Lecture – 7

Date: 25.01.2016

- Smith Chart – Examples
- Admittance Transformation

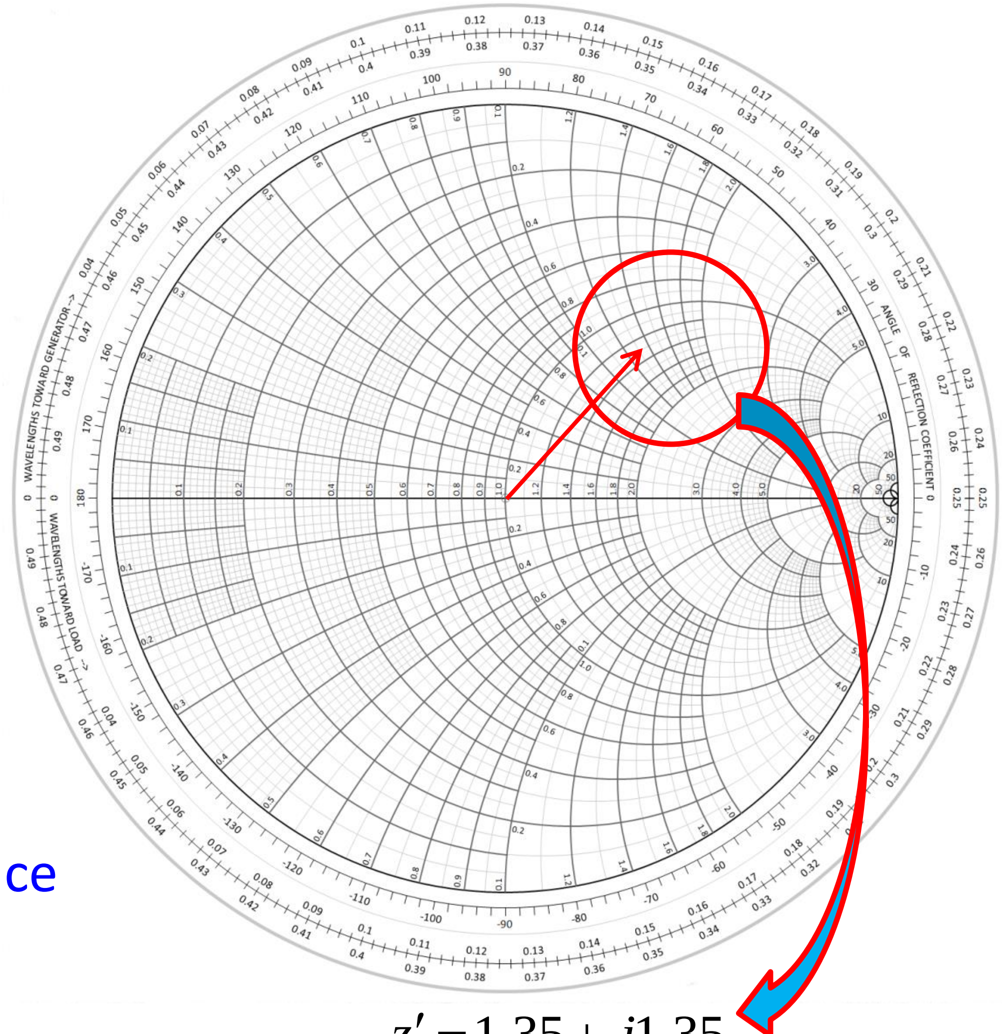
Example-1

Given:

$\Gamma_0 = 0.5\angle 45°$

$Z_0 = 50\Omega$

What is load impedance, Z_L?



$z' = 1.35 + j1.35$

- Locate Γ_0 on the smith chart
- Read the normalized impedance
- Then multiply the identified normalized impedance by Z_0

$$\therefore Z_L = 50\Omega * (1.35 + j1.35) = 67.5\Omega + j67.5\Omega$$

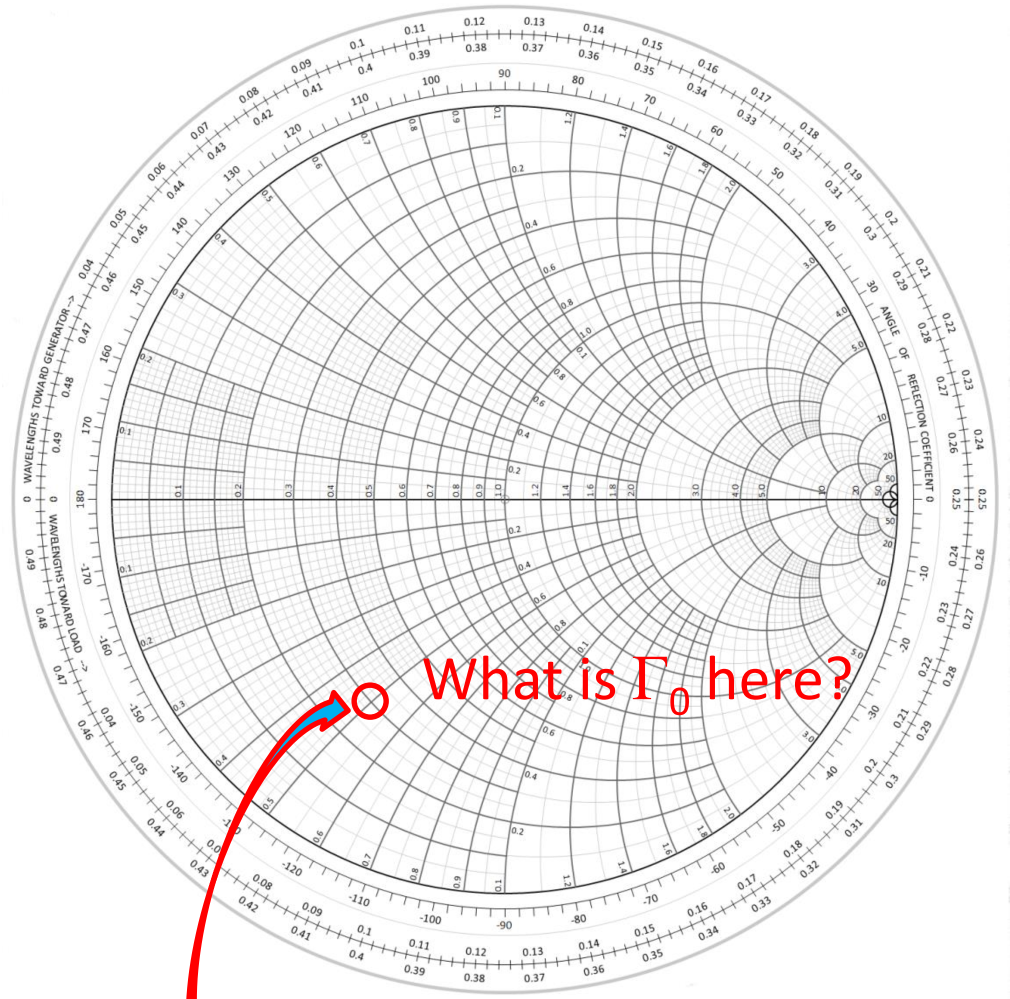
Example-2

Given:

$Z_L = (15 - j25)\Omega$

$Z_0 = 50\Omega$

What is load impedance, Γ_0?

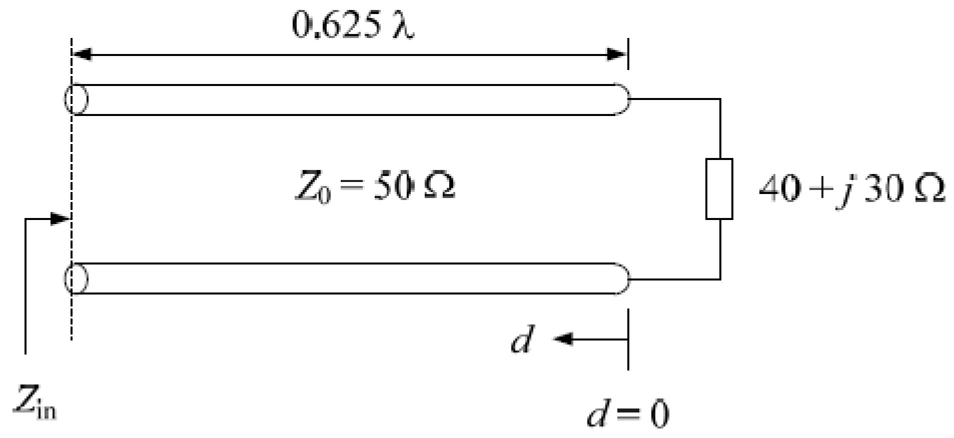


- Normalize the given Z_L
- Mark the normalized impedance Smith chart
- Read the value of Γ_0 from Smith chart

$$z' = \frac{15\Omega - j25\Omega}{50\Omega} = 0.3 - j0.5$$

Example-3

- Using Smith chart, determine the voltage reflection coefficient at the load and the input impedance of the following TL



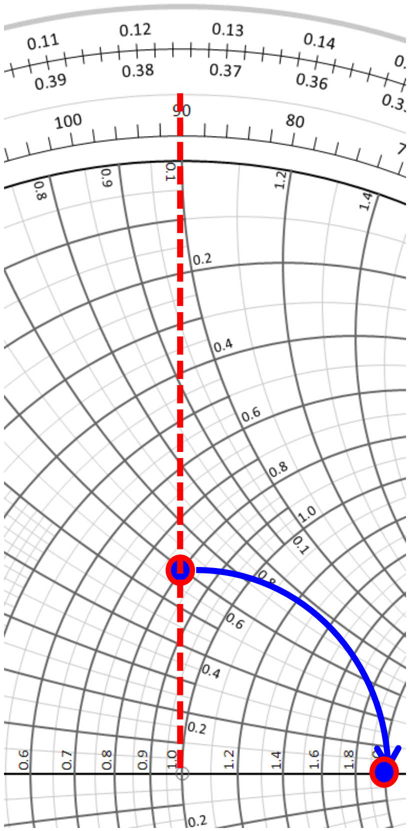
1. $z_L^{'}(d=0) = \dfrac{Z(d=0)}{Z_0} = \dfrac{Z_L}{Z_0} = 0.8 + j0.6$ ⟸ **Mark this on Smith chart**

2. What is Γ_0? Read this directly from Smith chart.

$$|\Gamma_0| = 0.33 \qquad \angle\Gamma_0 = 90^o$$

Example-3 (contd.)

3. For Z_{in}, rotate the load reflection coefficient point clockwise (towards generator) by d = 0.625λ (it is full rotation and then additional rotation of 0.125λ) → Then read normalized input impedance from Smith chart



$$z_{in}^{'} = 2 + j0$$

Therefore the input impedance of the TL is:

$$Z_{in} = 50 * z_{in}^{'} = 100\Omega$$

Example – 4

- A load impedance Z_L = (30 + j60)Ω is connected to a 50Ω TL of 2cm length and operated at 2 GHz. Use the reflection coefficient concept and find the input impedance Z_{in} under the assumption that the phase velocity is 50% of the speed of light

First Approach

- We first determine the load reflection coefficient:

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 + j60 - 50}{30 + j60 + 50} = 0.2 + j0.6 = \sqrt{.40}e^{j71.56^o}$$

- Next we compute Γ (l = 2cm) based on the fact that:

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_p} = \frac{2\pi f}{0.5c} = 83.77m^{-1}$$

$$\Rightarrow 2\beta l = 192^o$$

How?

Example – 4 (contd.)

- Therefore, reflection coefficient at the other end of the TL is:

$$\Gamma = \Gamma_0 e^{-j2\beta l} = \sqrt{.40} e^{-120.4^o} = -0.32 - j0.55$$

- The corresponding input impedance is:

$$Z_{in} = Z_0 \frac{1+\Gamma}{1-\Gamma} = R + jX = (14.7 - j26.7)\Omega$$

Second Approach

Using Smith chart

Example – 4 (contd.)

Using Smith Chart

1. The normalized load impedance is:

$$z_L^{'} = (30 + j60)\Omega / 50\Omega = 0.6 + j1.2$$

2. This point on the Smith chart can be identified as the intersection of the circle of constant resistance $r = 0.6$ with the circle of constant reactance $x = 1.2$

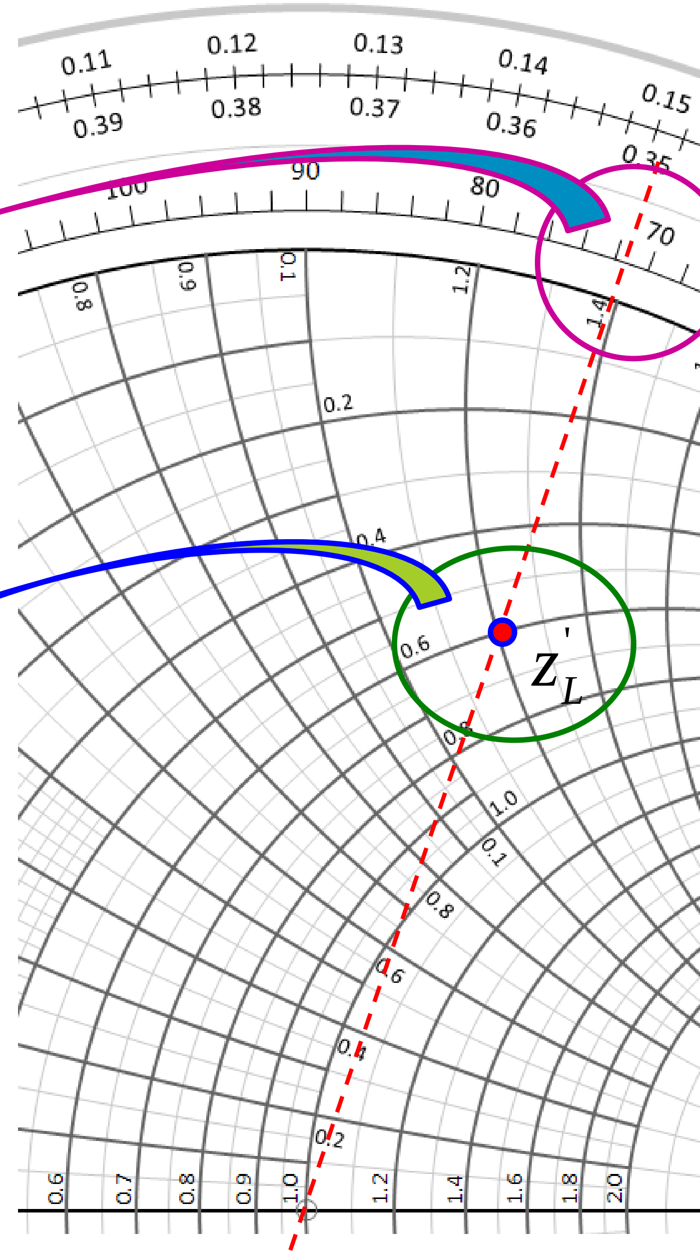
3. The straight line connecting the origin to *normalized load impedance* determines the load reflection coefficient Γ_0. The associated angle is recorded with respect to the positive real axis. From Smith chart we can find that $|\Gamma_0| = 0.6325$ and phase of $\Gamma_0 = 71.56^o$.

4. Rotate clockwise this by $2\beta l = 192^o$ to obtain Γ_{in}

Example – 4 (contd.)

phase (Γ_0)= 71.56°

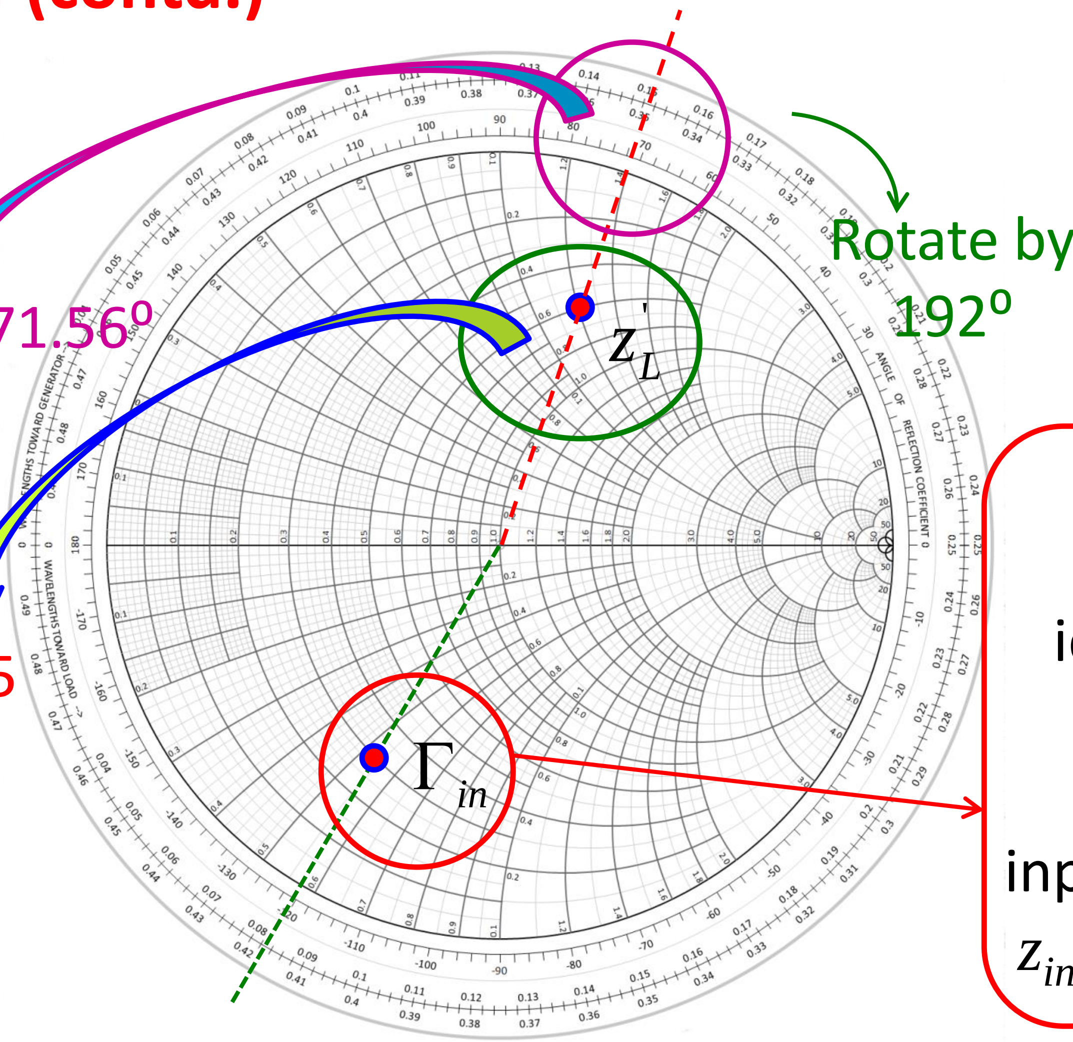
| Γ_0 | = 0.6325



Example – 4 (contd.)

phase (Γ_0)= 71.56°

$|\Gamma_0|$ = 0.6325



This point uniquely identifies the associated normalized input impedance $z_{in}' = 0.3 - j0.53$

Example – 4 (contd.)

5. The Γ_{in} uniquely identifies the associated normalized input impedance $z_{in}' = 0.3 - j0.53$
6. The preceding normalized impedance can be converted back to actual input impedance values by multiplying it by Z_0 = 50Ω, resulting in the final solution $Z_{in} = (15 - j26.5)\Omega$

The exact value of Z_{in} computed earlier was (14.7 – j26.7)Ω. The small anomaly is expected considering the approximate processing of graphical data in Smith chart

Special Transformation Conditions in Smith Chart

- The rotation angle of the normalized TL impedance around the Smith chart is regulated by the length of TL or operating frequency
- Thus, both capacitive and inductive impedances can be generated based on the length of TL and the termination conditions at a given frequency
- The open- and short-circuit terminations are very popular in generating inductive and capacitive elements

Open Circuit Transformations

- **For an arbitrary terminated line the input impedance is:**

$$Z_{in}(z) = Z_0 \frac{Z_L + jZ_0 \tan(\beta z)}{Z_0 + jZ_L \tan(\beta z)}$$

For an open circuit →

$$Z_{in}(z) = -jZ_0 \cot(\beta z)$$

- **For a capacitive impedance of $X_C = 1/j\omega C$ we get:**

$$\frac{1}{j\omega C} \cdot \frac{1}{Z_0} \equiv z_{in}^{'} = -j\cot(\beta z_1)$$

→

$$z_1 = \frac{1}{\beta}\left[\cot^{-1}\left(\frac{1}{\omega C Z_0}\right) + n\pi\right]$$

Special Transformation Conditions in Smith Chart (contd.)

Open Circuit Transformations

- **For an inductive impedance of X_L = jωL we get:**

$$j\omega L\frac{1}{Z_0}\equiv z_{in}^{'}=-j\cot(\beta z_2) \Rightarrow z_2=\frac{1}{\beta}\left[\pi-\cot^{-1}\left(\frac{\omega L}{Z_0}\right)+n\pi\right]$$

Short Circuit Transformations

- **For an arbitrary terminated line the input impedance is:**

$$Z_{in}(z)=Z_0\frac{Z_L+jZ_0\tan(\beta z)}{Z_0+jZ_L\tan(\beta z)}$$

For a short circuit

$$Z_{in}(z)=jZ_0\tan(\beta z)$$

- **For a capacitive impedance of X_C = 1/jωC we get:**

$$\frac{1}{j\omega C}\cdot\frac{1}{Z_0}\equiv z_{in}^{'}=j\tan(\beta z_1) \Rightarrow z_1=\frac{1}{\beta}\left[\pi-\tan^{-1}\left(\frac{1}{\omega CZ_0}\right)+n\pi\right]$$

- **For an inductive impedance of X_L = jωL we get:**

$$j\omega L\frac{1}{Z_0}\equiv z_{in}^{'}=j\tan(\beta z_2) \Rightarrow z_2=\frac{1}{\beta}\left[\tan^{-1}\left(\frac{\omega L}{Z_0}\right)+n\pi\right]$$

Example – 5

- For an open-circuited 50Ω TL operated at 3GHz and with a phase velocity of 77% of speed of light, find the line lengths to create a 2pF capacitor and 5.3nH inductor. Use Smith Chart for solving this problem.

➢ For the given phase velocity, the propagation constant is:

$$\beta = \frac{2\pi f}{v_p} = \frac{2\pi f}{0.77c} = 81.6m^{-1}$$

➢ We know that an open-circuit can create a capacitor as per following equation:

$$z_1 = \frac{1}{\beta}\left[\cot^{-1}\left(\frac{1}{\omega C Z_0}\right) + n\pi\right]$$

$\beta = 81.6m^{-1}$, $C = 2pF$, $f = 3GHz$ ⇒ $z_1 = 13.27 + n38.5$

➢ We know that an open-circuit can create an inductor as per following equation:

$$z_2 = \frac{1}{\beta}\left[\pi - \cot^{-1}\left(\frac{\omega L}{Z_0}\right) + n\pi\right]$$

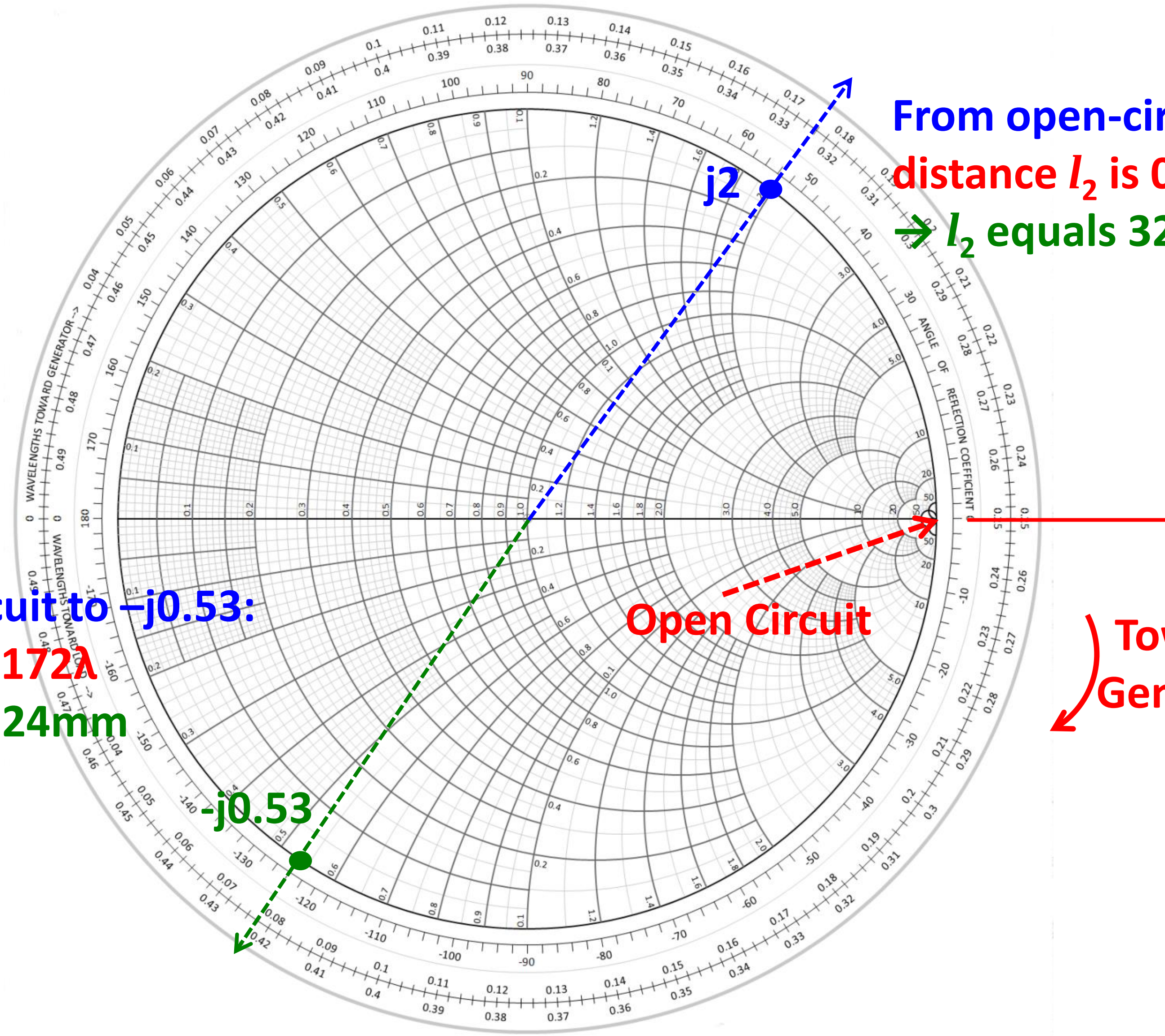
$\beta = 81.6m^{-1}$, $L = 5.3nH$, $f = 3GHz$ ⇒ $z_2 = 32.81 + n38.5$

Example – 5 (contd.)

Using Smith Chart

- At 3GHz, the reactance of a 2pF capacitor is: $X_C = \frac{1}{j\omega C} = -j26.5\Omega$
- Therefore, the normalized capacitive reactance is: $z_c^{'} = \frac{X_C}{Z_0} = -j0.53$
- At 3GHz, the reactance of a 5.3nH inductor is: $X_L = j\omega L = j100\Omega$
- Therefore, the normalized inductive reactance is: $z_L^{'} = \frac{X_L}{Z_0} = j2$
- The wavelength is: $\lambda = \frac{v_p}{f} = 77mm$

Example – 5 (contd.)

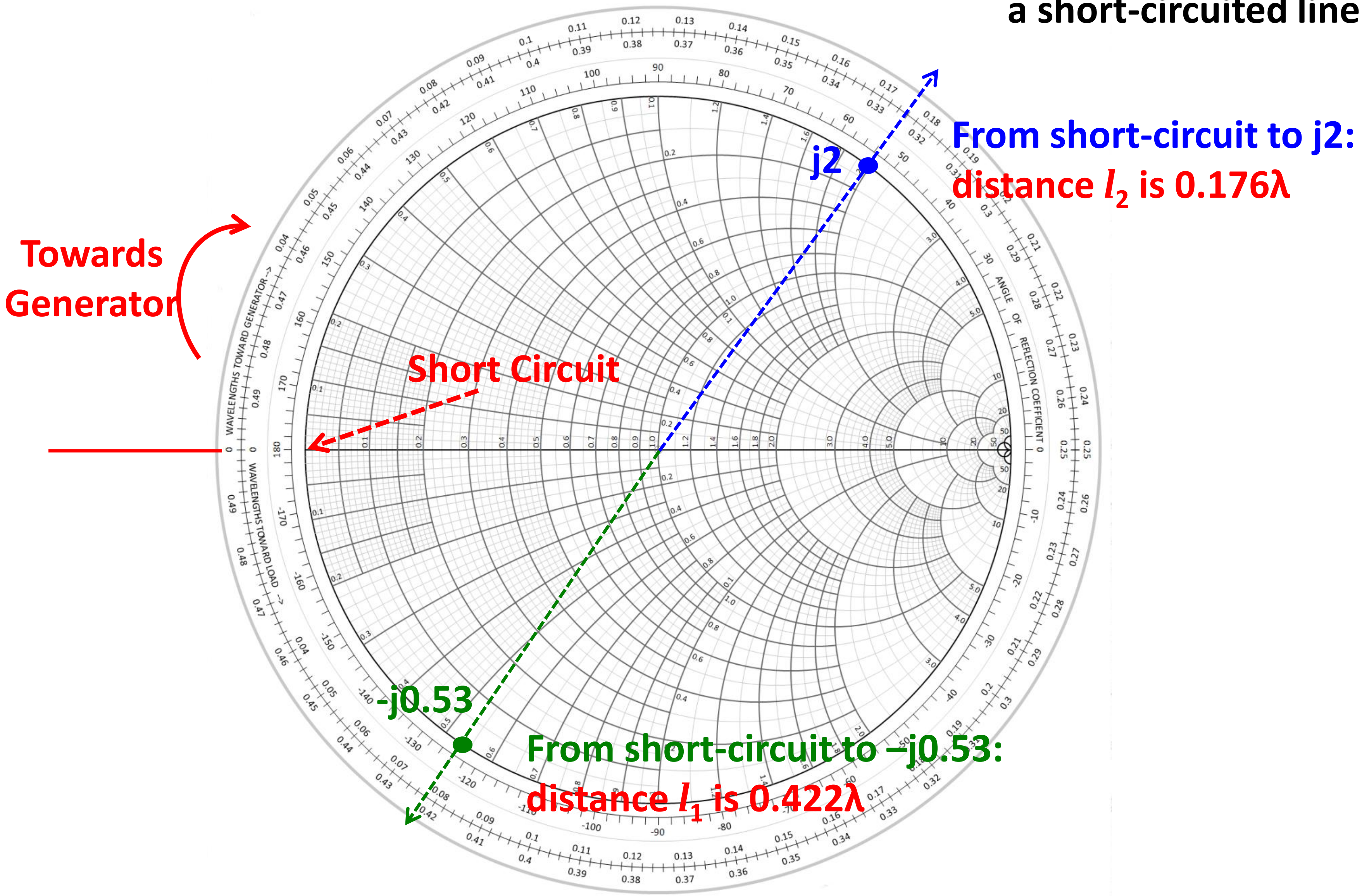


From open-circuit to j2:
distance l_2 is 0.426λ
→ l_2 equals 32.8mm

From open-circuit to –j0.53:
distance l_1 is 0.172λ
→ l_1 equals 13.24mm

Example – 6

- **Same problem but for a short-circuited line**



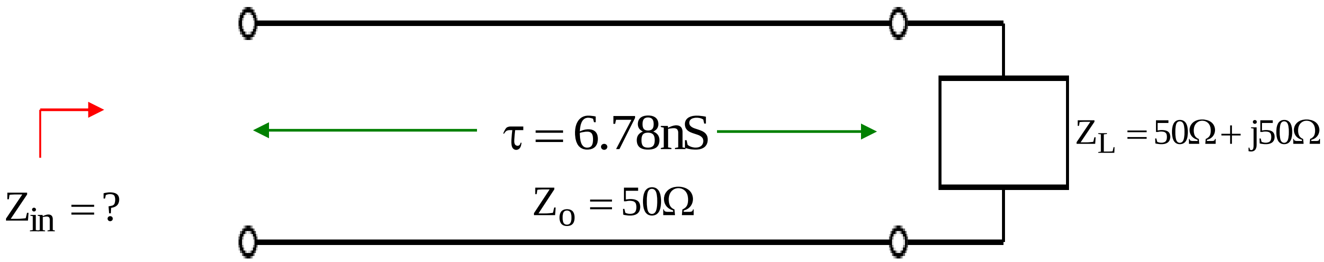
Special Transformation Conditions in Smith Chart (contd.)

Summary

- It is apparent that both open-circuit and short-circuit TLs can achieve desired capacitance or inductance. Which configuration is more useful?
- At high frequencies, its difficult to maintain perfect open-circuit conditions → due to changing temperatures, humidity, and other parameters of the medium surrounding the open TL → short-circuit TLs are, therefore, more popular
- However, short-circuit TL is problematic at higher frequencies → through-hole short connections create parasitic inductances (why?)
- Sometimes board size regulates the choice of open or short TL → for example, an open-circuit TL will always require smaller TL segment for realizing any specified capacitance as compared to a short-circuit TL segment

Example – 7

What is Z_{in} at 50 MHZ for the following circuit?



1. Normalized Impedance: $z_L^{'} = \dfrac{50\Omega + j50\Omega}{50\Omega} = 1.0 + j1.0$
2. Mark the normalized impedance on the Smith chart
3. Read reflection coefficient from Smith Chart: $\Rightarrow \Gamma_0 = 0.445\angle 64°$
4. Transform the load reflection coefficient to the input:

$$\Gamma_{in} = \Gamma_0 e^{-j2\beta l} = \Gamma_0 e^{-j2\omega\tau}$$

$$2\omega\tau = 244°$$

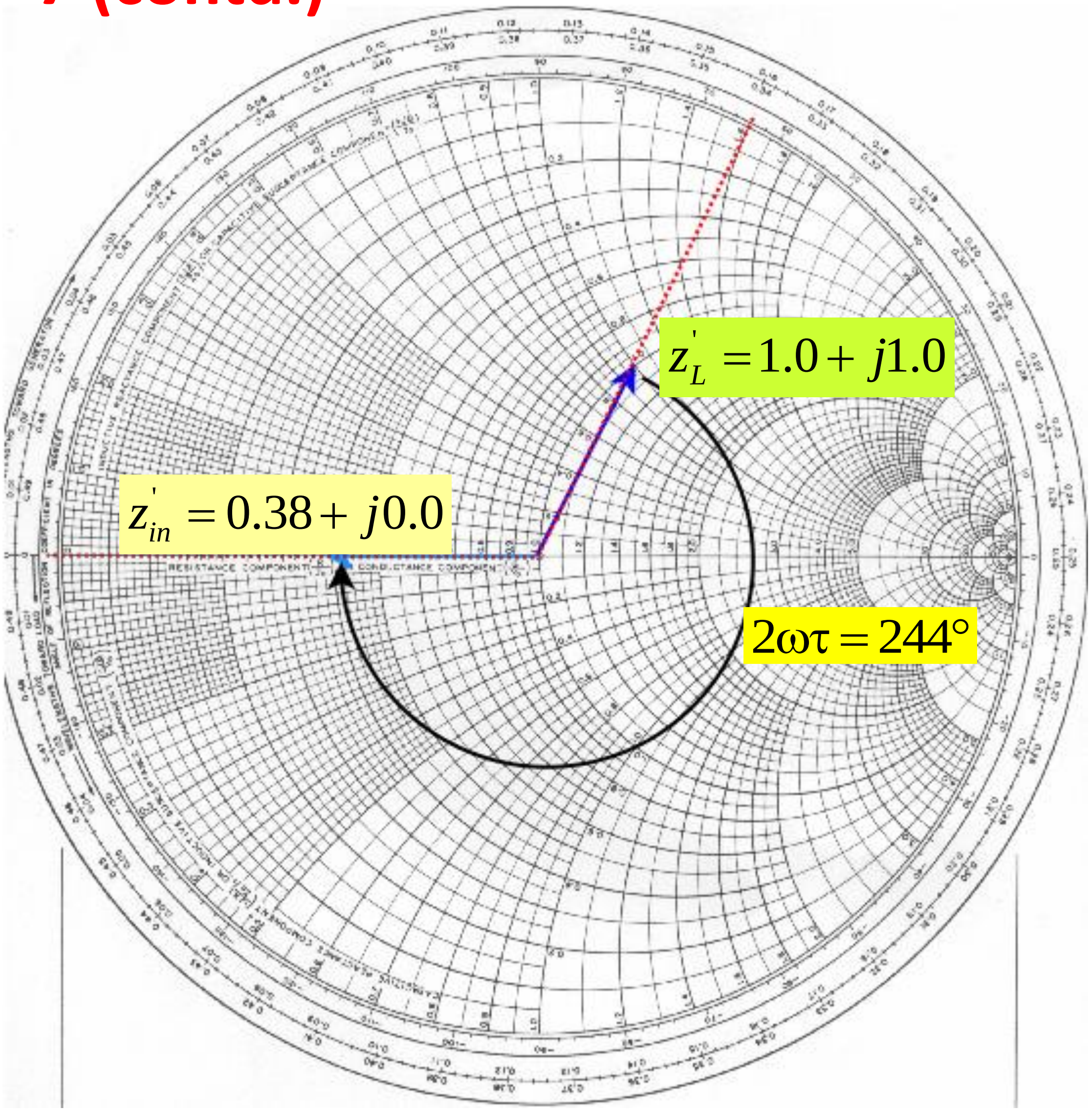
Rotate clockwise (towards generator)

$$\Rightarrow \Gamma_{in} = 0.445\angle 180°$$

Read the normalized input impedance in the Smith chart

$$z_{in}^{'} = 0.38 + j0.0$$

Example – 7 (contd.)

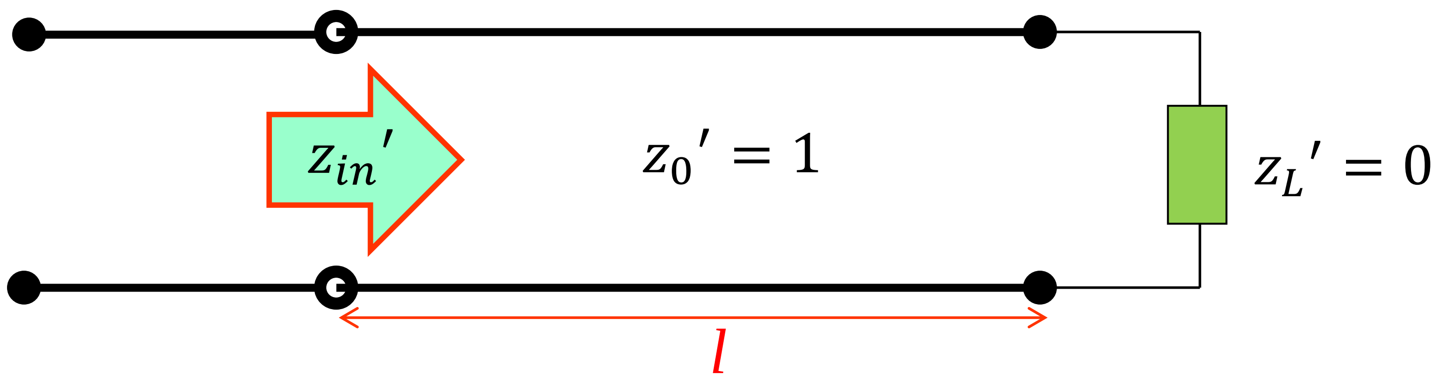


Example – 8

- determine the input impedance of a transmission line that is terminated in a **short circuit**, and whose length is:

$$a)\ l = \lambda/8 = 0.125\lambda \quad \Rightarrow \quad 2\beta l = 90°$$

$$\text{b)}\ l = 3\lambda/8 = 0.375\lambda \quad \Rightarrow \quad 2\beta l = 270°$$



- **Solution:**

a) Rotate **clockwise** $90°$ from $\Gamma = -1.0 = e^{j180°}$ and find z_{in}'. $z_{in}' = j$

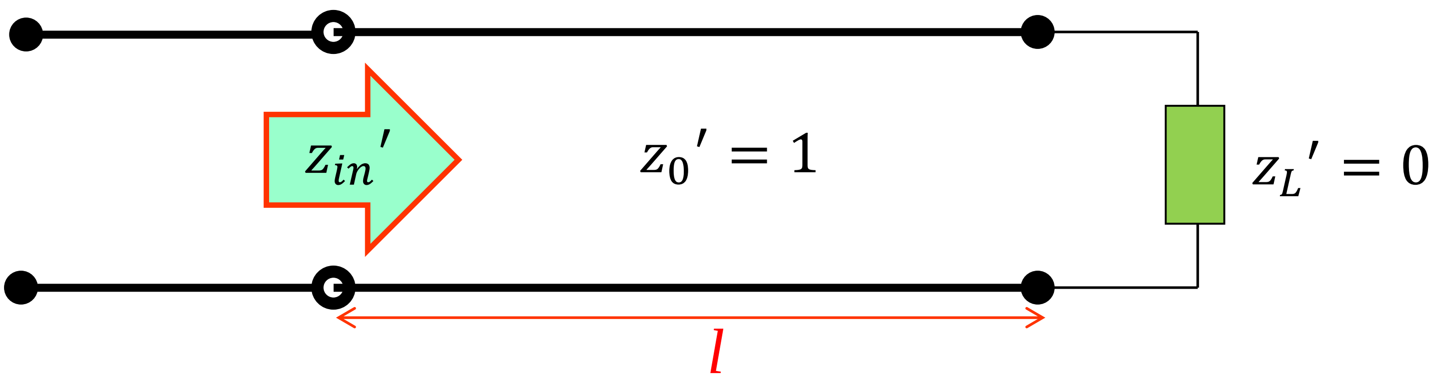
b) Rotate **clockwise** $270°$ from $\Gamma = -1.0 = e^{j180°}$ and find z_{in}'. $z_{in}' = -j$

Example – 9

- determine the input impedance of a transmission line that is terminated in a **short circuit**, and whose length is:

$$a)\ l = {}^{\lambda}/_{8} = 0.125\lambda \quad \Rightarrow \quad 2\beta l = 90°$$

$$\text{b) } l = {}^{3\lambda}/_{8} = 0.375\lambda \quad \Rightarrow \quad 2\beta l = 270°$$



- **Solution:**

a) Rotate **clockwise** $90°$ from $\Gamma = -1.0 = e^{j180°}$ and find z_{in}'. $\boxed{z_{in}' = j}$

b) Rotate **clockwise** $270°$ from $\Gamma = -1.0 = e^{j180°}$ and find z_{in}'. $\boxed{z_{in}' = -j}$

Example – 10

- A load **terminating** at transmission line has a normalized impedance $z_L{}' = 2.0 + j2.0$. What should the **length** l of transmission line be in order for its input impedance to be:
  a) Purely **real** (i.e., $X_{in} = 0$)
  b) Have a real (resistive) part equal to **one** (i.e., $r_{in} = 1.0$)
- **Solution:**

a) Find $z_L{}' = 2.0 + j2.0$ on your Smith Chart, and then rotate **clockwise** until you "bump into" the contour $x = 0$ (recall this contour lies on the $\Gamma_r -$ **axis!**).

- When you reach the $x = 0$ contour—**stop!** Lift your pen and note that the impedance value of this location is **purely real** (after all, $x = 0!$).
- Now, measure the **rotation angle** that was required to move clockwise from $z_L{}' = 2.0 + j2.0$ to an impedance on the $x = 0$ contour—this **angle** is equal to $2\beta l$!

You can now **solve** for l, or alternatively use the **electrical length scale** surrounding the Smith Chart.

Example – 10 (contd.)

One more important point—there are **two** possible solutions!

$z_{in}{}' = 4.2 + j0$ → $2\beta l = 30°$ → $l = 0.042\lambda$

$z_{in}{}' = 0.24 + j0$ → $2\beta l = 210°$ → $l = 0.292\lambda$

b) Find $z_L{}' = 2.0 + j2.0$ on your Smith Chart, and then rotate **clockwise** until you "bump into" the **circle** $r = 1$ (recall this circle intersects the **center** point of the Smith Chart!).

- When you reach the $r = 1$ circle—**stop**! Lift your pencil and note that the impedance value of this location has a real value equal to **one** (after all, $r = 1$!).
- Now, measure the **rotation angle** that was required to move clockwise from $z_L{}' = 2.0 + j2.0$ to an impedance on the $r = 1$ circle—this **angle** is equal to $2\beta l$!

Example – 10 (contd.)

- Thus, for impedances where $r = 1$ (i.e., $z' = 1 + jx$):

$$\Gamma = \frac{z'-1}{z'+1} = \frac{(1+jx)-1}{(1+jx)+1} = \frac{jx}{2+jx}$$

- and therefore:

$$|\Gamma|^2 = \frac{|jx|^2}{|2+jx|^2} = \frac{x^2}{4+x^2}$$

$$x^2 = \frac{4|\Gamma|^2}{1-|\Gamma|^2}$$

$$x = \pm\frac{2|\Gamma|}{\sqrt{1-|\Gamma|^2}}$$

there are **two** equal by opposite solutions!

for **this** example gives us solutions $x = \pm 1.6$.

Admittance Transformation

- RF/Microwave network, similar to any electrical network, has impedance elements in series and parallel
- Impedance Smith chart is well suited while working with series configurations while admittance Smith chart is more useful for parallel configurations
- The impedance Smith chart can easily be used as an admittance calculator

$$z_{in}(z) = \frac{1+\Gamma(z)}{1-\Gamma(z)} \quad \Leftarrow \quad y_{in}(z) = \frac{Y_{in}(z)}{Y_0} = \frac{1/Z_{in}(z)}{1/Z_0} = \frac{1}{Z_{in}(z)/Z_0} = \frac{1}{z_{in}(z)}$$

- Hence, $y_{in}(z) = \frac{1-\Gamma(z)}{1+\Gamma(z)}$ $\Rightarrow$ $y_{in}(z) = \frac{1+e^{-j\pi}\Gamma(z)}{1-e^{-j\pi}\Gamma(z)}$

It means, to obtain normalized admittance → take the normalized impedance and multiply associated reflection coefficient by -1 = $e^{-j\pi}$ → it is equivalent to a 180^o rotation of the reflection coefficient in complex Γ-plane

Example – 11

- Convert the following normalized input impedance z_{in}' into normalized input admittance y_{in}' using the Smith chart:

$$z_{in}' = 1 + j1 = \sqrt{2}e^{j(\pi/4)}$$

First approach: The normalized admittance can be found by direct inversion as:

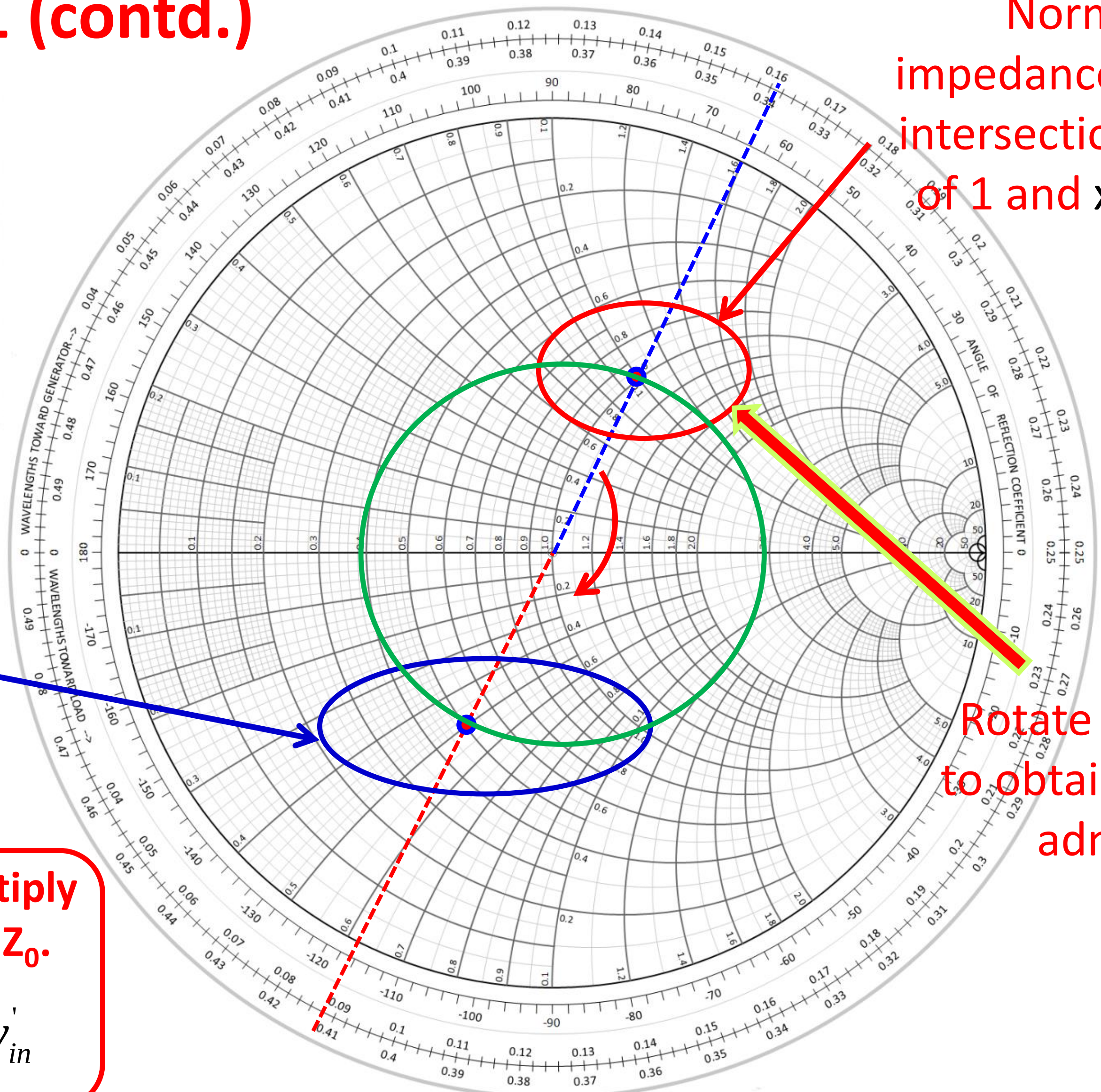
$$y_{in}' = \frac{1}{z_{in}'} = \frac{1}{1+j1} = \frac{1}{\sqrt{2}}e^{-j(\pi/4)} = \frac{1}{2} - j\frac{1}{2}$$

Alternative approach:

- Mark the normalized impedance on Smith chart
- Identify phase angle and magnitude of the associated reflection coefficient
- Rotate the reflection coefficient by 180^{o}
- Identify the x-circle and r-circle intersection of the rotated reflection coefficient

Example – 11 (contd.)

Quick investigation show that the normalized impedance (y_{in}') is the intersection of r-circle of 1/2 and x-circle of -1/2



Normalized impedance (z_{in}') is the intersection of r-circle of 1 and x-circle of 1

Rotate this by 180° to obtain normalized admittance

To denormalize, multiply with the inverse of Z_0.

$$Y_{in} = y_{in}' \frac{1}{Z_0} = Y_0 y_{in}'$$

Example – 12

Given: $z_{in}^{'} = 1 + j2$

- Find the normalized admittance $\lambda/8$ away from the load

Steps:

1. Mark the normalized impedance on Smith Chart
2. Clockwise rotate it by 180^o
3. Identify the normalized impedance and the phase angle of the associated reflection coefficient
4. Clockwise rotate the reflection coefficient (associated with the normalized admittance) by $2\beta l$ (here $l = \lambda/8$)
5. The new location gives the required normalized admittance

Example – 12 (contd.)

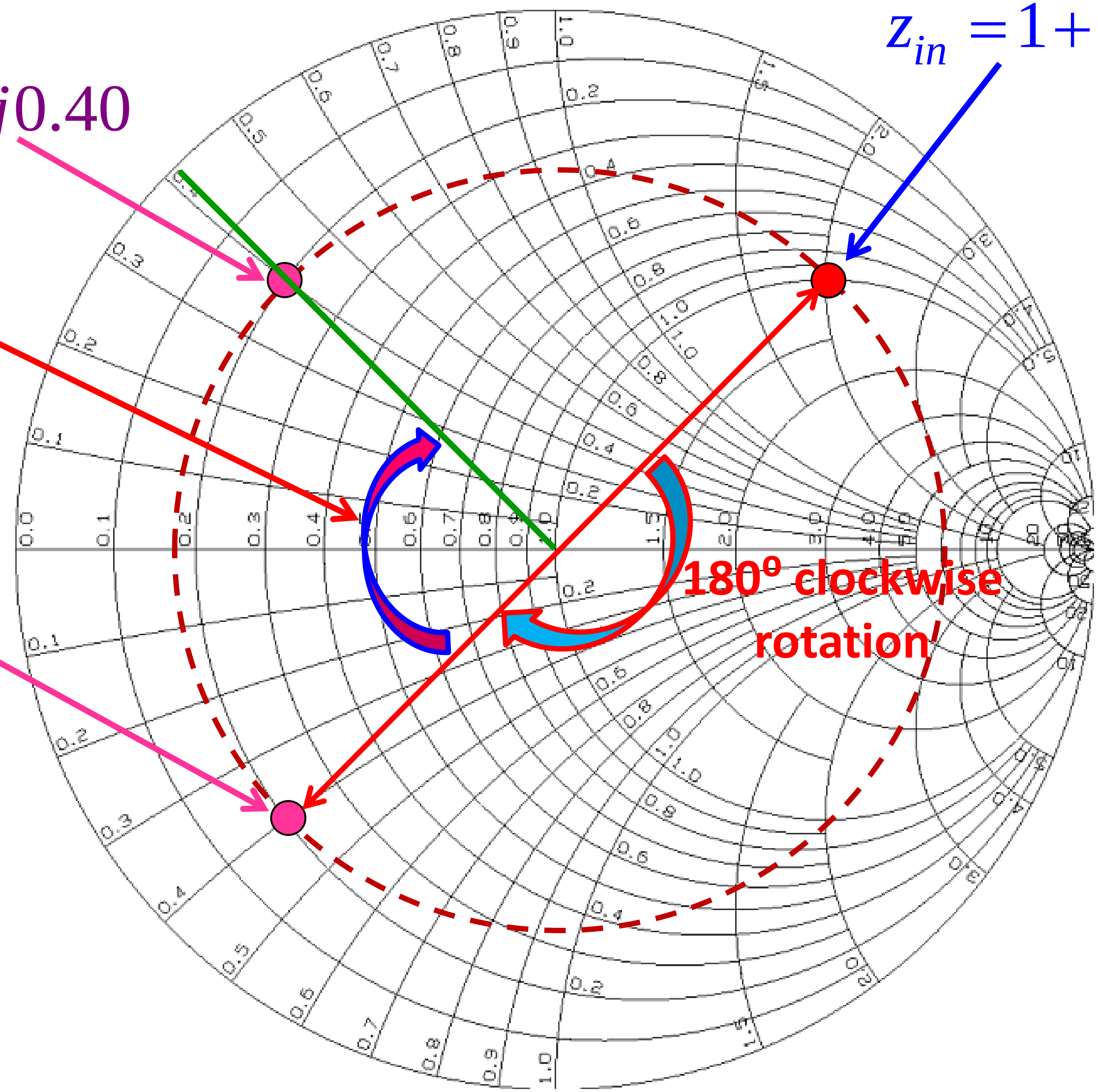
$$y_{in}^{'} = 0.20 + j0.40$$

Clockwise rotation by

$$2\beta l = 4\pi\left(\frac{l}{\lambda}\right)$$

$$y_{in}^{'} = 0.2 - j0.4$$

$$l = \lambda/8 \rightarrow 2\beta l = 90^{o}$$



Admittance Smith chart

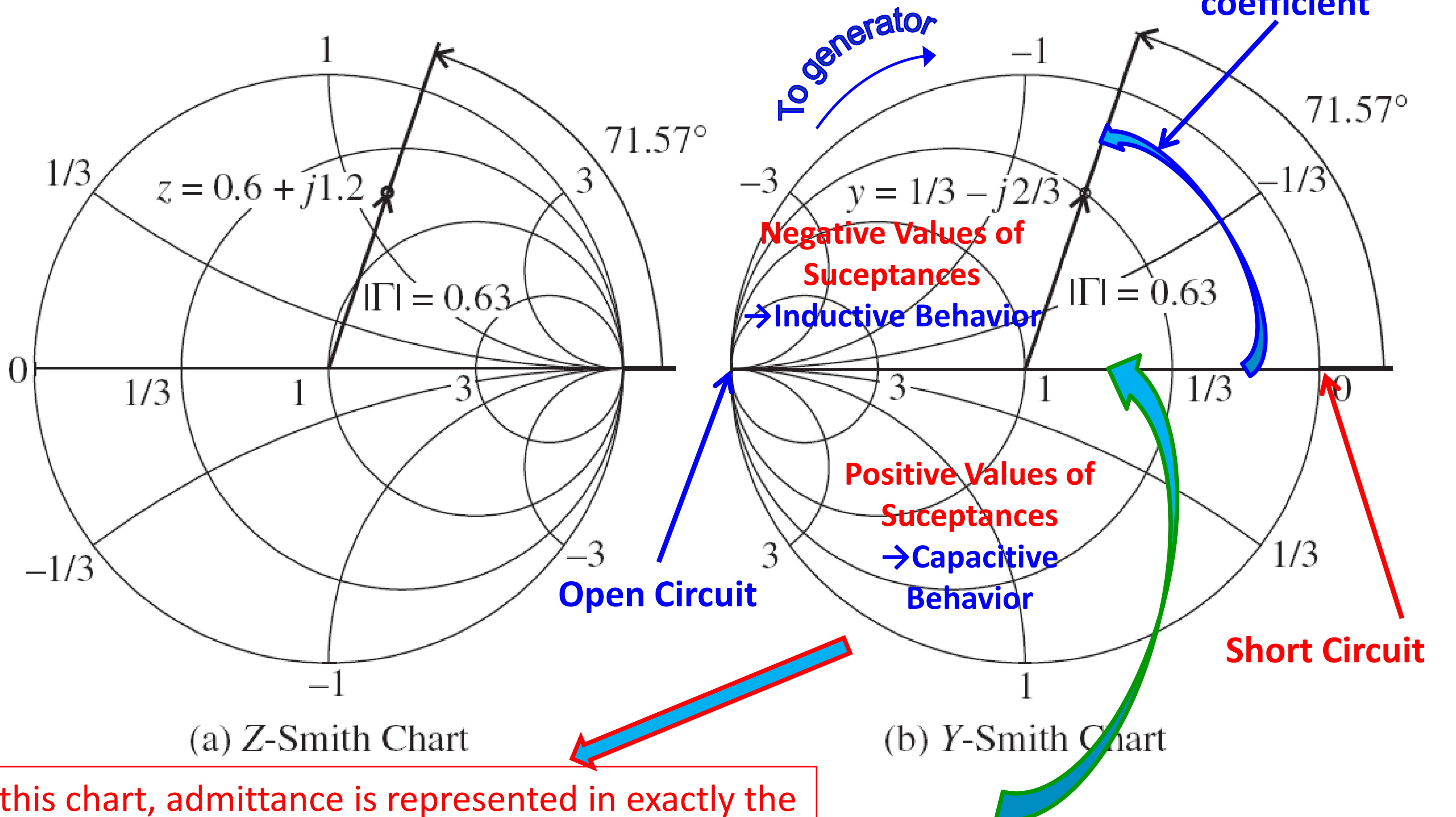
- Alternative approach to solve parallel network elements is through 180^o rotated Smith chart
- This rotated Smith chart is called **admittance Smith chart or Y-Smith chart**
- The corresponding normalized resistances become normalized conductances & normalized reactances become normalized suceptances

$$r = \frac{R}{Z_0} \Rightarrow g = \frac{G}{Y_0} = Z_0 G$$

$$x = \frac{X}{Z_0} \Rightarrow b = \frac{b}{Y_0} = Z_0 B$$

- **The Y-Smith chart preserves:**
  - The direction in which the angle of the reflection coefficient is measured
  - The direction of rotation (either toward or away from the generator)

Admittance Smith chart (contd.)

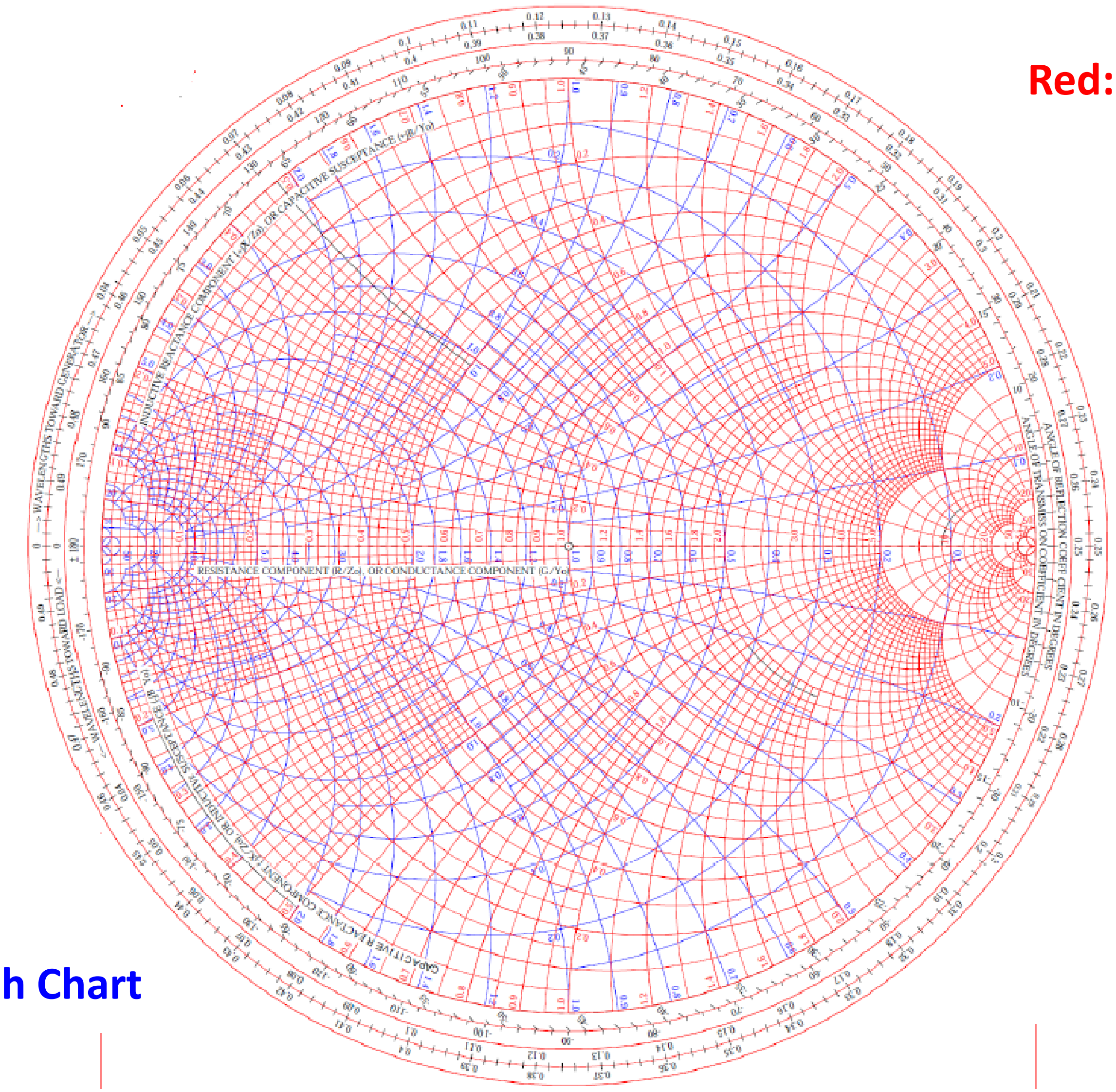


(a) Z-Smith Chart (b) Y-Smith Chart

In this chart, admittance is represented in exactly the same manner as the impedance in the Z-smith Chart → without 180^0 rotation

Combined Z- and Y- Smith Charts

Red: Z – Smith Chart



Blue: Y – Smith Chart

Example – 13

- Identify (a) the normalized impedance $z' = 0.5 + j0.5$, and (b) the normalized admittance value $y' = 1 + j2$ in the combined ZY-Smith Chart and find the corresponding values of normalized admittance and impedance

Example – 13 (contd.)