Ques 1 - a) Find the angle between the vectors

$$A = \sqrt{3}\, i_x + i_y \quad , \quad B = 2 i_x$$

b) Find unit vector perpendicular in right sense to vectors

$$A = -i_x + i_y + i_z \quad , \quad B = i_x - i_y + i_z$$

Ques 2. Express the vector

$A = 3\cos\phi\, \hat{a_\rho} - 2\rho\, \hat{a_\phi} + 5\, \hat{a_z}$ in cartesian coordinates

$$\cancel{\vec{A} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}}$$

Ques 3. The position P in spherical coordinate system is

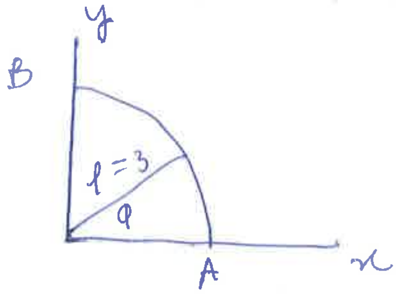
$(8, 120°, 330°)$

Specify its location in

a) Cartesian coordinates

b) Cylindrical coordinates

Ques 4. Given $F = \hat{a}_x\, xy - \hat{a}_y\, 2x$, evaluate the scalar line integral $\int_A^B F \cdot dl$ along quater circle



Ques 5 Find gradient of following scalar field:

$$W = 10r \sin^2\theta \cos\phi$$

Ques 6 Given $W = x^2y^2 + xyz$ compute ∇W and directional derivative dW/dl in the direction $3a_x + 4a_y + 12a_z$ at $(2, -1, 0)$

Ques 7 Determine divergence of vector field

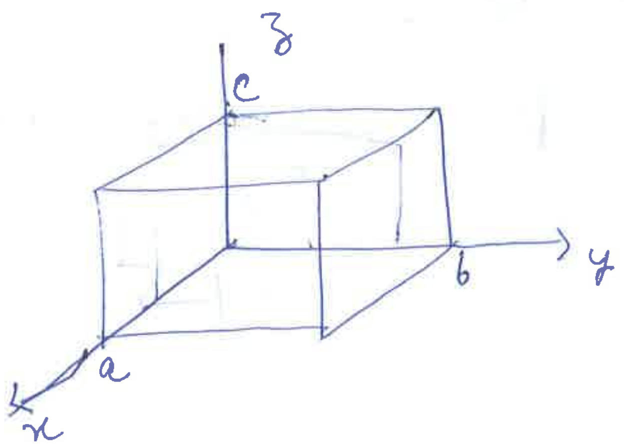
$$T = \frac{1}{r^2}\cos\theta\, \hat{a_r} + r\sin\theta\cos\phi\, \hat{a_\theta} + \cos\theta\, \hat{a_\phi}$$

Ques 8 Verify divergence theorem for vector

$$A = x\, i_x + y\, i_y + z\, i_z = r\, i_r$$ by

evaluating both sides of

$$\phi = \oint_S A \cdot ds = \int_V \nabla \cdot A\, dv$$ for rectangular volume



Ques ~~#~~9) Determine curl of

$$P = x^2 y z \, a_x + x z \, a_z$$

Ques 10) Given $F = a_x xy - a_y 2x$ verify stokes theorem ~~f~~. over
~~Ques 10~~ a quater circular disk with radius 3 in the 1st quadrant.

Solu 5

Solu 1

a) $A \cdot B = |A||B| \cos\theta$

$$\cos\theta = \frac{A \cdot B}{|A||B|}$$

$$A \cdot B = (\sqrt{3}\, i_x + i_y) \cdot 2\, i_x$$

$$= 2\sqrt{3}$$

$$|A| = \sqrt{3+1} = 2$$

$$|B| = 2$$

$$\Rightarrow \cos\theta = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \cos^{-1}\frac{\sqrt{3}}{2} = 30^\circ$$

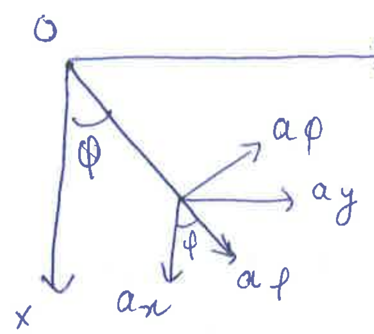
b) $$A \times B = \det\begin{vmatrix} i_x & i_y & i_z \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 2(i_x + i_y)$$

$$i_n = \frac{A \times B}{|A \times B|} = \frac{1}{\sqrt{2}}(i_x + i_y)$$

Solu 2

$$\vec{A} = A_\rho \hat{a_\rho} + A_\phi \hat{a_\phi} + A_z \hat{a_z}$$

$$\vec{A} = (\vec{A} \cdot \hat{a_x})\hat{a_x} + (\vec{A} \cdot \hat{a_y}) a_y + (\vec{A} \cdot \hat{a_z})\hat{a_z}$$



$A_x = A \cdot a_x$

$= A_\rho a_\rho \cdot a_x + A_\phi a_\phi \cdot a_x + A_z a_z \cdot a_x$

$a_\rho \cdot a_x = \cos\phi$

$a_\phi \cdot a_x = \cos(\pi/2 + \phi) = -\sin\phi$

$a_z \cdot a_x = 0$

$\Rightarrow A_x = A_\rho \cos\phi - A_\phi \sin\phi \quad -(1)$

$A_y = A \cdot a_y = A_\rho a_\rho \cdot a_y + A_\phi a_\phi \cdot a_y + A_z a_z \cdot a_y$

$= A_\rho \sin\phi + A_\phi \cos\phi \quad -(2)$

$A_z = A_z \cdot a_z \quad -(3)$

Using (1), (2), (3)

$A = a_x(3\cos^2\phi + 2\rho\sin\phi) + a_y(3\sin\phi\cos\phi - 2\rho\cos\phi) + a_z 5$

$\cos\phi = \frac{x}{\sqrt{x^2+y^2}}, \quad \sin\phi = \frac{y}{\sqrt{x^2+y^2}}$

$\Rightarrow A = \hat{a}_x\left(\frac{3x^2}{x^2+y^2} + 2y\right) + \hat{a}_y\left(\frac{3xy}{x^2+y^2} - 2x\right) + \hat{a}_z 5$ Ans

Solu 3

a) $x = r\sin\theta\cos\phi = 8\sin 120^\circ \cos 330^\circ = 6$

$y = r\sin\theta\sin\phi = 8\sin 120 \sin 330^\circ = -2\sqrt{3}$

$z = r\cos\theta = 8\cos 120^\circ = -4$

$\Rightarrow P(6, -2\sqrt{3}, -4)$ and position vector is

$OP = \hat{a}_x 6 - \hat{a}_y 2\sqrt{3} - \hat{a}_z 4$

6)

$\rho = r \sin\theta$

$\phi = \phi$

$z = r\cos\theta$

$P(4\sqrt{3}, 330°, -4)$

Position vector in cylindrical coordinates

$\overline{OP} = a_r 4\sqrt{3} - a_z 4$

~~Ques 4 Given $F = a_x xy - a_y 2x$, evaluate the scalar line~~
~~inte~~

Solu. 4

$F \cdot dl = xy\, dx - 2x\, dy$

$$\int_A^B F\cdot dl = \int_3^0 x\sqrt{9-x^2}\,dx - 2\int_0^3 \sqrt{9-y^2}\,dy$$

$$= -\frac{1}{3}(9-x^2)^{3/2}\Big]_3^0 - \left[y\sqrt{9-y^2} + 9\sin^{-1}\frac{y}{3}\right]_0^3$$

$$= -9(1+\pi/2)$$

Solu5: $\nabla W = \frac{\partial W}{\partial r} a_r + \frac{1}{r}\frac{\partial W}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\partial W}{\partial \phi} a_\phi$

$= 10 \sin 2\theta \cos\phi\, a_r + 10 \sin 2\theta \cos\phi\, a_\theta - 10 \sin\theta \sin\phi\, a_\phi$

Soln 6

$$\nabla w = \frac{\partial w}{\partial x} a_x + \frac{\partial w}{\partial y} a_y + \frac{\partial w}{\partial z} a_z$$

$$= (2xy^2 + yz)\hat{a_x} + (2x^2y + xz)a_y + (xy)a_z$$

at $(2,-1,0)$, $\nabla w = 4\hat{a_x} - 8\hat{a_y} - 2\hat{a_z}$

$$\frac{dw}{dl} = \nabla w \cdot \hat{a_l} = (4,-8,-2) \cdot \frac{(3,4,12)}{13} = \frac{-44}{13}$$

Soln 7:

$$\nabla \cdot T = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(T_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(T_\phi)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\cancel{\cos\theta}^{\,0} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(r\sin^2\theta\cos\phi) + \frac{1}{r\sin\theta}\cancel{\frac{\partial}{\partial\phi}\cos\theta}_{\,0}$$

$$= \frac{1}{r\sin\theta} 2r\sin\theta\cos\theta\cos\phi$$

$$= 2\cos\theta\cos\phi$$

Soln 8:

$$\int_v \nabla \cdot A\, dv = 3abc$$

$$\Phi = \oint_S A \cdot ds = \int_1 A_x(a)\, dy\, dz - \int_{1'} A_x(0)\, dy\, dz + \int_2 A_y(b)\, dx\, dz - \int_{2'} A_y(0)\, dx\, dz + \int_3 A_z(c)\, dx\, dy - \int_{3'} A_z(0)\, dx\, dy = 3abc$$

Solu 9:

a) $a_x + y\, a_y + (4y - z) a_z$

Solu 10: (Fig. Ques 4)

$$\nabla \times F = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & -2x & 0 \end{vmatrix}$$

$$= -a_z(2+x)$$

$$\therefore \int_S (\nabla \times F) \cdot ds = \int_0^3 \int_0^{\sqrt{9-y^2}} (\nabla \times F) \cdot (a_z\, dx\, dy)$$

$$= \int_0^3 \left[\int_0^{\sqrt{9-y^2}} -(2+x)\, dx \right] dy$$

$$= -9(1 + \pi/2)$$

Now from B to O: $x = 0$ $F \cdot dl = F \cdot (a_y\, dy) = -2x\, dy = 0$

O to A: $y = 0$ $F \cdot dl = F \cdot (a_x\, dx) = xy\, dx = 0$

$$\Rightarrow \oint_{ABOA} F \cdot dl = \int_A^B F \cdot dl = -9(1 + \pi/2)$$

So, Stokes theorem is verified