Lecture – 19 Date: 31.03.2016

- Filter Realization
- Richard’s Transformation
- Kuroda Identities
- Stepped Impedance Method

Filter Implementations

Q: So, we now know how to make any and all filters with **lumped** elements—but this is a **RF/microwave** engineering course!

You said that lumped elements where difficult to make and implement at high frequencies. **You** said that distributed elements were used to make microwave components. So **how** do we make a filter with **distributed elements**!?!

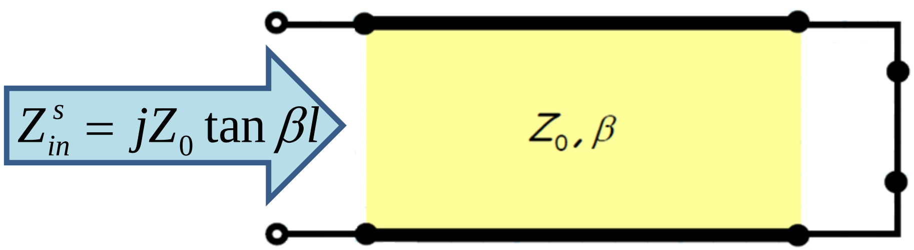
A: There are **many** ways to make RF/microwave filters with distributed elements. Perhaps the most straightforward is to "**realize**" each individual lumped element with transmission line sections, and then insert these **approximations** in our lumped element solutions.

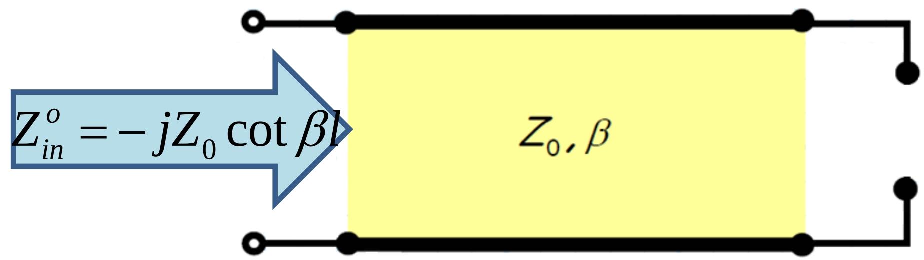
The **first** of these realizations is: Richard's Transformations

To easily **implement** Richard's Transforms in a microstrip or stripline circuit, we must apply one of **Kuroda's Identities**.

Richard’s Transformations

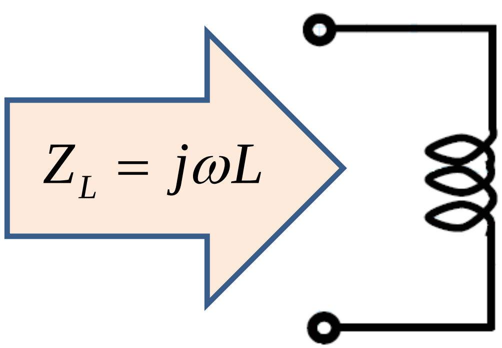
- Recall the input impedances of short-circuited and open-circuited transmission line **stubs**.

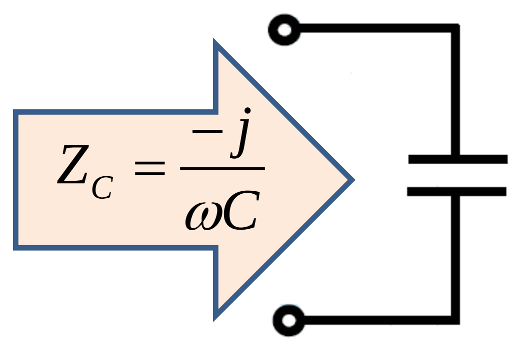




Note that the input impedances are purely **reactive**—just like **lumped** elements!

- However, the reactance of lumped inductors and capacitors have a **much** different mathematical form to that of transmission line stubs:





Richard's Transformations (contd.)

- In other words, the impedance of transmission line stubs and lumped elements (capacitors and inductors) are different functions with respect to **frequency**. Therefore, we can say in general that, for example:

$$Z_{in}^{s} \neq Z_L$$

$$Z_{in}^{o} \neq Z_C$$

However, for a given lumped element (L or C) and a given stub (with a given Z_0 and length l) the functions **will** be equal at precisely **one frequency**!

- For example, there is one frequency—let's call it ω_c —that satisfies **this** equation for a given L, Z_0, and l:

$$j\omega_c L = jZ_0 \tan \beta_c l = jZ_0 \tan\left[\frac{\omega_c}{v_p} l\right]$$

- Similarly:

$$\frac{-j}{\omega_c C} = -jZ_0 \cot \beta_c l = -jZ_0 \cot\left[\frac{\omega_c}{v_p} l\right]$$

- To make things easier, let's set the **length** of our transmission line stub to $\lambda_c/8$, where:

$$\lambda_c = \frac{v_p}{\omega_c} = \frac{2\pi}{\beta_c}$$

Richard's Transformations (contd.)

Q: Why $l = \lambda_c/8$?

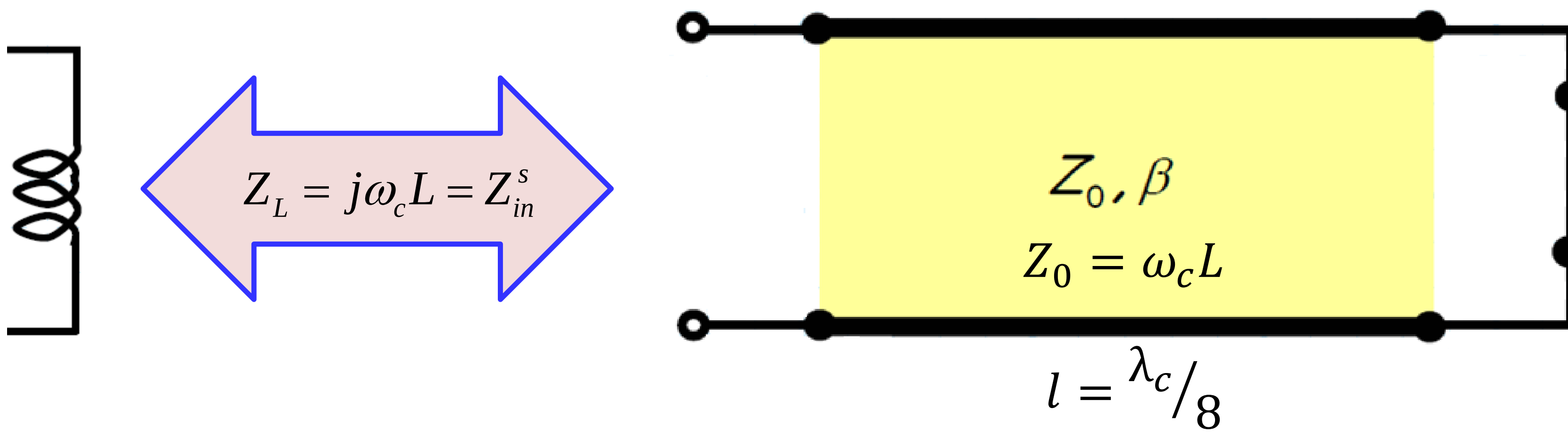
A: Well, for **one** reason, $\beta_c l = \pi/4$ and therefore tan (π /4) = 1.0!

- This greatly **simplifies** our earlier results:

$$j\omega_c L = jZ_0 \tan\left(\frac{\pi}{4}\right) = jZ_0$$

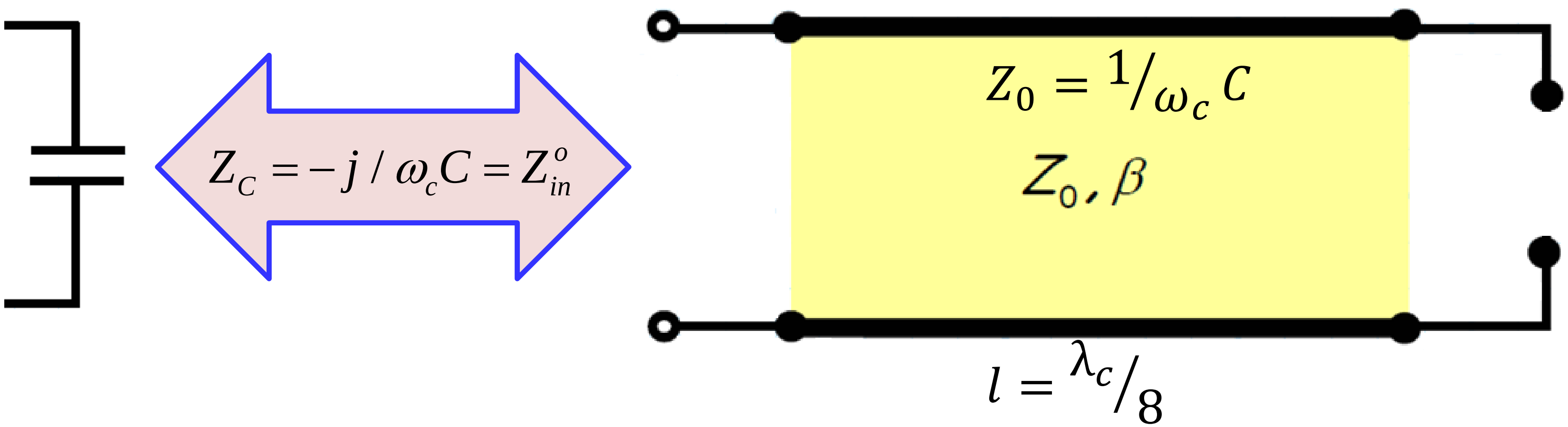
$$\frac{-j}{\omega_c C} = -jZ_0 \cot\left(\frac{\pi}{4}\right) = -jZ_0$$

- Therefore, if we wish to build a **short-circuited** stub with the **same** impedance as an **inductor** L at frequency ω_c, we set the **characteristic impedance** of the stub transmission line to be $Z_0 = \omega_c L$:



Richard's Transformations (contd.)

- Similarly, if we wish to build **open-circuited** stub with the **same** impedance as a **capacitor** C at ω_c, we set the **characteristic impedance** of the stub transmission line to be $Z_0 = {}^{1}/_{\omega_c C}$:



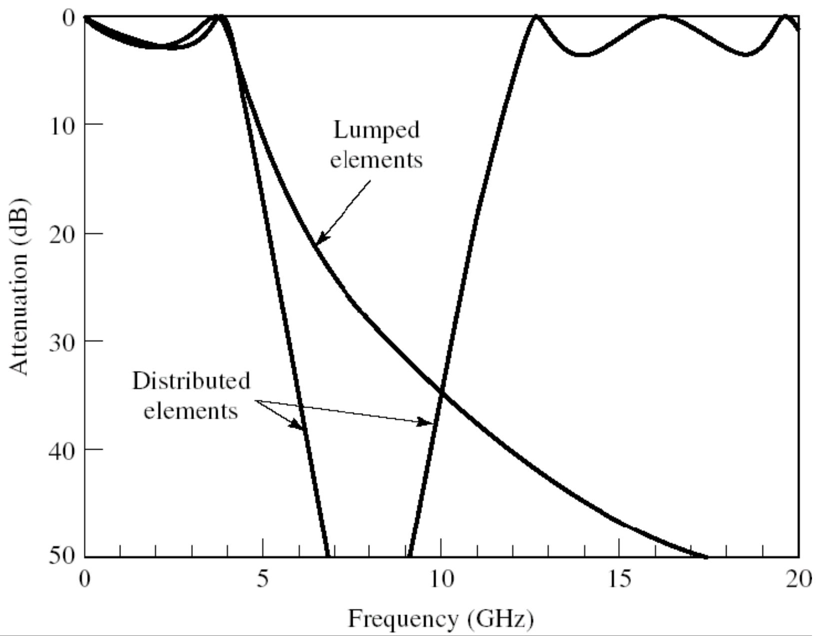
We call these two results as **Richard's Transformation.**

However, it is important to remember that Richard's Transformations do **not** result in **perfect** replacements for lumped elements—the stubs **do not** behave like capacitors and inductors!

Richard's Transformations (contd.)

- Instead, the transformation is **perfect**—the impedances are **equal**—at **only one frequency** (ω_c).
- We can use Richard's transformations to replace the inductors and capacitors of a lumped element filter design. In fact, for **low-pass filter design**, the frequency ω_c is the filter's **cut-off frequency**.
- Using these stubs to **replace** inductors and capacitors will result in a filter response **similar** to that of the lumped element design—a low pass filter with cut-off frequency ω_c.
- However, the behavior of the filter in the **stop-band** will be **very** different from the lumped element design. For example, at the (high) frequencies where the stub length becomes a **multiple** of $\lambda/2$, the filter response will be that of $\omega = 0$—near perfect **transmission**!

Richard's Transformations (contd.)



Q: So **why** does the filter response match the lumped element response so **well** in the **pass-band**?

A: To see why, we first note that the **Taylor Series approximation** for $tan\varphi$ and $cot\varphi$ when φ is small (i.e., $\varphi \ll 1$) is:

$$\boxed{tan\varphi \approx \varphi} \quad \text{and} \quad \boxed{cot\varphi \approx \frac{1}{\varphi}} \quad \text{for } \varphi \ll 1$$

φ is expressed in **radians**.

- The **impedance** of Richard's transformation shorted stub at some **arbitrary frequency** ω is therefore:

$$Z_{in}^{s}(\omega) = jZ_0 \tan\left(\beta \frac{\lambda_c}{8}\right) = j(\omega_c L)\tan\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right)$$

Richard's Transformations (contd.)

- Therefore, when $\omega \ll \omega_c$(i.e., frequencies in the **pass-band** of a low-pass filter!), we can **approximate** this impedance as:

$$Z_{in}^{s}(\omega) = j(\omega_c L)\tan\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right) \approx j(\omega_c L)\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right) = j\omega L\left(\frac{\pi}{4}\right)$$

$$Z_L = j\omega L$$

Compare this to a **lumped inductor** impedance

Since the value π/4 is **relatively** close to one, we find that the Richard's Transformation shorted stub has an input impedance **very** close to the lumped element inductor for **all** frequencies **less than** ω_c (i.e., all frequencies of the low-pass filter pass-band)!

Richard’s Transformations (contd.)

- Similarly, we find that the Richard’s transformation **open-circuit** stub, when $\omega \ll \omega_c$, has an input impedance of **approximately**:

$$Z_{in}^{o}(\omega) = \frac{-j}{\omega_c C}\cot\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right) \approx \frac{-j}{\omega_c C}\left(\frac{\omega_c}{\omega}\frac{4}{\pi}\right) = \frac{1}{j\omega C}\left(\frac{4}{\pi}\right)$$

$$Z_C = \frac{1}{j\omega C}$$

Compare this to a **lumped capacitor** impedance

we find that results are approximately the **same** for all pass-band frequencies (i.e., when $\omega \ll \omega_c$).

Kuroda's Identities

- We will find that **Kuroda's Identities** can be very useful in making the implementation of Richard's transformations more **practicable.**
- Kuroda's Identities essentially provide a list of **equivalent** two port networks. By equivalent, we mean that they have **precisely** the same scattering/impedance/admittance/transmission matrices.
- In other words, we can **replace** one two-port network with its equivalent in a circuit, and the behavior and characteristics (e.g., its scattering matrix) of the circuit will **not** change!

Q: Why would we want to do this?

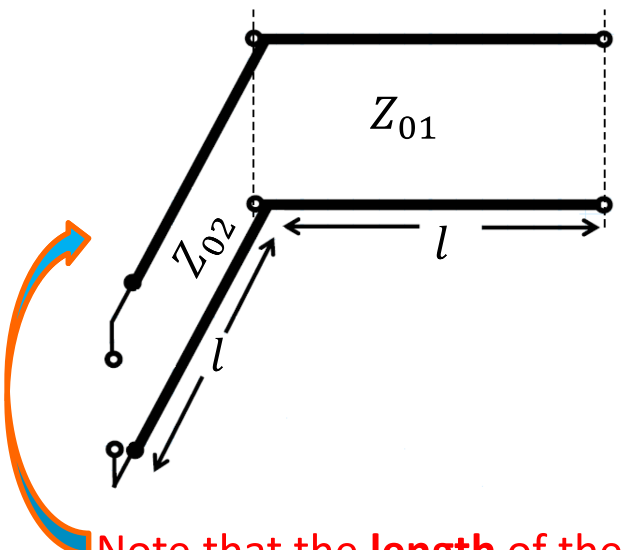
A: Because one of the equivalent may be more **practical** to implement!

For example, we can use Kuroda's Identities to:

1. Physically **separate** transmission line stubs.
2. Transform series stubs into **shunt** stubs.
3. Change impractical **characteristic impedances** into more realizable ones.

Kuroda's Identities (contd.)

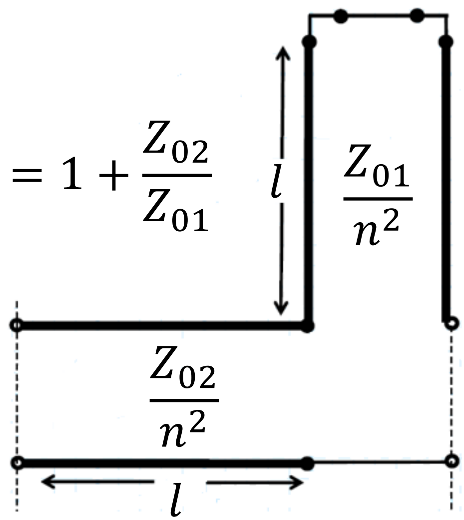
- **Four** Kuroda's identities are provided in a very **ambiguous** and confusing table (Table 8.7) in your **book**. We will find the **first two** identities to be the most useful.
- Consider the following two-port network, constructed with a length of transmission line, and an **open**-circuit **shunt** stub:



Note that the **length** of the stub and the transmission line are **identical**, but the characteristic **impedance** of each are **different**.

- The **first Kuroda identity** states that this two-port network is **precisely** the same two-port network as **the following**:

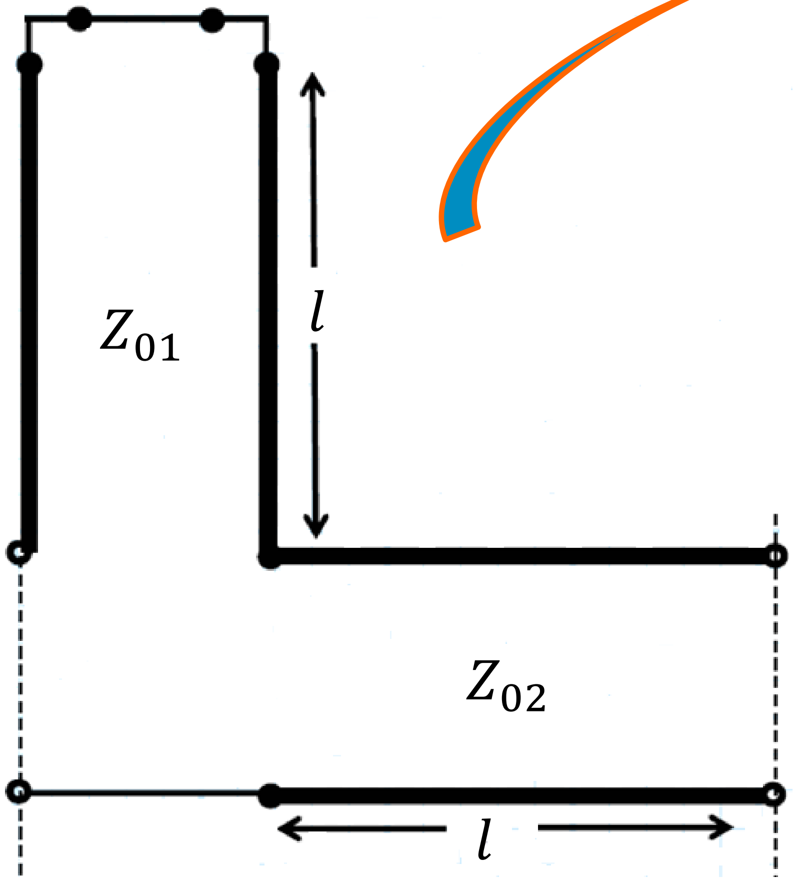
$$n^2 = 1 + \frac{Z_{02}}{Z_{01}}$$



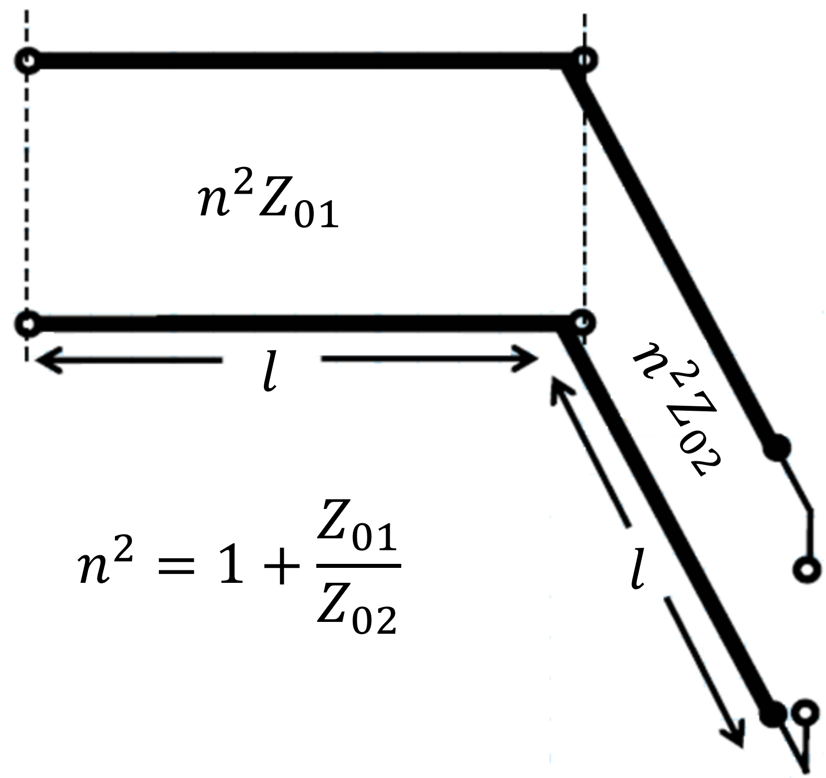
- Thus, we can **replace** the first structure in some circuit with this, and the behavior of that circuit will **not change** in the least!
- Note this equivalent circuit uses a **short-circuited series** stub.

Kuroda's Identities (contd.)

- The **second** of Kuroda's Identities states that this two port network:



Is **precisely identical** to this two-port network:

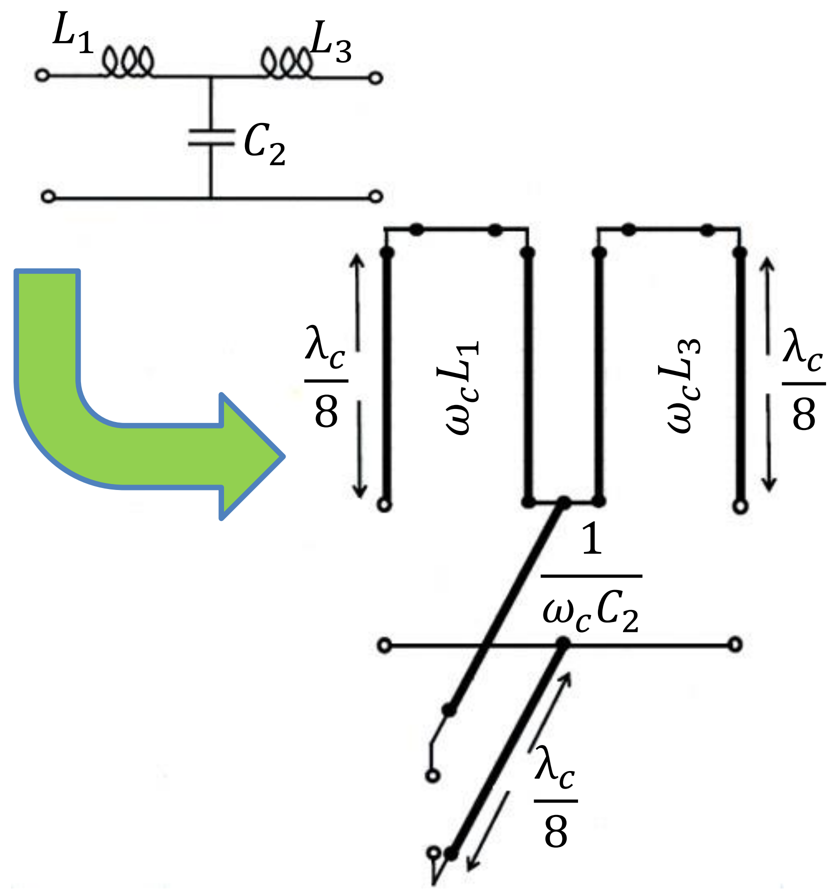


$$n^2 = 1 + \frac{Z_{01}}{Z_{02}}$$

Kuroda's Identities (contd.)

- With regard to **Richard's Transformation**, these identities are useful when we replace the series **inductors** with **shorted stubs**.

- To see **why** this is useful when implementing a **lowpass filter** with distributed elements, consider this third order filter example, realized using Richard's Transformations:



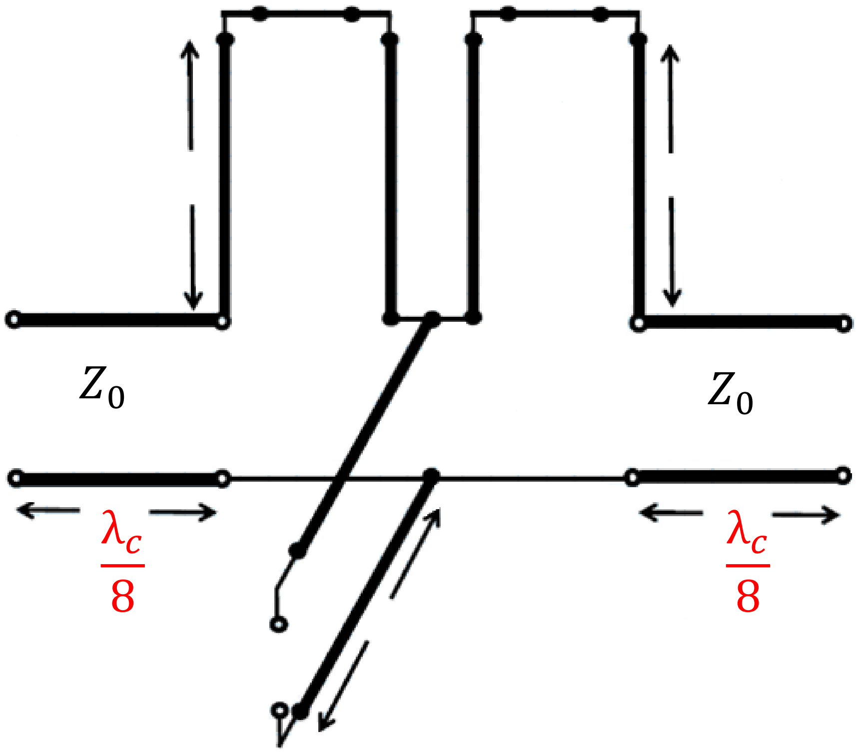
Note that we have a few **problems** in terms of implementing this design!

Kuroda's Identities (contd.)

- First of all the stubs are ideally **infinitely close** to each other— how do we build that? We could physically **separate** them, but this would introduce some transmission **line length** between them that would **mess up** our filter response!

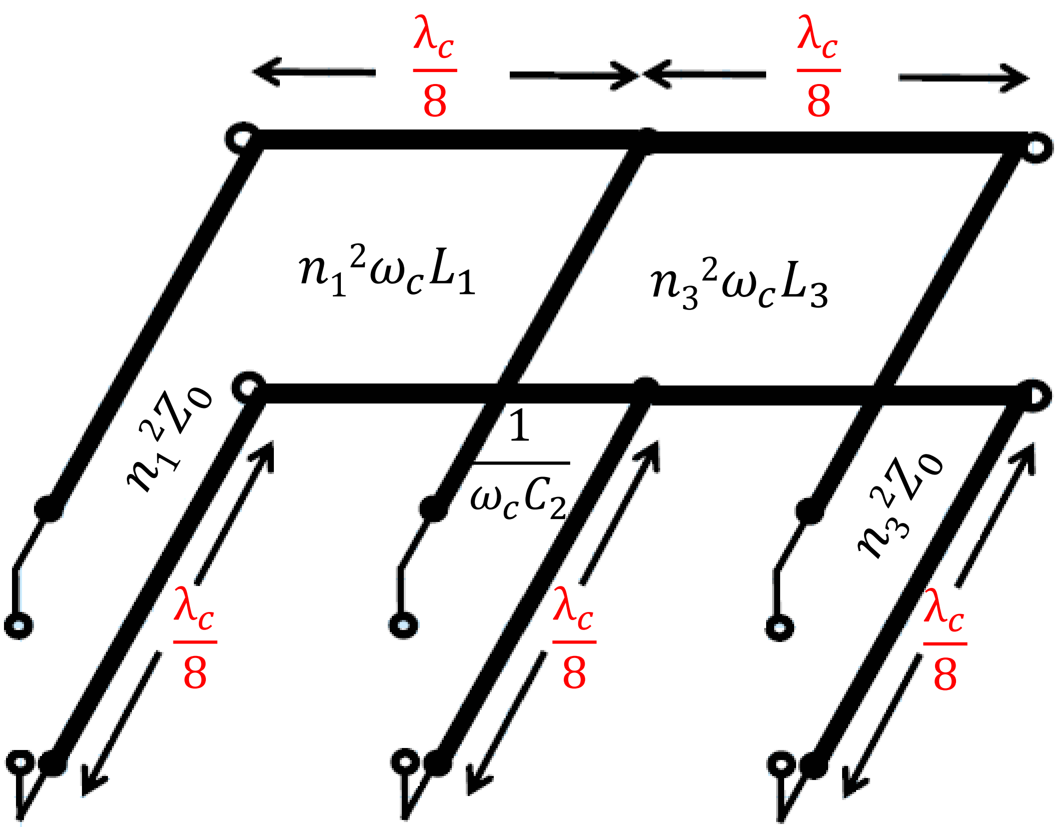
- Secondly, **series** stubs are difficult to construct in microstrip/ stripline—we like **shunt** stubs **much** better!

- To solve these problems, we first **add** a short length of transmission line (Z_0 and l= $\lambda_c/8$) to the **beginning** and **end** of the filter:



Kuroda's Identities (contd.)

- Note adding these lengths only results in a **phase shift** in the filter response—the transmission and reflection functions will remain **unchanged**.
- Then we can use the second of **Kuroda's Identities** to replace the **series** stubs with **shunts**:



Where:

$$n_1^2 = 1 + \frac{Z_0}{\omega_c L_1}$$

$$n_3^2 = 1 + \frac{Z_0}{\omega_c L_3}$$

Now **this** is a realizable filter! Note the **three stubs** are separated, and they are all **shunt** stubs.

Stepped-Impedance Low-Pass Filters

- Another distributed element realization of a lumped element low-pass filter designs is the **stepped-impedance** low-pass filter.
- These filters are also known as **“hi-Z, low-Z”** filters, and we’re about to find out why!

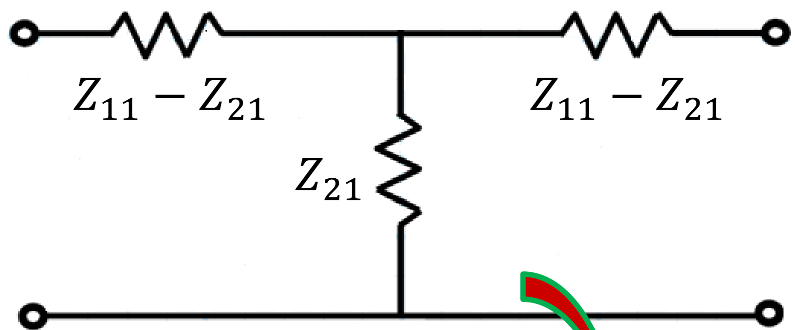
All distributed elements (e.g., transmission lines, coupled lines, resonators, stubs) exhibit **some** frequency dependency. If we are clever, we can construct these structures in a way that their frequency dependency (i.e., $S_{21}(\omega)$) conforms to a desirable function of ω.

Stepped-Impedance Low-Pass Filters (contd.)

- Say we know the impedance matrix of a **symmetric** two-port device:

$$Z = \begin{bmatrix} Z_{11} & Z_{21} \\ Z_{21} & Z_{11} \end{bmatrix}$$

- **Regardless** of the construction of this two port device, we can **model** it as a simple "T-circuit", consisting of three impedances:

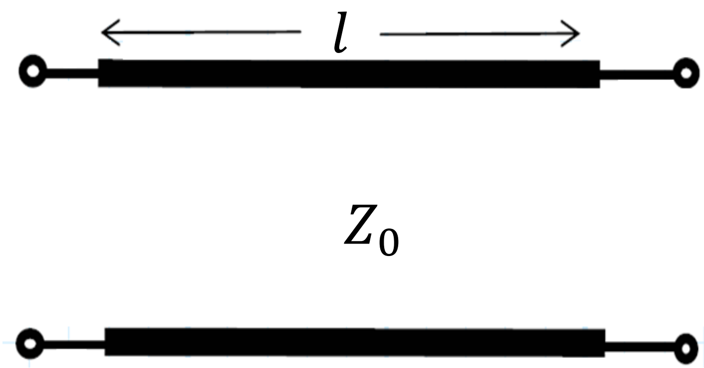


- In other words, if the two **series impedances** have an impedance value equal to $Z_{11} - Z_{21}$, and the **shunt impedance** has a value equal to Z_{21}, the impedance matrix of this "T-circuit" is:

$$Z = \begin{bmatrix} Z_{11} & Z_{21} \\ Z_{21} & Z_{11} \end{bmatrix}$$

Thus, **any** symmetric two-port network can be modeled by this "T-circuit"!

Stepped-Impedance Low-Pass Filters (contd.)



- For example, consider a length l of **transmission line** (a symmetric two-port network!):

- Recall (or determine for yourself!) that the **impedance parameters** of this two port network are:

$$\boxed{Z_{11} = Z_{22} = -jZ_0 \cot \beta l}$$

$$\boxed{Z_{12} = Z_{21} = -jZ_0 \operatorname{cosec} \beta l}$$

- With a little **trigonometry:**

$$\boxed{Z_{11} - Z_{12} = jZ_0 \tan\left(\frac{\beta l}{2}\right)}$$

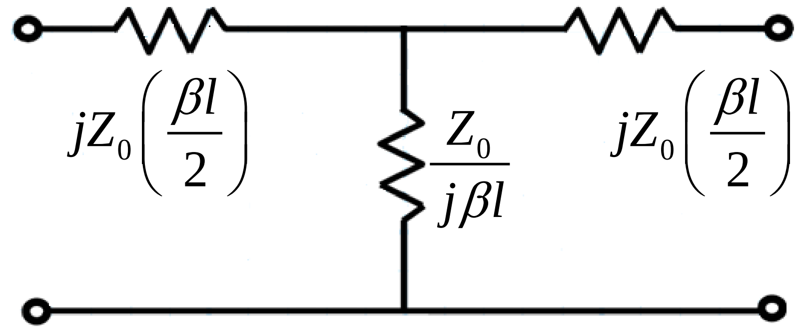
- For small βl:

$$\boxed{Z_{11} - Z_{12} \approx jZ_0\left(\frac{\beta l}{2}\right)}$$

$$\boxed{Z_{12} = Z_{21} = -jZ_0 \operatorname{cosec} \beta l \approx \frac{Z_0}{j\beta l}}$$

Stepped-Impedance Low-Pass Filters (contd.)

- Thus, an **electrically short** ($\beta l \ll 1$) transmission line can be **approximately modeled** with a "T-circuit" as:

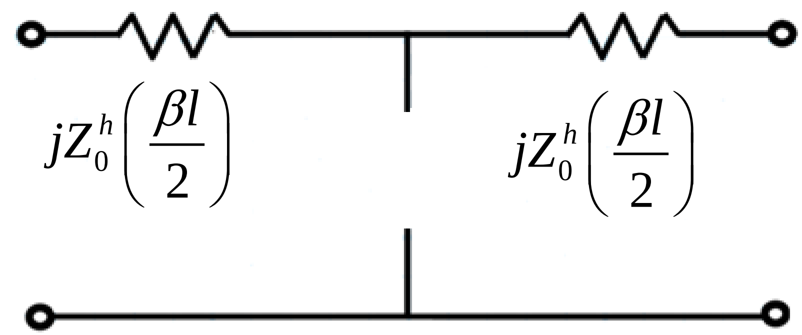


- Now, consider also the case where the **characteristic impedance** of the transmission line is **relatively large**. We'll **denote** this large characteristic impedance as ${Z_0}^h$.
- Note the **shunt** impedance value $\frac{{Z_0}^h}{j\beta l}$. Since the **numerator** (${Z_0}^h$) is relatively **large**, and the **denominator** ($j\beta l$) is **small**, the impedance of shunt device is **very large**.
- So large, in fact, that we can approximate it as an **open circuit**!

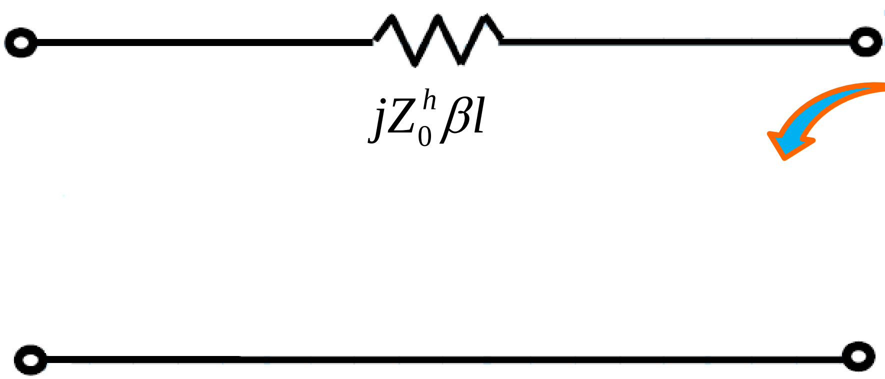
$$\boxed{\frac{Z_0^h}{j\beta l} \approx \infty} \qquad \text{For } \beta l \ll 1 \text{ and } {Z_0}^h \gg Z_0$$

Stepped-Impedance Low-Pass Filters (contd.)

- So now we have a further **simplification** of our **model**:



- The remaining impedances are now in **series**, so the circuit can be further simplified to:

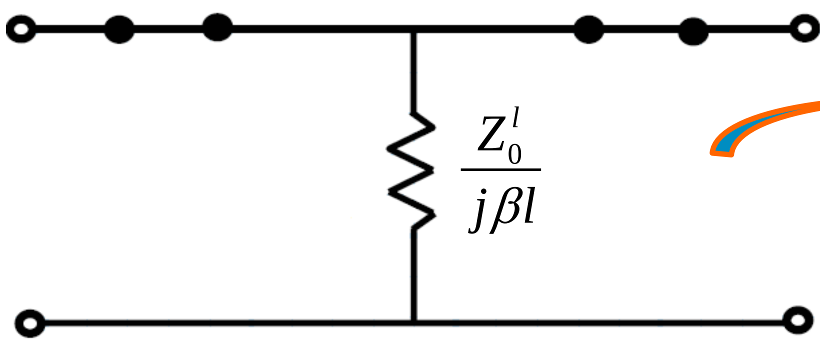


The equivalent circuit for transmission line with short electrical length ($\beta l \ll 1$) and large characteristic impedance ($Z_0{}^h \gg Z_0$)

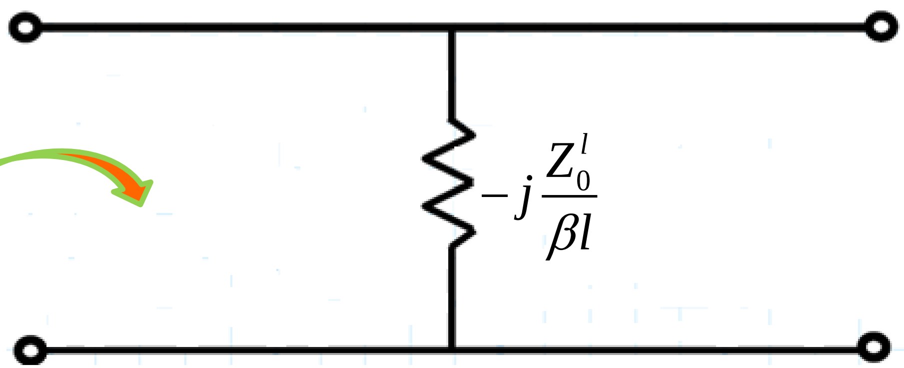
- Now, consider the case where the **characteristic impedance** of the transmission line has a relatively **low value**, denoted as $Z_0{}^l$, where $Z_0{}^l \ll Z_0$.

Stepped-Impedance Low-Pass Filters (contd.)

- In such a case: $\boxed{Z_{11} - Z_{12} \approx jZ_0^l\left(\frac{\beta l}{2}\right) \approx 0}$ For $\beta l \ll 1$ and $Z_0{}^l \ll Z_0$
- So now we have **another simplification** of our model:



Which **of course** further simplifies to



The equivalent circuit for transmission line with short electrical length ($\beta l \ll 1$) and small characteristic impedance ($(Z_0{}^l \ll Z_0)$).

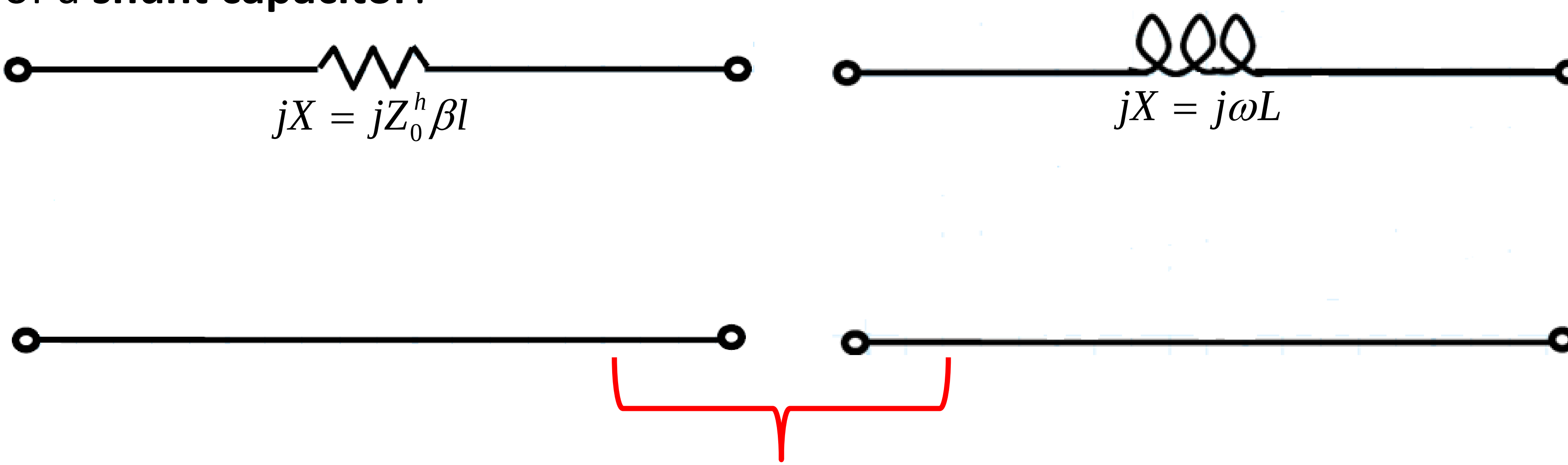
Q: But, **what** does all this have to do with constructing a **low-pass filter**?

A: Plenty! Recall that a lossless low-pass filter constructed with **lumped** elements consists of a "circuit ladder" of **series inductors** and **shunt capacitors**!

Stepped-Impedance Low-Pass Filters (contd.)

Q: So?

A: Look at the two **equivalent circuits** for an electrically short transmission line. The one with **large** characteristic impedance $Z_0{}^h$ has the form of a **series inductor**, and the one with **small** characteristic impedance $Z_0{}^l$ has the form of a **shunt capacitor**!



are identical **if**:

$$\boxed{jZ_0^h\beta l = j\omega L} \Rightarrow \boxed{Z_0^h\beta l = \omega L}$$

Stepped-Impedance Low-Pass Filters (contd.)

- Thus, the "**series inductance**" of our **transmission line** length is:

$$\boxed{L = \frac{Z_0^h \beta l}{\omega}}$$

Q: Yikes! Our inductance appears to be a function of **frequency** ω. I **assume** we simply **set** this value to **cut-off frequency** ω_c, just like we did for Richard's transformation?

A: Nope! We can **simplify** the result a bit more. Recall that $\beta = {}^{\omega}/_{v_p}$, so that:

$$\boxed{L = \frac{Z_0^h \beta l}{\omega} = \frac{Z_0^h l}{v_p}}$$

- In other words, the **series impedance** resulting from our short transmission line is:

$$\boxed{Z = j\omega L = j\omega\left(\frac{Z_0^h l}{v_p}\right)}$$

Q: Wow! This realization seems to give us a result that **precisely** matches an inductor at **all** frequencies—**right**?

A: Not quite! Recall this result was obtained from applying a few **approximations**—the result is **not** exact!

Stepped-Impedance Low-Pass Filters (contd.)

Moreover, one of the approximations was that the **electrical length** of the transmission line be **small**. This obviously **cannot** be true at **all** frequencies. As the signal frequency **increases**, so does the **electrical length**—our **approximate** solution will **no longer be valid**.

Thus, this realization is accurate **only** for "**low** frequencies"— recall that was **likewise** true for **Richard's transformations**!

Q: Low-frequencies? How **low** is **low**?

A: Well, for our filter to provide a response that **accurately** follows the **lumped element** design, our approximation should be valid for frequencies **up to** (and including!) the **filter cut-off frequency** ω_c.

- A general "**rule-of-thumb**" is that a **small electrical length** is defined as being **less than** π/4 radians. Thus, to maintain this small electrical length at frequency ω_c, our realization **must** satisfy the relationship:

$$\beta_c l = \frac{\omega L}{Z_0^h} < \frac{\pi}{4}$$

Stepped-Impedance Low-Pass Filters (contd.)

- Note that this criterion is **difficult** to satisfy if the **filter cut-off frequency** and/or the **inductance value L** that we are trying to realize is **large**.
- Our **only** recourse for these challenging conditions is to **increase** the value of **characteristic impedance** ${Z_0}^h$.

Q: Is there some particular difficulty with increasing ${Z_0}^h$?

A: Could be! There is always a **practical** limit to how large (or small) we can make the **characteristic impedance** of a transmission line.

For example, a **large** characteristic impedance implemented in **microstip/stripline** requires a **very narrow** conductor **width W.** But manufacturing tolerances, power handling capability and/or line loss (line resistance R increases as W decreases) place a **lower bound** on how **narrow** we can make these conductors!

- However, assuming that we **can** satisfy the above constraint, we can approximately "**realize**" a **lumped inductor** of inductance **value L** by selecting the correct **characteristic impedance** ${Z_0}^h$ and **line length** l of our short transmission line.

$$L = \frac{Z_0^h l}{v_p}$$

Stepped-Impedance Low-Pass Filters (contd.)

Q: For **Richard's Transformation**, we **first** set the stub length to a **fixed** value (i.e., $l = \lambda_c/8$), and **then** determined the **specific characteristic impedance** necessary to realize a **specific inductor value** L. I **assume** we follow the same procedure here?

A: Nope! When constructing stepped-impedance low-pass filters, we typically do the **opposite**!

1. **First**, we select the value of $Z_0{}^h$, making sure that the short electrical length inequality is **satisfied** for the **largest inductance value** L in our lumped element filter:

$$Z_0^h > \frac{4\omega_c L}{\pi}$$

This characteristic impedance value is typically used to realize **all** inductor values L in our low-pass filter, **regardless** of the actual value of inductance L.

2. Then, we determine the **specific lengths** l_n of the transmission line required to realize **specific** filter **inductors values** L_n:

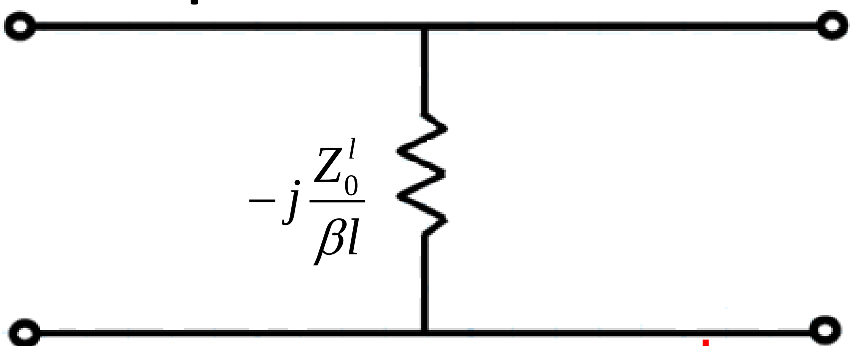
$$l_n = \left(\frac{v_p}{Z_0^h}\right) L_n$$

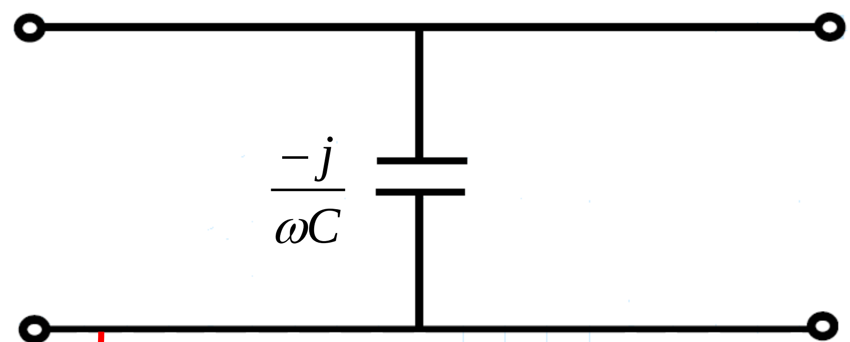
Stepped-Impedance Low-Pass Filters (contd.)

Q: What about the **shunt capacitors**?

A: Almost forgot!

- Recall the **low-impedance** transmission line provided a **shunt impedance** that matched a shunt capacitor:





$$-j\frac{Z_0^l}{\beta l} = \frac{-j}{\omega C}$$

are identical **if**:

$$\frac{\beta l}{Z_0^l} = \omega C$$

- Thus, the "**shunt capacitance**" of our transmission line length is:

$$C = \frac{\beta l}{\omega Z_0^l}$$

$$C = \frac{l}{v_p Z_0^l}$$

Stepped-Impedance Low-Pass Filters (contd.)

- And thus the **shunt reactance** of our transmission line realization is:

$$Z = \frac{-j}{\omega}\left(\frac{v_p Z_0^l}{l}\right)$$

Although this again **appears** to provide **exactly** the same behavior as a **capacitor** (as a function of frequency), it is likewise accurate **only** for **low frequencies**, where $\beta l < \frac{\pi}{4}$.

- Thus from our realization **equality**: $\frac{\beta l}{Z_0^l} = \omega C$

- We can conclude that for our approximations to be valid at all frequencies **up to** the filter **cut-off frequency**, the following inequality **must** be valid:

$$\beta_c l = \omega_c C Z_0^l < \frac{\pi}{4}$$

Note that for **difficult** design cases where ω_c and/or C is **very large**, the line **characteristic impedance** $Z_0{}^l$ must be made **very small**.

Stepped-Impedance Low-Pass Filters (contd.)

Q: I suppose there is **likewise** a problem with making $Z_0{}^l$ **very small**?

A: Yes! In microstrip and stripline, making $Z_0{}^l$ **small** means making conductor width W **very large**. In other words, it will take up **lots of space** on our substrate. For most applications the **surface area** of the substrate is both **limited** and precious, and thus there is generally a **practical limit** on how wide we can make width W (i.e., how **low** we can make $Z_0{}^l$).

- However, assuming that we **can** satisfy the above constraint, we can approximately **"realize"** a **lumped capacitor** of inductance value C by selecting the correct **characteristic impedance** $Z_0{}^l$ and **line length** l of our short transmission line:

$$\boxed{C = \frac{l}{v_p Z_0^l}}$$

Stepped-Impedance Low-Pass Filters (contd.)

- The **design rules** for **shunt capacitor realization** are:

1. **First**, we select the value of $Z_0{}^l$, making sure that the short electrical length inequality is **satisfied** for the **largest** capacitance value C in our lumped element filter:

$$\boxed{Z_0^l < \frac{\pi}{4\omega C}}$$

This characteristic impedance value is typically used to realize **all** capacitor values C in our low-pass filter, **regardless** of the actual value of capacitance C.

2. Then, we determine the **specific lengths** l_n of the transmission line required to realize **specific** filter capacitor values C_n:

$$\boxed{l_n = \left(v_p Z_0^l\right) C_n}$$

Stepped-Impedance Low-Pass Filters (contd.)

- An **example** of a low-pass, stepped-impedance filter design is provided on page 414-416 of **your** book