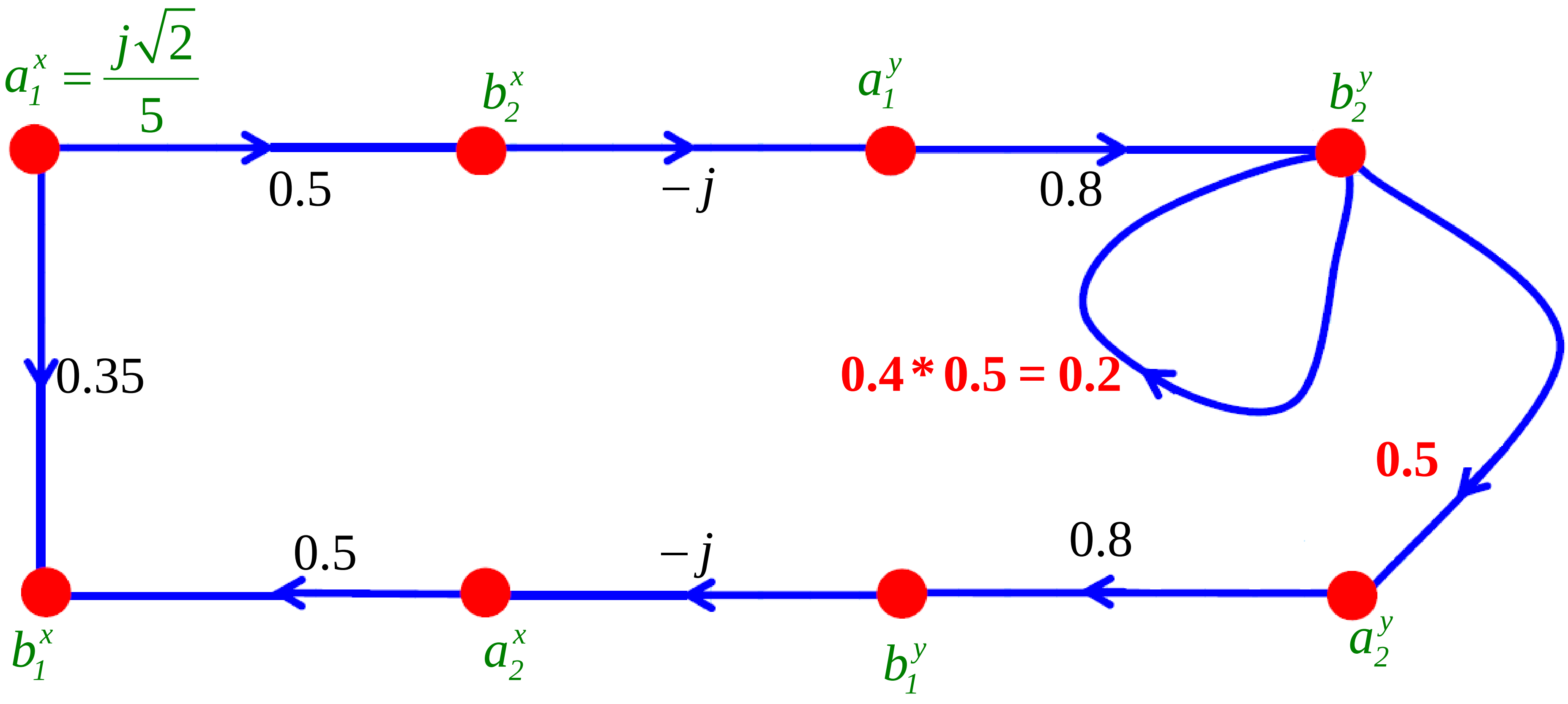
Example – 2 (contd.)

- Remove the zero valued branches:



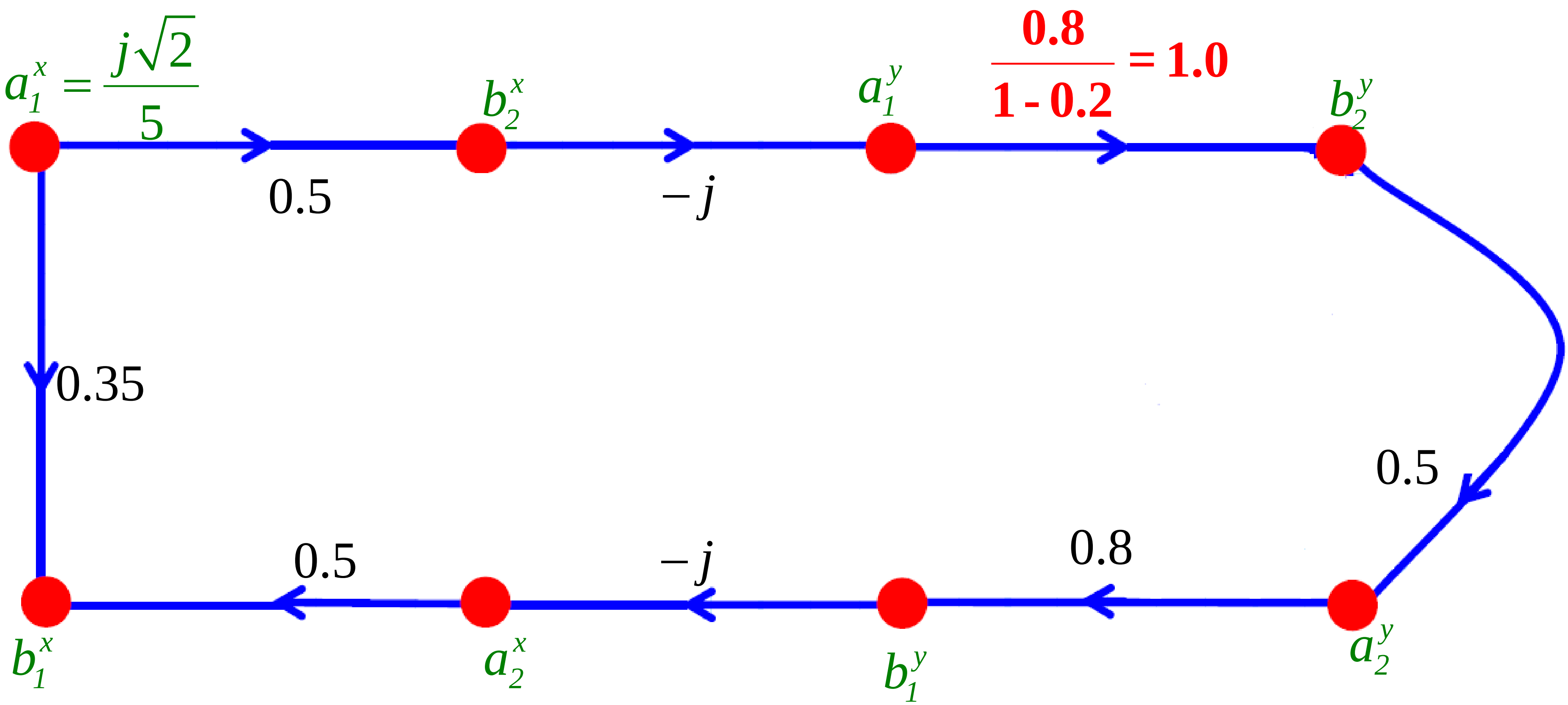
Example – 2 (contd.)

Now apply "splitting" rule at node a_{2y}



Example – 2 (contd.)

Then apply "self-loop" rule at node b_{2y}



Example – 2 (contd.)

let's use this simplified signal flow graph to find the solutions to our questions!

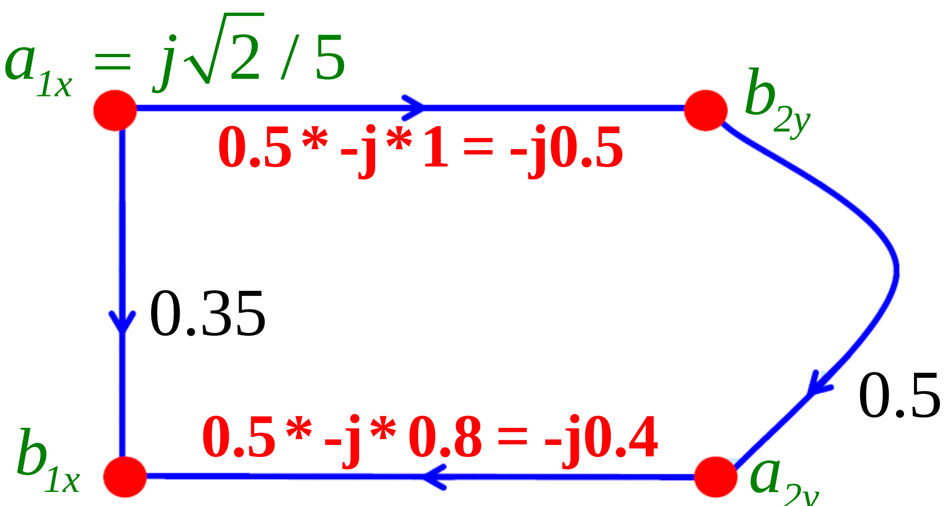
a) The total current through load Γ_L

$$I_L = -I\left(z_{2y} = z_{2yP}\right) = -\frac{V_{2y}^{+}\left(z_{2y} = z_{2yP}\right) - V_{2y}^{-}\left(z_{2y} = z_{2yP}\right)}{Z_0}$$

$$\boxed{\Rightarrow I_L = -\frac{a_{2y} - b_{2y}}{\sqrt{Z_0}} = \frac{b_{2y} - a_{2y}}{\sqrt{50}}}$$

Thus, we need to determine the value of nodes a_{2y} and b_{2y}

- <u>**Using the "series" rule on the SFG gives**</u>



<u>**Therefore,**</u>

$$b_{2y} = -j0.5 * a_1^x = -j0.5 * \frac{j\sqrt{2}}{5} = 0.1\sqrt{2}$$

$$a_{2y} = 0.5 * b_{2y} = 0.05\sqrt{2}$$

Example – 2 (contd.)

Thus the total current through Γ_L is:

$$\boxed{I_L = \frac{b_{2y} - a_{2y}}{\sqrt{50}} = \frac{(0.1 - 0.05)\sqrt{2}}{\sqrt{50}} = \frac{0.05}{5} = 10mA}$$

b) The **power** delivered to (i.e., absorbed by) port 1x is:

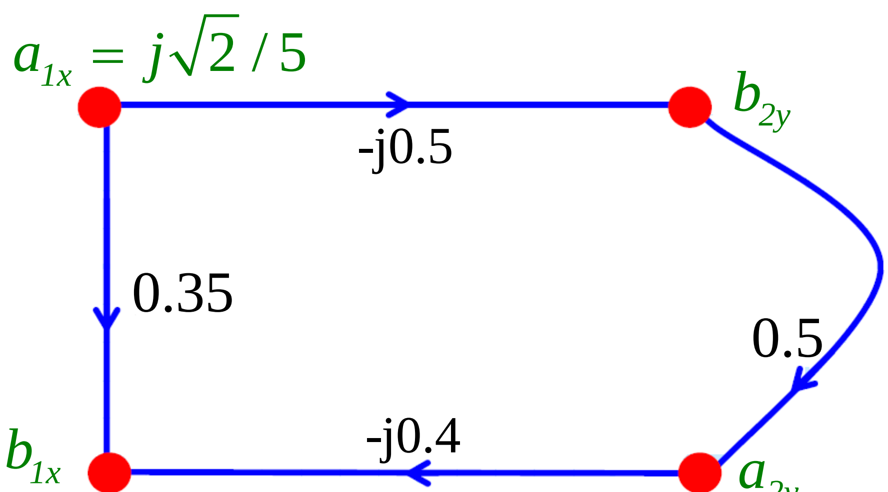
$$P_{abs} = P^+ - P^- = \frac{\left|V_{1x}^+\left(z_{1x} = z_{1xP}\right)\right|^2}{2Z_0} - \frac{\left|V_{1x}^-\left(z_{1x} = z_{1xP}\right)\right|^2}{2Z_0}$$

$$\boxed{\Rightarrow P_{abs} = \frac{\left|a_{1x}\right|^2 - \left|b_{1x}\right|^2}{2}}$$

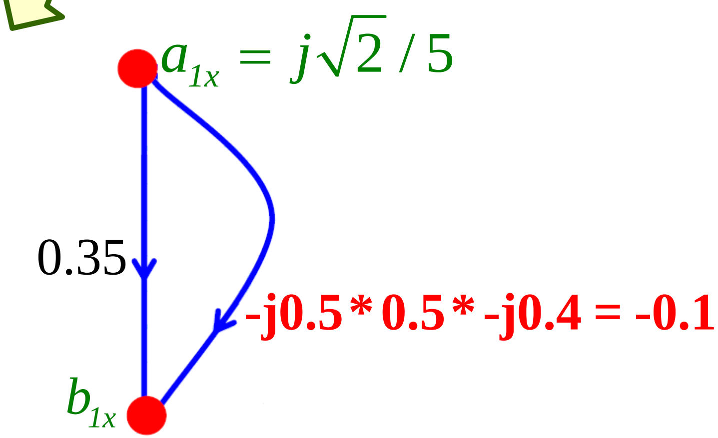
Requires knowledge of nodes a_{1x} and b_{1x}

Example – 2 (contd.)

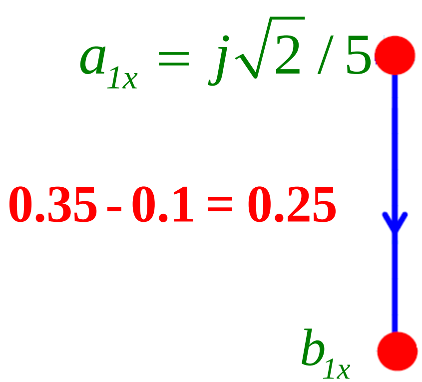
We have:



apply “series” rule



apply “parallel” rule



Therefore: $b_{1x} = 0.25 a_{1x} = \left(0.25 * j\sqrt{2}/5\right) = j0.05\sqrt{2}$

Therefore, the power delivered to (i.e., absorbed by) port 1x is:

$$\boxed{\Rightarrow P_{abs} = \frac{\left|j\sqrt{2}/5\right|^2 - \left|j0.05\sqrt{2}\right|^2}{2} = \frac{0.08 - 0.005}{2} = 37.5mW}$$