Lecture – 3

Date: 11.01.2016

- Transmission Lines (TL) Introduction
- TL Equivalent Circuit Representation
- Definition of Some TL Parameters
- Examples of Transmission Lines

Line Impedance (Z) – contd.

It appears to me that Z_0 is a transmission line parameter, depending only on the transmission line values R, L, C and G.

Whereas, $Z(z)$ depends on the magnitude and the phase of the two propagating waves $V^+(z)$ and $V^-(z)$ → values that depend not only on the transmission line, but also on the two things attached to either end of the transmission line.

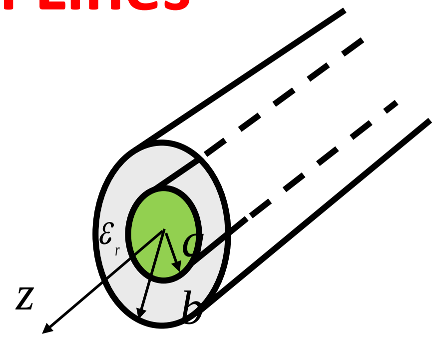
Right?

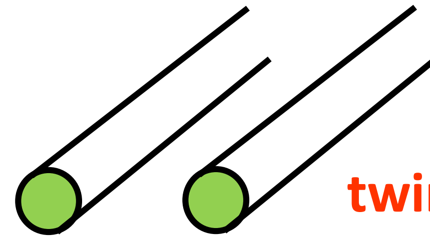
Exactly!!!

Example of Transmission Lines

Two common examples:

coaxial cable



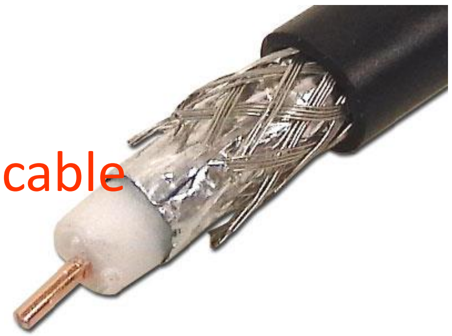


twin line

A transmission line is normally used in the balanced mode, meaning equal and opposite currents (and charges) on the two conductors.

Here's what they look like in real-life:

coaxial cable



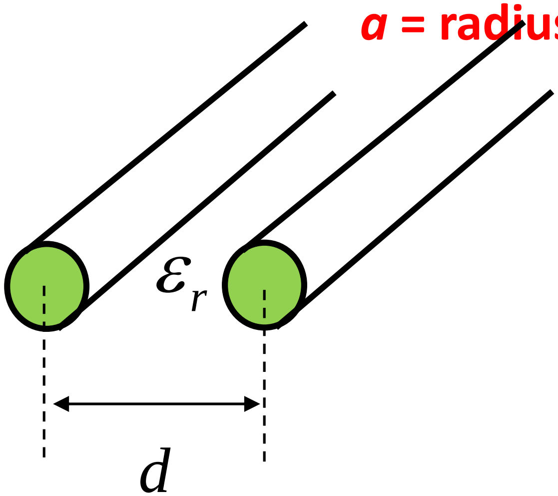
twin line

coax to twin line matching section

Example of Transmission Lines (contd.)

Twin Line

a = radius of wires



$$C = \frac{\pi \varepsilon_0 \varepsilon_r}{\cosh^{-1}\left(\frac{d}{2a}\right)} \quad [\text{F/m}]$$

$$L = \frac{\mu_0}{\pi} \cosh^{-1}\left(\frac{d}{2a}\right) \quad [\text{H/m}]$$

$$Z_0 = \frac{1}{\pi} \eta_0 \frac{1}{\sqrt{\varepsilon_r}} \cosh^{-1}\left(\frac{d}{2a}\right) \quad [\Omega]$$

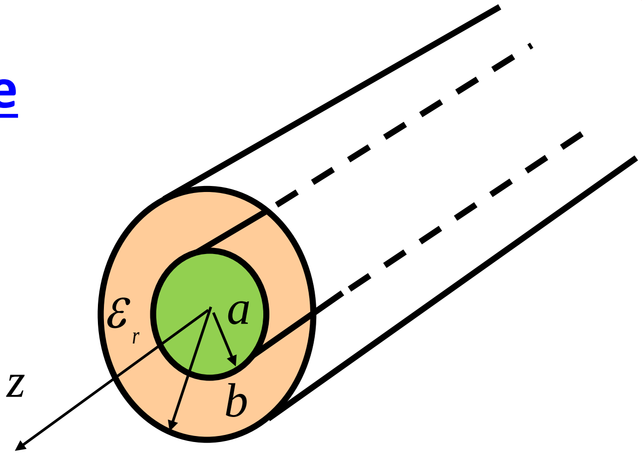
$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right) \xrightarrow{x \to \infty} \ln 2x$$

$$Z_0 \approx \frac{1}{\pi} \eta_0 \frac{1}{\sqrt{\varepsilon_r}} \ln\left(\frac{d}{a}\right) \quad [\Omega]$$

$$a \ll d$$

Example of Transmission Lines (contd.)

Coaxial Cable



σ_d = conductivity of dielectric [S/m].

σ_m = conductivity of metal [S/m].

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\text{F/m}]$$

$$G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad [\text{S/m}]$$

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) \quad [\text{H/m}]$$

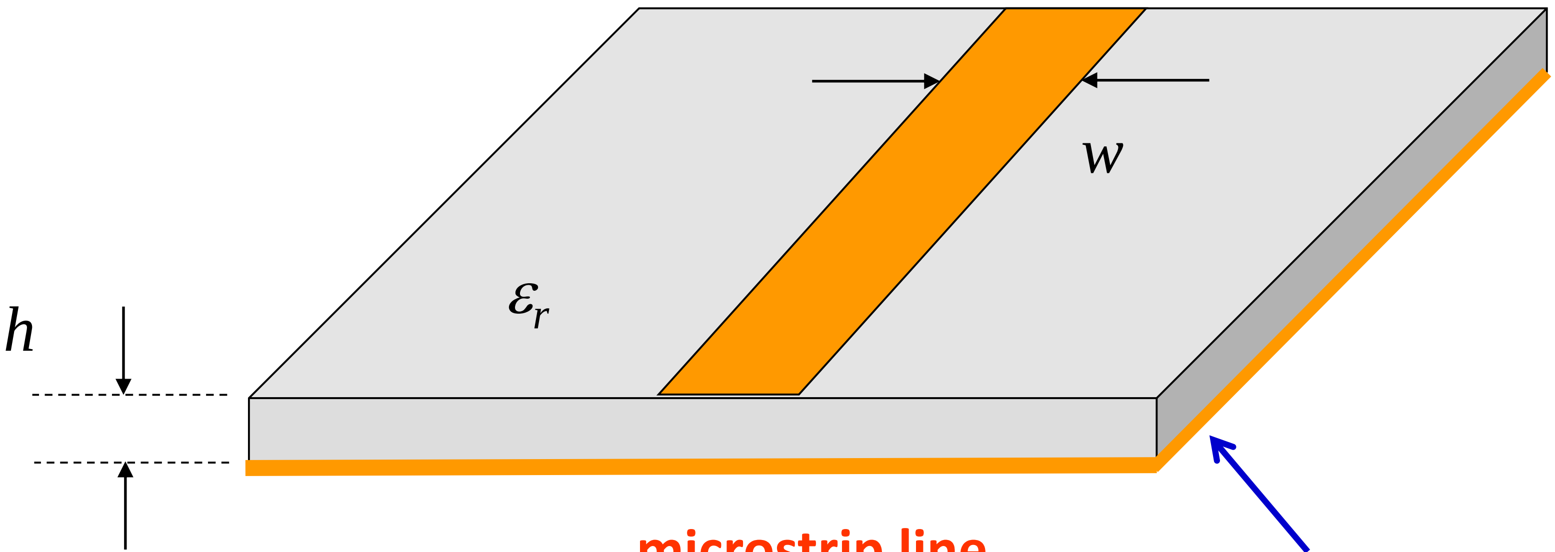
$$R = \frac{1}{\sigma_m\delta}\left(\frac{1}{2\pi a} + \frac{1}{2\pi b}\right) \quad [\Omega/\text{m}]$$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma_m}}$$

(skin depth of metal)

Example of Transmission Lines (contd.)

Another common example (for printed circuit boards):

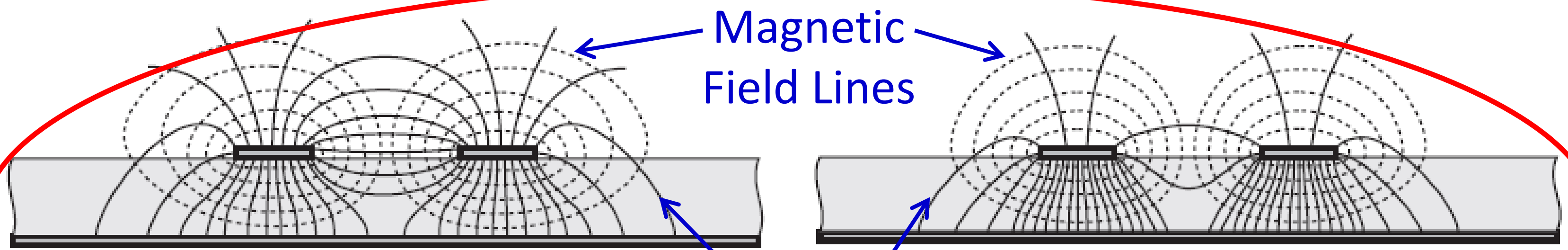


microstrip line

Ground plane helps in preventing the field leakage and thus reduces the radiation loss

Microstrip Line (contd.)

- The severity of field leakage also depends on the relative dielectric constants (ε_r).



(a) Teflon epoxy ($\varepsilon_r = 2.55$)

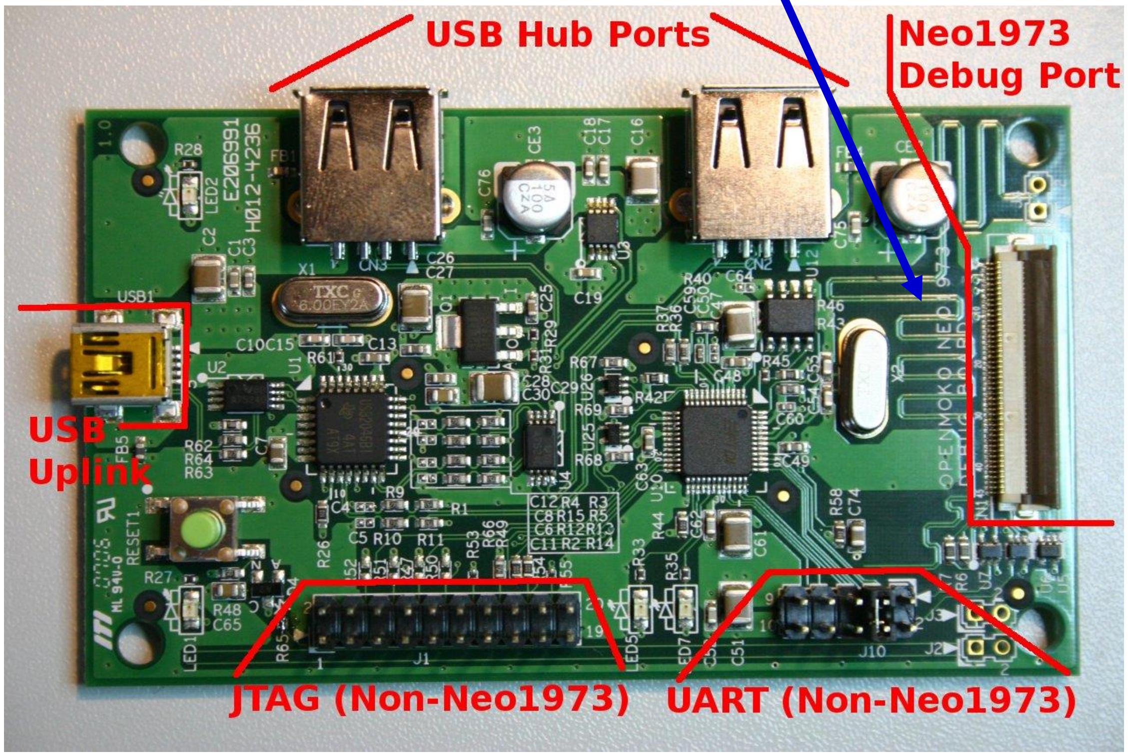
(b) Alumina ($\varepsilon_r = 10.0$)

It is apparent that the radiation loss could be minimized by using substrates with high dielectric constants

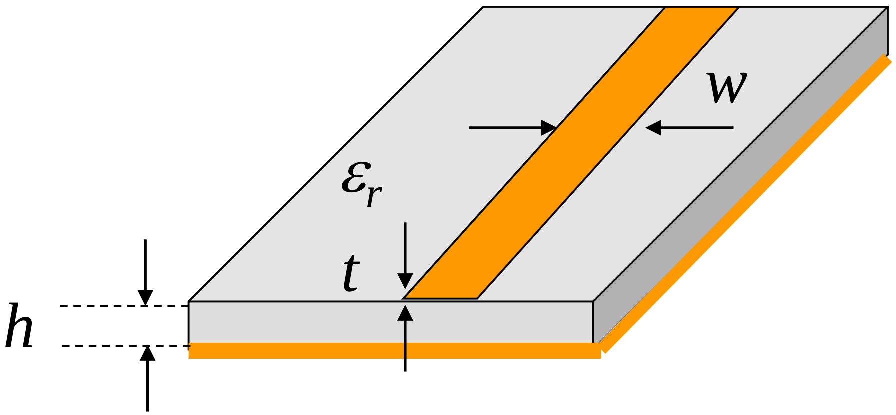
Alternative approaches to reduce radiation loss and interference are shielded microstrip line and multi-layer boards

Microstrip Line (contd.)

microstrip line



Microstrip Transmission Lines Design



- Simple parallel plate model can not accurately define this structure.
- Because, if the substrate thickness increases or the conductor width decreases then fringing field become more prominent (and therefore need to be incorporated in the model).

Case-I: thickness (t) of the line is negligible

- **For narrow microstrips ($^w/_h \leq 1$):** $$Z_0 = \frac{Z_f}{2\pi\sqrt{\varepsilon_{eff}}}\ln\left(8\frac{h}{w}+\frac{w}{4h}\right)$$

Where, $Z_f = \sqrt{\mu_0 / \varepsilon_0} = 377\Omega$ **wave impedance in free space**

$$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2}+\frac{\varepsilon_r-1}{2}\left[\left(1+12\frac{h}{w}\right)^{-1/2}+0.004\left(1-\frac{w}{h}\right)^2\right]$$ **Effective Dielectric Constant**

Microstrip Transmission Lines Design (contd.)

- For wide microstrips ($^{w}/_{h} \geq 1$):

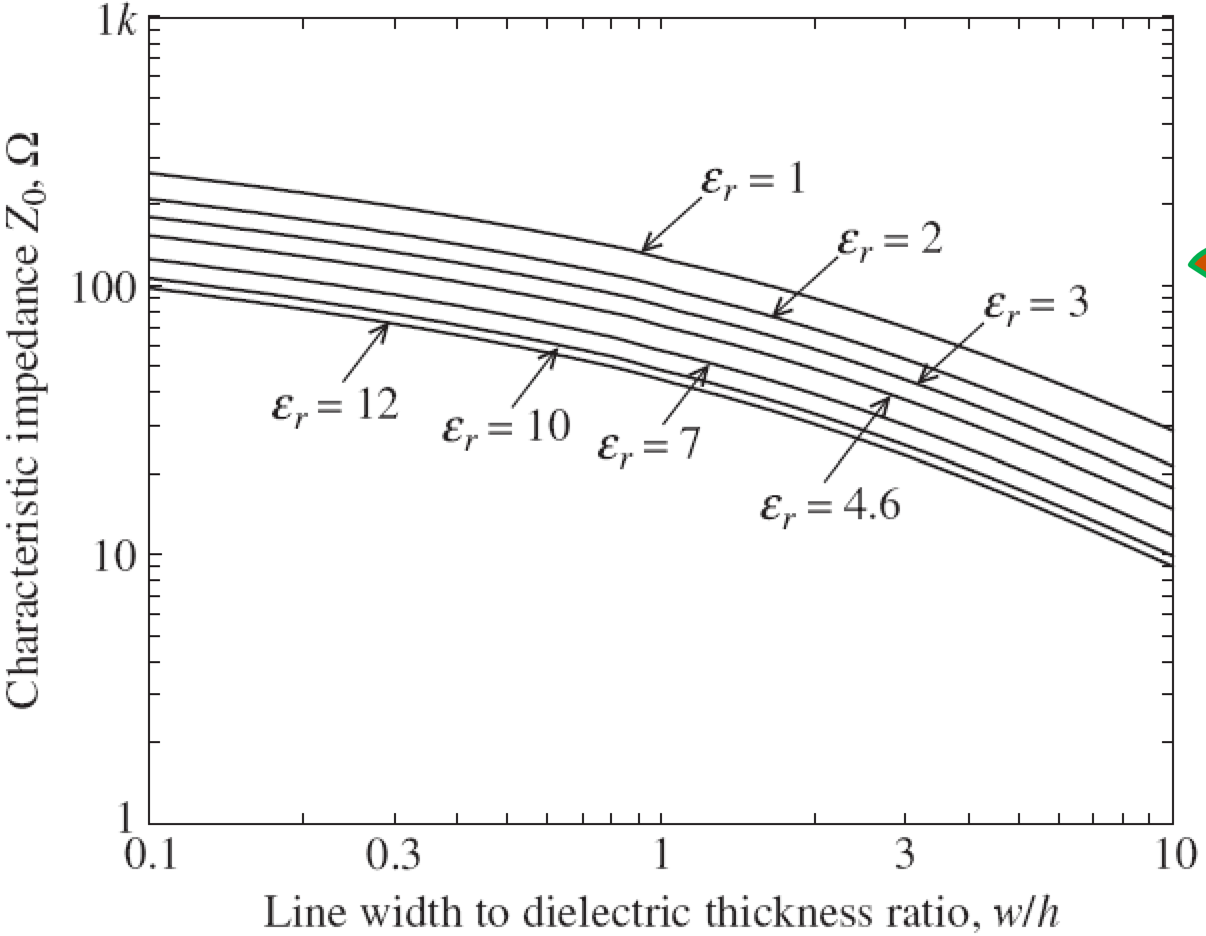
$$Z_0 = \frac{Z_f}{\sqrt{\varepsilon_{eff}}\left(1.393 + \frac{w}{h} + \frac{2}{3}\ln\left(\frac{w}{h} + 1.444\right)\right)}$$

- Where the effective dielectric constant is expressed as:

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 12\frac{h}{w}\right)^{-1/2}$$

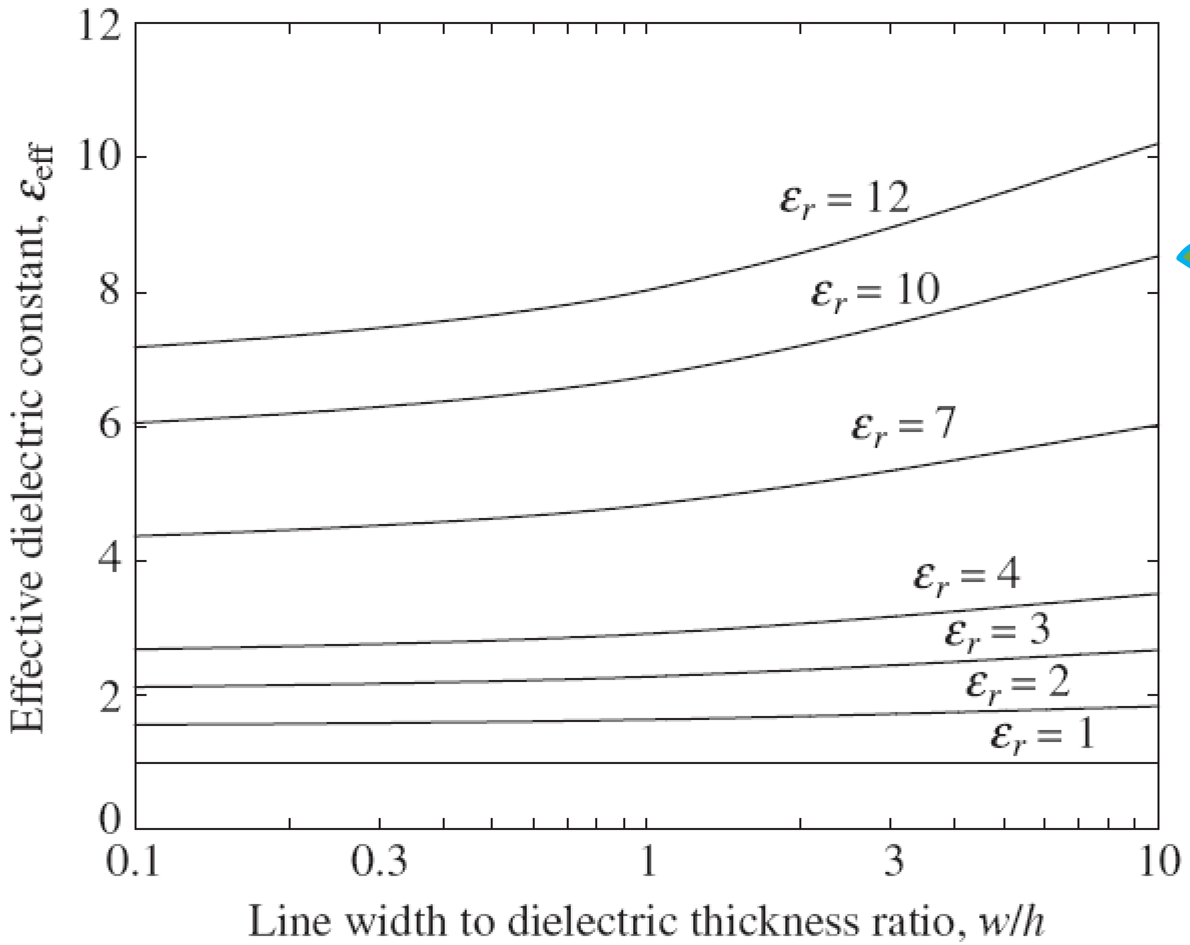
- The two distinct expressions give approximate values of characteristic impedance and effective dielectric constant for narrow and wide strip microstrip lines → these can be used to plot Z_0 and ε_{eff} as a function of $^{w}/_{h}$.

Microstrip Transmission Lines Design (contd.)



For a desired characteristic impedance using known substrate, the dimension w/h can be identified from this curve

Microstrip Transmission Lines Design (contd.)



Once the line dimensions are known, the effective dielectric constant can be identified

Microstrip Transmission Lines Design (contd.)

- The effective dielectric constant (ε_{eff}) is viewed as the dielectric constant of a homogenous material that fills the entire space around the line. Therefore:

Free Space Wavelength

$$\lambda = \frac{v_p}{f} = \frac{c}{f\sqrt{\varepsilon_{eff}}} = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}}$$

Speed of Light

- The wavelength in the microstrip line for $W/h \geq 0.6$ is:

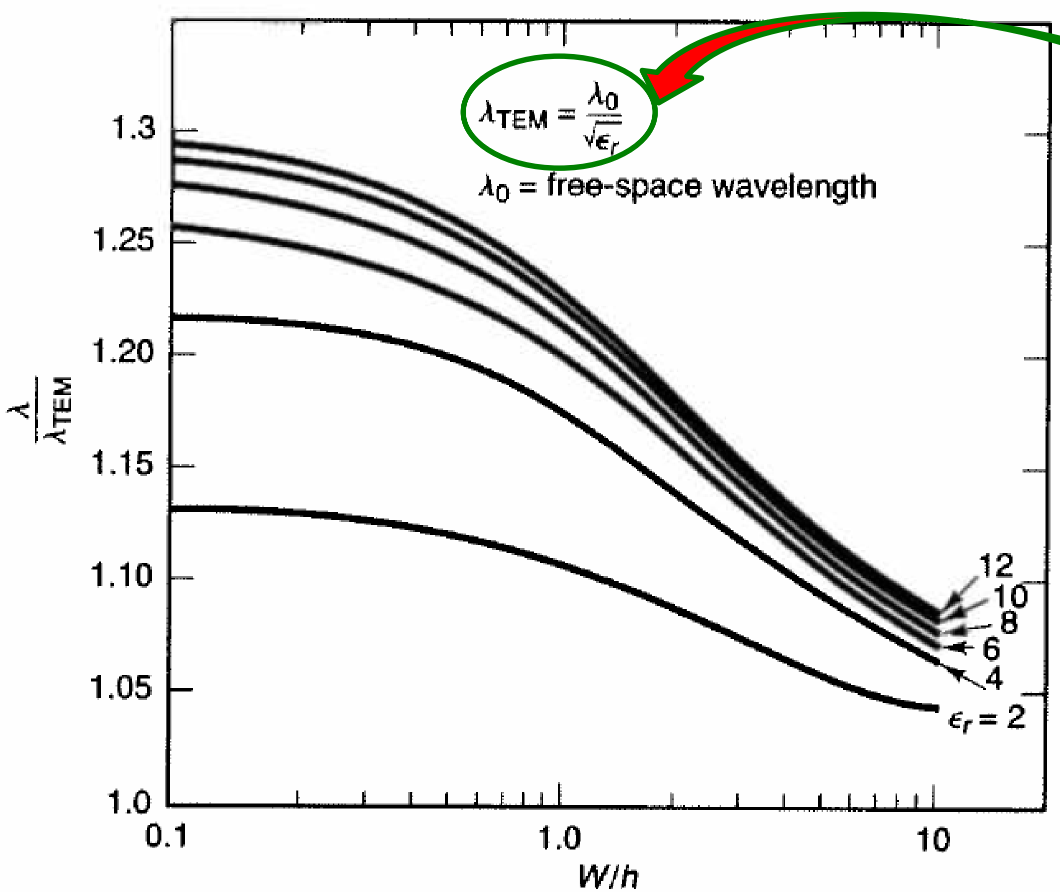
$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\left[\frac{\varepsilon_r}{1+0.63(\varepsilon_r-1)(W/h)^{0.1255}}\right]^{1/2}$$

- The wavelength in the microstrip line for $W/h \leq 0.6$ is:

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\left[\frac{\varepsilon_r}{1+0.6(\varepsilon_r-1)(W/h)^{0.0297}}\right]^{1/2}$$

Microstrip Transmission Lines Design (contd.)

- In some specifications, wavelength is known. In that case following curve can be used to identify the *w/h* ratio.



It is a good approximation at lower microwave frequencies. However, at higher microwave frequencies this assumption is no more valid.

Microstrip Transmission Lines Design (contd.)

- If Z_0 and ε_r is specified or known, following expression can be used to determine w/h:

For w/h≤2: $$\frac{w}{h} = \frac{8e^A}{e^{2A} - 2}$$

Where: $$A = 2\pi \frac{Z_0}{Z_f}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

For w/h≥2: $$\frac{w}{h} = \frac{2}{\pi}\left(B - 1 - \ln(2B - 1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(B - 1) + 0.39 - \frac{0.61}{\varepsilon_r}\right]\right)$$

Where: $$B = \frac{Z_f \pi}{2Z_0\sqrt{\varepsilon_r}}$$

Case-II: thickness (t) of the line is not negligible → in this scenario all the formulas are valid with the assumption that the effective width of the line increases as:

$$\boxed{w_{eff} = w + \frac{t}{\pi}\left(1 + \ln\frac{2x}{t}\right)}$$

Where $x = h$ if $w > {}^{h}/_{2\pi}$ or $x = 2\pi w$ if ${}^{h}/_{2\pi} > w > 2t$

Example – 1

A microstrip material with ε_r = *10* and *h* = *1.016 mm* is used to build a narrow transmission line. Determine the width for the microstrip transmission line to have a characteristic impedance of 50Ω. Also determine the wavelength and the effective relative dielectric constant of the microstrip line.

Using the Formulas:

Let us consider the first formula: $$\frac{w}{h} = \frac{8e^{A}}{e^{2A} - 2}$$

$$A = 2\pi \frac{Z_0}{Z_f}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right) = 2\pi \frac{50}{377}\sqrt{\frac{10+1}{2}} + \frac{10-1}{10+1}\left(0.23 + \frac{0.11}{10}\right)$$

$$\Rightarrow A = 2.1515$$

Therefore: $$\frac{w}{h} = \frac{8e^{2.1515}}{e^{2(2.1515)} - 2} = 0.9563$$

Now: h = 1.016 mm = 0.1016 cm = 0.1016(1000/2.54) mils = 40 mils

$$\therefore w = 0.9563 * 40 mils = 38.2 mils$$

Example – 1 (contd.)

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\left[\frac{\varepsilon_r}{1+0.63(\varepsilon_r-1)(w/h)^{0.1255}}\right]^{1/2}$$

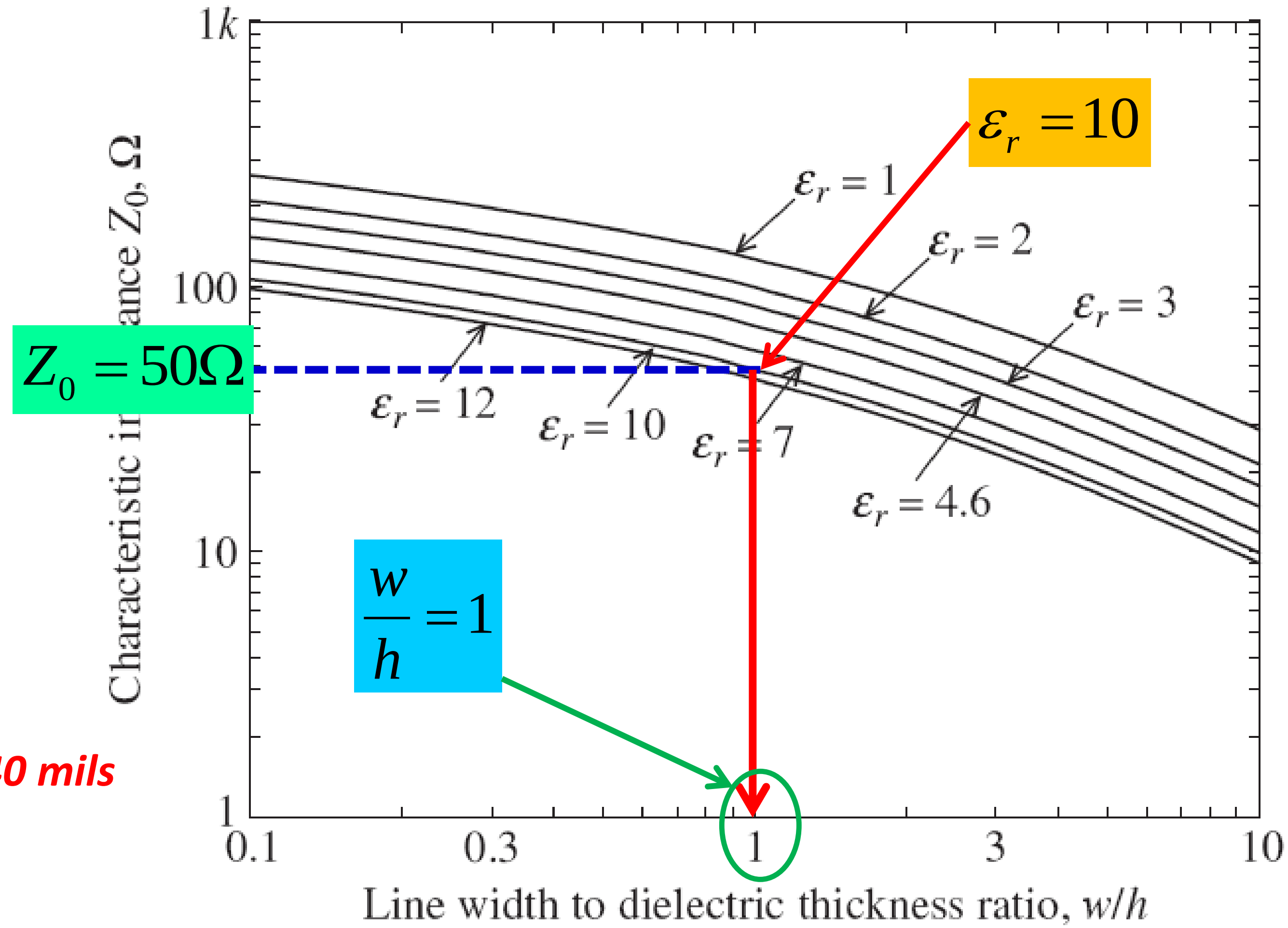
$$\therefore \lambda = \frac{\lambda_0}{\sqrt{10}}\left[\frac{10}{1+0.63(10-1)(0.9563)^{0.1255}}\right]^{1/2} = 0.387\lambda_0$$

$$\lambda = \frac{v_p}{f} = \frac{c}{f\sqrt{\varepsilon_{eff}}} = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}} \quad \Rightarrow \varepsilon_{eff} = \left(\frac{\lambda_0}{\lambda}\right)^2$$

$$\therefore \varepsilon_{eff} = \left(\frac{1}{0.387}\right)^2 = 6.68$$

Example – 1 (contd.)

Using the Design Curves

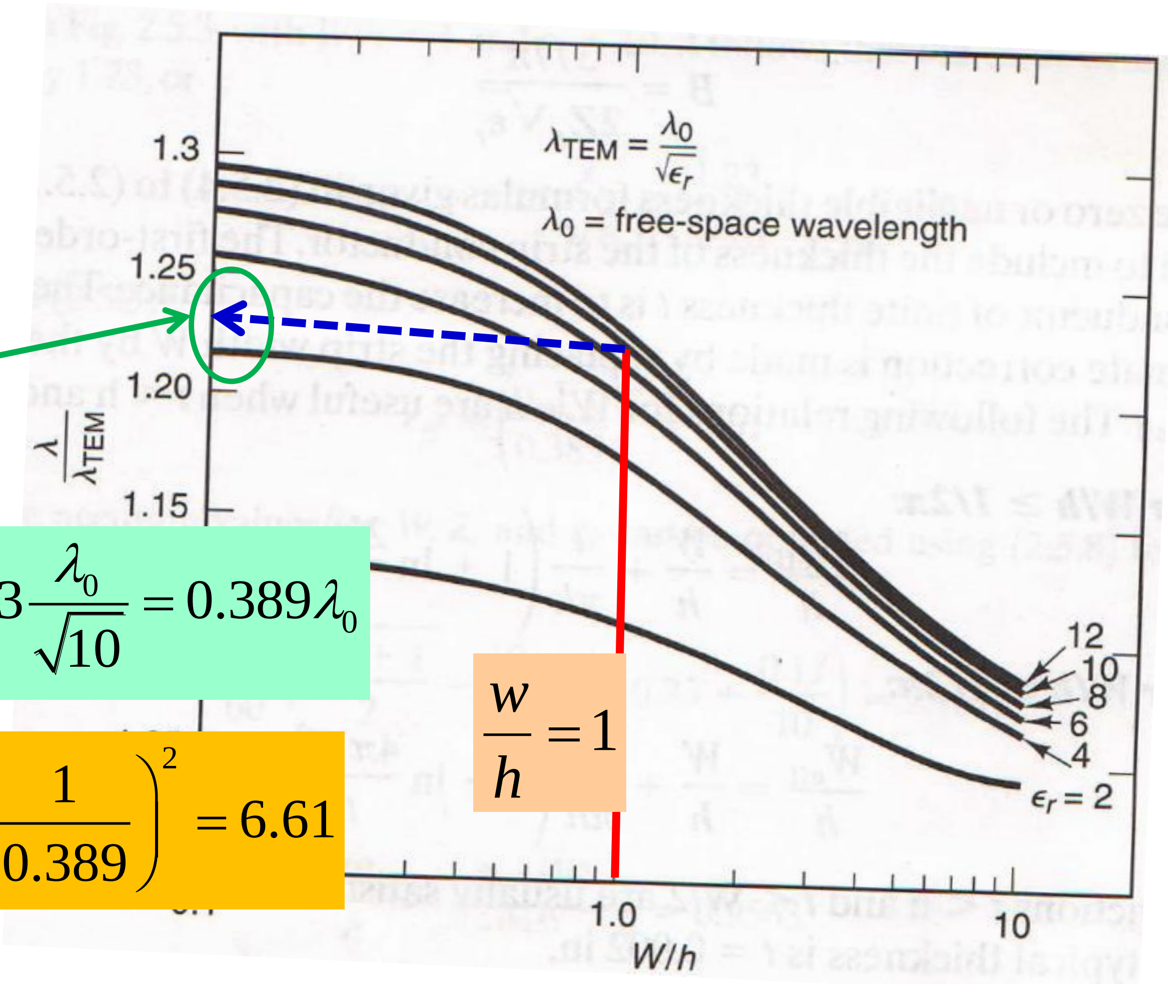


h = 1.016 mm = 40 mils

=> w = 40 mils

Example – 1 (contd.)

Using the Design Curves



$$\frac{\lambda}{\lambda_{TEM}} = 1.23$$

$$\frac{w}{h} = 1$$

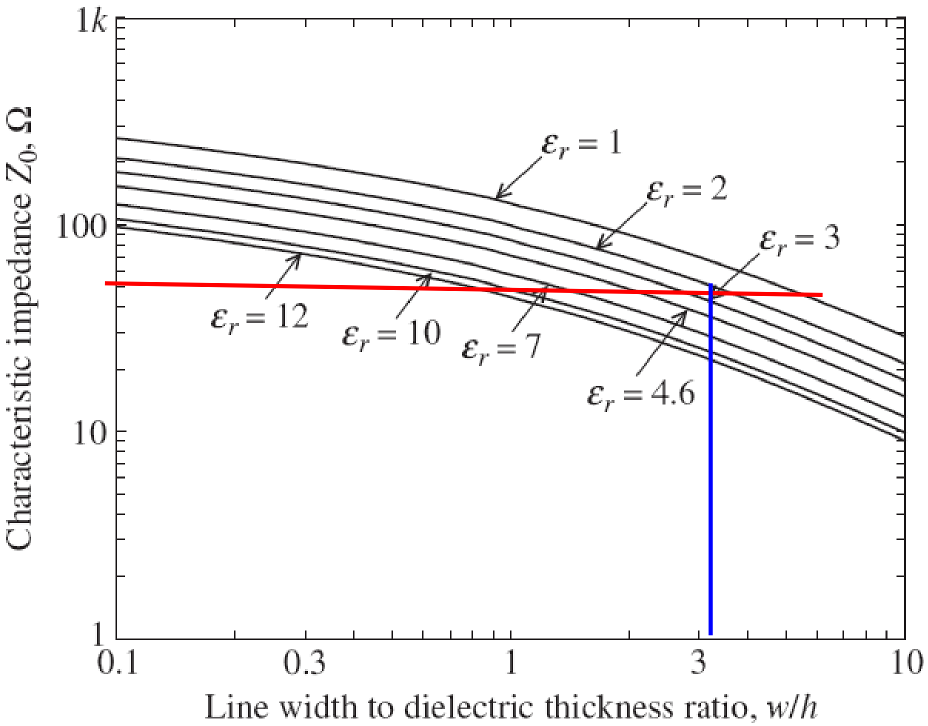
$$\lambda = 1.23\frac{\lambda_0}{\sqrt{\varepsilon_r}} = 1.23\frac{\lambda_0}{\sqrt{10}} = 0.389\lambda_0$$

$$\Rightarrow \varepsilon_{eff} = \left(\frac{\lambda_0}{\lambda}\right)^2 = \left(\frac{1}{0.389}\right)^2 = 6.61$$

Example – 2

a. Using the design curves, calculate W, λ, and ε_{eff} for a characteristic impedance of 50Ω using RT/Duroid with $\varepsilon_r = 2.23$ and $h = 0.7874\ mm$.

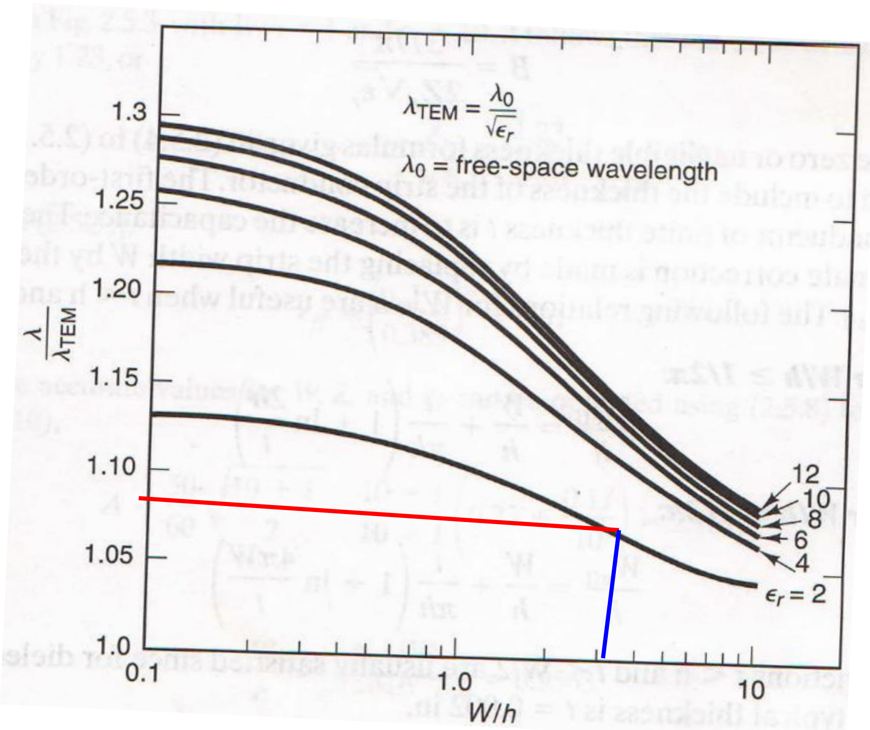
b. Use design equations to show that for RT/Duroid with $\varepsilon_r = 2.23$ and $h = 0.7874\ mm$, a 50Ω-characteristic impedance is obtained with $W/_h = 3.073$. Also show, $\varepsilon_{eff} = 1.91$ and $\lambda = 0.7236\lambda_0$.



$$\frac{W}{h} \approx 3.1$$

W$=3.1 \times h = 3.1 \times 0.784 = 2.44mm$

Example – 2 (contd.)



For $\frac{W}{h} \approx 3.1$ and $\varepsilon_r = 2.23$

$$\frac{\lambda}{\lambda_{TEM}} = 1.08 \Rightarrow \lambda = 1.08\lambda_{TEM}$$

We know: $\lambda_{TEM} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$

$$\therefore \lambda = 0.723\lambda_0$$

Also: $\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}} \Rightarrow \varepsilon_{eff} = 1.91$

Example – 2 (contd.)

For w/h≥2: $\frac{w}{h} = \frac{2}{\pi}\left(B - 1 - \ln(2B-1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(B-1) + 0.39 - \frac{0.61}{\varepsilon_r}\right]\right)$

Where: $B = \frac{Z_f \pi}{2Z_0\sqrt{\varepsilon_r}}$

Therefore: $\frac{w}{h} = \frac{2}{\pi}\left(B - 1 - \ln(2B-1) + \frac{2.23-1}{2\times 2.23}\left[\ln(B-1) + 0.39 - \frac{0.61}{2.23}\right]\right)$

Where: $B = \frac{377\pi}{2\times 50\times\sqrt{2.23}} = 7.931$

$\therefore \frac{w}{h} = 3.073$

- For $W/h \geq 0.6$: $\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\left[\frac{\varepsilon_r}{1 + 0.63(\varepsilon_r - 1)\left(W/h\right)^{0.1255}}\right]^{1/2}$

$$\therefore \lambda = \frac{\lambda_0}{\sqrt{2.23}}\left[\frac{2.23}{1 + 0.63(2.23-1)\left(3.073\right)^{0.1255}}\right]^{1/2} = 0.724\lambda_0$$

Lossless Transmission Line

- For a lossless transmission line: $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$

$$\beta = \omega\sqrt{LC}$$

- Similarly the current phasor for a lossless line can be described:

$$\therefore I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

$$Z_0 = \frac{\omega L}{\beta} = \frac{\omega L}{\omega\sqrt{LC}} = \sqrt{\frac{L}{C}}$$

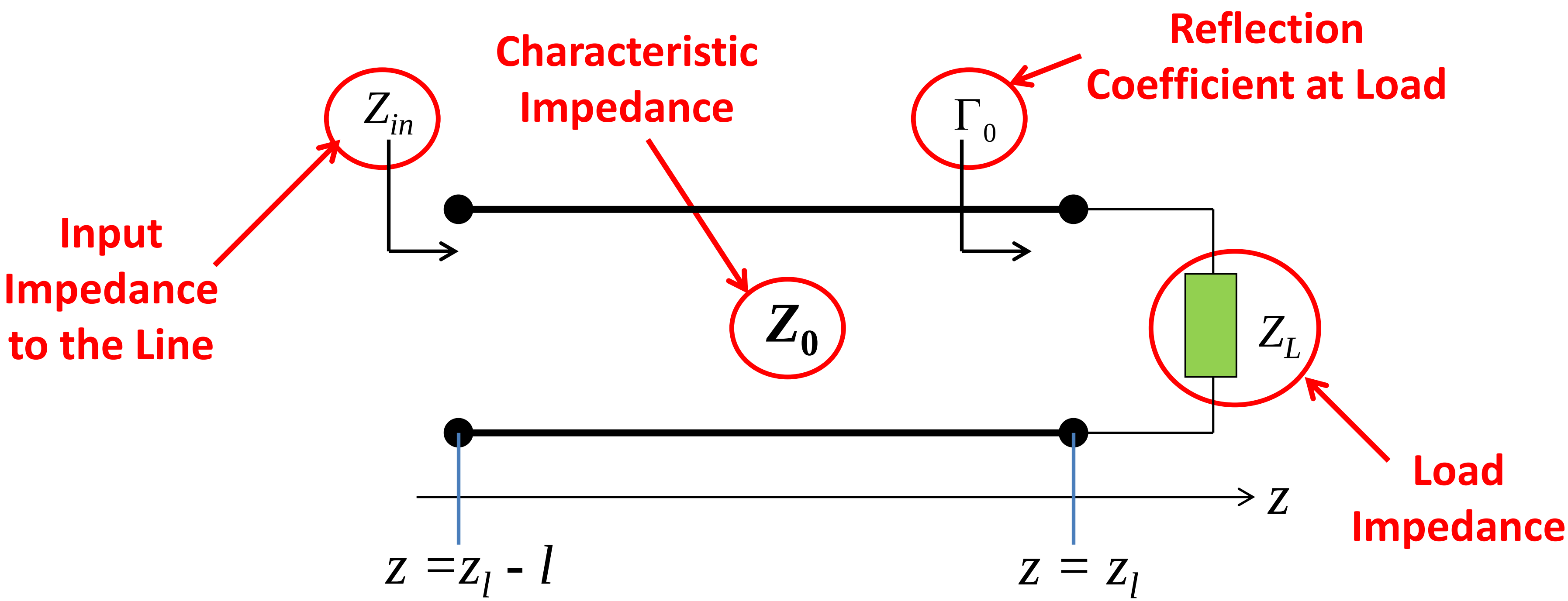
Q: Z_0 and β are determined from L, C, and ω. How do we find V_0^+ and V_0^-?
A: Apply **Boundary Conditions**!

Every transmission line has **2** "boundaries":
1) At one end of the transmission line.
2) At the **other** end of the trans line!

Typically, there is a **source** at one end of the line, and a **load** at the other.

Terminated Lossless Transmission Line

- Now let's **attach** something to our transmission line. Consider a **lossless** line, length l, terminated with a **load** Z_l.

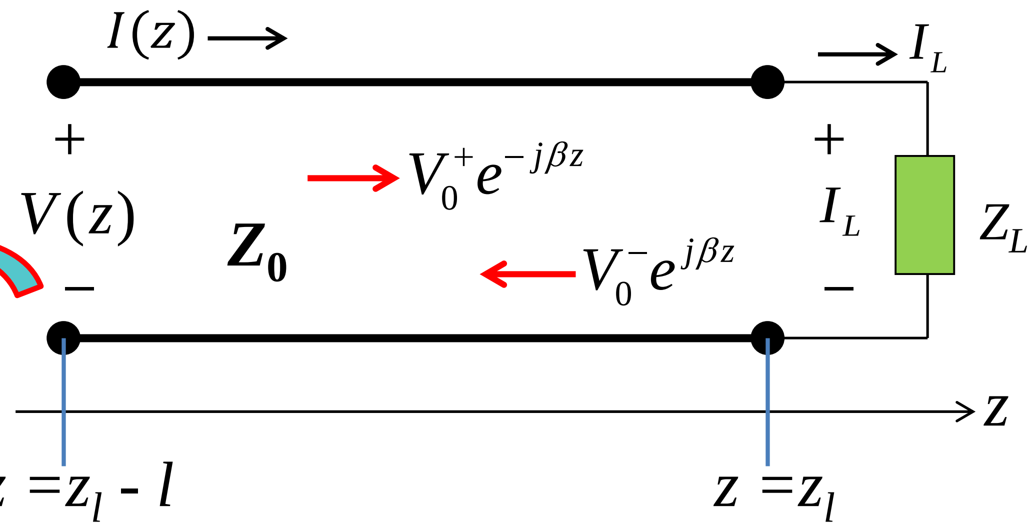


Q: What is the **current** and **voltage** at each and **every** point on the transmission line (i.e., what is $I(z)$ and $V(z)$ for **all** points z where $z_l - l < z < z_l$.

A: To find out, we must apply **boundary conditions**!

Terminated Lossless Transmission Line (contd.)

- The load is assumed at $z = z_l$
- The voltage wave couples into the line at $z = z_l - l$



$$\boxed{V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}}$$

Incident Wave Reflected Wave

- **At the load:** $$\boxed{V(z = z_l) = V^+(z = z_l) + V^-(z = z_l) = V_0^+ e^{-j\beta z_l} + V_0^- e^{j\beta z_l}}$$

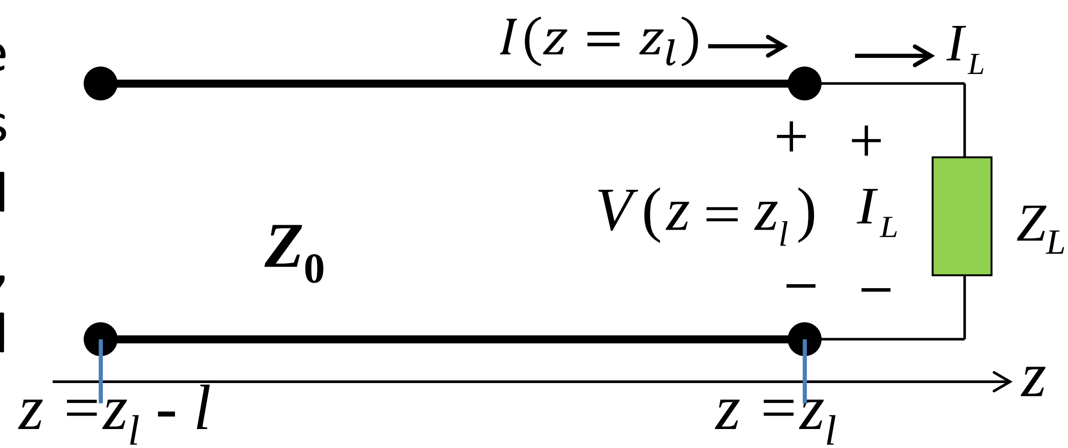
$$\boxed{I(z = z_l) = \frac{V^+(z = z_l)}{Z_0} - \frac{V^-(z = z_l)}{Z_0} = \frac{V_0^+}{Z_0} e^{-j\beta z_l} - \frac{V_0^-}{Z_0} e^{j\beta z_l}}$$

Terminated Lossless Transmission Line (contd.)

- Furthermore, the load voltage and current must be related by **Ohm's law**:

$$\boxed{V_L = Z_L I_L}$$

- Most importantly, we recognize that the values $I(z = z_l), V(z = z_l)$ and I_L, V_L are **not** independent, but in fact are strictly related by **Kirchoff's Laws**!



$$\boxed{I(z = z_l) = I_L} \qquad \boxed{V(z = z_l) = V_L}$$

So now we have the **boundary conditions** for **this** particular problem.

Careful! Different transmission line problems lead to **different** boundary conditions—**you** must assess each problem **individually** and **independently**!

Terminated Lossless Transmission Line (contd.)

- **Combining** these equations and boundary conditions, we find that:

$$V(z=z_l)=V_L=Z_L I_L=Z_L I(z=z_l)$$

$$V^{+}(z=z_l)+V^{-}(z=z_l)=\frac{Z_L}{Z_0}\left(V^{+}(z=z_l)-V^{-}(z=z_l)\right)$$

- Rearranging, we can conclude: $$\frac{V^{-}(z=z_l)}{V^{+}(z=z_l)}=\frac{Z_L-Z_0}{Z_L+Z_0}$$

Voltage Reflection Coefficient $\Gamma(z=z_l)$

also holds true for current wave but with opposite sign

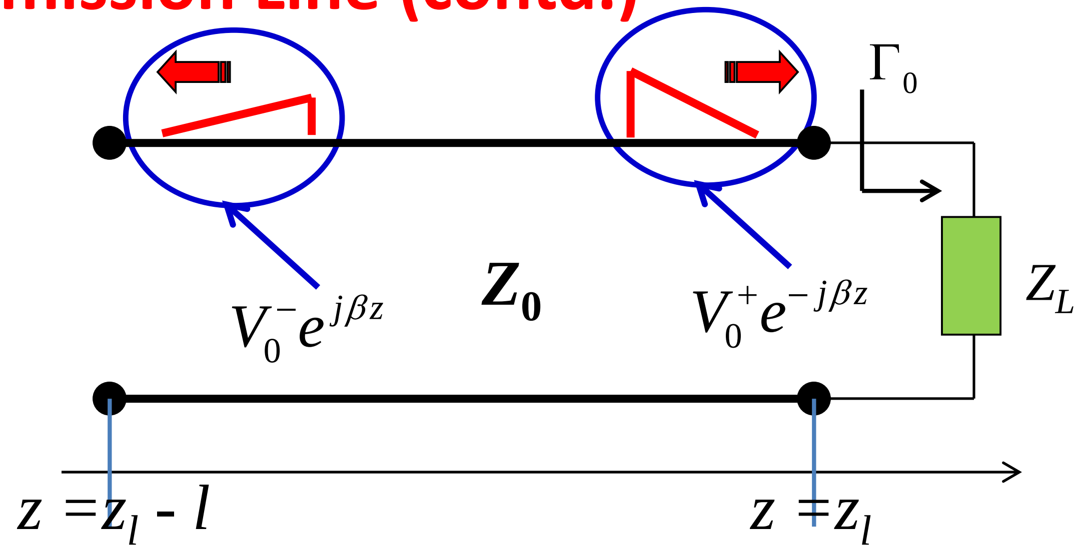
This value is of **fundamental** importance for the terminated transmission line problem, so we provide it with its **own** special symbol (Γ_0)!

Terminated Lossless Transmission Line (contd.)

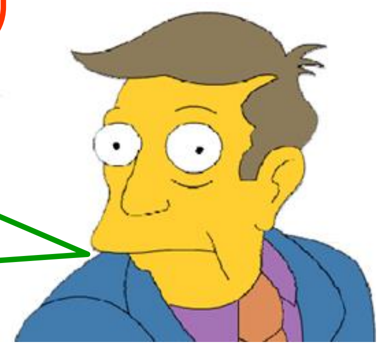
$$\Gamma_0 = \frac{V^-(z=z_l)}{V^+(z=z_l)}$$

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

More useful representation as it involves known circuit/system quantities



Q: I'm confused! Just what are we trying to accomplish in this handout?



A: We are trying to find V(z) and I(z) when a lossless transmission line is terminated by a load Z_L!

- We can express the reflected voltage wave as:

$$\Gamma_0 = \frac{V^-(z=z_l)}{V^+(z=z_l)} = \frac{V_0^- e^{+j\beta z_l}}{V_0^+ e^{-j\beta z_l}}$$

$$V_0^- = \Gamma_0 V_0^+ e^{-j2\beta z_l}$$

Terminated Lossless Transmission Line (contd.)

- Therefore: $V^{-}(z)=\left(\Gamma_0 V_0^{+} e^{-j2\beta z_l}\right)e^{+j\beta z}$

$$V(z)=V^{+}(z)+V^{-}(z)=V_0^{+}\left[e^{-j\beta z}+\left(\Gamma_0 e^{-j2\beta z_l}\right)e^{+j\beta z}\right]$$

$$I(z)=\frac{V^{+}(z)-V^{-}(z)}{Z_0}=\frac{V_0^{+}}{Z_0}\left[e^{-j\beta z}-\left(\Gamma_0 e^{-j2\beta z_l}\right)e^{+j\beta z}\right]$$

- **Simplify** by **arbitrarily** assigning the end point a **zero** value (i.e., $z_l = 0$)

$$V(z=0)=V^{+}(z=0)+V^{-}(z=0)=V_0^{+}e^{-j\beta(0)}+V_0^{-}e^{+j\beta(0)}=V_0^{+}+V_0^{-}$$

$$I(z=0)=\frac{V_0^{+}-V_0^{-}}{Z_0}$$

$$Z(z=0)=\frac{V(z=0)}{I(z=0)}=Z_0\left[\frac{V_0^{+}+V_0^{-}}{V_0^{+}-V_0^{-}}\right]=Z_L$$

- The current and voltage along the line in this case are:

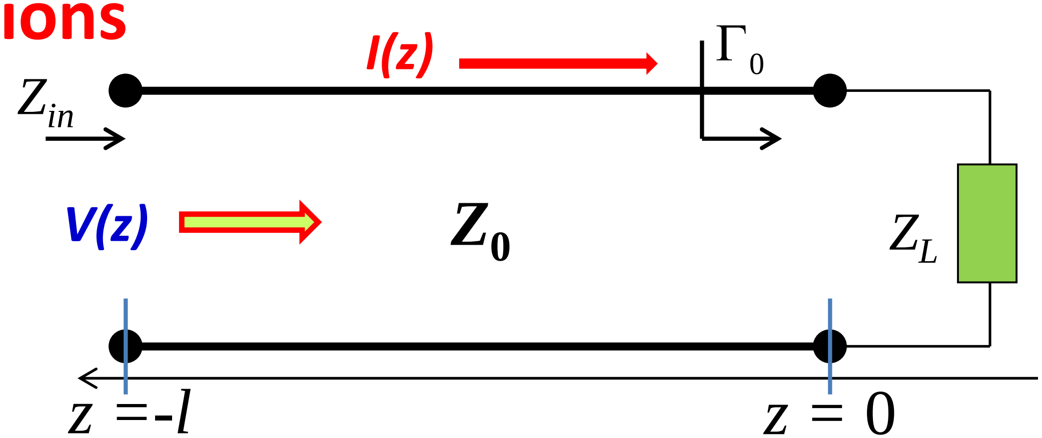
$$V(z)=V_0^{+}\left[e^{-j\beta z}+\Gamma_0 e^{+j\beta z}\right]$$

$$I(z)=\frac{V_0^{+}}{Z_0}\left[e^{-j\beta z}-\Gamma_0 e^{+j\beta z}\right]$$

Q: But, how do we determine $V_0{}^{+}$??

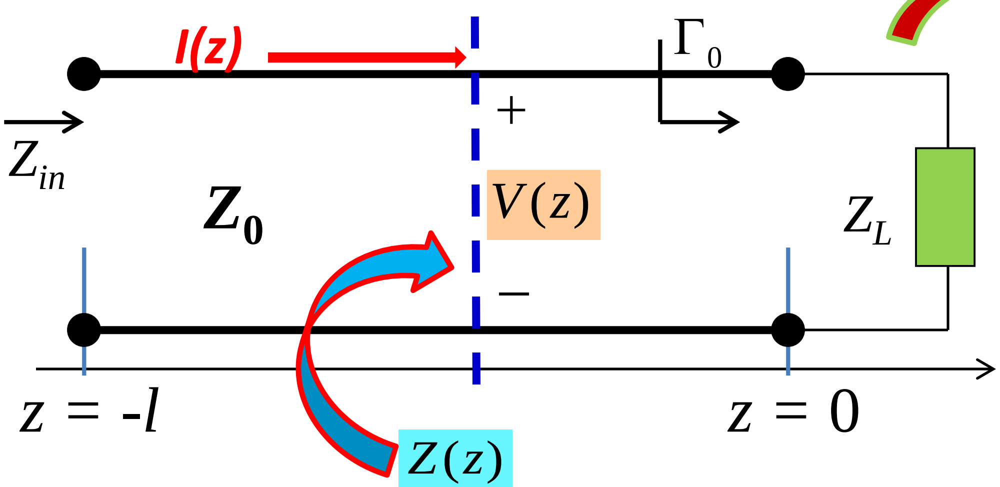
Special Termination Conditions

- Let us once again consider a generic TL terminated in arbitrary impedance Z_L



- It's interesting to note that **Z_L** enforces a boundary condition that explicitly determines neither **V(z)** nor **I(z)**—but **completely** specifies **line impedance** Z(z)!

$$Z(z) = \frac{V(z)}{I(z)}$$



$$Z(z) = \frac{V(z)}{I(z)} = \frac{V^+ e^{+j\beta z}\left(1+\Gamma_0 e^{-j(2\beta z)}\right)}{\dfrac{V^+ e^{+j\beta z}}{Z_0}\left(1-\Gamma_0 e^{-j(2\beta z)}\right)}$$

Special Termination Conditions (contd.)

$$Z(z) = Z_0 \frac{e^{-j\beta z} + \Gamma_0 e^{+j\beta z}}{e^{-j\beta z} - \Gamma_0 e^{+j\beta z}} = Z_0 \frac{Z_L \cos(\beta z) - jZ_0 \sin(\beta z)}{Z_0 \cos(\beta z) - jZ_L \sin(\beta z)}$$

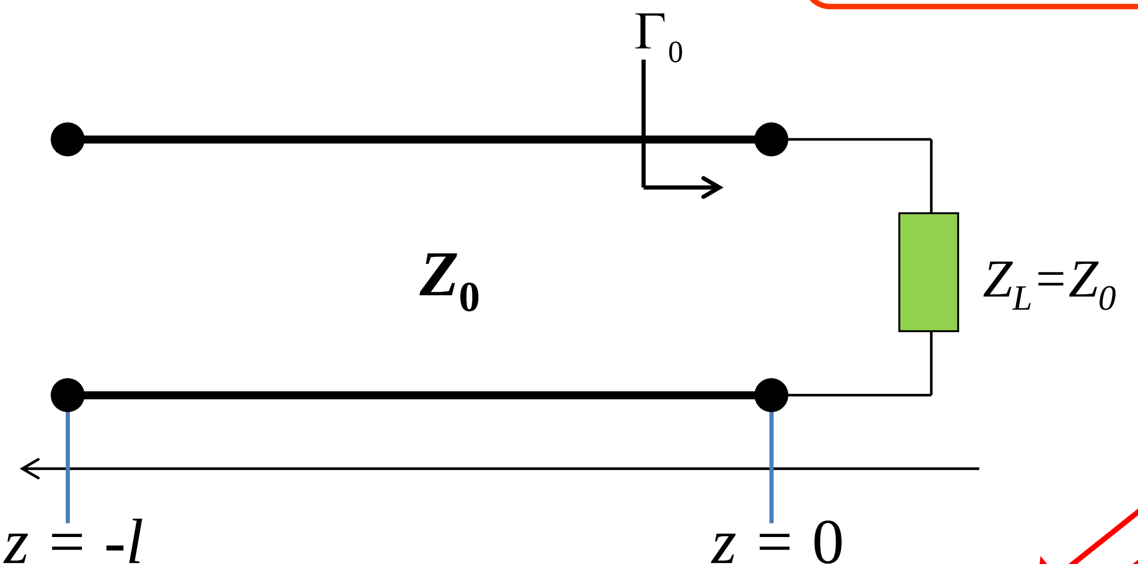
- Likewise, the load boundary condition leaves $V^+(z)$ and $V^-(z)$ undetermined, but **completely** determines **reflection coefficient function** $\Gamma(\boldsymbol{z})$!

$$\Gamma(z) = \frac{V^-(z)}{V^+(z)} = \Gamma_0 e^{+j2\beta z} = \frac{Z_L - Z_0}{Z_L - Z_0} e^{+j2\beta z}$$

Let's look at some **specific** values of load impedance $Z_L = R_L + jX_L$ and see what functions $Z(z)$ and $\Gamma(\boldsymbol{z})$ result!

Special Termination Conditions (contd.)

- $Z_L = Z_0$ ⟸ **Matched Line** ⟹ the load impedance equals the characteristic impedance of the TL



means no reflected wave $V^{-}(z)$

The load reflection coefficient:

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

reflection coefficient is zero at all points along the line

The impedance at position z:

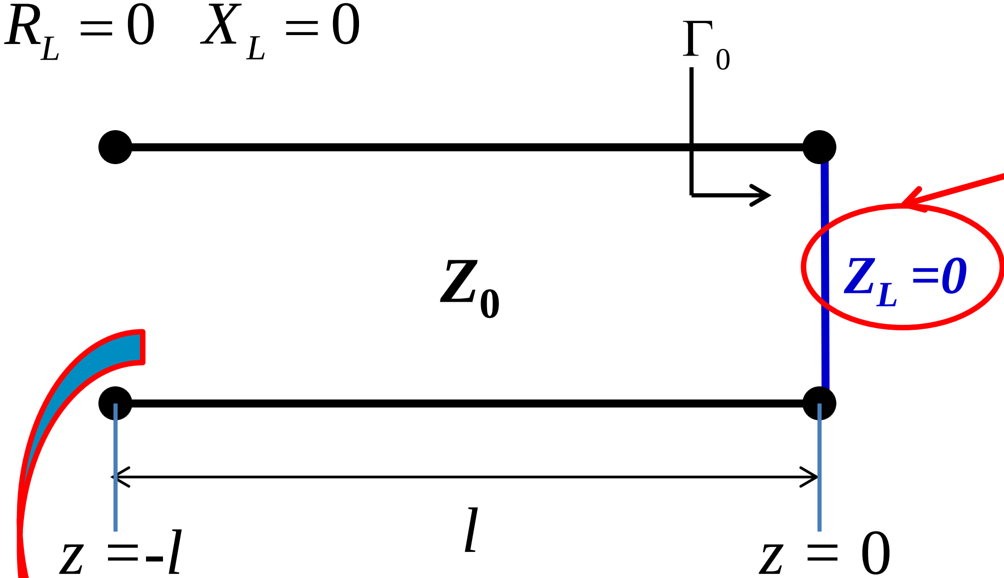
$$\boxed{Z(z) = Z_0}$$

The line impedance equals Z_0 → matched condition

Special Termination Conditions (contd.)

- $Z_L = 0$ ⟸ **Short-Circuited Line** ⟹ A device with no load is called short circuit

$R_L = 0 \quad X_L = 0$



Short-circuit entails setting this impedance to zero

$$\Gamma_0 = \frac{0 - Z_0}{0 + Z_0} = -1$$

$$Z(z) = -jZ_0 \tan(\beta z)$$

Alternatively

$$Z(z) = -jZ_0 \tan\left(\frac{2\pi z}{\lambda}\right)$$

Note that this impedance is **purely reactive**. This means that the current and voltage on the transmission line will be everywhere 90° **out of phase**.

Special Termination Conditions (contd.)

- **Short-Circuited Line**

- The current and voltage along the TL is:

$$V(z) = V_0^+\left[e^{-j\beta z} - e^{+j\beta z}\right] = -j2V_0^+\sin(\beta z)$$

$$I(z) = \frac{2V_0^+}{Z_0}\cos(\beta z)$$

- Finally, the reflection coefficient **function** is:

$$\Gamma(z) = \frac{V^-(z)}{V^+(z)} = \frac{-V_0^+e^{+j\beta z}}{V_0^+e^{-j\beta z}} = -e^{j2\beta z}$$

$$|\Gamma(z)| = 1$$

$$\left|V^-(z)\right| = \left|V^+(z)\right|$$

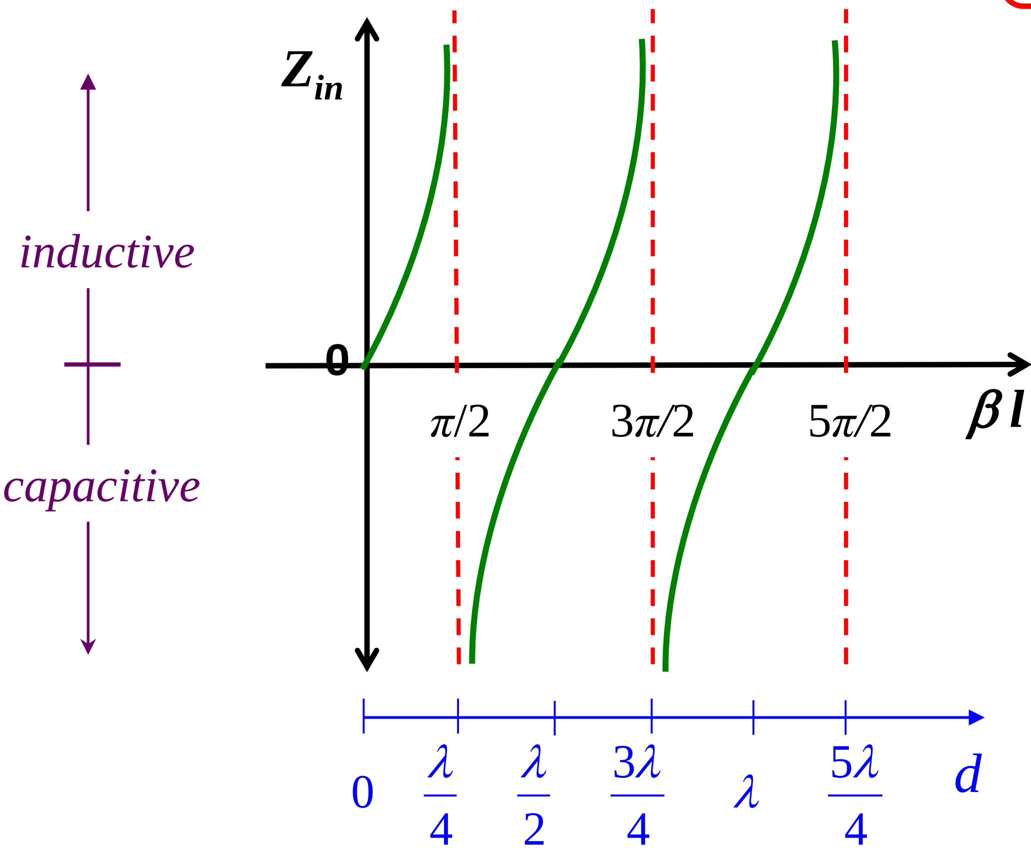
In other words, the **magnitude** of each **wave** on the transmission line is the **same**—the reflected wave is **just** as big as the incident wave!

- **Short-Circuited Line:** $$Z(-l) = jZ_0\tan(\beta l)$$

Special Termination Conditions (contd.)

- **Short-Circuited Line**

$$Z(-l) = jZ_0 \tan(\beta l)$$



It can be observed:

- **At -*l*=0,** the impedance is zero (short-circuit condition)
- **Increase in -*l*** leads to inductive behavior
- **At -*l*=λ/4,** the impedance equals infinity (open-circuit condition)
- **Further increase in -*l*** leads to capacitive behavior
- **At -*l*=λ/2,** the impedance becomes zero (short-circuit condition)
- **The entire periodic sequence repeats for -*l*>λ/2 and so on…**

Example – 3

For a short-circuited TL of $l = 10$ cm, compute the magnitude of the input impedance when the frequency is swept from f = 1 GHz to 4 GHz. Assume the line parameters L = 209.4 nH/m and C = 119.5 pF/m.

Solution:

$$Z_0 = \sqrt{L/C} = \sqrt{(209.4*0.1)/(119.5*0.5)} = 41.86\Omega$$

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(209.4*0.1)*(119.5*0.5)} = 41.86\Omega} = 1.99*10^8 m/s$$

$$Z(z=-l) = jZ_0 \tan(\beta l) = jZ_0 \tan\left(\frac{2\pi f}{v_p} l\right)$$

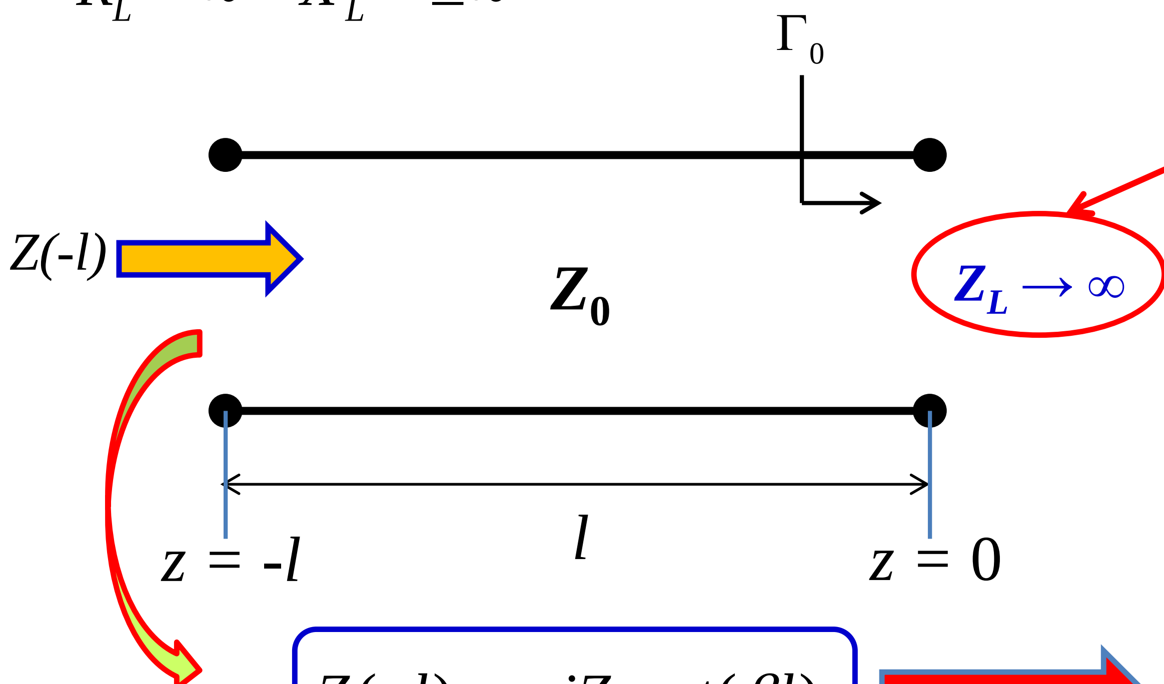
Set $l = 10$ cm and then write a MATLAB program to obtain the Z_{in} curve

Compare the MATLAB results to that obtained from ADS simulation

Special Termination Conditions (contd.)

- $Z_L \to \infty$ ⟸ **Open-Circuited Line** ⟹ A device with infinite load is called open-circuit

$$R_L = \infty \quad X_L = \pm\infty$$



Open-circuit entails setting this impedance to infinite

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0} = 1$$

$$Z(-l) = -jZ_0 \cot(\beta l)$$

Alternatively

$$Z(-l) = -jZ_0 \cot\left(\frac{2\pi l}{\lambda}\right)$$

Again note that this impedance is **purely reactive**. current and voltage on the transmission line are 90° **out of phase**.

Special Termination Conditions (contd.)

- **Open-Circuited Line**

- The current and voltage along the TL is:

$$V(z) = V_0^+\left[e^{-j\beta z} + e^{+j\beta z}\right] = 2V_0^+\cos(\beta z)$$

$$I(z) = -j\frac{2V_0^+}{Z_0}\sin(\beta z)$$

- At the load, $z = 0$, therefore: $V(0) = 2V_0^+$ $\quad I(0) = 0$

As expected, the current is zero at the end of the transmission line (i.e. the current through the open). Likewise, the voltage at the end of the line (i.e., the voltage across the open) is at a maximum!

- Finally, the reflection coefficient **function** is:

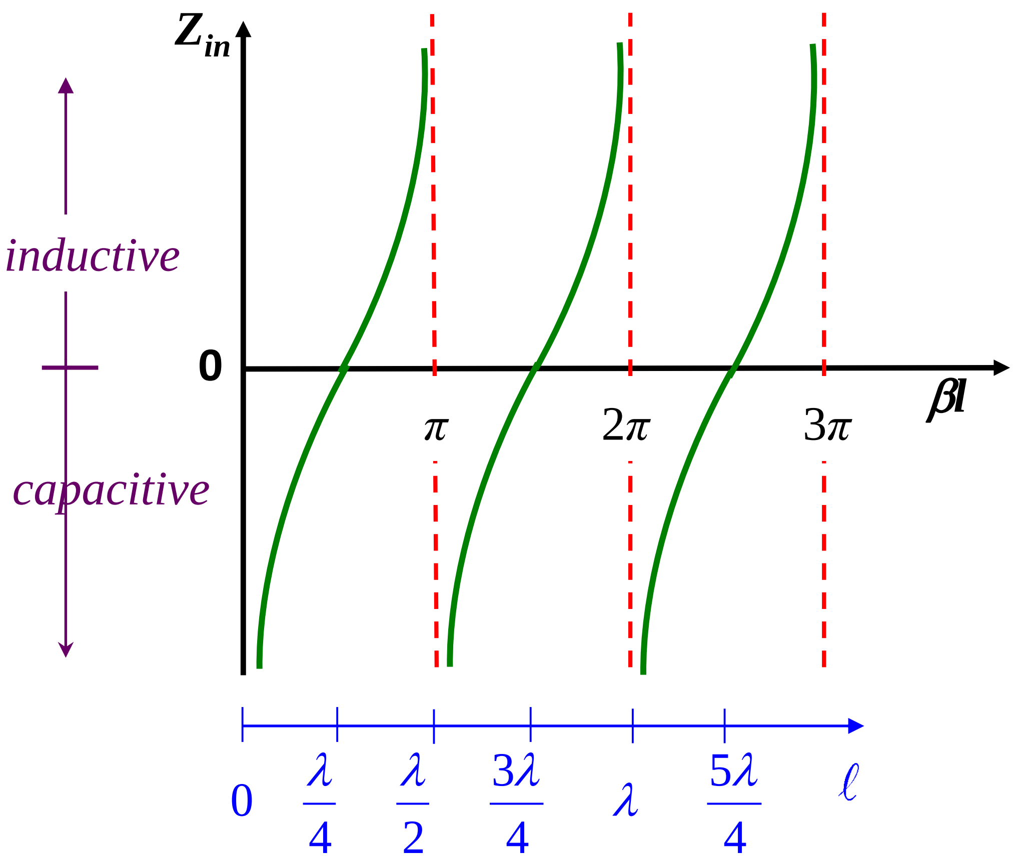
$$\Gamma(z) = \frac{V^-(z)}{V^+(z)} = \frac{V_0^+e^{+j\beta z}}{V_0^+e^{-j\beta z}} = e^{j2\beta z} \Rightarrow |\Gamma(z)| = 1 \Rightarrow \left|V^-(z)\right| = \left|V^+(z)\right|$$

In other words, the **magnitude** of each **wave** on the transmission line is the **same**—the reflected wave is **just** as big as the incident wave!

Special Termination Conditions (contd.)

- **Open-Circuited Line**

$$Z(-l) = -jZ_0 \cot(\beta l)$$



It can be observed:

- **At -*l*=0,** the impedance is infinite (open-circuit condition)
- **Increase in -*l*** leads to capacitive behavior
- **At -*l* = λ/4,** the impedance equals zero (short-circuit condition)
- **Further increase in -*l*** leads to inductive behavior
- **At -*l*=λ/2,** the impedance becomes infinite (open-circuit condition)
- **The entire periodic sequence repeats for -*l* >λ/2 and so on…**

Transmission Line Input Impedance

Q: Just what do you mean by **input** impedance?

A: The input impedance is simply the line impedance seen at the **beginning** $(\boldsymbol{z = -l})$ of the transmission line, i.e.:

$$Z_{in} = Z(z = -l) = \frac{V(z = -l)}{I(z = -l)}$$

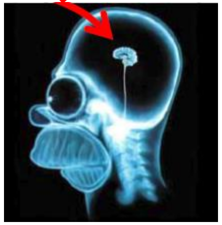
Note Z_{in} equal to **neither** the load impedance Z_L **nor** the characteristic impedance Z_0!

$$Z_{in} \neq Z_L \qquad\qquad Z_{in} \neq Z_0$$

- We know the line impedance of a lossless TL loaded with an arbitrary load impedance is:

$$Z(z) = Z_0 \frac{Z_L \cos(\beta z) - jZ_0 \sin(\beta z)}{Z_0 \cos(\beta z) - jZ_L \sin(\beta z)}$$

The input impedance can be radically different from load impedance (Z_L) and you should commit it to memory

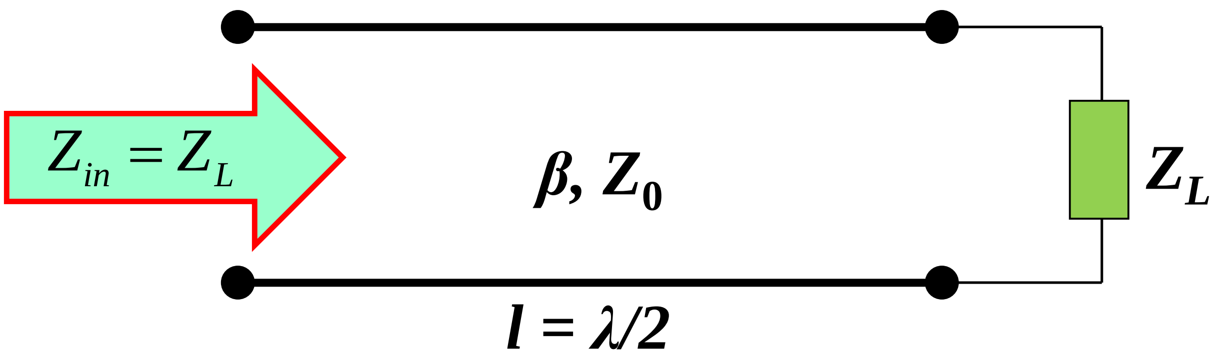


Transmission Line Input Impedance – Special Cases

1. length of the line is $l = m(\lambda/2)$

$$Z_{in} = Z(z = \lambda/2) = Z_0 \frac{Z_L + jZ_0 \tan\left(\frac{2\pi}{\lambda}.\frac{\lambda}{2}\right)}{Z_0 + jZ_L \tan\left(\frac{2\pi}{\lambda}.\frac{\lambda}{2}\right)} = Z_L$$

- For a transmission line of half wavelength long the input impedance equals the load impedance irrespective of the characteristic impedance of the line
- It means it is possible to design a circuit segment where the transmission line's characteristic impedance plays no role (obviously the length of line segment has to equal half wavelength at the operating frequency)

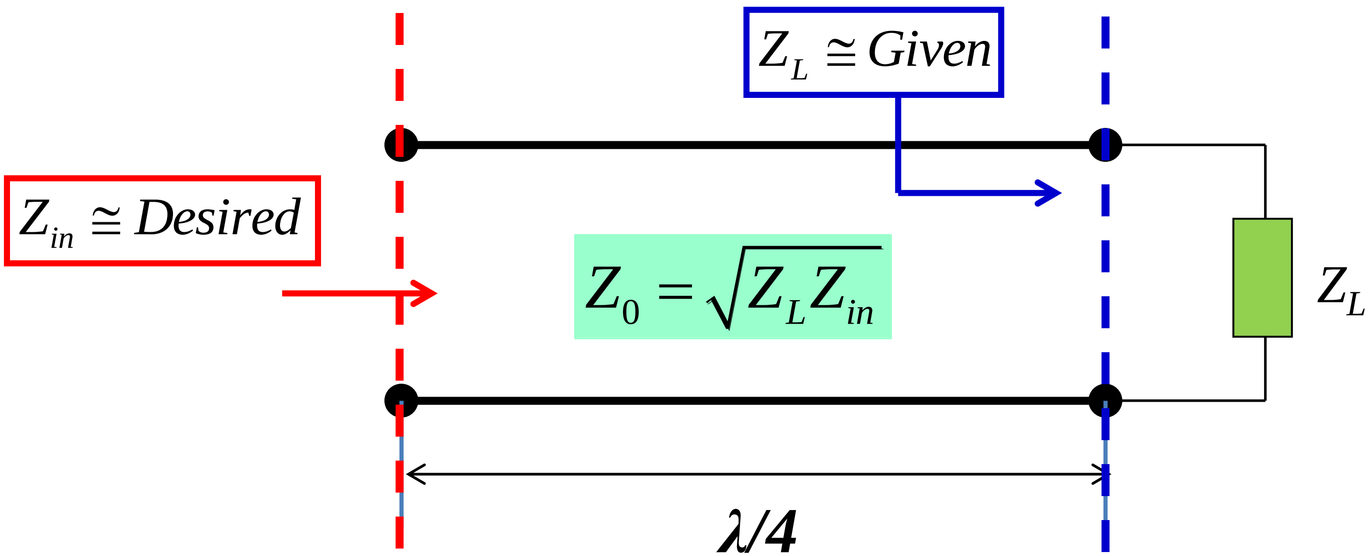


Transmission Line Input Impedance – Special Cases (contd.)

2. **length of the line is $l = \lambda/4$ or $\lambda/4 + m(\lambda/2)$**

$$Z_{in} = Z(l=\lambda/4) = Z_0 \frac{Z_L + jZ_0 \tan\left(\frac{2\pi}{\lambda}.\frac{\lambda}{4}\right)}{Z_0 + jZ_L \tan\left(\frac{2\pi}{\lambda}.\frac{\lambda}{4}\right)} = \frac{Z_0^2}{Z_L}$$

- This result implies that a transmission line segment can be used to synthesize matching of any desired real input impedance (Z_{in}) to the specified real load impedance (Z_L)



This is known as quarter-wave impedance transformer

Transmission Line Input Impedance – Special Cases (contd.)

2. length of the line is $l = \lambda/4$ or $\lambda/4 + m(\lambda/2)$

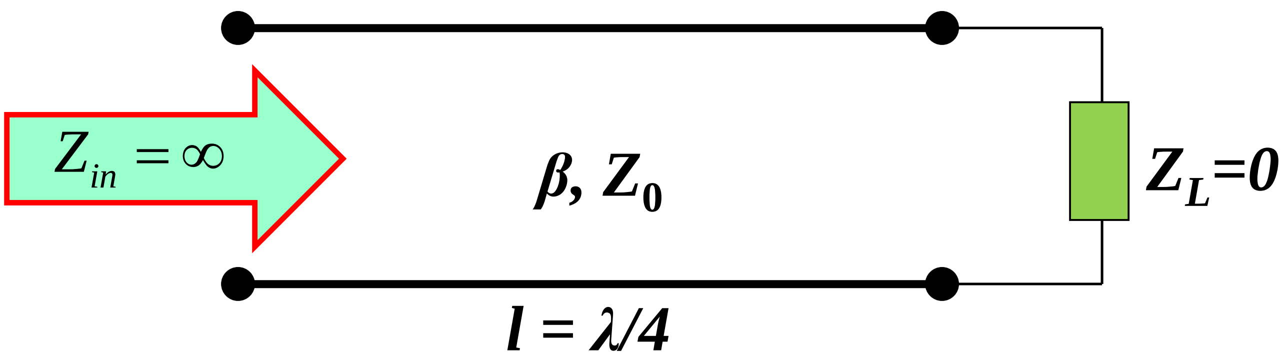
$$Z_{in} = \frac{Z_0^2}{Z_L}$$

input impedance of a quarter-wave line is inversely proportional to the load impedance

→ Think about what this means! Say the load impedance is a short circuit then:

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{Z_0^2}{0} = \infty$$

$Z_{in} = \infty$! This is an open circuit ! The quarter wave TL transforms a short-circuit into open-circuit and vice-versa



Example – 4

- Consider a load resistance $R_L = 100\Omega$ to be matched to a 50Ω line with a quarter-wave transformer. Find the characteristic impedance of the matching section and plot the magnitude of the reflection coefficient versus normalized frequency, f/f_0, where f_0 is the frequency at which the line is $\lambda/4$ long.

  - the necessary characteristic impedance is:

$$Z_0 = \sqrt{Z_L Z_{in}} \Rightarrow \boxed{\therefore Z_0 = \sqrt{Z_L Z_{in}} = \sqrt{50 \times 100} = 70.71\Omega}$$

  - The reflection coefficient magnitude is given as

$$\boxed{|\Gamma_0| = \left|\frac{Z_0 - Z_{in}}{Z_0 + Z_{in}}\right|} \Leftarrow$$

Z_{in} is dependent on frequency

Example – 4 (contd.)

$$Z_{in} = Z(l = \lambda/4) = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda_0}{4}\right) = \left(\frac{2\pi f}{v_p}\right)\left(\frac{v_p}{4 f_0}\right) = \frac{\pi f}{2 f_0}$$

For higher frequencies the matching section looks electrically longer, and for lower frequencies it looks shorter.

Plot the magnitude of the reflection coefficient versus ${}^{f}/_{f_0}$ using these two equations

HW # 0