Lecture – 4

Date: 14.01.2016

- Terminated Lossless Transmission Line (Contd.)
- TL Input Impedance
- Solved Examples
- Return Loss and Insertion Loss
- Standing Wave and SWR

Example

- Consider a load resistance $R_L = 100\Omega$ to be matched to a 50Ω line with a quarter-wave transformer. Find the characteristic impedance of the matching section and plot the magnitude of the reflection coefficient versus normalized frequency, f/f_0, where f_0 is the frequency at which the line is $\lambda/4$ long.

  - the necessary characteristic impedance is:

$$Z_0 = \sqrt{Z_L Z_{in}} \Rightarrow \boxed{\therefore Z_0 = \sqrt{Z_L Z_{in}} = \sqrt{50 \times 100} = 70.71\Omega}$$

  - The reflection coefficient magnitude is given as

$$\boxed{|\Gamma_0| = \left| \frac{Z_0 - Z_{in}}{Z_0 + Z_{in}} \right|} \Leftarrow Z_{in} \text{ is dependent on frequency}$$

Example – (contd.)

$$Z_{in} = Z(l = \lambda/4) = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda_0}{4}\right) = \left(\frac{2\pi f}{v_p}\right)\left(\frac{v_p}{4 f_0}\right) = \frac{\pi f}{2 f_0}$$

For higher frequencies the matching section looks electrically longer, and for lower frequencies it looks shorter.

Plot the magnitude of the reflection coefficient versus f/f_0 using these two equations

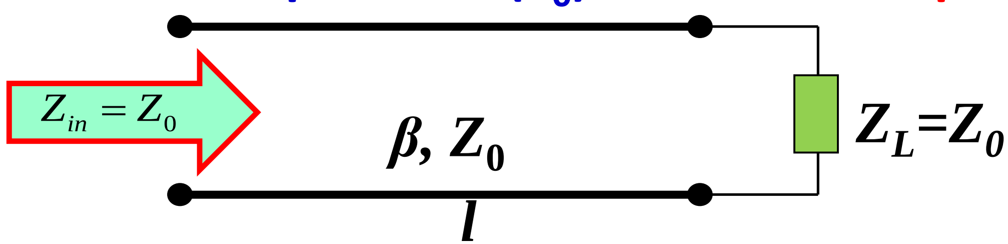
HW # 0

Transmission Line Input Impedance – Special Cases (contd.)

- $Z_L = Z_0$ ← the load is **numerically equal** to the characteristic impedance of the transmission line (a real value).

$$Z_{in} = Z_0 \frac{Z_0 + jZ_0 \tan(\beta l)}{Z_0 + jZ_0 \tan(\beta l)} = Z_0$$

In other words, if the **load impedance (Z_L)** is **equal** to the TL **characteristic impedance (Z_0)**, the **input impedance (Z_{in})** likewise will be equal to **characteristic impedance (Z_0)** of the TL **irrespective of its length**

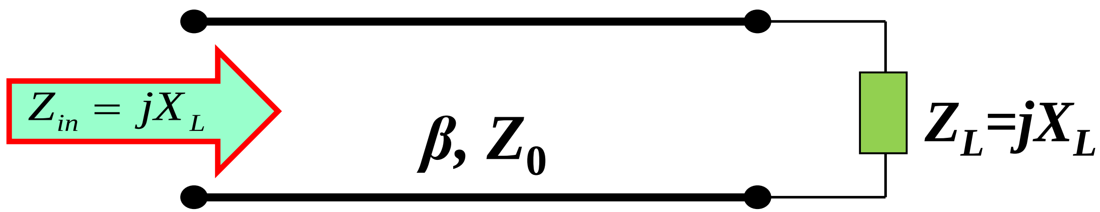


- $Z_L = jX_L$ ← the load is **purely reactive** (i.e., the resistive component is zero)

$$Z_{in} = Z(z=-l) = Z_0 \frac{jX_L + jZ_0 \tan(\beta l)}{Z_0 - X_L \tan(\beta l)} = jZ_0 \frac{X_L + Z_0 \tan(\beta l)}{Z_0 - X_L \tan(\beta l)}$$ → **Purely Reactive**

Transmission Line Input Impedance – Special Cases (contd.)

In other words, if the **load impedance (Z_L) is purely reactive** then the **input impedance likewise will be purely reactive irrespective of the line length (*l*)**



Note that the **opposite is not true: even if the load is purely resistive (Z_L = R), the input impedance will be complex** (both resistive and reactive components).

- ***l* << λ** ⟵ the transmission line is **electrically small**

- If length l is small with respect to signal wavelength λ then: $$\beta l = \frac{2\pi}{\lambda} l = 2\pi \frac{l}{\lambda} \approx 0$$

- Therefore: $\cos(\beta l) = 1$, $\sin(\beta l) = 0$

- Thus the input impedance is: $$Z_{in} = Z_0 \frac{Z_L \cos(\beta l) + jZ_0 \sin(\beta l)}{Z_0 \cos(\beta l) + jZ_L \sin(\beta l)} = Z_0 \frac{Z_L(1) + jZ_0(0)}{Z_0(1) + jZ_L(0)} = Z_0$$

Transmission Line Input Impedance – Special Cases (contd.)

In other words, if the transmission line length is much smaller than a wavelength, the **input** impedance Z_{in} will **always** be equal to the **load** impedance Z_L.

This is the assumption we used in all previous circuits courses (e.g., Linear Circuits, Digital Circuits, Integrated Electronics, Analog Circuit Design etc.)! In those courses, we assumed that the signal frequency ω is relatively **low**, such that the signal wavelength λ is **very large** $(\lambda \gg l)$.

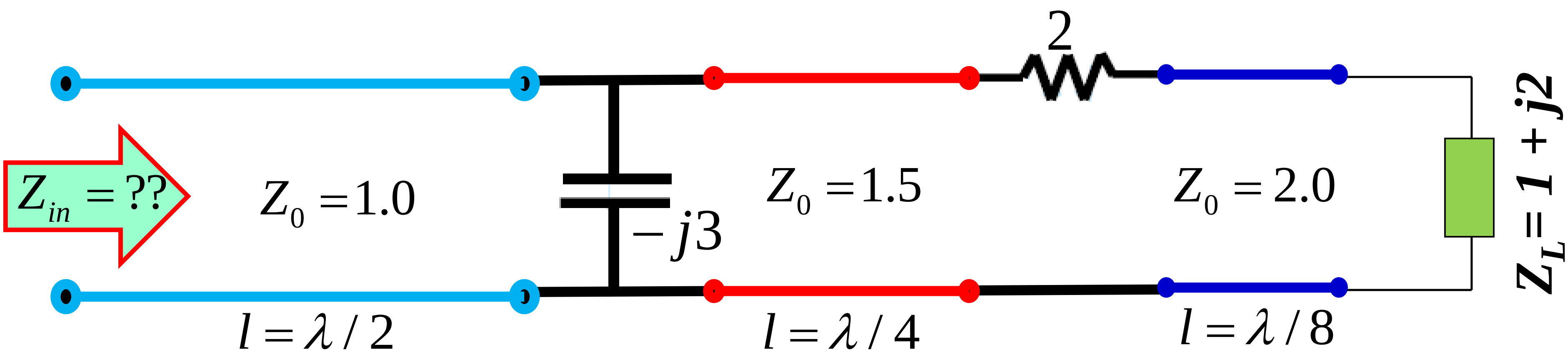
- Note also for this case (the electrically short transmission line), the voltage and current at each end of the transmission line are approximately the **same**!

$$\boxed{V(z=-l) \approx V(z=0)} \qquad \boxed{I(z=-l) \approx I(z=0)}$$

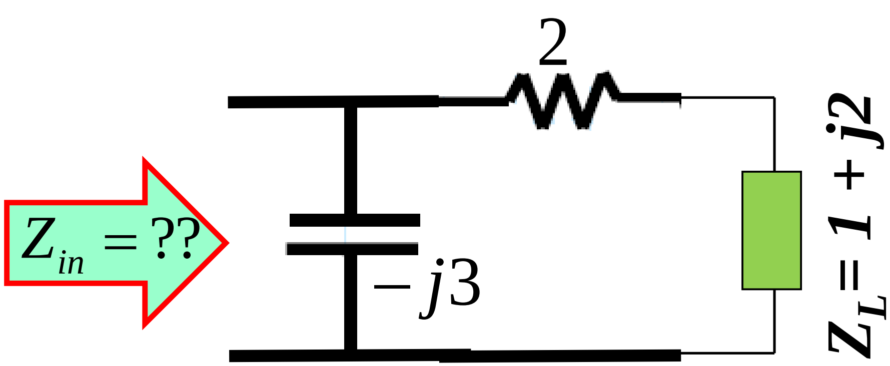
If $l \ll \lambda$, our "wire" behaves **exactly** as it did in *Linear Circuits* course!

Example – 1

Determine the input impedance of the following circuit:



How about the following solution?

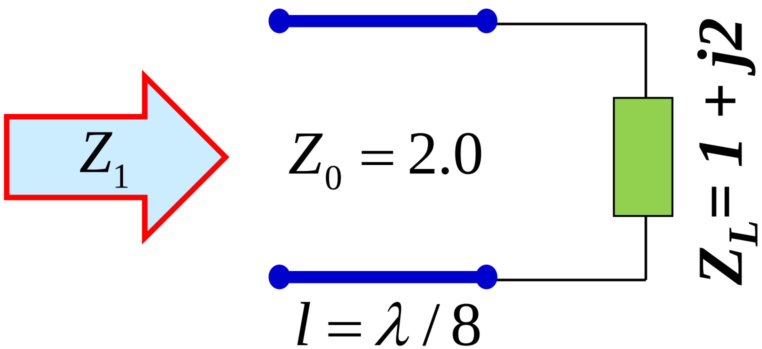


$$Z_{in} = \frac{-j3 * (2 + 1 + j2)}{-j3 + (2 + 1 + j2)} = 2.7 - j2.1$$

Where are the contributions of the TL??

Example – 1 (contd.)

- **Let us define Z_1 as the input impedance of the last section as:**



Then the impedance Z_1 is:

$$Z_1 = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)}$$

Where: $\beta d = \frac{2\pi}{\lambda} * \frac{\lambda}{8} = \frac{\pi}{4}$

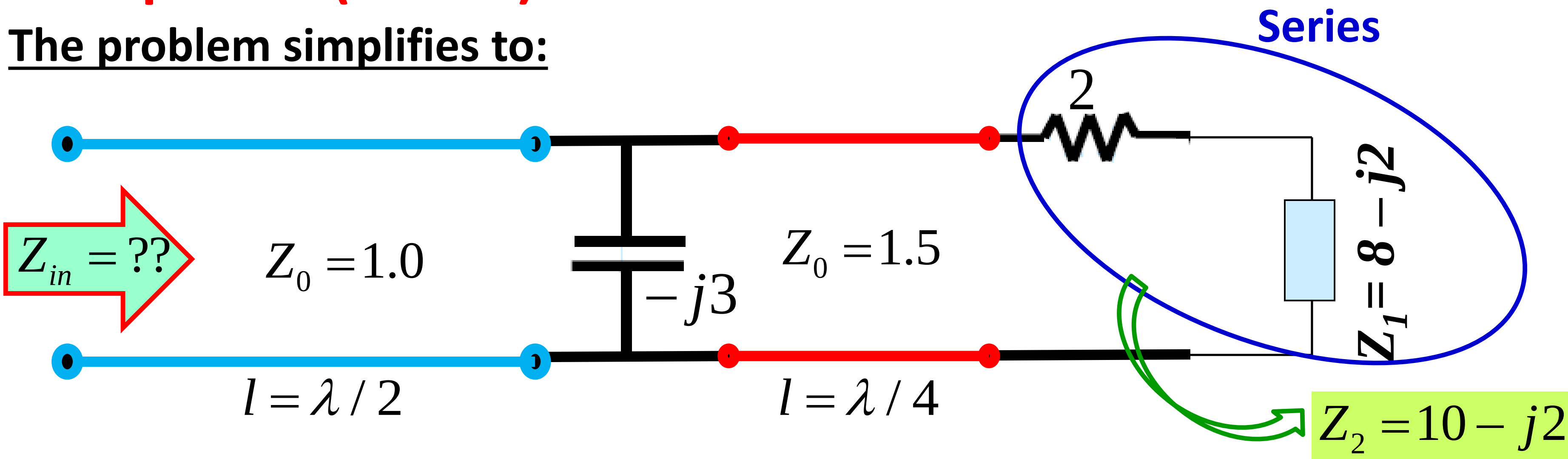
Therefore:

$$Z_1 = 2\left(\frac{(1+j2) + j2\tan(\pi/4)}{2 + j(1+j2)\tan(\pi/4)}\right) \Rightarrow Z_1 = 2\left(\frac{1+j4}{j}\right)$$

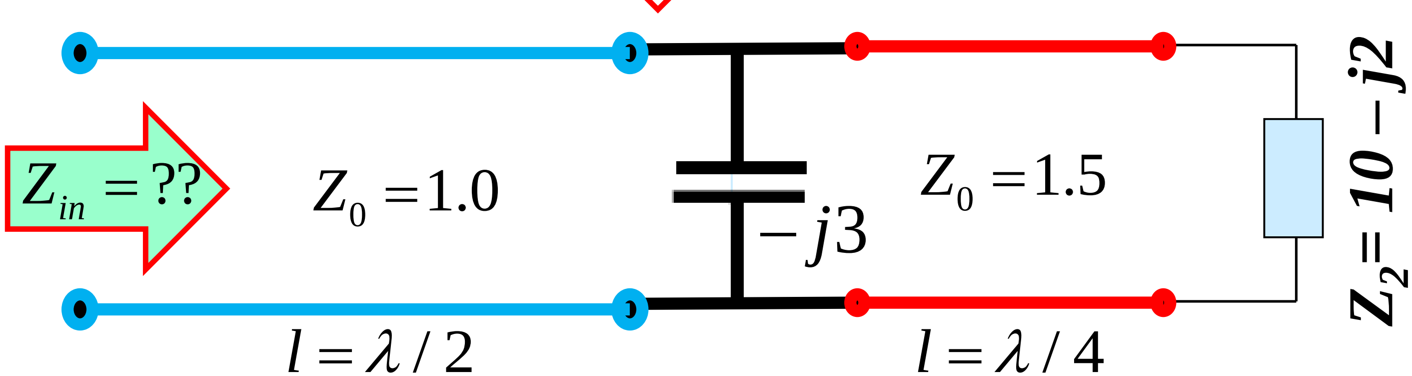
$$\boxed{\therefore Z_1 = 8 - j2}$$

Example – 1 (contd.)

The problem simplifies to:

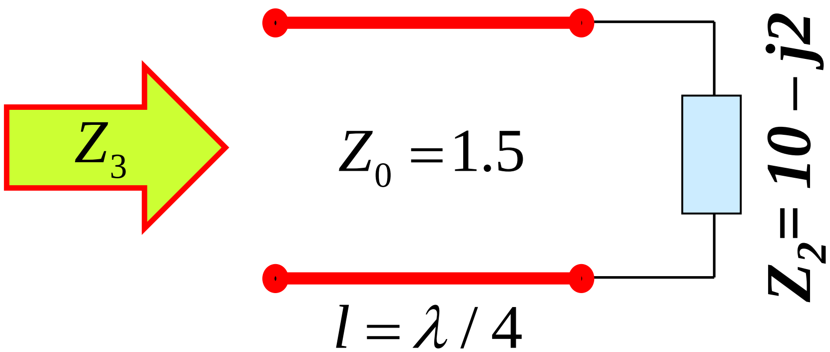


Simplification of the problem



Example – 1 (contd.)

- **Now let us define the input impedance of the middle TL as Z_3:**

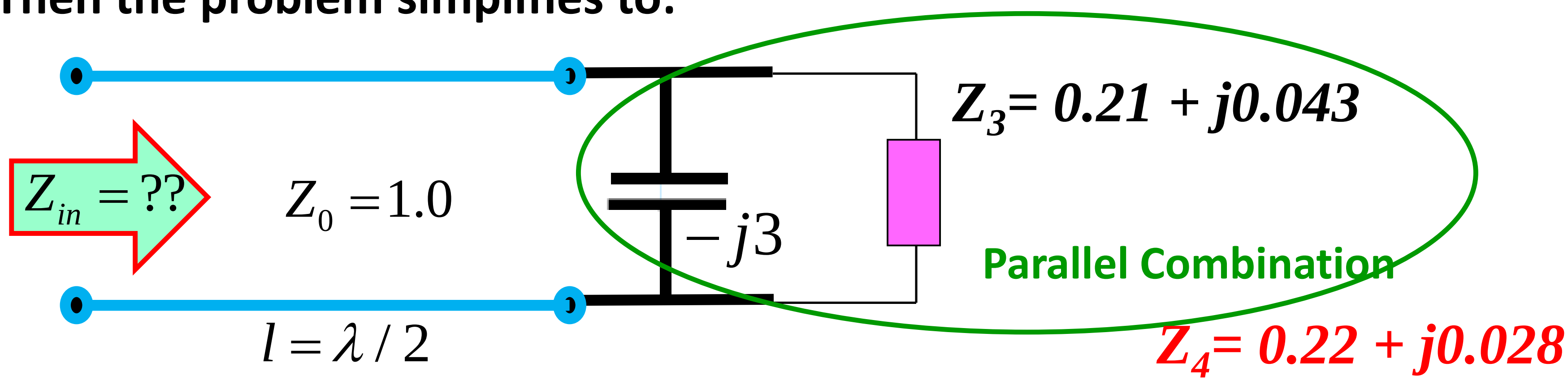


This is a quarter-wave TL → one of the special cases we considered earlier → where the input impedance is:

$$\boxed{Z_3 = \frac{Z_0^2}{Z_2}}$$

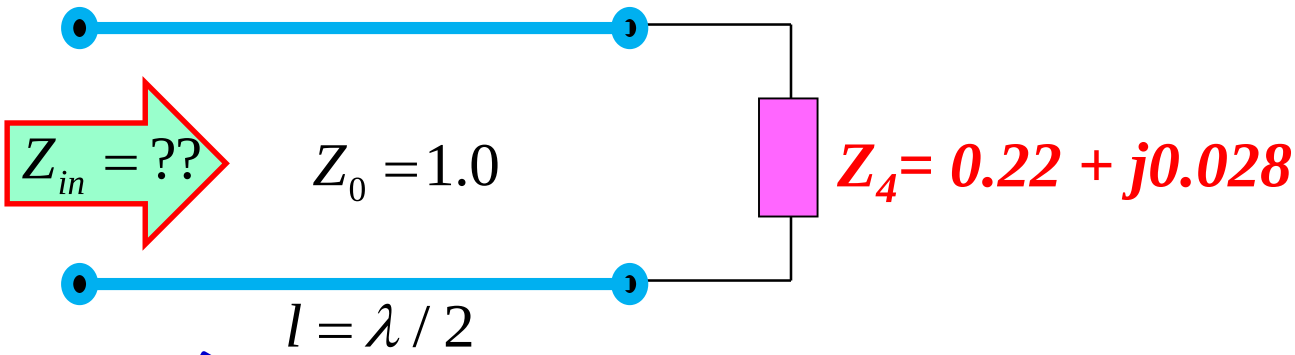
Therefore: $Z_3 = \dfrac{(1.5)^2}{10 - j2}$ → $\boxed{\therefore Z_3 = 0.21 + j0.043}$

- **Then the problem simplifies to:**



Example – 1 (contd.)

- Finally the simplified problem is:

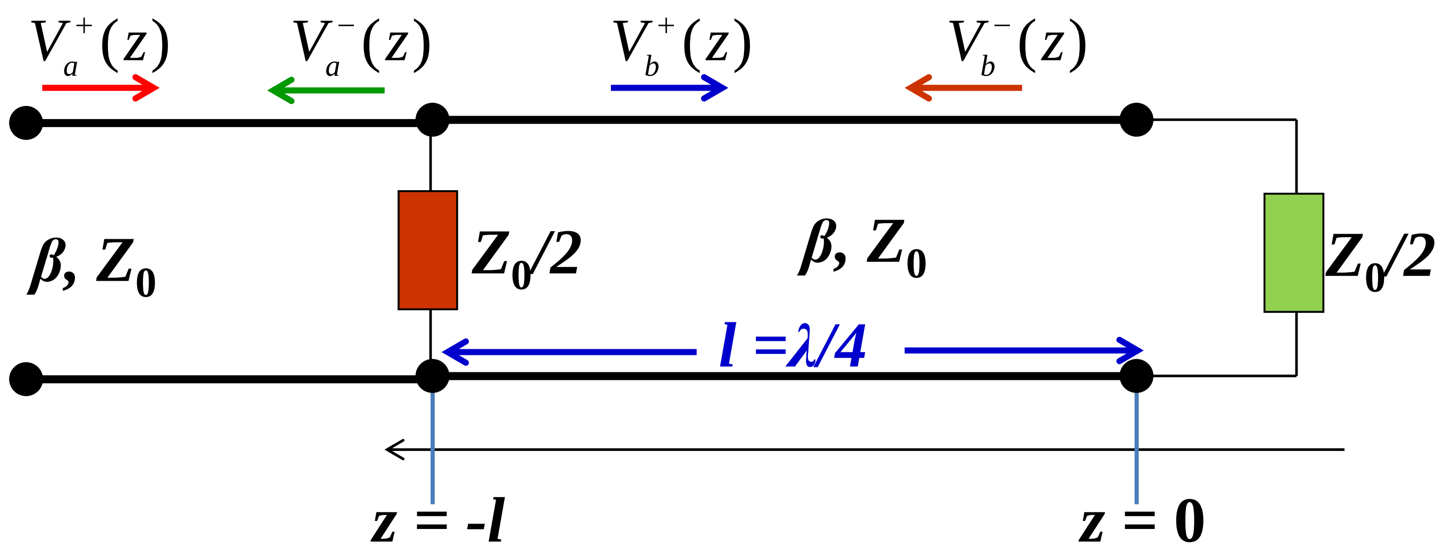


TL is a half wavelength → special case we discussed earlier → input impedance equals the load impedance

$$\boxed{\therefore Z_{in} = Z_4 = 0.22 + j0.028}$$

Example – 2

For the following circuit determine: $\frac{V_a^-}{V_a^+}$ $\frac{V_b^+}{V_a^+}$ $\frac{V_b^-}{V_a^+}$



Given:

$$V(z) = V_a^+(z) + V_a^-(z) = V_a^+ e^{-j\beta z} + V_a^- e^{+j\beta z} \qquad \text{For } z < -l$$

$$V(z) = V_b^+(z) + V_b^-(z) = V_b^+ e^{-j\beta z} + V_b^- e^{+j\beta z} \qquad \text{For } -l < z < 0$$

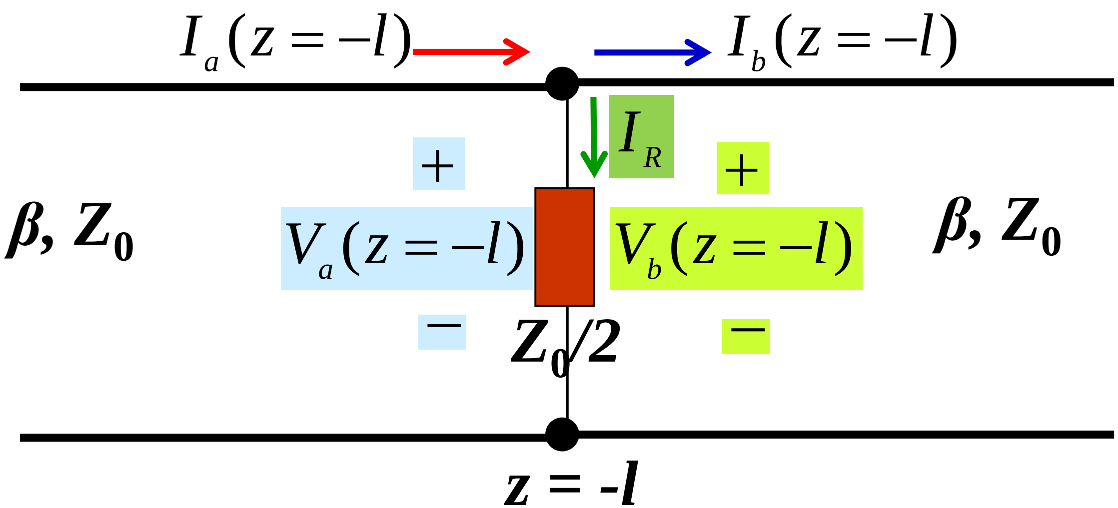
Example – 2 (contd.)

- **We can write current equations as:**

$$I(z) = \frac{V_a^+(z)}{Z_0} - \frac{V_a^-(z)}{Z_0} = \frac{V_a^+}{Z_0}e^{-j\beta z} - \frac{V_a^-}{Z_0}e^{+j\beta z}$$ **For $z < -l$**

$$I(z) = \frac{V_b^+(z)}{Z_0} - \frac{V_b^-(z)}{Z_0} = \frac{V_b^+}{Z_0}e^{-j\beta z} - \frac{V_b^-}{Z_0}e^{+j\beta z}$$ **For $-l < z < 0$**

- **At $z = -l$:**



KVL gives:

$$V_a(z=-l) = V_b(z=-l)$$

KCL gives:

$$I_a(z=-l) = I_b(z=-l) + I_R$$

Ohm's Law gives: $$I_R = \frac{V_a(z=-l)}{Z_0/2} = \frac{2V_a(z=-l)}{Z_0} = \frac{2V_b(z=-l)}{Z_0}$$

Example – 2 (contd.)

- **At $z = -l$:**

$$\boxed{V_a(z=-l) = V_a^+(z=-l) + V_a^-(z=-l) = V_a^+ e^{-j\beta(-l)} + V_a^- e^{+j\beta(-l)} = V_a^+ e^{+j\beta l} + V_a^- e^{-j\beta l}}$$

It is given: $l = \frac{\lambda}{4}$ $\Rightarrow$ $\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$

$$\boxed{\therefore V_a(z=-l) = V_a^+ e^{+j(\pi/2)} + V_a^- e^{-j(\pi/2)} = j\left(V_a^+ - V_a^-\right)}$$

Similarly:

$$V_b(z=-l) = j\left(V_b^+ - V_b^-\right)$$

$$I_a(z=-l) = j\left(\frac{V_a^+ + V_a^-}{Z_0}\right)$$

$$I_b(z=-l) = j\left(\frac{V_b^+ + V_b^-}{Z_0}\right)$$

Example – 2 (contd.)

- Now let us revisit the expressions achieved from KVL, KCL and Ohm's Law

KVL

$$V_a(z=-l)=V_b(z=-l)$$

$$\Rightarrow j\left(V_a^+-V_a^-\right)=j\left(V_b^+-V_b^-\right)$$

$$\boxed{\therefore 1-\frac{V_a^-}{V_a^+}=\frac{V_b^+}{V_a^+}-\frac{V_b^-}{V_a^+}}$$

Ohm's Law

$$I_R=\frac{2V_a(z=-l)}{Z_0}=\frac{2j\left(V_a^+-V_a^-\right)}{Z_0}$$

$$I_R=\frac{2V_b(z=-l)}{Z_0}=\frac{2j\left(V_b^+-V_b^-\right)}{Z_0}$$

KCL

$$I_a(z=-l)=I_b(z=-l)+I_R$$

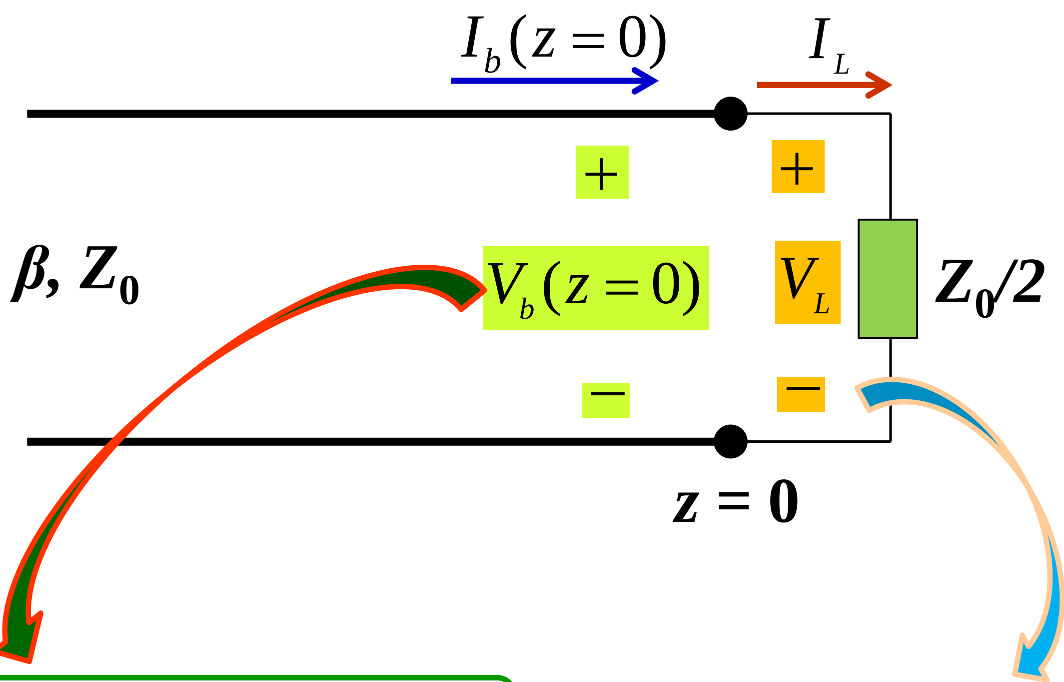
$$\Rightarrow j\left(\frac{V_a^++V_a^-}{Z_0}\right)=j\left(\frac{V_b^++V_b^-}{Z_0}\right)+I_R$$

$$V_a^++V_a^-=V_b^++V_b^--jI_RZ_0$$

$$\boxed{\therefore 1+\frac{V_a^-}{V_a^+}=3\frac{V_b^+}{V_a^+}-\frac{V_b^-}{V_a^+}}$$

Example – 2 (contd.)

- **At $z = 0$:**



$$V_b(z=0) = V_b^+ e^{-j\beta(0)} + V_b^- e^{+j\beta(0)} = V_b^+ + V_b^-$$

$$I_b(z=0) = \frac{V_b^+}{Z_0} e^{-j\beta(0)} - \frac{V_b^-}{Z_0} e^{+j\beta(0)} = \frac{V_b^+ - V_b^-}{Z_0}$$

KVL: $V_b(z=0) = V_L$

KCL: $I_b(z=0) = I_L$

Ohm's Law: $I_L = \dfrac{V_L}{Z_0/2} = \dfrac{2V_L}{Z_0}$

Example – 2 (contd.)

At $z = 0$:

$$I_L = \frac{V_L}{Z_0/2} = \frac{2V_L}{Z_0}$$

$$= I_b(z=0) = \frac{V_b^+ - V_b^-}{Z_0}$$

$$= V_b(z=0) = V_b^+ + V_b^-$$

$$\frac{V_b^+ - V_b^-}{Z_0} = \frac{2\left(V_b^+ + V_b^-\right)}{Z_0}$$

simplify

$$\boxed{V_b^- = -\frac{1}{3}V_b^+}$$

You can also achieve this result by writing the expression for reflection coefficient

Example – 2 (contd.)

Let us bring all the three simplified equations together

$$\boxed{1-\frac{V_a^-}{V_a^+}=\frac{V_b^+}{V_a^+}-\frac{V_b^-}{V_a^+}}$$

(1)

$$\boxed{1+\frac{V_a^-}{V_a^+}=3\frac{V_b^+}{V_a^+}-\frac{V_b^-}{V_a^+}}$$

(2)

$$\boxed{V_b^-=-\frac{1}{3}V_b^+}$$

(3)

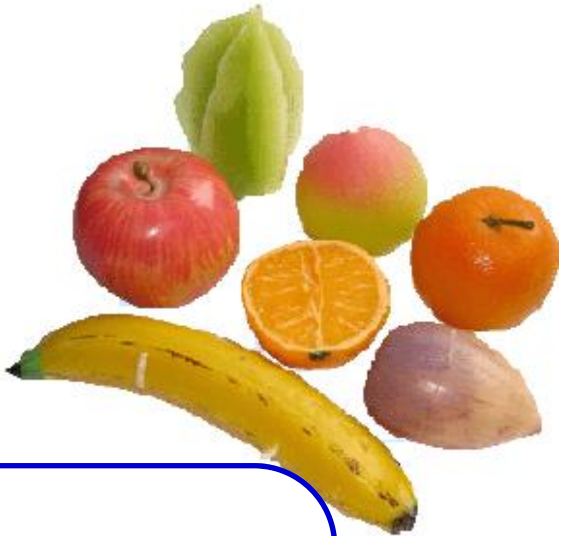
Simplification of (1) and (3) results in: $1-\frac{V_a^-}{V_a^+}=\frac{4}{3}\frac{V_b^+}{V_a^+}$ **(4)**

Simplification of (2) and (3) results in: $1+\frac{V_a^-}{V_a^+}=\frac{10}{3}\frac{V_b^+}{V_a^+}$ **(5)**

Simplify all of these to obtain the values of

$$\frac{V_a^-}{V_a^+} \qquad \frac{V_b^+}{V_a^+} \qquad \frac{V_b^-}{V_a^+}$$

Example – 2 (contd.)



Let us now summarize the fruits of our effort

$$\frac{V_a^-}{V_a^+} = \frac{3}{7} \qquad \frac{V_b^+}{V_a^+} = \frac{3}{7} \qquad \frac{V_b^-}{V_a^+} = -\frac{1}{7}$$

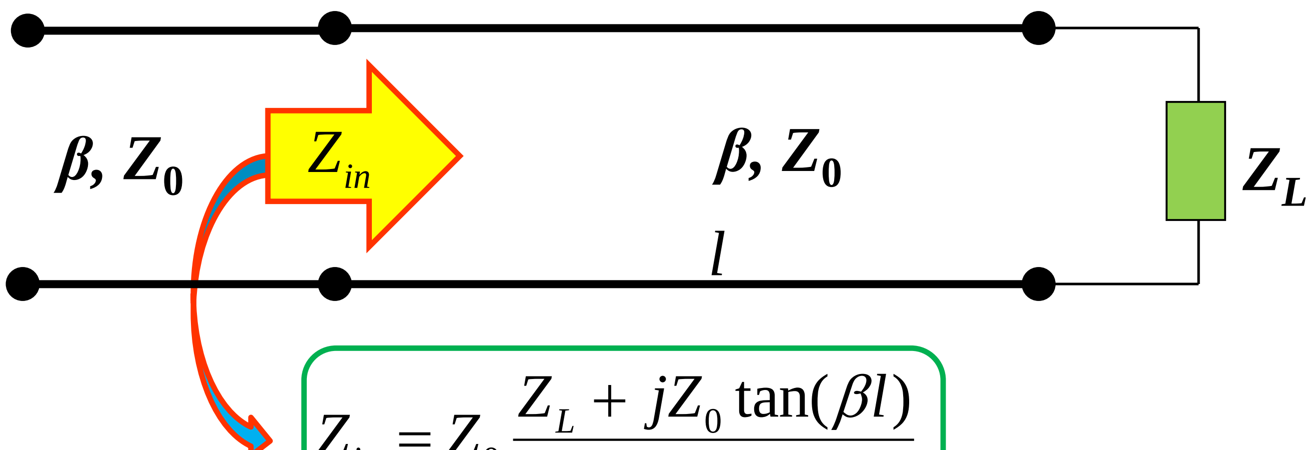
Reflection Coefficient Transformation

- We know that the **load** at the end of some length of a transmission line (with characteristic impedance Z_0) can be specified in terms of its impedance Z_L **or** its reflection coefficient Γ_0.
- Note **both** values are complex, and **either one** completely specifies the load—if you know **one**, you know the **other**!

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_L = Z_0\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right)$$

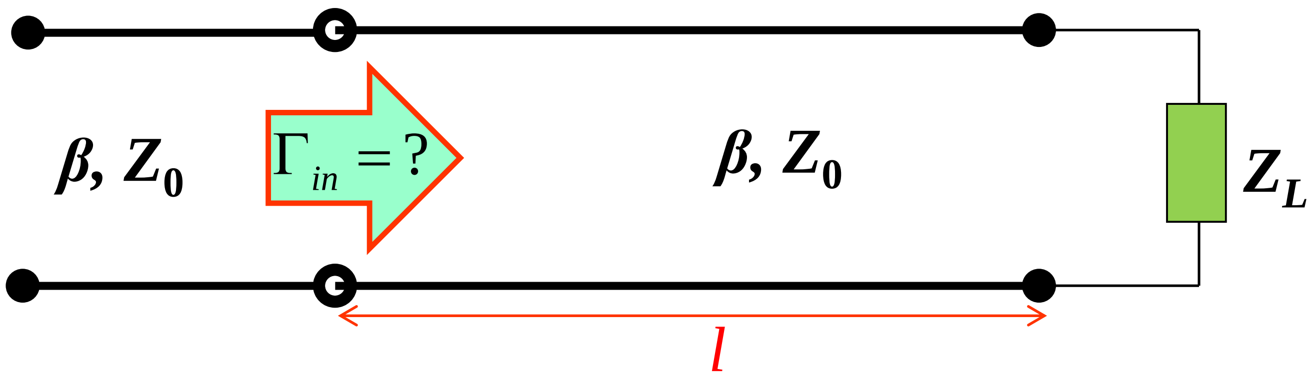
- Recall that we determined how a length of transmission line **transformed** the load **impedance** into an input **impedance** of a (generally) different value:



$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

Reflection Coefficient Transformation (contd.)

Q: Say we know the load in terms of its **reflection coefficient**. How can we express the **input** impedance in terms its **reflection coefficient** (call this Γ_{in})?



A: Well, we **could** execute these **three** steps:

1. Convert Γ_0 to Z_L:

$$Z_L = Z_0 \left(\frac{1+\Gamma_0}{1-\Gamma_0} \right)$$

2. Transform Z_L down the line to Z_{in}:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

3. Convert Z_{in} to Γ_{in}:

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

Reflection Coefficient Transformation (contd.)

Q: Yikes! This is a **ton** of complex arithmetic—isn't there an **easier** way?

A: Actually, there **is**!

- Recall that the input impedance of a transmission line length l, terminated with a load Γ_0, is:

$$\boxed{Z_{in} = Z(z=-l) = \frac{V(z=-l)}{I(z=-l)} = Z_0\left(\frac{e^{j\beta l} + \Gamma_0 e^{-j\beta l}}{e^{j\beta l} - \Gamma_0 e^{-j\beta l}}\right)}$$

Directly insert this into:

Note this **directly** relates Γ_0 to Z_{in} (steps 1 and 2 combined!).

$$\boxed{\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}}$$

directly relates Γ_0 to Γ_{in}.

$$\boxed{\Gamma_{in} = \Gamma_0 e^{-j2\beta l}}$$

Q: Hey! This result looks **familiar**.

A: Absolutely! Recall that we found the reflection coefficient **function** $\Gamma(z)$:

$$\boxed{\Gamma(z) = \Gamma_0 e^{j2\beta z}} \Rightarrow \boxed{\Gamma(z=-l) = \Gamma_0 e^{-j2\beta l}}$$

Reflection Coefficient Transformation (contd.)

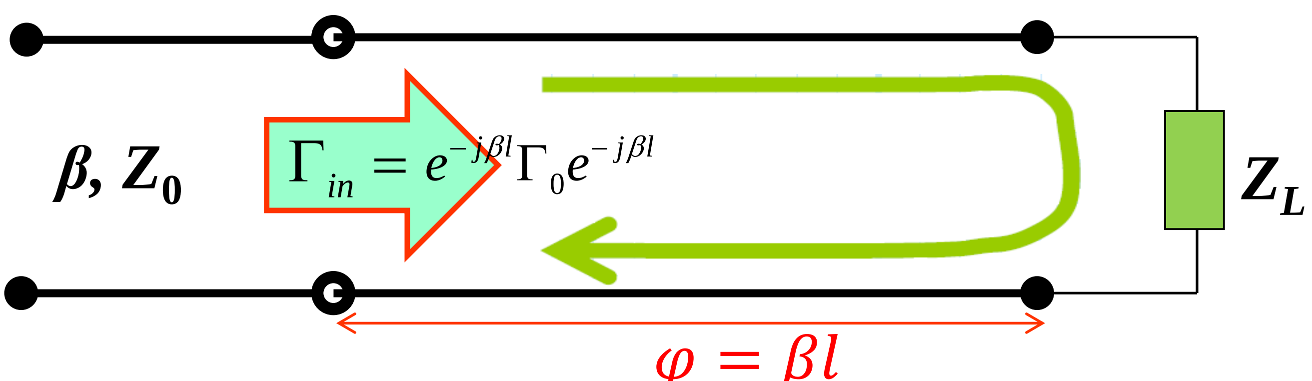
$$\Gamma_{in} = \Gamma_0 e^{-j2\beta l}$$

the **magnitude** of Γ_{in} is the **same** as the magnitude of Γ_0!

$$|\Gamma_{in}| = |\Gamma_0 e^{-j2\beta l}| = |\Gamma_0|$$

The reflection coefficient at the input is simply related to Γ_0 by a **phase shift** of $2\beta l$.

Finally, the **phase shift** associated with transforming Γ_0 down a transmission line can be attributed to the phase shift associated with the wave propagating a length l down the line, reflecting from load Z_L, and then propagating a length l back up the line.



Terminated Lossless Transmission Line (contd.)

Which relationship to use:

$$V(z), I(z), Z(z)$$

or

$$V^{+}(z), V^{-}(z), \Gamma(z)$$

Terminated Lossless Transmission Line (contd.)

Based on your **circuits** experience, you might well be **tempted** to always use **V(z)**, **I(z)** and **Z(z)**.

However, it is **useful (as well as simple)** to describe activity on a transmission line in terms of $V^{+}(z)$, $V^{-}(z)$ and $\Gamma(z)$

Terminated Lossless Transmission Line (contd.)

- The solution of Telegrapher equations (the equations defining the current and voltages along a TL) boils down to determination of complex coefficients V^+, V^-, I^+ and I^-. Once these are known, we can describe all the quantities along the TL.

- For example, the wave representations are:

$$V^+(z) = V_0^+ e^{-j\beta z}$$

$$V^-(z) = V_0^- e^{j\beta z}$$

$$\Gamma(z) = \frac{V_0^-(z)}{V_0^+(z)} = \frac{V_0^-}{V_0^+} e^{j2\beta z}$$

Magnitudes

$$\left|V^+(z)\right| = V_0^+$$

$$\left|V^-(z)\right| = V_0^-$$

$$\left|\Gamma(z)\right| = \left|\frac{V_0^-}{V_0^+}\right|$$

Relative Phases

$$\arg\{V^+(z)\} = -\beta z$$

$$\arg\{V^-(z)\} = +\beta z$$

$$\arg\{\Gamma(z)\} = +2\beta z$$

Terminated Lossless Transmission Line (contd.)

- Contrast the wave functions with complex voltage, current and impedance

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}}{Z_0}$$

$$Z(z) = Z_0 \frac{V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}}$$

Magnitudes

$$|V(z)| = \left|V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}\right| = ??$$

$$|I(z)| = \frac{\left|V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}\right|}{Z_0} = ??$$

$$|Z(z)| = Z_0 \frac{\left|V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}\right|}{\left|V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}\right|} = ??$$

Relative Phases

$$\arg\{V(z)\} = \arg\left\{V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}\right\} = ??$$

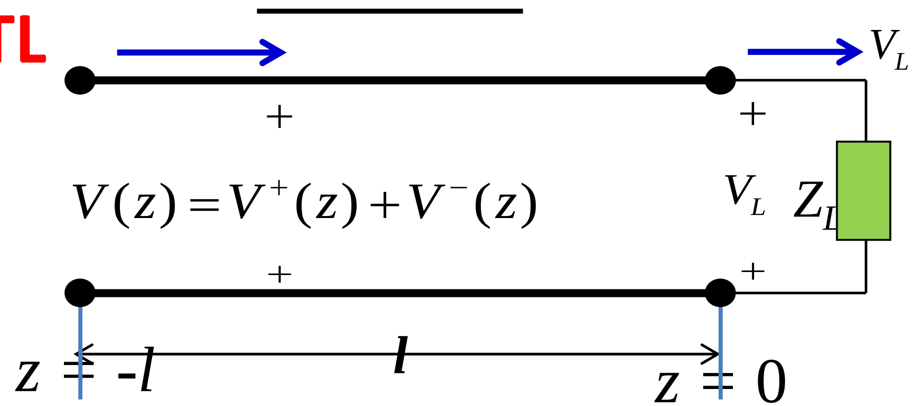
$$\arg\{I(z)\} = \arg\left\{V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}\right\} = ??$$

$$\arg\{Z(z)\} = \arg\left\{V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}\right\} - \arg\left\{V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}\right\} = ??$$

- It is, thus, much **easier** and more **straightforward** to use the **wave** representation → However, V(z), I(z), or Z(z) are still **fundamental** and very important—particularly at each **end** of the transmission line!

Power Considerations on a TL

- We have discovered that **two waves propagate** along a transmission line, one in each direction ($V^{+}(z)$ and $V^{-}(z)$).



The result is that electromagnetic energy flows along the transmission line at a given rate (i.e., **power**).

Q: How much power flows along a transmission line, and where does that power go?

A: We can answer by determining the power **absorbed** by the **load**!

$$P_{abs} = \frac{1}{2}\mathrm{Re}\left(V_L I_L^*\right) = \frac{1}{2}\mathrm{Re}\left(V(0)I(0)^*\right) = \frac{\left|V_0^+\right|^2}{2Z_0}\left(1-\left|\Gamma_0\right|^2\right)$$

$$P_{abs} = \frac{\left|V_0^+\right|^2}{2Z_0} - \frac{\left|V_0^+\Gamma_0\right|^2}{2Z_0} = \frac{\left|V_0^+\right|^2}{2Z_0} - \frac{\left|V_0^-\right|^2}{2Z_0}$$

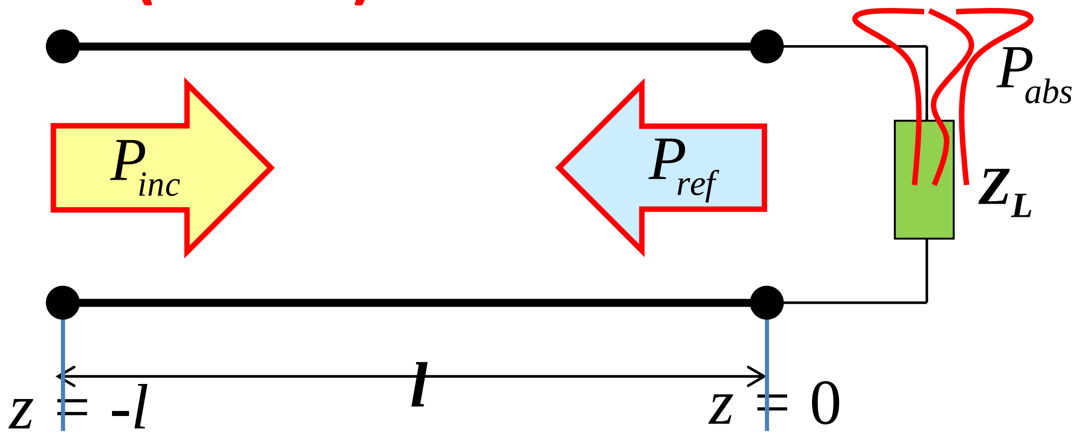
$$P_{ref} = \frac{\left|V_0^+\Gamma_0\right|^2}{2Z_0} = \left|\Gamma_0\right|^2 P_{inc}$$

Incident Power, P_{inc}

Reflected Power, P_{ref}

Power Considerations on a TL (contd.)

- It is thus apparent that the power flowing towards the load (P_{inc}) is either absorbed by the load (P_{abs}) or reflected back from the load (P_{ref})

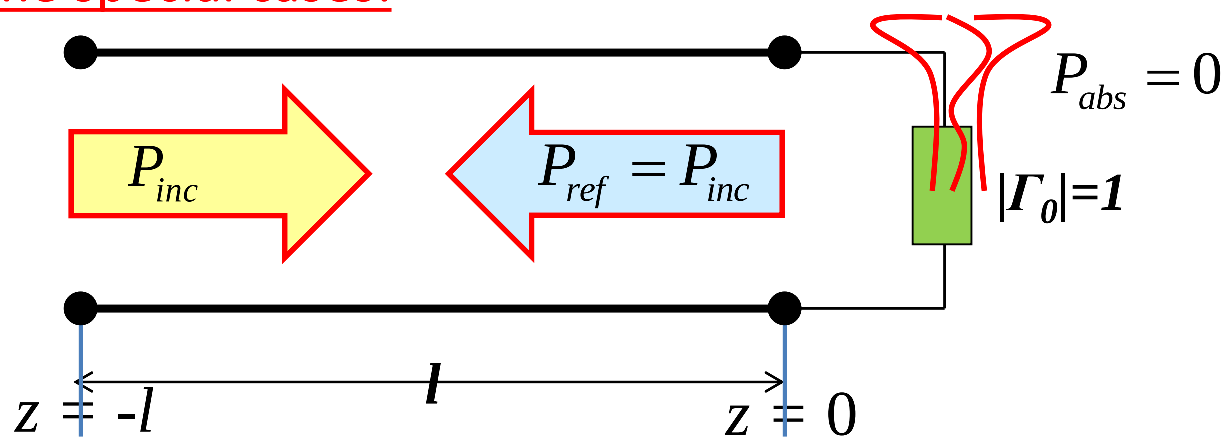


Now let us consider some special cases:

1. $|\Gamma_0| = 1$

$$P_{ref} = |\Gamma_0|^2 P_{inc} = P_{inc}$$

$$\Rightarrow P_{abs} = 0$$



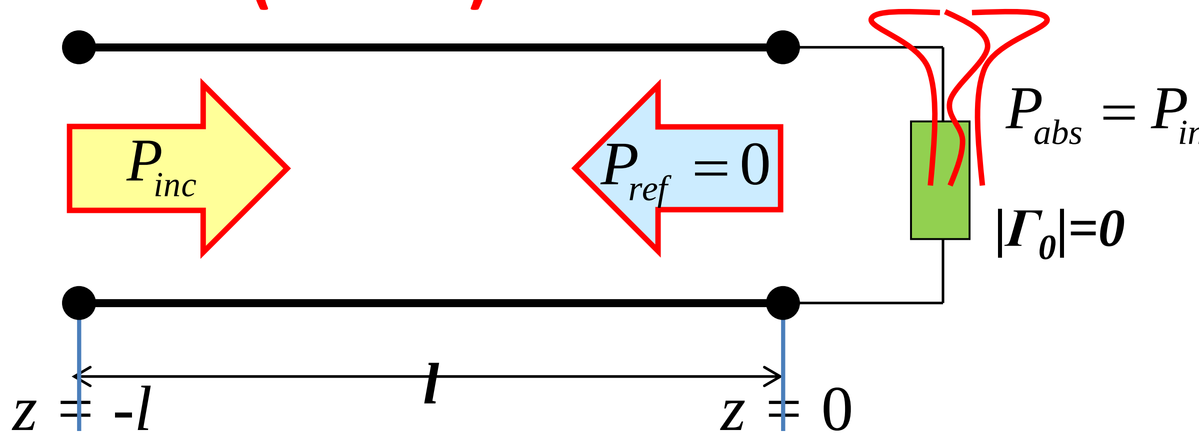
There is no power absorbed by the load → all the incident power is reflected

Power Considerations on a TL (contd.)

2. $|\Gamma_0| = 0$

$$P_{ref} = |\Gamma_0|^2 P_{inc} = 0$$

$$\Rightarrow P_{abs} = P_{inc}$$



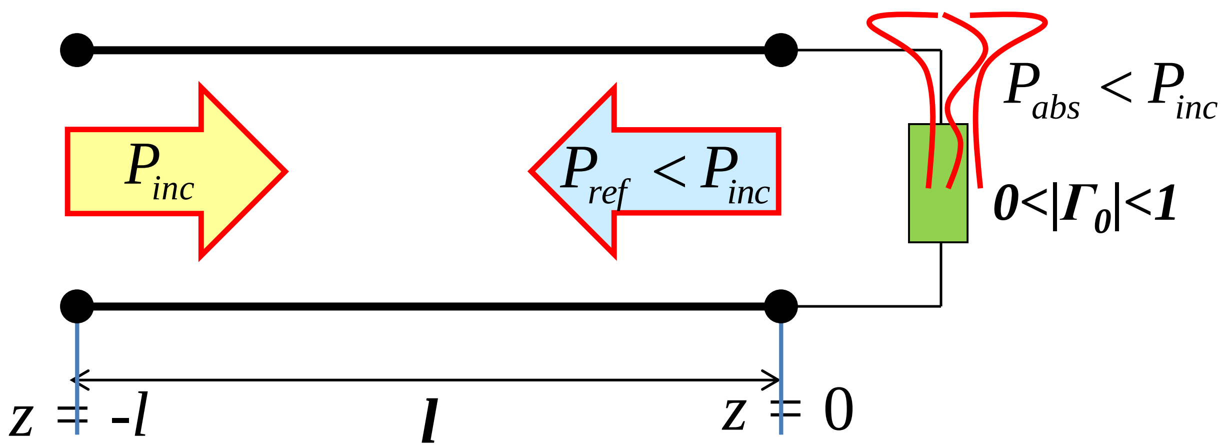
all the incident power is absorbed by the load

None of the incident power is **reflected**

3. $0 < |\Gamma_0| < 1$

$$0 < P_{ref} = |\Gamma_0|^2 P_{inc} < 0$$

$$\Rightarrow 0 < P_{abs} = P_{inc}\left(1 - |\Gamma_0|^2\right) < P_{inc}$$



Power Considerations on a TL (contd.)

Power Absorbed is Negative

4. $|\Gamma_0| > 1$ $\quad P_{ref} = |\Gamma_0|^2 P_{inc} > P_{inc}$ $\quad \Rightarrow P_{abs} = P_{inc}\left(1 - |\Gamma_0|^2\right) < 0$

What type of load it could be?

Alternatively, we can say that the load creates extra power → i.e, acts as a power source and not a sink!

Definitely not a passive load → A passive device can't produce power

Therefore: $|\Gamma_0| \leq 1$ **For all passive loads**

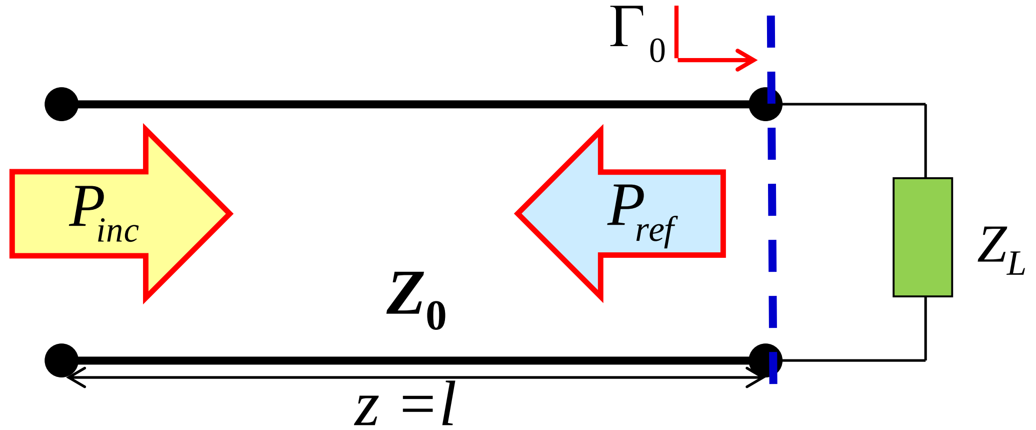
Q: Can Γ_0 every be **greater** than one?

A: Sure, if the "load" is an **active** device. In other words, the load must have some **external power** source connected to it.

Q: What about the case where $|\Gamma_0| < 0$, shouldn't we examine **that** situation as well?

A: That would be just plain **silly**; do **you** see why?

Return Loss



- The **ratio** of the reflected power from a load, to the incident power on that load, is known as **return loss**. Typically, return loss is expressed in **dB:**

Return Loss (R.L.): $$RL[dB] = -10\log\left(\frac{P_{ref}}{P_{inc}}\right) = -10\log\left(|\Gamma_0|^2\right)$$

- The return loss tells us the percentage of the incident power reflected at the point of mismatch
- For example, if the return loss is **10dB**, then **10%** of the power is **reflected** while the **90%** is **absorbed/transmitted** → i.e, we lose 10% of the incident power
- For the **return loss of 30dB**, the **reflected power is 0.1%** of the incident power → **we lose only 0.1% of the incident power**
- A larger numeric value of return loss actually indicates smaller lost power → **An ideal return loss would be ∞ → matched condition**

Return Loss (contd.)

- A **return loss** of **0dB** indicates that **reflection coefficient** is **ONE** → reactive termination
- Return Loss **(RL)** is very helpful as it provides **real-valued** measures of mismatch (unlike the **complex-valued** Z_L and Γ_0)

A match is good if the return loss is high. A high return loss is desirable and results in a lower insertion loss.

Insertion Loss

- This is another parameter to address the mismatch problem and is defined as:

$$IL[dB] = -10\log\left(\frac{P_{transmitted}}{P_{incident}}\right) = -10\log\left(\frac{P_{incident} - P_{reflected}}{P_{incident}}\right) - 10\log\left(1 - |\Gamma_{in}|^2\right)$$

For open- and short-circuit conditions

$$IL \to \infty$$

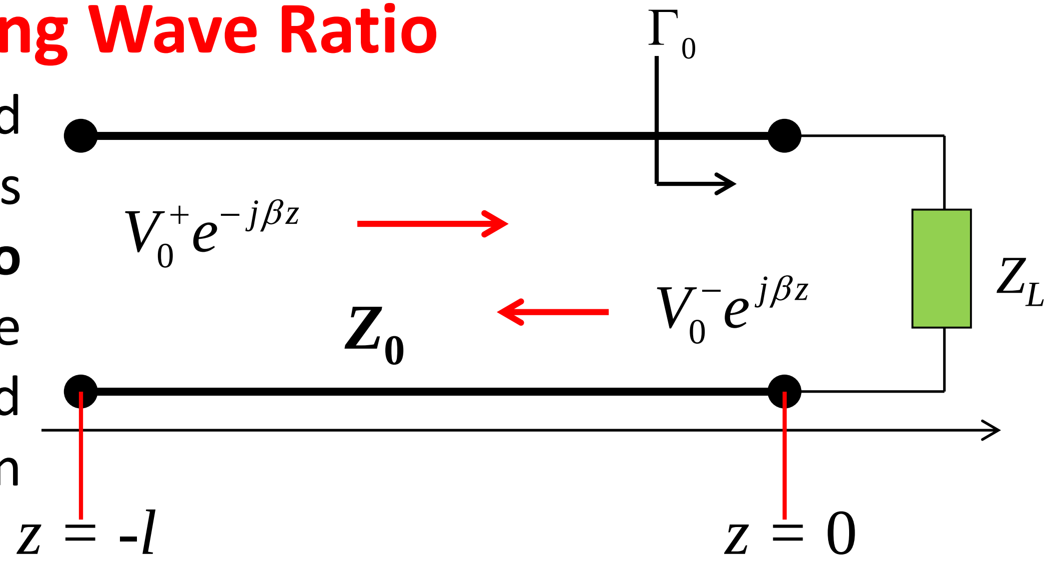
For perfectly matched conditions

$$IL = 0$$

insertion loss signifies the loss of signal power resulting from the insertion of a device in a transmission line.

Standing Wave and Standing Wave Ratio

- Another traditional real-valued measure of load match is **Voltage Standing Wave Ratio** (VSWR). Consider again the **voltage** along a terminated transmission line, as a function of **position** z.



$$V(z) = V_0^+\left[e^{-j\beta z} + \Gamma_0 e^{+j\beta z}\right] \Rightarrow V(-l) = V_0^+\left[e^{j\beta l} + \Gamma_0 e^{-j\beta l}\right]$$

- For a short circuited line: $\mathbf{\Gamma_0 = -1}$ $\Rightarrow$ $V(-l) = V_0^+\left(e^{+j\beta l} - e^{-j\beta l}\right)$

2*j*sin(β*l*)

$$\Rightarrow \quad v(-l,t) = \mathrm{Re}\left(V(-l)e^{j\omega t}\right) = \mathrm{Re}\left(2jV_0^+(z)\sin(\beta l)e^{j\omega t}\right)$$

Standing Wave and Standing Wave Ratio (contd.)

$$\therefore v(-l,t) = 2V_0^+ \sin(\beta l)\cos(\omega t + (\pi/2))$$

Always zero for $-l=0$ i.e., the point of short-circuit

Definitely not a traveling wave!!

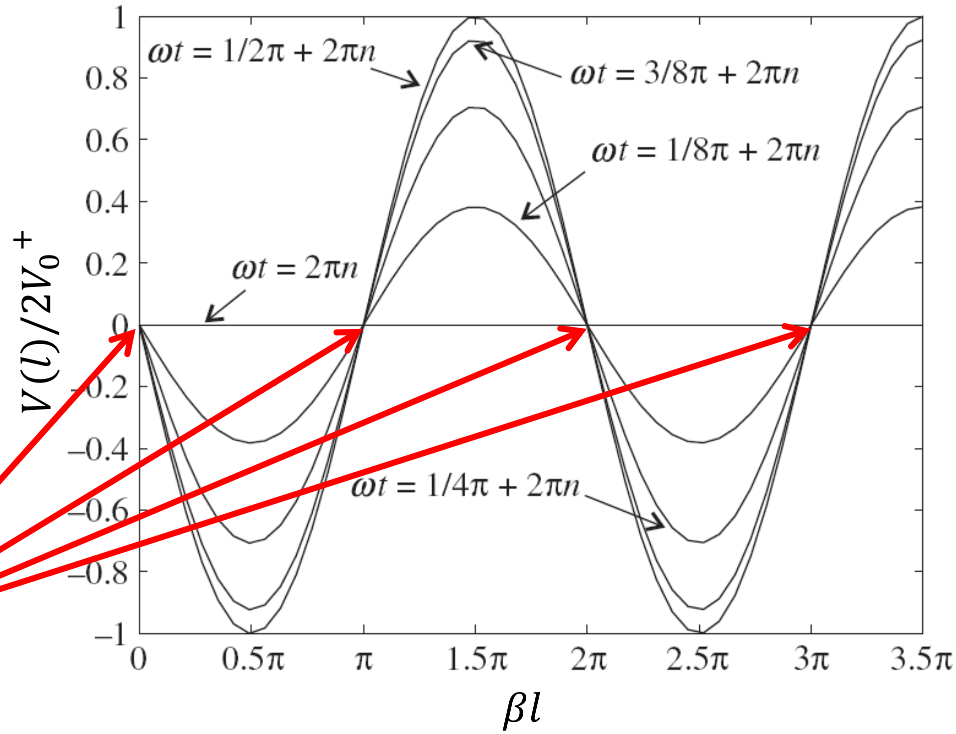
Where has the traveling wave V(z) gone?

- As the time and space are decoupled → No wave propagation takes place
- The incident wave is 180º out of phase with the reflected wave → gives rise to zero crossings of the wave at 0, λ/2, λ, 3λ/2, and so on → standing wave pattern!!!

Standing Wave and Standing Wave Ratio (contd.)

Corresponding Electrical Length (βl):
$0, \pi, 2\pi, 3\pi$

Spatial Location:
$0, \lambda/2, \lambda, 3\lambda/2$



Standing Wave Pattern for Various Instances of Time

- **for arbitrarily terminated line:** $$V(-l) = V_0^+\left(e^{+j\beta l} + \Gamma_0 e^{-j\beta l}\right) = V_0^+ e^{+j\beta l}\left(1 + \Gamma_0 e^{-j2\beta l}\right)$$

$$\boxed{\Rightarrow V(-l) = A(-l)\left(1 + \Gamma(-l)\right)}$$

Valid anywhere on the line

Standing Wave and Standing Wave Ratio (contd.)

Similarly: $$I(-l) = \frac{A(-l)}{Z_0}\left(1 - \Gamma(-l)\right)$$

Valid anywhere on the line

- Under the matched condition, $\Gamma_0 = 0$ and therefore $\Gamma(-l) = 0$ → as expected, only positive traveling wave exists.
- For other arbitrary impedance loads: Standing Wave Ratio (SWR) or Voltage Standing Wave Ratio (VSWR) is the measure of mismatch.
- SWR is defined as the ratio of maximum voltage (or current) amplitude and the minimum voltage (or current) amplitude along a line → therefore, for an arbitrarily terminated line:

$$VSWR = ISWR = SWR = \left|\frac{V(-l)_{\max}}{V(-l)_{\min}}\right| = \left|\frac{I(-l)_{\max}}{I(-l)_{\min}}\right|$$

We have: $V(-l) = V_0^+ e^{+j\beta l}\left(1 + \Gamma_0 e^{-j2\beta l}\right)$

- Two possibilities for extreme values: $\Gamma_0 e^{-j\beta l} = 1$ $\quad$ $\Gamma_0 e^{-j\beta l} = -1$

Standing Wave and Standing Wave Ratio (contd.)

Max. voltage: $|V(-l)|_{max} = |V_0^+|(1+|\Gamma_0|)$ **Min. voltage:** $|V(-l)|_{min} = |V_0^+|(1-|\Gamma_0|)$

$\therefore VSWR = \dfrac{1+|\Gamma_0|}{1-|\Gamma_0|}$ **Apparently:** $0 \le \Gamma_0 \le 1$ ⇒ $\therefore 1 \le VSWR < \infty$

- Note if $|\Gamma_0| = 0$ (i.e., $Z_L = Z_0$), then VSWR = 1. We find for this case: $|V(z)|_{max} = |V(z)|_{min} = |V_0^+|$

In other words, the voltage magnitude is a **constant** with respect to position z.

- Conversely, if $|\Gamma_0| = 1$ (i.e., $Z_L = Z_0$), then VSWR = ∞. We find for **this** case: $|V(z)|_{max} = 2|V_0^+|$ $|V(z)|_{min} = 0$

In other words, the voltage magnitude varies **greatly** with respect to position z.

Standing Wave and Standing Wave Ratio (contd.)

- **Similarly,** **We have:** $$I(-l)=\frac{V^+}{Z_0}\left(e^{+j\beta l}+\Gamma_0 e^{-j\beta l}\right)$$

$$\left|\mathrm{I}(d)\right|_{\max}=\left(\frac{V^+}{Z_0}\right)\left(1+\left|\Gamma_0\right|\right)$$ **and** $$\left|\mathrm{I}(d)\right|_{\min}=\left(\frac{V^+}{Z_0}\right)\left(1-\left|\Gamma_0\right|\right)$$

$$\therefore ISWR=\frac{1+\left|\Gamma_0\right|}{1-\left|\Gamma_0\right|}$$

$$\therefore 1\leq ISWR<\infty$$

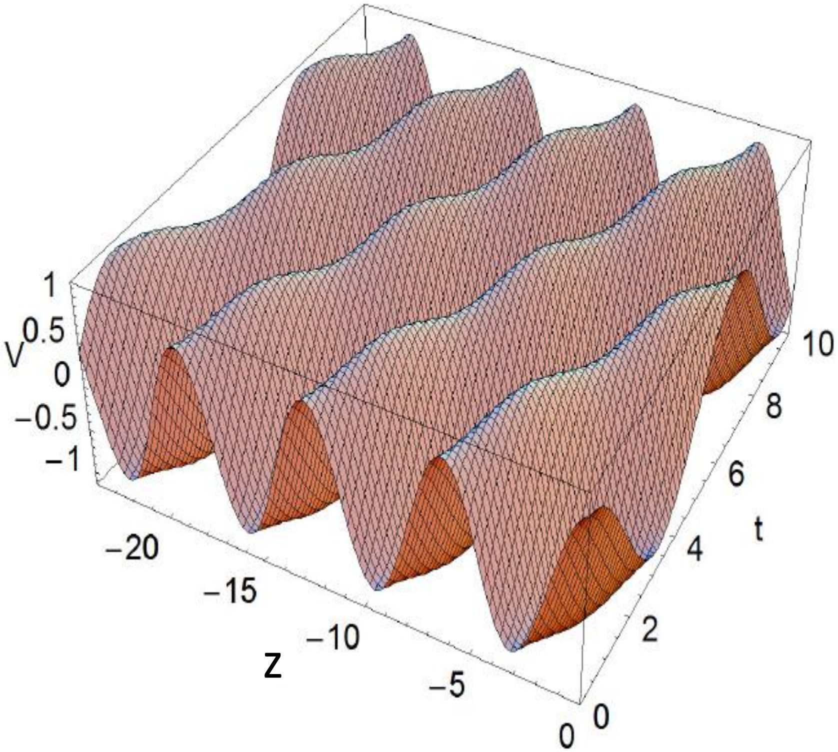
Thus: VSWR=ISWR=SWR

In our course we will mention both as VSWR

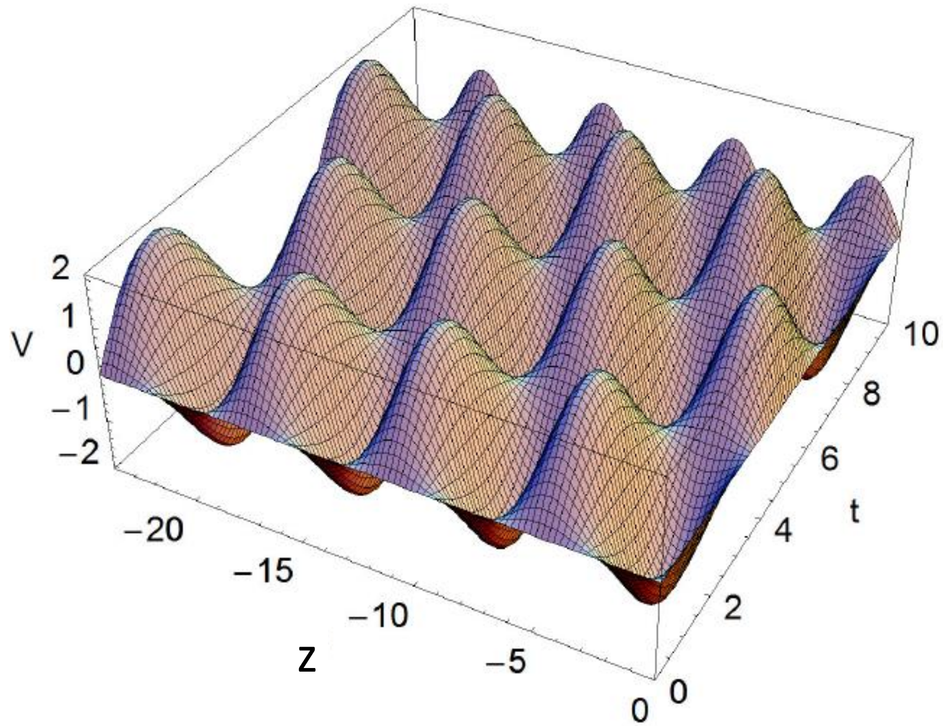
As with **return loss**, VSWR is dependent on the **magnitude** of $|\Gamma_0|$ (i.e, $|\Gamma_0|$) **only** !

<u>In practice, SWR can only be defined for lossless line as the SWR equation is not valid for attenuating voltage and current</u>

Standing Wave and Standing Wave Ratio (contd.)



Standing Wave Pattern at Γ_0=0.1



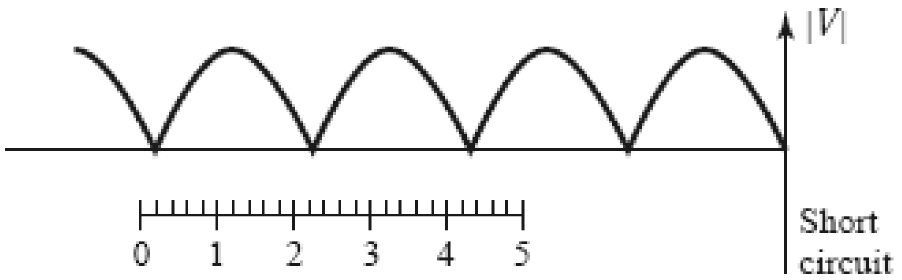
Standing Wave Pattern at Γ_0=1

- It is apparent that the maximum and minimum repeats periodically and its values can be used to identify the degree of mismatch by calculating the Standing Wave Ratio

Example – 3

- The following two-step procedure has been carried out with a 50Ω coaxial slotted line to determine an unknown load impedance:

1. short circuit is placed at the load plane, resulting in a standing wave on the line with infinite SWR and sharply defined voltage minima, as shown in Figure.

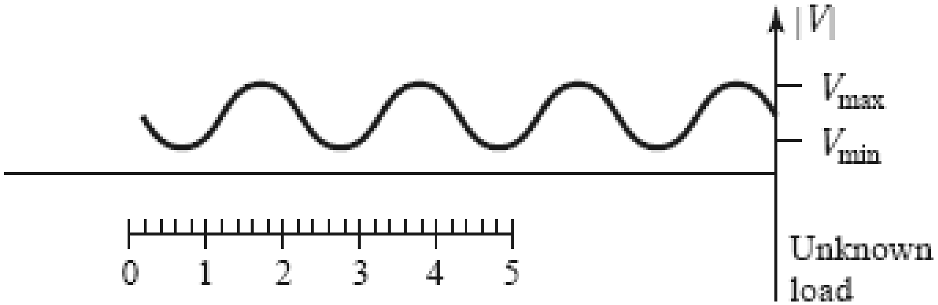


On the arbitrarily positioned scale on the slotted line, voltage minima are recorded at:

$$z = 0.2cm, \qquad 2.2cm, \qquad 4.2cm$$

Example – 3 (contd.)

2. The short circuit is removed and replaced with the unknown load. The standing wave ratio is measured as SWR = 1.5, and voltage minima, which are not as sharply defined as those in step 1, are recorded at:



$$z = 0.72cm, \qquad 2.72cm, \qquad 4.72cm$$

Find the load impedance.

Example – 3 (contd.)

- Knowing that voltage minima repeat every $\lambda/2$, we have from the data of step 1 that λ = 4.0 cm.
- In addition, because the reflection coefficient and input impedance also repeat every $\lambda/2$, we can consider the load terminals to be effectively located at any of the voltage minima locations listed in step 1.
- Thus, if we say the load is at $4.2cm$, then the data from step 2 show that the next voltage minimum away from the load occurs at $2.72cm$.

- It gives: $\boxed{l_{min} = 4.2 - 2.72 = 1.48cm = 0.37\lambda}$

- Now: $\boxed{|\Gamma_0| = \frac{SWR-1}{SWR+1}} \Rightarrow \boxed{|\Gamma_0| = \frac{1.5-1}{1.5+1} = 0.2}$

$$\boxed{\theta_\Gamma = \pi + 2\beta l_{\min}} \Rightarrow \boxed{\theta_\Gamma = \pi + \left(2 \times \frac{2\pi}{\lambda} l_{\min}\right) = 86.4^\circ}$$

Example – 3 (contd.)

- Therefore:

$$\Gamma_0 = 0.2e^{j86.4^\circ} = 0.0126 + j0.1996$$

- The unknown impedance is then:

$$Z_L = Z_0\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right) \Rightarrow Z_L = 50\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right) = 47.3 + j19.7\Omega$$

Potential Projects

- Dual-band impedance-matching networks based on split-ring resonators
- A multiband reconfigurable matching network
- T-section dual-band impedance transformer for frequency-dependent complex loads
- Dual-band matching technique based on dual-characteristic impedance transformers
- Multi-band frequency transformations matching networks and amplifiers
- Analytical design of dual-band impedance transformer with extra transmission zero
- A T-section dual-band matching network for frequency-dependent complex loads incorporating coupled line with dc-block property
- Techniques to measure port impedances
- Pi -model dual-band impedance transformer for unequal complex impedance loads, and its use in Power Divider, Coupler, Crossover etc.
- triple-frequency matching network for FDCLs