Test - 3

(1) (a) $\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z\right]$

$\therefore \vec{E} = -\left[2\rho z \sin\phi\, \hat{a}_\rho + \rho z \cos\phi\, \hat{a}_\phi + \rho^2 \sin\phi\, \hat{a}_z\right]$

Then, $W_E = \frac{1}{2}\epsilon_0 \int_V |\vec{E}|^2\, dv = \int_1^4 \int_{-2}^{2} \int_0^{\pi/3} \left(4\rho^2 z^2 \sin^2\phi \,\hat{a}_\rho + \rho^2 z^2 \cos^2\phi + \rho^4 \sin^2\phi\right)\rho\, d\phi\, dz\, d\rho$

$\therefore W_E = \frac{1507.67}{2}\left[\frac{10^{-9}}{36\pi}\right]$ marks 0.5

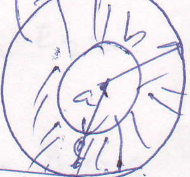
(b) $\vec{J} = 10 z \sin^2\phi\, \phi\, \hat{a}_\rho \quad A/m^2$

$\vec{ds} = \rho\, d\phi\, dz\, \hat{a}_\rho \Rightarrow I = \int_S \vec{J}\cdot\vec{ds}$ marks 0.5

$\Rightarrow I = \int_{\phi=0}^{2\pi}\int_{z=1}^{5} 10 z \sin^2\phi\, \rho\, dz\, d\phi \Big|_{\rho=2} \quad \therefore I = 240\pi = 754\,A$

(2) (a)

$\epsilon = \epsilon_0 \epsilon_r$



$0 < r < a$: $\boxed{\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r}$ from Gauss's Law

$\therefore \boxed{\vec{E} = \frac{Q}{4\pi \epsilon_0 r^2}\hat{a}_r}$ & $\boxed{\vec{P} = \vec{D} - \epsilon_0\vec{E} = 0}$

$\underline{a < r < b}$

$$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r \quad \text{from Gauss}$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0\epsilon_r r^2}\hat{a}_r$$

$$\vec{P} = \chi_e \epsilon_0 \vec{E} = \frac{\epsilon_r - 1}{\epsilon_r}\frac{Q}{4\pi r^2}\hat{a}_r$$

$\underline{r > b}$

$$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r \qquad \therefore \vec{P} = 0$$

So,

$$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r, \quad r > 0.$$

$$\vec{E} = \begin{cases} \dfrac{Q}{4\pi\epsilon_0\epsilon_r r^2}\hat{a}_r, & a < r < b \\ \dfrac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r & \text{elsewhere} \end{cases}$$

1.5 marks

$$\vec{P} = \begin{cases} \dfrac{\epsilon_r - 1}{\epsilon_r}\dfrac{Q}{4\pi r^2}\hat{a}_r & a < r < b \\ 0 & \text{elsewhere} \end{cases}$$

(2)

$$\vec{P} = (\epsilon - \epsilon_0)\vec{E}$$

$$\vec{D} = \frac{\epsilon_r}{\epsilon_r - 1}\vec{P}$$

marks: 0.5

we know:

$$\chi_e = \epsilon_r - 1$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E} = \epsilon_0(\epsilon_r - 1)\vec{E}$$

$$= (\epsilon_0 \epsilon_r \vec{E} - \epsilon_0 \vec{E}) = (\epsilon - \epsilon_0)\vec{E}$$

$$\vec{D} = \cancel{\epsilon_0 \vec{E}}$$

$$\vec{D} = \epsilon \vec{E} = \boxed{\epsilon_0 \epsilon_r \vec{E}} = \frac{\epsilon_0 \epsilon_r}{\epsilon_0 \chi_e}\vec{P}$$

$$\Rightarrow \vec{D} = \frac{\epsilon_r}{\chi_e}\vec{P} \quad \therefore \vec{D} = \left(\frac{\epsilon_r}{\epsilon_r - 1}\right)\vec{P}$$

(3)

Apply boundary conditions:

$$E_{1x} = E_{2x} = 2 \quad , \quad E_{1y} = E_{2y} = -3$$

for the z-component:

$$\epsilon_1 E_{1z} = \epsilon_2 E_{2z} \quad \Rightarrow E_{1z} = \frac{8\epsilon_0}{2\epsilon_0} E_{1z} = 12$$

$$\therefore \vec{E}_1 = 2\hat{a}_x - 3\hat{a}_y + 12\hat{a}_z \text{ V/m}$$

marks: 1

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$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

1 marks

No y- and z- component.

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} = 0 \quad \Rightarrow V(x) = c_1 x + c_2$$

$$\Rightarrow \cancel{V(x) = c_1 x + c_2}$$

$x = 0$, $V(x) = 0$

$x = 10$ cm, $V(10) = -100$ v.

$$\Rightarrow 0 = c_2$$

$$-100 = c_1 \frac{10}{100} + 0$$

$$\Rightarrow c_1 = -1000$$

$$\Rightarrow \boxed{V = -1000x} \; V$$

$$\boxed{\vec{E} = -\nabla \vec{V} = 1000\, \hat{a}_x \;\; V/m}$$