Quiz-1 Sol^n

Q:(1)

$$S = \int_{\theta=30^\circ}^{60^\circ}\int_{\phi=0}^{2\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$$= r^2 \int_{\theta=30^\circ}^{60^\circ} \sin\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi$$

1

$$= (3\,cm)^2 * \left[-\cos\theta\right]_{30^\circ}^{60^\circ} * \left[\phi\right]_0^{\pi}$$

$$= 18\pi\left(\cos 30^\circ - \cos 60^\circ\right) cm^2$$

$$\boxed{\therefore S = 20.7\ cm^2}$$

Q: 2 $\vec{E} = rA\,\hat{a}_r$

(a) $\oint_S \vec{E}\cdot\bar{ds} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}(rA\hat{a}_r)\cdot(r^2\sin\theta\, d\theta d\phi\,\hat{a}_r)$

$= A r^3 \int_{\theta=0}^{\pi}\sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi$

0.5

$= 4\pi A r^3$

for $r = a$: $\boxed{\oint_S \vec{E}\cdot\bar{ds} = 4\pi A a^3}$

(b) $\int_V \nabla\cdot\vec{E}\, dv = \oint_S \vec{E}\cdot\bar{ds}$

Here :

$\nabla\cdot\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r) = \frac{1}{r^2}\frac{\partial}{\partial r}(Ar^3)$

$\therefore \nabla\cdot\vec{E} = 3A$

$\therefore$ LHS $= \int_V \nabla \cdot \vec{E}\, dv$

$$= \int_0^a \int_0^\pi \int_0^{2\pi} 3A\ r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= \left[\frac{3Ar^3}{3}\right]_0^a * \left[-\cos\theta\right]_0^\pi * \left[\phi\right]_0^{2\pi}$$

$$= \underline{\underline{4\pi A a^3}}$$

0.5

from part (a): RHS $= 4\pi A a^3$

$\therefore$ Divergence Theorem verified.

Q:- 3

(a) $\vec{E} = 3x^2\,\hat{a}_x + 2Z\,\hat{a}_y + x^2 z\,\hat{a}_z$

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

0.5

$$= \frac{\partial}{\partial x}(3x^2) + \frac{\partial}{\partial y}(2z) + \frac{\partial}{\partial z}($$

$$\therefore \nabla \cdot \vec{E} = 6x + 0 + x^2 = x^2 + 6x$$

At $(2, -2, 0)$: $\boxed{\nabla \cdot \vec{E} = 16}$

(b) $\vec{E} = \left(\frac{a^3 \cos\theta}{r^2}\right)\hat{a}_r - \left(\frac{a^3 \sin\theta}{r^2}\right)\hat{a}_\theta$

$$\nabla \cdot \vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(E_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial E_\phi}{\partial \phi}$$

$$\Rightarrow \nabla \cdot \vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(a^3\cos\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(-\frac{a^3 \sin^2\theta}{r^2}\right)$$

$$= 0 - \frac{2a^3\cos\theta}{r^3}$$

$$\therefore \nabla \cdot \vec{E} = -\frac{2a^3\cos\theta}{r^3}$$

...t $\left(\frac{a}{2}, 0, \pi\right)$:

0.5

$$\boxed{\nabla \cdot \vec{E} = -16}$$

Q:4 :-

$$\vec{R} = (x-6)\hat{a}_x + (y-8)\hat{a}_y$$

$$\cancel{|\vec{R}|} \; \cancel{=}$$

$$|\vec{R}| = \sqrt{(x-6)^2 + (y-8)^2}$$

1

$$\therefore \hat{a}_\rho = \frac{(x-6)\hat{a}_x + (y-8)\hat{a}_y}{\sqrt{(x-6)^2 + (y-8)^2}}$$

$$\therefore \vec{E} = \frac{\rho_l}{2\pi\epsilon_0} \frac{(x-6)\hat{a}_x + (y-8)\hat{a}_y}{(x-6)^2 + (y-8)^2}$$

Q:5

$$V = \rho^2 z \sin\phi$$

$$\therefore \vec{E} = -\nabla V = \cancel{-\frac{\partial V}{\partial x}\hat{a}_x - \frac{\partial V}{\partial y}\hat{a}_y - \frac{\partial V}{\partial z}\hat{a}_z}$$

$$= -\left[\frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z\right]$$

$$\therefore \vec{E} = -\left(2\rho z \sin\phi \, \hat{a}_\rho + \rho z \cos\phi \, \hat{a}_\phi + \rho^2 \sin\phi \, \hat{a}_z\right)$$

Therefore :

1

$$W_E = \frac{1}{2}\epsilon_0 \int_V |\vec{E}|^2 dv$$

$$= \frac{1}{2}\epsilon_0 \iiint_V \left(4\rho^2 z^2 \sin^2\phi + \rho^2 z^2 \cos^2\phi + \rho^4 \sin^2\phi\right) \rho \, d\phi \, dz \, d\rho$$

$$\boxed{\therefore W_E = \frac{1507.67}{2}\left(\frac{10^{-9}}{36\pi}\right)}$$