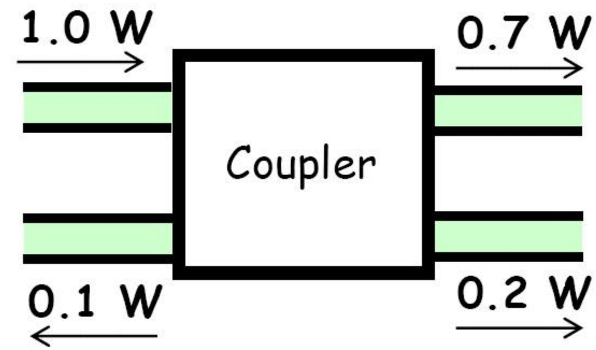
Lecture – 14

Date: 03.03.2016

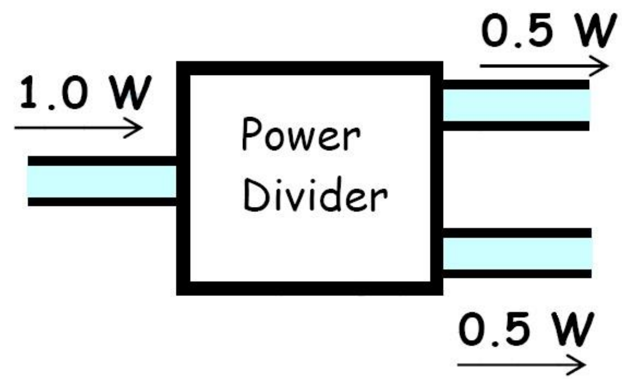
- Power Dividers and Couplers
- Basic Properties
- Power Divider – Design Aspect
- Circulator

Power Dividers and Couplers

- One of the most fundamental problems in RF/microwave engineering is how to efficiently **divide** signal power.



- The **simplest** RF/microwave problem would seemingly be to **equally** divide signal power in two:



However, building these devices is **more difficult** than you might think!

First let's examine four-port networks called **directional couplers**, and explain some fundamental values that characterize them

Directional Coupler

- A **directional coupler** is a 4-port network that is designed to **divide** and **distribute** power.
- Although this would seem to be a particularly **mundane** and simple task, these devices are both very **important** in high frequency systems, and at the same time very **difficult** to design and construct.
- Two of the **reasons** for this difficulty are our desire for the device to be:

1. **Matched**
2. **Lossless**

Thus, we require a **matched**, **lossless**, and (to make it simple) **reciprocal** 4-port device!

Recall that a matched, lossless, reciprocal, 4-port device was difficult to even **mathematically** determine, as the resulting scattering matrix must be (among other things) **unitary**.

Directional Coupler (contd.)

- However, we were able to determine two possible mathematical solutions, which we called the **symmetric** and **asymmetric** solutions respectively:

Symmetric

$$\mathbf{S} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

Asymmetric

$$\mathbf{S} = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

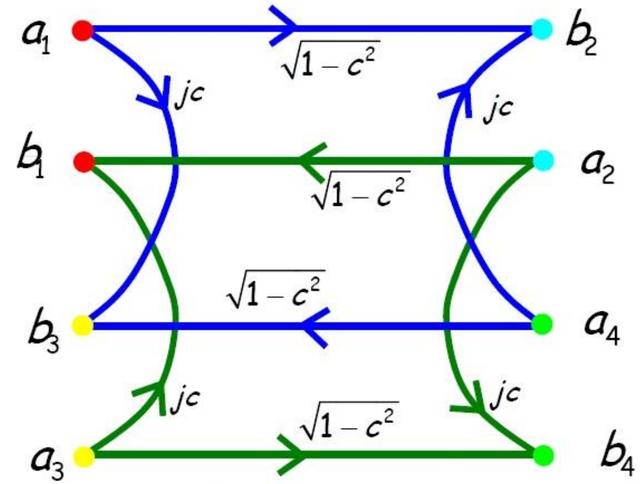
- For both cases, the relationship $|\alpha|^2 + |\beta|^2 = 1$ must be true in order for the device to be lossless (i.e, for **S** to be unitary)
- For most couplers it can be found that α and β can (at least ideally) be represented by a real value **c** , known as the **coupling coefficient**.

$$\beta = c \qquad\qquad \alpha = \sqrt{1-c^2}$$

Directional Coupler (contd.)

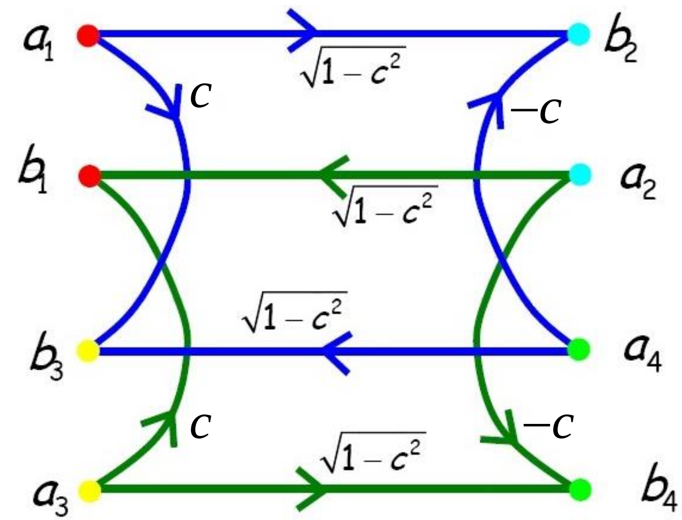
- The **symmetric** solution is thus described as:

$$\boldsymbol{S} = \begin{bmatrix} 0 & \sqrt{1-c^2} & jc & 0 \\ \sqrt{1-c^2} & 0 & 0 & jc \\ jc & 0 & 0 & \sqrt{1-c^2} \\ 0 & jc & \sqrt{1-c^2} & 0 \end{bmatrix}$$



- And the **asymmetric** solution is described as:

$$\boldsymbol{S} = \begin{bmatrix} 0 & \sqrt{1-c^2} & c & 0 \\ \sqrt{1-c^2} & 0 & 0 & -c \\ c & 0 & 0 & \sqrt{1-c^2} \\ 0 & -c & \sqrt{1-c^2} & 0 \end{bmatrix}$$



Directional Coupler (contd.)

- Additionally, for a directional coupler, the coupling coefficient c will be **always** less than $1/\sqrt{2}$. Therefore, we can express:

$$0 \le c \le \frac{1}{\sqrt{2}} \qquad \frac{1}{\sqrt{2}} \le \sqrt{1-c^2} \le 1$$

- Let's see what this means in terms of the **physical behavior** of a directional coupler.
- First, consider the case where some signal is incident on **port 1**, with power $\mathbf{P_1^+}$.
- If all other ports are matched, the power flowing out of **port 1** is:

$$P_1^- = |S_{11}|^2 P_1^+ = 0^2 * P_1^+ = 0$$

- While the power out of **port 2** is:

$$P_2^- = |S_{21}|^2 P_1^+ = (1-c^2) P_1^+$$

- and the power out of **port 3** is:

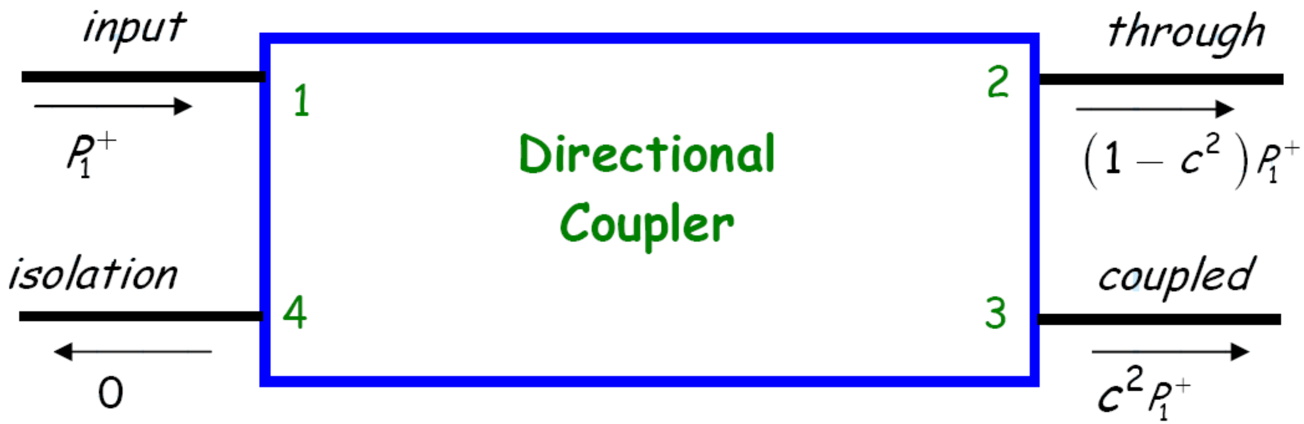
$$P_3^- = |S_{31}|^2 P_1^+ = c^2 P_1^+$$

- Finally, we find there is **no power** flowing out of **port 4**:

$$P_4^- = |S_{41}|^2 P_1^+ = 0^2 * P_1^+ = 0$$

Directional Coupler (contd.)

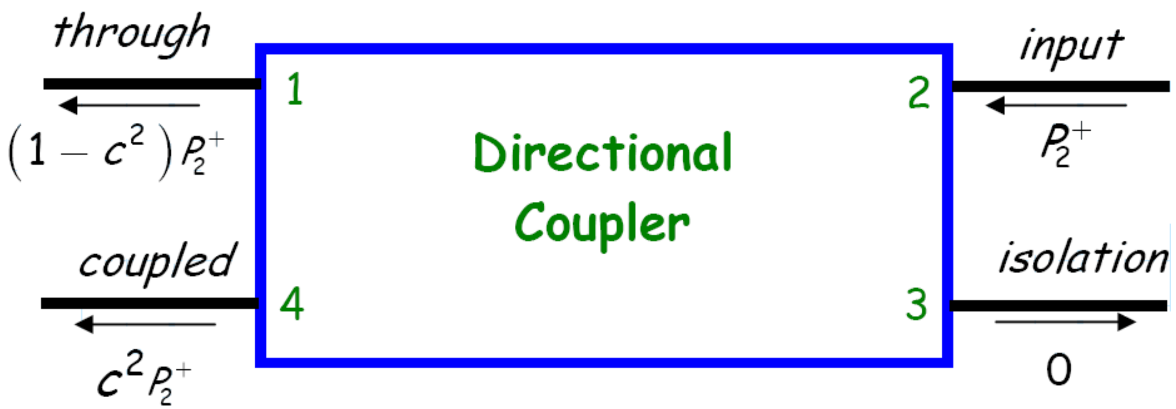
- In the terminology of the directional coupler, we say that **port 1** is the **input** port, **port 2** is the **through** port, **port 3** is the **coupled** port, and **port 4** is the **isolation** port



Note however, that **any** of the coupler ports can be an input, with a **different** through, coupled and isolation port for each case

Directional Coupler (contd.)

- For example, **if** a signal is incident on **port 2**, while all other ports are matched, we find that:



- Thus, from the scattering matrix of a directional coupler, we can form the following table:

Input	Through	Coupled	Isolation
Port 1	Port 2	Port 3	Port 4
Port 2	Port 1	Port 4	Port 3
Port 3	Port 4	Port 1	Port 2
Port 4	Port 3	Port 2	Port 1

Directional Coupler (contd.)

- **Typically**, the coupling coefficients for a directional coupler are in the range of approximately:

$$0.25 > c^2 > 0.0001$$

- As a result, we find that: $\sqrt{1-c^2} \approx 1$

What this means is that the power out of the **through** port is just **slightly smaller** (typically) than the power incident on the input port

Similarly, the power out of the **coupling** port is typically a **small fraction** of the power incident on the input port

Directional Coupler (contd.)

Q: Pfft! **Coupling Port Power**, Just a **small fraction** of the input power! What is the use in doing that??

A: A directional coupler is often used for **sampling** a small portion of the signal power. For example, we might **measure** the output power of the **coupled** port (e.g., P_3^-) and then we can determine the amount of signal power flowing through the device (e.g., $P_1^+ = P_3^-/c^2$).

Directional Coupler (contd.)

Unfortunately, the **ideal** directional coupler **cannot** be built! For example, the input match is never **perfect**, so that the diagonal elements of the scattering matrix, although **very small**, are not zero.

Similarly, the isolation port is never **perfectly** isolated, so that the values S_{41}, S_{32}, S_{23} and S_{14} are also non-zero—some **small** amount of power leaks out!

As a result, the through port will be **slightly less** than the value $\sqrt{1-c^2}$. The scattering matrix for a **non-ideal coupler** would therefore look like as:

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{21} & jc & S_{41} \\ S_{21} & S_{11} & S_{41} & jc \\ jc & S_{41} & S_{11} & S_{21} \\ S_{41} & jc & S_{21} & S_{11} \end{bmatrix}$$

From **this** scattering matrix, we can extract **some important parameters** about directional couplers

Directional Coupler (contd.)

Coupling Coefficient, C

The **coupling coefficient** is the ratio of the coupled output power (P_3^-) to the input power (P_1^+), expressed in decibels as:

$$C(dB) = 10\log_{10}\left[\frac{P_3^-}{P_1^+}\right] = -10\log_{10}|jc|^2$$

This is the **primary** specification of a directional coupler!

- Note the **larger** the coupling value, the **smaller** the coupled power! For example:
  - A **6 dB** coupler couples out **25%** of the input power
  - A **10 dB** coupler couples out **10%** of the input power
  - A **20 dB** coupler couples out **1.0%** of the input power
  - A **30 dB** coupler couples out **0.1%** of the input power

Directional Coupler (contd.)

Directivity, D

The **directivity** is the ratio of the power **out** of the coupling port (P_3^-) to the power **out** of the isolation port (P_4^-), expressed in decibels.

$$D(dB) = 10\log_{10}\left[\frac{P_3^-}{P_4^-}\right] = 10\log_{10}\left[\frac{|jc|^2}{|S_{41}|^2}\right]$$

This value indicates how effective the device is in "**directing**" the coupled energy into the correct port (i.e., into the coupled port, **not** the isolation port)

Ideally this is infinite (i.e.,P_4^- =0) , so the **higher** the directivity, the **better**

Directional Coupler (contd.)

Isolation, I

Isolation is the ratio of the **input power** (P_1^+) to the power out of the **isolation** port (P_4^-), expressed in decibels.

$$I(dB) = 10\log_{10}\left[\frac{P_1^+}{P_4^-}\right] = -10\log_{10}\left[|S_{41}|^2\right]$$

This value indicates how "isolated" the isolation port actually is. **Ideally** this is infinite (i.e., $P_4^- = 0$), so the **higher** the isolation, the better

- Note that isolation, directivity, and coupling are **not** independent values! **You** should be able to quickly show that:

$$I(dB) = C(dB) + D(dB)$$

Directional Coupler (contd.)

Mainline Loss, ML

The **mainline loss** is the ratio of the **input** power (P_1^+) to the power out of the **through** port (P_2^-), expressed in decibels.

$$ML(dB) = 10\log_{10}\left[\frac{P_1^+}{P_2^-}\right] = -10\log_{10}\left[\left|S_{21}\right|^2\right]$$

It indicates how much power the signal **loses** as it travels from the input to the through port

Coupling Loss, CL

The **coupling loss** indicates the **portion** of the mainline loss that is due to coupling some of the input power into the coupling port.

$$CL(dB) = 10\log_{10}\left[\frac{P_1^+}{P_1^+ - P_3^-}\right] = -10\log_{10}\left[1 - \left|jc\right|^2\right]$$

Directional Coupler (contd.)

$$CL(dB) = 10\log_{10}\left[\frac{P_1^+}{P_1^+ - P_3^-}\right] = -10\log_{10}\left[1 - |jc|^2\right]$$

Conservation of energy conveys that this loss is **unavoidable**

- Note this value can be **very small**, for example:
  - The coupling loss of a **10dB** coupler is **0.44 dB**
  - The coupling loss of a **20dB** coupler is **0.044 dB**
  - The coupling loss of a **30dB** coupler is **0.0044 dB**

Directional Coupler (contd.)

Insertion Loss, IL

Q: But wait, shouldn't $(P_1^+ - P_3^-) = P_2^-$, meaning the coupling loss and the mainline loss will be the **same exact value**?

A: Ideally this would be true.

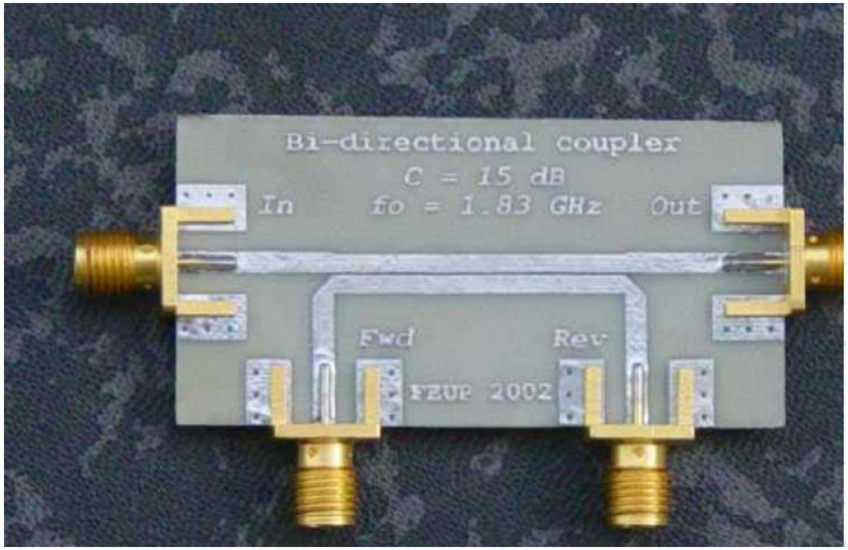
- But, the reality is that couplers are **not perfectly lossless**, so there will additionally be loss due to **absorbed** energy (i.e., heat). This loss is called **insertion loss** and is simply the **difference** between the mainline loss and coupling loss:

$$IL(dB) = ML(dB) - CL(dB)$$

The insertion loss thus indicates the portion of the mainline loss that is **not** due to coupling some input power to the coupling port. This insertion loss **is** avoidable, and thus the **smaller** the insertion loss, the better.

Directional Coupler (contd.)

- For couplers with **very small coupling** coefficients (e.g., C(dB) > 20) the coupling loss is so small that the mainline loss is almost entirely due to insertion loss (i.e., ML = IL) → often then, the two terms are used **interchangeably**.



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The T – Junction Power Divider

- Three-port couplers are also known as **T – Junction** Couplers, or T – Junction Dividers.
- Let us say that we desire a **matched** and **lossless** 3-port coupler.

Wait a minute! You already told that a matched, lossless, reciprocal **3-port** device of **any** kind is a **physical impossibility**!

Absolutely true! Our desire in this case will be **unfulfilled**. There are, however, a few designs that come **close**.

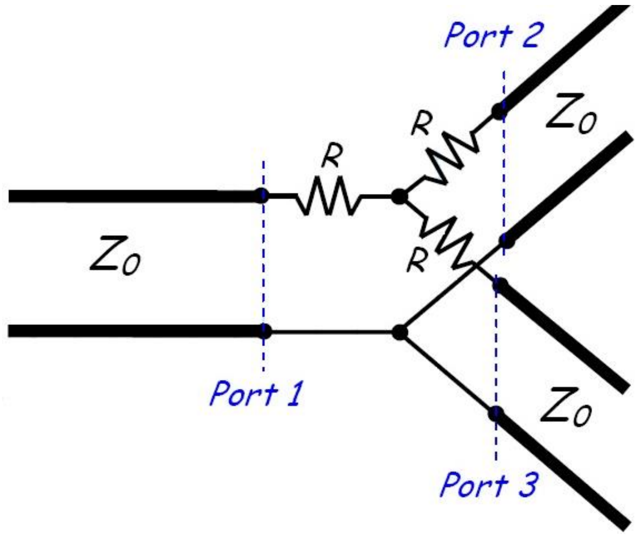
The T – Junction Power Divider (contd.)

1. **The Lossless Divider** – As the name states, this divider is lossless. It is likewise reciprocal, and thus is **not matched.**
2. **The Resistive Divider** – As the name implies, this divider **is lossy**. However, it is both matched and reciprocal.
3. **The Circulator** – This three-port divider is both matched and (ideally) lossy. This of course means that it is **not reciprocal**!
4. **The Wilkinson Divider** - Like the resistive divider, it is matched and reciprocal, and thus **is lossy**. However, it is lossy in a way that is not apparent when power is **divided** (i.e., power can be divided **without loss**).

As a result, the Wilkinson Power Divider is in most ways as **ideal** a T-junction as there is. Accordingly, it has its very **own importance** in RF/microwave applications !

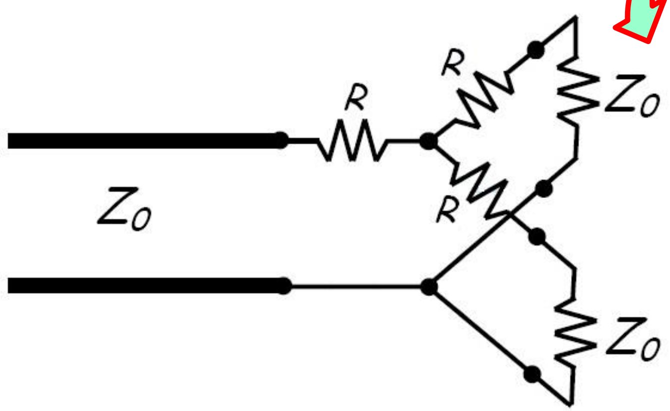
The Resistive Divider

- Let us consider the following **resistive power divider**:



This symmetric power divider will be matched at port 1 **if** R is selected as:

$$Z_0 = R + (R + Z_0) \| (R + Z_0)$$



$$R = \frac{Z_0}{3}$$

Condition for port 1 to be matched

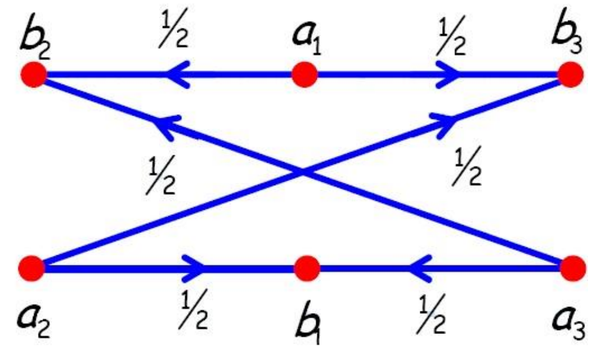
The Resistive Divider (contd.)

- From the **symmetry** of the circuit, we find that all the **other** ports will be matched as well (i.e., $S_{11} = S_{22} = S_{33} = 0$).
- Furthermore, it can be shown that:

$$S_{12} = S_{21} = S_{13} = S_{31} = S_{23} = S_{32} = \frac{1}{2}$$

- Therefore:

$$S = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$



- Note the magnitude of each column is less than one. e.g.:

$$|S_{21}|^2 + |S_{31}|^2 = \frac{1}{2} < 1$$

Therefore this power divider is **lossy**!

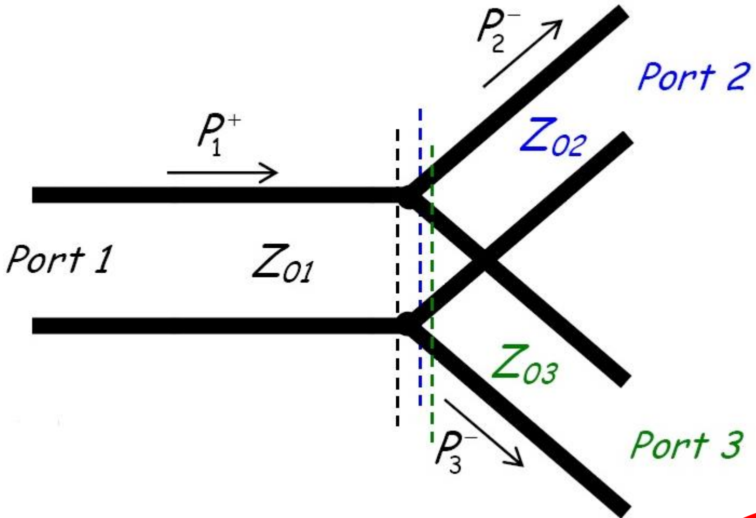
- In fact, we find that the power out of each port is just **one quarter** of the input power:

$$P_2^- = P_3^- = \left(P_1^+ / 4\right)$$

In other words, **half** the input power is **absorbed** by the divider!

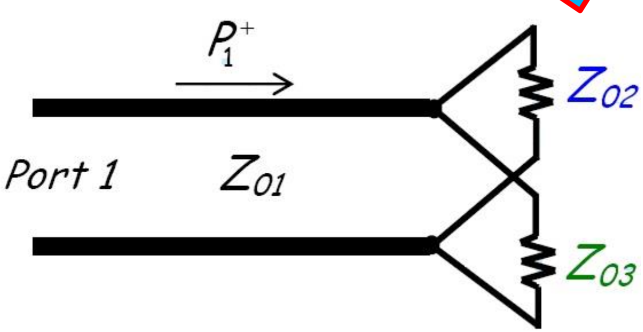
The Lossless Divider

- Now let us consider the following **lossless power divider**:



To be ideal, we want S_{11} = 0. Thus, when **ports 2** and **port 3** are **terminated** in matched loads, the input impedance at **port 1** (Z_{01}) must be related to impedances Z_{02} and Z_{03} as:

$$Z_{01} = \left(\frac{1}{Z_{02}} + \frac{1}{Z_{03}} \right)^{-1} = \frac{Z_{02} Z_{03}}{Z_{02} + Z_{03}}$$



Note, however, that this circuit configuration is **not** symmetric, thus we find that $S_{22} \neq 0$ and $S_{33} \neq 0$!

The Lossless Divider (contd.)

- As the divider is **lossless** (no resistive components), we can write:

$$P_1^+ = P_2^- + P_3^-$$

where P_1^+ is the power incident (and absorbed if $S_{11} = 0$) on port 1, and P_2^- and P_3^- is the power absorbed by the matched loads of ports 2 and 3.

- Unless $Z_{02} = Z_{03}$, the power will not divide equally between P_2^- and P_3^-. With a little high frequency circuit analysis, it can be shown that the **division ratio *k*** is:

$$k = \frac{P_2^-}{P_3^-} = \frac{Z_{03}}{Z_{02}}$$

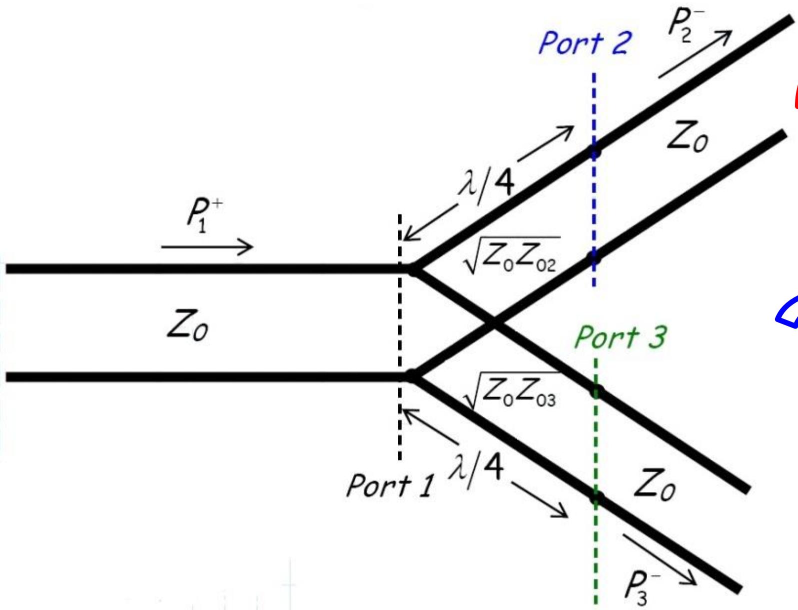
- Thus, if we desire an **ideal** ($S_{11} = 0$) divider with a specific division ratio ***k***, we will find that:

$$Z_{02} = Z_{01}\left(1 + \frac{1}{k}\right) \qquad Z_{03} = Z_{01}\left(1 + k\right)$$

The Lossless Divider (contd.)

Q: I don't understand how this is helpful. Don't we typically want the characteristic impedance of all three ports to be equal to the **same** value (e.g., $Z_{01} = Z_{02} = Z_{03} = Z_0$)?

A: True ! A more practical way to implement this divider is to use a **matching network**, such as a quarter wave transformer, on ports 2 and 3:



But beware! Recall that this matching network will work perfectly at only **one** frequency.

This lossless divider has a scattering matrix (at the design frequency) of this form:

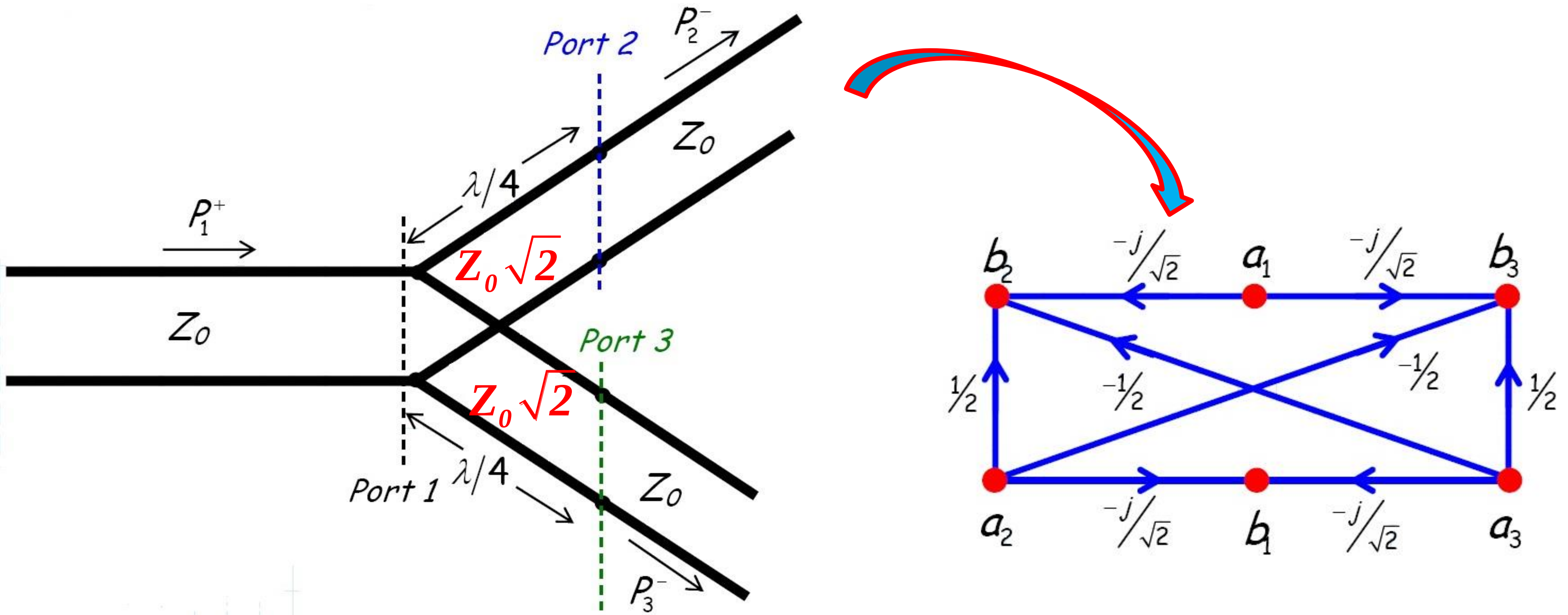
$$S = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & S_{22} & S_{23} \\ -j/\sqrt{2} & S_{32} & S_{33} \end{bmatrix}$$

The Lossless Divider (contd.)

- Where the (non-zero!) values of S_{22}, S_{23}, S_{32}, and S_{33} depend on the division ratio ***k***.
- Note that if we desire a **3 dB** divider (i.e., ***k*** = 1), then:

$$Z_{02} = Z_{03} = 2Z_{01}$$

- This **3dB** lossless divider (where $Z_{02} = Z_{03} = 2Z_{01}$), would have this design:

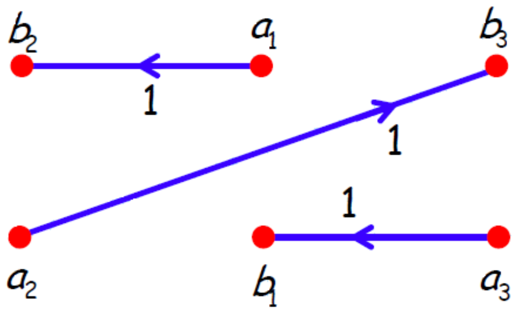


Circulators

- Circulator is a matched, lossless but **non-reciprocal 3-port** device, whose scattering matrix is **ideally:**

$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

SFG →

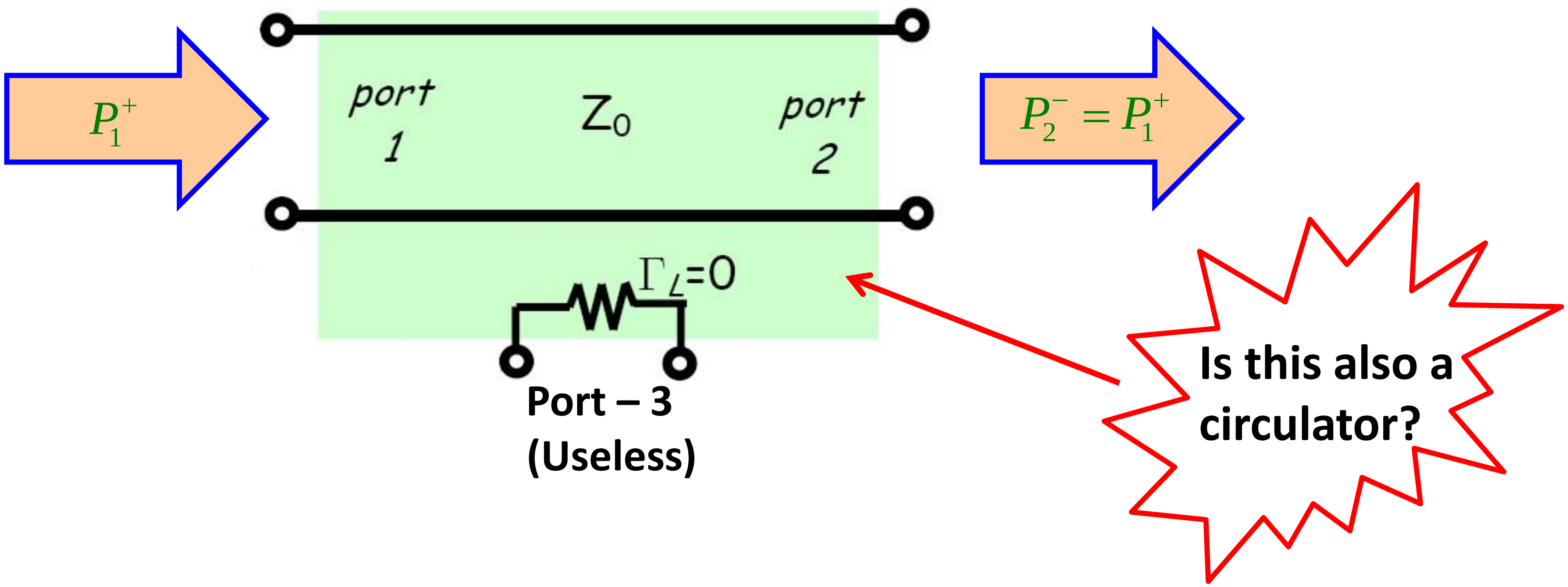


- Circulators use anisotropic **ferrite materials, which are often** "biased" by a permanent magnet! → The result is a **nonreciprocal** device!
- First, we note that for a circulator, the power incident on port 1 will exit **completely from port 2:**

$$P_2^- = P_1^+$$

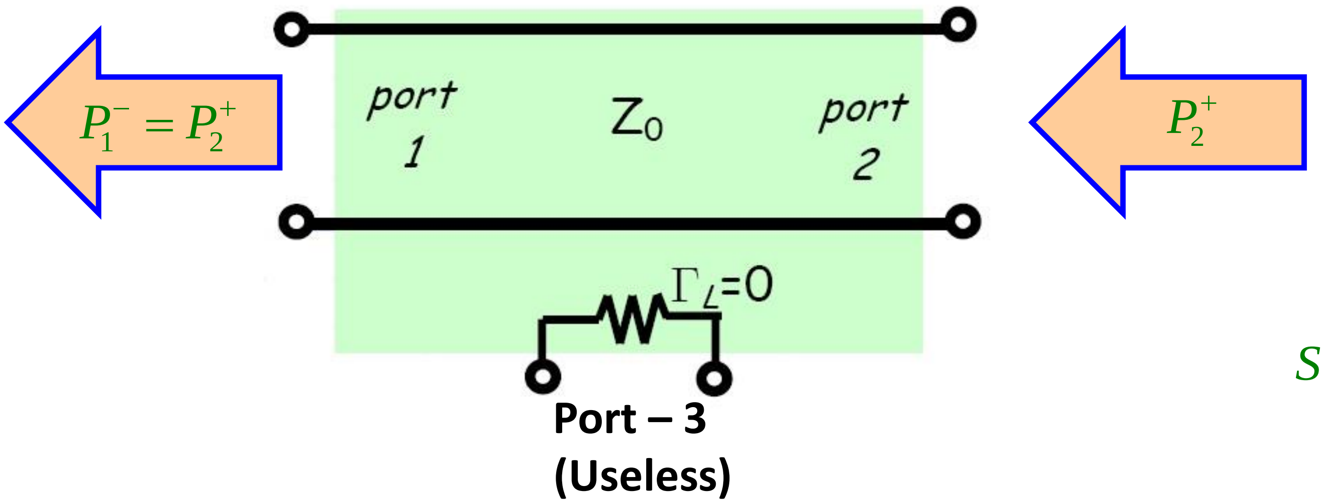
Circulators (contd.)

Pardon me while I feign **ignorance. This unremarkable** behavior is likewise true for the simple circuit below, which requires just a length of **transmission line. Oh please, continue to** waste our valuable time.



Circulators (contd.)

- True! But a transmission line, being a **reciprocal device, will** likewise result in the power **incident on port 2 of your simple** circuit to **exit completely from port 1 ($P_1^- = P_2^+$):**



$$S = e^{-j\beta l}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

But, this is **not true for a circulator! If power is incident on** port 2, then **no power will exit port 1 !**

Circulators (contd.)

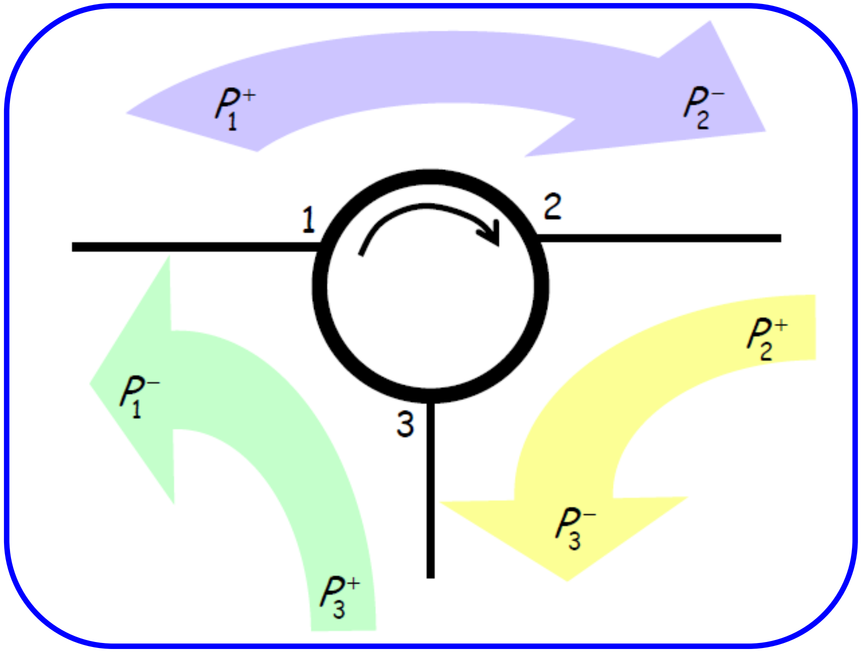
Q: **You have been surprisingly** successful in regaining my interest. Please tell us then, just **where does** the power incident on port 2 **go?**

A: It will exit from port 3 !

Likewise, power flowing into **port 3 will exit—port 1!**

It is evident, then how the circulator gets its **name: power** appears to **circulate around the device, a behavior that is** emphasized by its device **symbol**

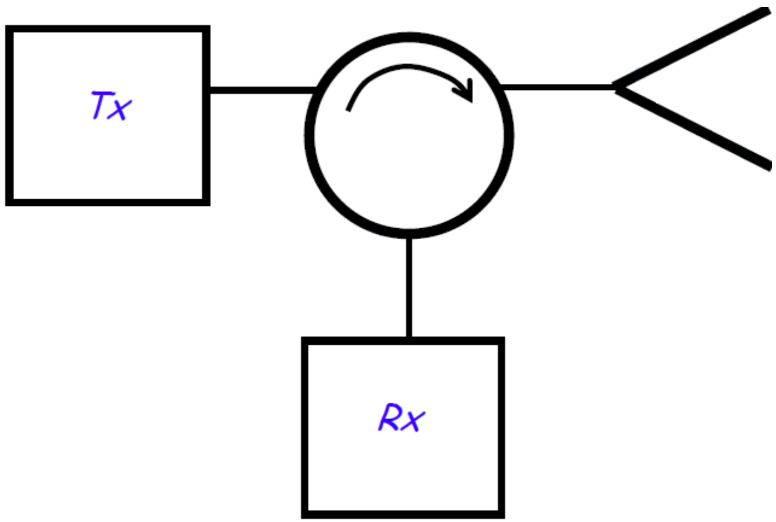
Circulators (contd.)



We can see that, for example, a **source at port 2 "thinks" it is** attached to a **load at port 3, while a load at port 2 "thinks" it is** attached to a **source at port 1!**

Circulators (contd.)

- These type of behavior is useful, for example, when we want to use **one antenna as both** the transmitter and receiver antenna. The transmit antenna (i.e., the load) at port 2 **gets its power from the transmitter at port 1. However, the receive antenna (i.e., the source) at port 2 delivers its power to the receiver at port 3!**



It is **particularly important to keep the transmitter power from** getting to the receiver. To accomplish this, the **antenna must** be **matched to the transmission line. Do you see why?**

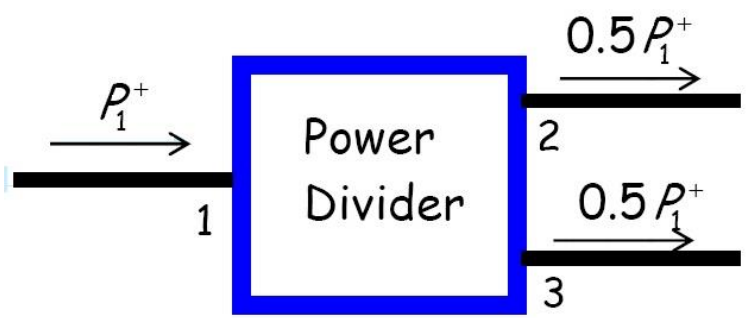
Circulators (contd.)

- It is important that we should note some major **drawbacks of a circulator:**

1. **They're expensive.**
2. **They're heavy.**
3. **The generally produce a large, static magnetic field.**
4. **They typically exhibit a large insertion loss (e.g.,** $|S_{21}|^2 = |S_{32}|^2 = |S_{13}|^2 \approx 0.75$).

The (Nearly) Ideal T- Junction Power Divider

- Recall that we **cannot build a matched, lossless** reciprocal **three-port device.**
- So, let's **mathematically try and determine the scattering** matrix of the best possible T-junction 3 dB **power divider.**



- To **efficiently divide the power incident on the input port, the** port (port 1) must first be **matched (i.e., all incident power** should be delivered to port 1): $S_{11} = 0$

- Likewise, this delivered power to port 1 must be divided efficiently (i.e., **without loss) between ports 2 and 3.**
- Mathematically, this means that the first column of the scattering matrix must have **magnitude of 1.0:**

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 1 \quad \xrightarrow{S_{11}=0} \quad |S_{21}|^2 + |S_{31}|^2 = 1$$

The (Nearly) Ideal T- Junction Power Divider (contd.)

- Provided that we wish to evenly divide the input power, we can conclude from the expression above that:

$$|S_{21}|^2 = |S_{31}|^2 = 1/2 \quad \Rightarrow \quad |S_{21}| = |S_{31}| = 1/\sqrt{2}$$

- Note that **this device would take the power into port 1 and** divide into **two equal parts—half exiting port 2, and half** exiting **port3 (provided ports 2 and 3 are terminated in** matched loads!).

$$P_2^- = |S_{21}|^2 P_1^+ = 0.5P_1^+ \qquad P_3^- = |S_{31}|^2 P_1^+ = 0.5P_1^+$$

- In addition, it is **desirable that ports 2 and 3 be matched** (the whole device is thus matched):

$$S_{22} = S_{33} = 0$$

- And also **desirable that ports 2 and 3 be isolated:**

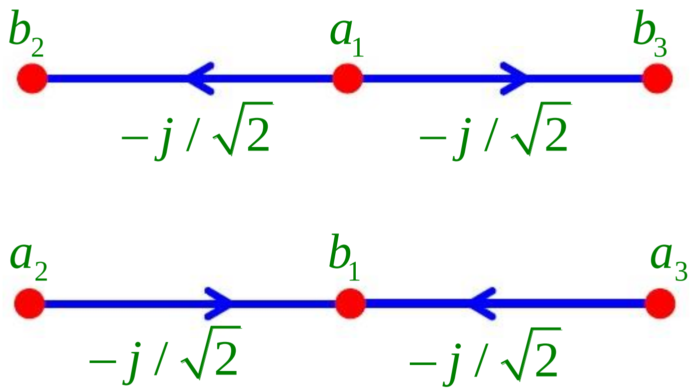
$$S_{23} = S_{32} = 0$$

This last requirement ensures that no signal incident on port 2 (e.g., reflected from a load) will **“leak” into port 3—and vice** versa.

The (Nearly) Ideal T- Junction Power Divider (contd.)

- The ideal 3 dB power divider **could therefore have the form:**

$$S = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 0 & 0 \\ -j/\sqrt{2} & 0 & 0 \end{bmatrix}$$



Since we can describe this ideal power divider **mathematically, we can potentially build it physically!**

Q: Huh!? I thought you said that a matched, lossless, reciprocal three-port device is **impossible?**

A: It is! This divider is clearly a lossy device. The magnitudes of both column 2 and 3 are less than one:

$$|S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 = \left|-j/\sqrt{2}\right|^2 + 0 + 0 = 0.5 < 1$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = \left|-j/\sqrt{2}\right|^2 + 0 + 0 = 0.5 < 1$$

The (Nearly) Ideal T- Junction Power Divider (contd.)

Note then that **half the power incident on port 2 (or port 3)** of this device would **exit port 1 (i.e., reciprocity), but no power** would exit port 3 (port2), since ports 2 and 3 are **isolated. i.e.,**

$$P_1^- = |S_{12}|^2 P_2^+ = 0.5P_2^+ \qquad P_3^- = |S_{32}|^2 P_2^+ = 0 * P_2^+ = 0$$

$$P_1^- = |S_{13}|^2 P_3^+ = 0.5P_3^+ \qquad P_2^- = |S_{23}|^2 P_3^+ = 0 * P_3^+ = 0$$

Q: Any ideas on how to build this thing?

A: Note that the first column of the scattering matrix is precisely the same as that of the **lossless 3 dB divider.**

Also note that since the device is **lossy, the design must** include some **resistors.**

Lossless Divider + resistors = The Wilkinson Power Divider

Topic of our next lecture!