

## <u>Lecture – 11</u>

## Date: 08.02.2016

• Impedance Matching using Lumped Components



## Introduction – Impedance Transformation

One of the most important and fundamental two-port networks that microwave engineers design is a **lossless matching network** (otherwise known as an **impedance transformer**).

**Q:** In high frequency circuits, a source and load are connected by a TL. Can we implement matching networks in transmission line circuits?

Furthermore, these matching networks seem too good to be true—can we **really** design and construct them to provide a **perfect** match?

A: We can easily provide a near perfect match at precisely one frequency

But, since lossless matching and transmission lines are made of entirely reactive elements (not to mention the reactive components of source and load impedance), we find that changing the frequency will typically "unmatch" our circuit!



## **Introduction – Impedance Transformation (contd.)**

Therefore, a difficult challenge for any RF/microwave design engineer is to design a **wideband** matching network—a matching network that provides an **"adequate"** match over a wide range of frequencies.

- Generally speaking, matching network design requires a **tradeoff** between following parameters for desirable attributes:
  - 1. Bandwidth
  - 2. Complexity
  - 3. Implementation
  - 4. Adjustability



## Matching with Lumped Components

- let's begin to examine how matching networks are **built**!
- we begin with the simplest solution: An L-network, consisting of a single capacitor and a single inductor.

Q: Just two elements! That seems simple enough. Do we always use these L-networks when constructing lossless matching networks?

A: Nope. L-networks have **two** major drawbacks:

- 1. They are narrow-band.
- 2. Capacitors and inductors are difficult to make at microwave frequencies!

Soon we will see how these L-networks actually **work** 



## Matching Network Analysis

Consider following circuit where a passive load is attached to an active source:





## Matching Network Analysis (contd.)

 We call the maximized power, the available power P<sub>avl</sub> from the source → it is, after all, the largest amount of power that the source can ever deliver!

$$P_{L}^{\max} \doteq P_{avl} = \frac{1}{2} \left| V_{g} \right|^{2} \frac{R_{g}}{\left| Z_{g} + Z_{g}^{*} \right|^{2}} = \frac{1}{2} \left| V_{g} \right|^{2} \frac{R_{g}}{\left| 2R_{g} \right|^{2}} = \frac{\left| V_{g} \right|^{2}}{8R_{g}}$$

- Note the available power of the source is dependent on source parameters only (i.e., V<sub>g</sub> and R<sub>g</sub>). This makes sense! Do you see why?
- Thus, we can say that to "take full advantage" of all the available power of the source, we must make the load impedance the complex conjugate of the source impedance.
- Otherwise, the power delivered to the load will be less than power made available by the source! In other "words":

$$P_L \leq P_{avl}$$

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## **Matching Networks and Transmission Lines**

**Note:** we can construct a network to transform the **input impedance** of the transmission line into the complex conjugate of **the source impedance**:



**Q:** But, do we **have** to place the matching network between the source and the transmission line?

A: Nope! We could **also** place a (different) matching network between the transmission line and the load.





## Matching Networks and Transmission Lines (contd.)

**Q:** So **which** method should we choose? Do engineers typically place the matching network between the source and the transmission line, **or** place it between the transmission line and the load?

A: Actually, the typical solution is to do **both**!

• We find that often there is a matching network between the a source and the transmission line, **and** between the line and the load.



**Q:** Yikes! Why would we want to build **two** separate matching networks, instead of just **one**?

## Matching Networks and Transmission Lines (contd.)

A: By using two separate matching networks, we can **decouple** the design problem. Recall again that the design of a **single** matching network solution would depend on four separate parameters:

- Alternatively, the design of the network matching the source and transmission line depends on only:
- In addition, the design of the network matching the load and transmission line depends on only:

- 1. the source impedance Z<sub>g</sub>
- 2. load impedance  $Z_L$
- 3. the TL characteristic impedance  $Z_0$
- 4. the TL length *l*
- **1.** the source impedance  $Z_g$

**2.** the transmission line characteristic impedance  $Z_0$ 

- **1.** the source impedance Z  $_{L}$
- **2.** the transmission line

characteristic impedance Z<sub>0</sub>

Note that **neither** design depends on the transmission line **Length** *l*!

**Q:** How is that possible?

A: Remember the case where  $Z_g = Z_0 = Z_L$ . For that special case, we found that a conjugate match was the result—regardless of the transmission line length.



## Matching Networks and Transmission Lines (contd.)

Thus, by matching the source to line impedance Z<sub>0</sub> and likewise matching the load to the line impedance, a conjugate match is assured—but the length of the transmission line does not matter!

In fact, the typical problem for microwave engineers is to match a load (e.g., device input impedance) to a **standard** transmission line impedance (typically  $Z_0 = 50\Omega$ ); **or** to independently match a source (e.g., device output impedance) to a **standard** line impedance.

A **conjugate match** is thus obtained by connecting the two with a transmission line of **any length**!





## **L-Network Analysis**

 Let us consider the first matching L-network, and denote it as matching network (A):



Note that this matching network consists of just **two** lumped elements, which must be **purely reactive** — in other words, a **capacitor** and an **inductor**!

 To make Γ<sub>in</sub> = 0, the input impedance of the network must be:

$$Z_{in} = Z_0$$

• Using **basic** circuit analysis we can find that:  

$$Z(z = 0) = Z_{in} = jX + \frac{(1/jB)Z_L}{(1/jB) + Z_L} = jX + \frac{Z_L}{1+jBZ_L}$$



## L-Network Analysis (contd.)

• Note that a **matched** network, with  $Z_{in} = Z_0$ ,  $\operatorname{Re}\{Z_{in}\} = Z_0$   $\operatorname{Im}\{Z_{in}\} = 0$  means that:

Gives two equations and aids in the determination of two unknowns (X, and B)

Essentially, the L-network matching network can be viewed as consisting of **two distinct parts**, each attempting to satisfy a specific requirement.



- $Y_1 \doteq Y + \frac{1}{Z_L} = jB + Y_L$
- The impedance of this element is therefore:  $Z_1 = \frac{1}{Y_1} = \frac{1}{\frac{jB + Y_L}{2}} = \frac{Z_L}{\frac{Z_L + jBZ_L}{2}}$





## L-Network Analysis (contd.)

• To achieve a perfect match, we must set the value of susceptance B such that:

$$\operatorname{Re}\left\{Z_{in}\right\} = \operatorname{Re}\left\{Z_{1}\right\} \quad \Longrightarrow Z_{0} = \operatorname{Re}\left\{\frac{Z_{L}}{Z_{L} + jBZ_{L}}\right\}$$

Thus if B is properly selected: 
$$Z = jX$$
  
 $Z_0, \beta$ 
 $\Gamma(0) = \Gamma_{in}$ 
 $Z(0) = Z_{in}$ 
 $Z_1 = Z_0 + jX_1$ 
reactive part of matching circuit

**Part 2: Selecting Z = jX:** Note that the ideal real impedance  $Z_1 = Z_L ||(1/jB)$  has the ideal real  $X_1 = \text{Im}\{Z_1\} = \text{Im}\{\frac{Z_L}{Z_1 + jBZ_L}\}$ value of  $Z_0$ . However, it also posses an **annoying** imaginary part of:

$$\left( Z_{L} + jBZ_{L} \right)$$

However, this imaginary component can be easily removed by setting the series element Z =jX to its equal but **opposite** value!

$$\boldsymbol{X} = -\boldsymbol{X}_1 = -\operatorname{Im}\left\{\frac{\boldsymbol{Z}_L}{\boldsymbol{Z}_L + j\boldsymbol{B}\boldsymbol{Z}_L}\right\}$$



**2)** For jB to be purely imaginary (i.e., reactive), B must be real  $\rightarrow$  R<sub>L</sub> must be greater than Z<sub>0</sub> (R<sub>L</sub> > Z<sub>0</sub>) to insure that B and thus X is real.

In other words, this matching network **(type-A)** can only be used when R<sub>L</sub> > Z<sub>0</sub>. Notice that this condition means that the normalized load z<sub>1</sub>' lies **inside** the r = 1 circle on the Smith Chart!



## L-Network Analysis (contd.)

Now let's consider the second of the two L-networks, which we call network (B). Note it is also formed with just two lumped elements.



• To make  $\Gamma_{in}$  = 0, the **input admittance** of the network must be:

$$Y_{in} = Y_0$$

- From circuit theory we can determine that the input  $Y_{in} = jB + \frac{1}{jX + Z_I}$ admittance for this network is:
- We can describe a **matched** network,  $\operatorname{Re}\{Y_{in}\} = Y_0$   $\operatorname{Im}\{Y_{in}\} = 0$  with  $Y_{in} = Y_0$  as:



## L-Network Analysis (contd.)

• For this design, we set the value of Z =jX such that:



 It is apparent that a perfect match will occur if the shunt element Y = jB is set to "cancel" the reactive component of Y<sub>1</sub>:

$$\mathbf{B} = -\operatorname{Im}\left\{Y_{1}\right\} = -\operatorname{Im}\left\{\frac{1}{jX + Z_{L}}\right\}$$



**Note:** 1) Because of the  $\pm$ , there are **two** solutions for B (and thus X) 2) For jB and jX to be purely imaginary (i.e., reactive), B and X must be **real**  $\rightarrow$  R<sub>L</sub> **must** be less than Z<sub>0</sub> (R<sub>L</sub> < Z<sub>0</sub>) to insure that B and X are real.

In other words, this matching network **(type-B)** can only be used when R<sub>L</sub> < Z<sub>0</sub>. Notice that this condition means that the normalized load z<sub>L</sub>' lies **outside** the r = 1 circle on the Smith Chart!



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## L-Network Analysis (contd.)



Smith Chart Regions Depicting the Validity Regions for Networks A and B Indraprastha Institute of Information Technology Delhi

and

## L – Type Matching Network (contd.)

• Once the values of X and B are found, we can determine the required values of inductance L and/or capacitance C, for the signal frequency  $\omega_0$ !



As a result, we see that the reactance or susceptance of the elements of our L-network will have the proper values for matching at precisely **one and only one frequency!** 

And this frequency **better be the signal frequency**  $\omega_0$  !



## L – Type Matching Network (contd.)

If the signal frequency changes from the design frequency, the reactance and susceptance of the matching network inductors and capacitors will change → As a result the circuit will **no longer** be matched

Therefore the L-Type matching network has a **narrow bandwidth!** 





## L – Type Matching Network (contd.)

 In addition; it becomes very difficult to build quality lumped elements with useful values past 1 or 2 GHz. Thus, L-network solutions are generally applicable only in the relatively low RF region (i.e., < 2GHz).</li>





## Example – 1

The output impedance of a transmitter operating at a frequency of **2GHz** is  $Z_T = (150 + j75)\Omega$ . Design an L–Type matching network, as shown below, such that maximum power is delivered to the antenna whose input impedance is  $Z_A = (75 + j15) \Omega$ .



Matching Circuit – determine C and L



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## Example – 1 (contd.)

Simplification gives:

$$\boldsymbol{R}_{T} = \boldsymbol{R}_{A}(1 - \boldsymbol{B}_{C}\boldsymbol{X}_{T}) + (\boldsymbol{X}_{A} + \boldsymbol{X}_{L})\boldsymbol{B}_{C}\boldsymbol{R}_{T}$$

$$\boldsymbol{X}_{T} = \boldsymbol{R}_{T}\boldsymbol{R}_{A}\boldsymbol{B}_{C} - (1 - \boldsymbol{B}_{C}\boldsymbol{X}_{T})(\boldsymbol{X}_{A} + \boldsymbol{X}_{L})$$

$$B_{C} = \frac{X_{T} \pm \sqrt{\frac{R_{T}}{R_{A}} \left(R_{T}^{2} + X_{T}^{2}\right) - R_{T}^{2}}}{R_{T}^{2} + X_{T}^{2}}$$

In this example, R<sub>T</sub> > R<sub>A</sub>; therefore the square root is +ve and therefore for positive B<sub>C</sub> (capacitor) we must choose the plus sign in this expression.

**Therefore:** 

$$X_{L} = \frac{1}{B_{C}} - \frac{R_{A}(1 - B_{C}X_{T})}{B_{C}R_{T}} - X_{A}$$

Insert the given values in the obtained expressions to get:

$$B_{C} = 9.3mS \Longrightarrow C = \frac{B_{C}}{\omega} = 0.73\,pF \qquad X_{L} = 76.9\Omega \Longrightarrow L = \frac{X_{L}}{\omega} = 6.1nH$$



## Example – 1 (contd.)

#### **Smith Chart Based Approach**

• Before we tread this path, let us have a look on Smith chart navigation when series/shunt reactances are added to any impedance (Z)

#### **Series Connection of Inductance to a given Impedance**





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## Example – 1 (contd.)

#### **Series Connection of Capacitance to a given Impedance**





## Example – 1 (contd.)

#### **Shunt Connection of Inductance to a given Impedance**





## Example – 1 (contd.)

#### **Shunt Connection of Capacitance to a given Impedance**





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## Example – 1 (contd.)

#### Normalize the transmitter and antenna impedances

• Since no  $Z_0$  is given, one can choose any. In this example,  $Z_0 = 75\Omega$  makes simplification easier.

$$z_T = Z_T / Z_0 = 2 + j1$$
  $z_A = Z_A / Z_0 = 1 + j0.2$ 



- C is in shunt with  $Z_T \rightarrow$  the movement will be downward on a constant conductance circle  $\rightarrow$  starting point will be  $z_T'$  and the end point will be its intersection with  $z_M = 1 - j0.2$  circle
- Then the movement will be upward on a constant resistance circle (from  $z_M = (z_A')^* = 1 j0.2$ )  $\rightarrow$  to account for the series inductance L



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# Example – 1 (contd.)

Therefore, the normalized susceptance  $jb_c$  is:

$$jb_c = y_{TC} - y_T = 0.4 + j0.49 - (0.4 - j0.2) = j0.69$$

Similarly, the normalized reactance  $jx_L$  is:

$$jx_L = z_A^{*'} - z_{TC} = 1 - j0.2 - (1 - j1.22) = j1.02$$

Finally,

$$L = \frac{x_L Z_0}{\omega} = 6.09 nH$$
$$C = \frac{b_c}{\omega Z_0} = 0.73 pF$$



## L – Type Matching Network (contd.)

#### **Various configurations**



Their usefulness is regulated by the specified source and load impedances and the associated matching requirements

## L – Type Matching Network (contd.)

#### **Design procedure for two element L – Type matching Network**

- 1. Find the normalized source and load impedances.
- 2. In the Smith chart, plot circles of constant resistance and conductance that pass through the point denoting the source impedance.
- 3. Plot circles of constant resistance and conductance that pass through the point of the complex conjugate of the load impedance  $(z_M = z_L^*)$ .
- 4. Identify the intersection points between the circles in steps 2 and 3. The number of intersection points determine the number of possible L-type matching networks.
- 5. Find the values of normalized reactances and susceptances of the inductors and capacitors by tracing a path along the circles from the source impedance to the intersection point and then to the complex conjugate of the load impedance.
- 6. Determine the actual values of inductors and capacitors for a given frequency.



## Example – 2

Using the Smith chart, design all possible configurations of discrete twoelement matching networks that match the source impedance  $Z_s = (50+j25)\Omega$ to the load  $Z_L = (25-j50)\Omega$ . Assume a characteristic impedance of  $Z_0 = 50\Omega$  and an operating frequency of f = 2 GHz

#### **Solution**

1. The normalized source and load impedances are:

$$z_{s} = Z_{s} / Z_{0} = 1 + j0.5 \qquad y_{s} = 0.8 - j0.4$$
$$z_{s} = Z_{L} / Z_{0} = 0.5 - j1 \qquad y_{s} = 3 + j0.8$$

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## Example – 2 (contd.)

 Mark z<sub>s</sub> and then plot circles of constant resistance and conductance that passes through z<sub>s</sub>.

3. Mark  $(z_L)^*$  and then plot circles of constant resistance and conductance that passes through  $(z_L)^*$ .



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## Example – 2 (contd.)

4. The intersection points of these circles are A, B, C and D with the normalized impedances and inductances as:

$$z_{A} = 0.5 + j0.6 \qquad y_{A} = 0.8 - j1$$

$$z_{B} = 0.5 - j0.6 \qquad y_{B} = 0.8 + j1$$

$$z_{C} = 1 - j1.2 \qquad y_{C} = 3 + j0.5$$

$$z_{D} = 1 + j1.2 \qquad y_{D} = 3 - j0.5$$

5. There are four intersection points and therefore four L-type matching circuit configurations are possible.

$$z_{S} \rightarrow z_{A} \rightarrow (z_{L})^{*}$$

$$z_{S} \rightarrow z_{B} \rightarrow (z_{L})^{*}$$

$$z_{S} \rightarrow z_{C} \rightarrow (z_{L})^{*}$$

$$z_{S} \rightarrow z_{D} \rightarrow (z_{L})^{*}$$
Shunt L, Series L  
Shunt C, Series L  
Series C, Shunt L  
Series L, Shunt L



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## Example – 2 (contd.)

6. Find the actual values of the components

#### In the first case:

 $z_S \rightarrow z_A$ : the normalized admittance is changed by

$$jb_{L_2} = y_A - y_S = -j0.6 \qquad \Rightarrow L_2 = -\frac{Z_0}{b_{L_2}\omega} = 6.63nH$$

 $z_A \rightarrow z_L$  : the normalized impedance is changed by

$$jx_{L_1} = (z_L)^* - z_A = j0.4 \qquad \Rightarrow L_1 = \frac{x_{L_1}Z_0}{\omega} = 1.59nH$$

Therefore the circuit is:





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## Example – 2 (contd.)

#### **Similarly:**





2.79 nH

