Lecture – 4 Date: 15.01.2015

- Electric Charge, Charge Density, Total Charge
- Coulomb's Law
- Electric Field
- Electric Field due to a Point Charge
- Examples
- Electric Field due to a Line Charge

Review

- Under the static conditions the Maxwell's equations become:

$$\nabla.\vec{D} = \rho_v \qquad \nabla \times \vec{E} = 0 \qquad \nabla.\vec{B} = 0 \qquad \nabla \times \vec{H} = \vec{J}$$

Electric and Magnetic fields become decoupled under static conditions

Enables us to study electricity and magnetism as distinct separate phenomena

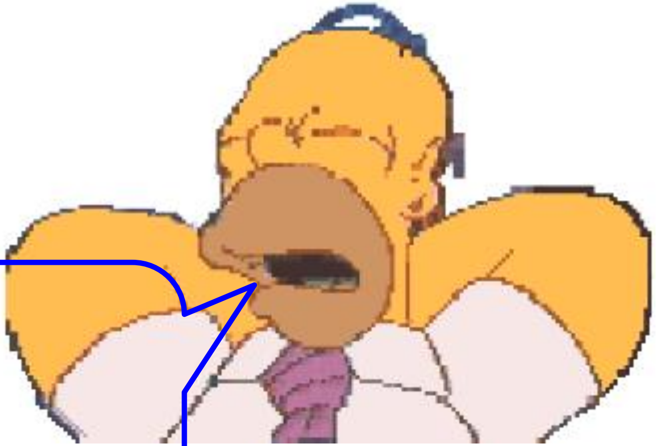
We refer the study of electric and magnetic phenomena under static conditions as ***electrostatics*** and ***magnetostatics***

The experience gained through studying electrostatics and magnetostatics phenomena will prove invaluable in tackling the more involved concepts which deal with time-varying fields

Review

- **Oh yes!** We do not study electrostatics just as a prelude to the study of time-varying fields.
- **Electrostatics** is an **important concept** in its own right.
- Many electronics devices and systems are based on the principles of electrostatics.
- **Examples include:** x-ray machines, oscilloscopes, ink-jet electrostatic printers, liquid crystal displays, copy machines, micro-electro-mechanical switches (MEMS), accelerometers, and solid-state-based control devices etc.
- **Electrostatic principles** also guide the design of medical diagnostic sensors, such as the electrocardiogram, which records the heart's pumping pattern, and electroencephalogram, which records brain activity.

Review



Q: **I see !** Electrostatics is important as a distinct phenomena but not Magnetostatics. Right?

A: **that is not correct!** Magnetostatics is equally important and this concept is utilized in design of systems such as Loudspeakers, Door Bells, Magnetic Relays, Maglev Trains etc.

Electric Charge

- Most of classical physics can be described in terms of three fundamental units, which define our physical “reality”.

Mass (e.g., Kg)

Distance (e.g., meters)

Time (e.g., seconds)

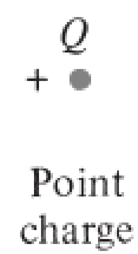
- From these fundamental units, we can define other important physical parameters such as Energy, Work, Pressure etc.
- However, these three fundamental units alone are insufficient for describing all of classical physics—we require one more to completely describe physical reality!
- This fourth fundamental unit is **Coulomb**, the unit of **electric charge**.

All **electromagnetic** phenomena can be attributed to electric charge!

Electric Charge (contd.)

- It should be apparent that electric charge is **somewhat** analogous to mass. However, one important difference between mass and charge is that charge can be either **positive** or **negative**!
- Essentially, charge (like mass) is a property of **atomic particles**. Specifically, it is important to note that:
    - The charge "on" a **proton** is $+1.602 \times 10^{-19}$ C
    - The charge "on" a **neutron** is 0.0 C
    - The charge "on" an **electron** is -1.602×10^{-19} C

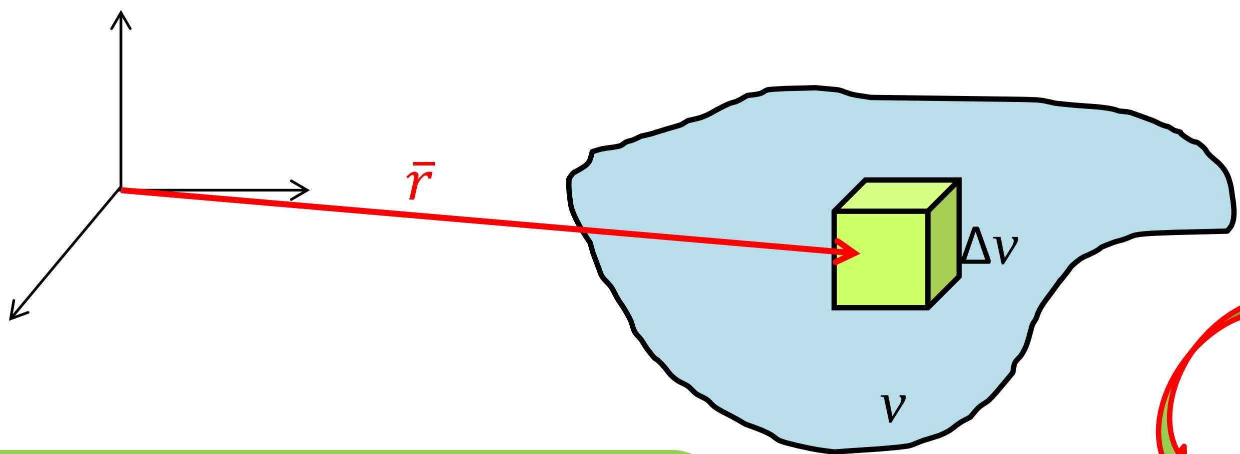
Example:



Charged particles (of all types) can be **distributed** (unevenly) across a volume, surface, or contour.

Charge Density

- In many cases, charged particles (e.g., electrons, protons, positive ions) are **unevenly distributed** throughout some volume v.
- We define **volume charge density** at a specific point $\bar{r}$ by evaluating the total net charge ΔQ in a small volume Δv surrounding the point.



Volume Charge Density

$$\rho_v(\bar{r}) = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v}$$

IMPORTANT NOTE: Volume charge density indicates the **net** charge density at each point $\bar{r}$ within volume v.

Volume charge density is a **scalar field**, and is expressed with units such as **coulombs/m³**.

Charge Density (contd.)

Q: What is meant by **net** charge density ?

A: Remember, there are positively charged particles and there are negatively charged particles, and **both** can exist at the same location $\bar{r}$.

Thus, a **positive** charge density does **not** mean that **no** negatively charged particles (e.g., electrons) are present, it simply means that there is **more** positive charge than there is negative!

- It is therefore more instructive to define: $\Delta Q = \Delta Q^{+} + \Delta Q^{-}$

  where ΔQ^{+} is the amount of **positive** charge (therefore a **positive number**) and ΔQ^{-} is the amount of **negative** charge (therefore a **negative number**). We can call ΔQ the net, or **total charge.**

Charge Density (contd.)

- **Volume** charge density can therefore be expressed as:

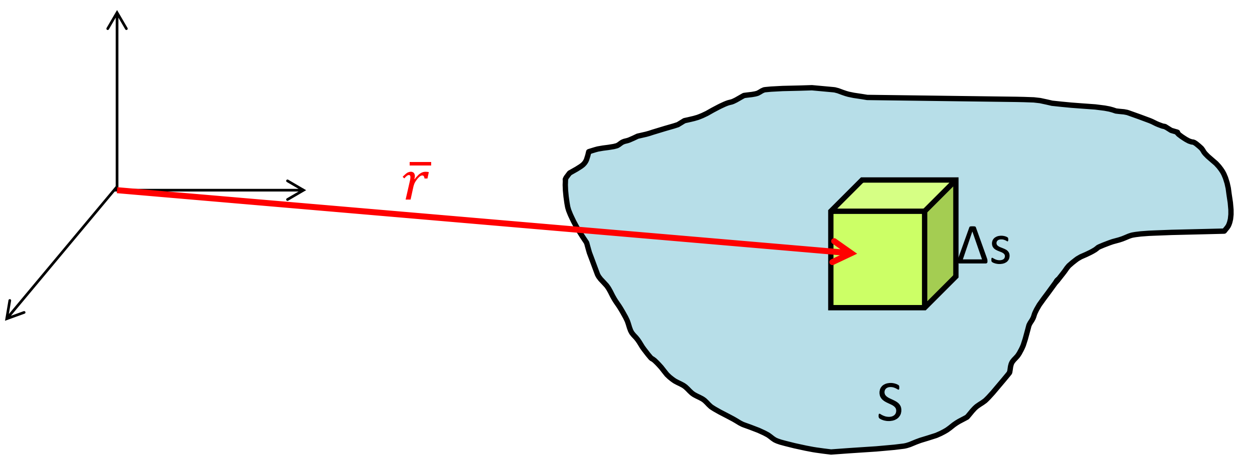
$$\rho_v(\bar{r}) \doteq \lim_{\Delta v \to 0} \frac{\Delta Q^+ + \Delta Q^-}{\Delta v} = \rho_v^+(\bar{r}) + \rho_v^-(\bar{r})$$

- **For example**, the charge density at some location $\bar{r}$ due to negatively charged particles might be 10.0 C/m^3, while that of positively charged particles might be 5.0 C/m^3. Therefore, the net, or **total** charge density is:

$$\rho_v^+(\bar{r}) + \rho_v^-(\bar{r}) = 5 + (-10) = -5.0 C/m^3$$

Surface Charge Density

- Another possibility is that charge is unevenly distributed across some surface S. In this case, we can define a **surface charge density** as by evaluating the total charge ΔQ on a small patch of surface Δs, located at point $\bar{r}$ on surface S:



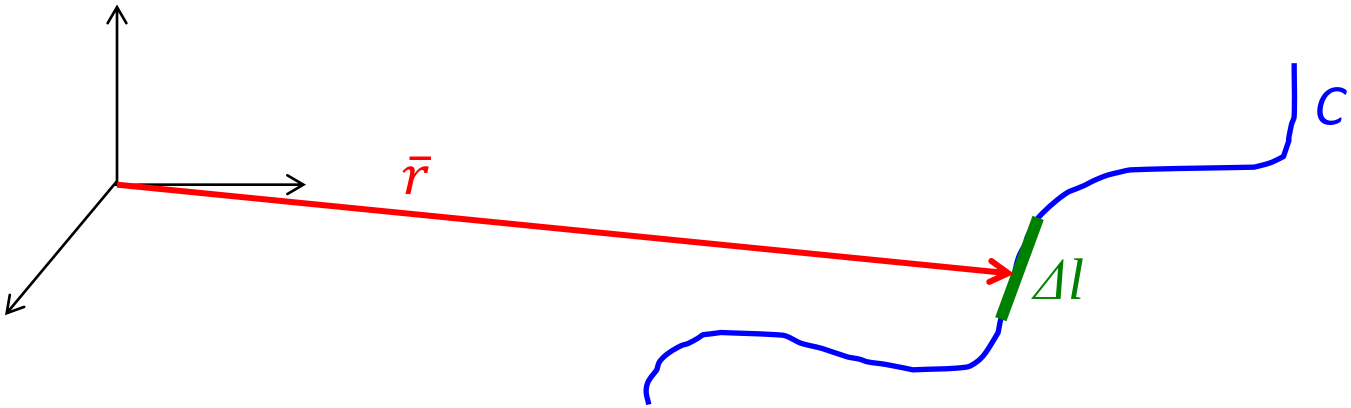
- **Surface** charge density $\rho_s(\bar{r})$ is defined as:

$$\rho_s(\bar{r}) \doteq \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s}$$

Note the **units** for surface charge density will be **charge/area** (e.g. **C/m²**).

Line Charge Density

- Finally, let us consider the case where charge is unevenly distributed across some **contour** C. We can therefore define a **line charge density** as the charge ΔQ along a small distance Δl, located at point $\bar{r}$ of contour C.



- **Line** charge density $\rho_l(\bar{r})$ is defined as: $$\rho_l(\bar{r}) \doteq \lim_{\Delta l \to 0} \frac{\Delta Q}{\Delta l}$$

As you might expect, the units of a line charge density is charge per length (e.g., **C/m**).

Total Charge

Q: If we know charge density $\rho_v(\bar{r})$, describing the charge distribution throughout a **volume** v, can we determine the **total charge** Q contained within this volume?

A: Yes definitely! Simply **integrate** the charge density over the entire volume, and you get the **total charge** Q contained within the volume.

In other words: $Q = \iiint_v \rho_v(\bar{r})\, dv$

- Likewise, we can determine the total charge distributed across a **surface** S by integrating the surface charge density:

$$Q = \iint_S \rho_s(\bar{r})\, ds$$

Q: Hey! is this **NOT** the surface integral we studied earlier.

A: True! This is a **scalar** integral; sort of a 2D version of the volume integral.

- The differential surface element ds in this integral is simply the **magnitude** of the differential surface vectors we studied earlier:

$$ds = \left|\overline{ds}\right|$$

Total Charge

- **For example**, if we integrate over the surface of a sphere, we would use the differential surface element:

$$ds = \left|\overline{ds_r}\right| = r^2 \sin\theta \, d\theta \, d\phi$$

- Finally, we can determine the total charge on **contour** C by integrating the **line charge density** $\rho_l(\bar{r})$ across the entire contour:

$$Q = \int_C \rho_l(\bar{\text{r}})\, dl$$

- The differential element *dl* is likewise related to the differential displacement vector we studied earlier:

$$dl = \left|\overline{dl}\right|$$

- **For example,** if the contour is a circle around the z-axis, then *dl* is:

$$ds = \left|\overline{d\phi}\right| = \rho\, d\phi$$

Example – 1

Find the total charge on a circular disc defined by $\rho \leq a$ and $z = 0$ if: $\rho_S = \rho_{S0} e^{-\rho}$ (C/m²).

$$Q = \int_{\rho=0}^{a} \int_{\phi=0}^{2\pi} \rho_{s0} e^{-\rho} \rho d\rho d\phi \quad \Rightarrow \quad Q = 2\pi\rho_{s0} \int_{0}^{a} \rho e^{-\rho} d\rho$$

$$\therefore Q = 2\pi\rho_{s0} \left[-\rho e^{-\rho} - e^{-\rho} \right]_{0}^{a}$$

Example – 2

A circular beam of charge of **radius** $\boldsymbol{a}$ consists of electrons moving with a **constant speed** $\boldsymbol{u}$ along the $+\boldsymbol{z}$ **direction**. The beam's axis is coincident with the z-axis and the electron **charge density is given by:** $\boldsymbol{\rho_v = -c\rho^2}$ **(C/m³),** where c is a constant and ρ is the radial distance from the axis of the beam. **Determine the charge density per unit length.**

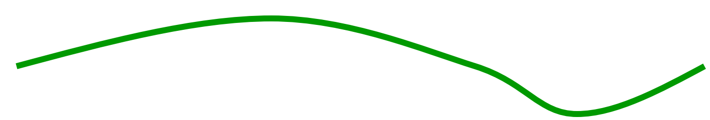
Ans: $\rho_l = -\frac{\pi c a^4}{2} C/m$

Electric Charge – Review

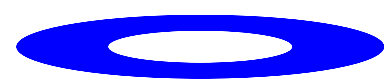
- Charge is measured in Coulombs (C). A Coulomb is a lot of charge.
- Charge comes in both positive and negative amounts.
- Charge is conserved – it can neither be created nor destroyed.

Charge can be spread out ….

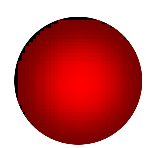
- Charge may be at a point, on a line, on a surface, or throughout a volume
- Linear charge density ρ_l units C/m
  - Multiply by length



- Surface charge density ρ_s units C/m^2
  - Multiply by area



- Charge density ρ_v units C/m^3
  - Multiply by volume

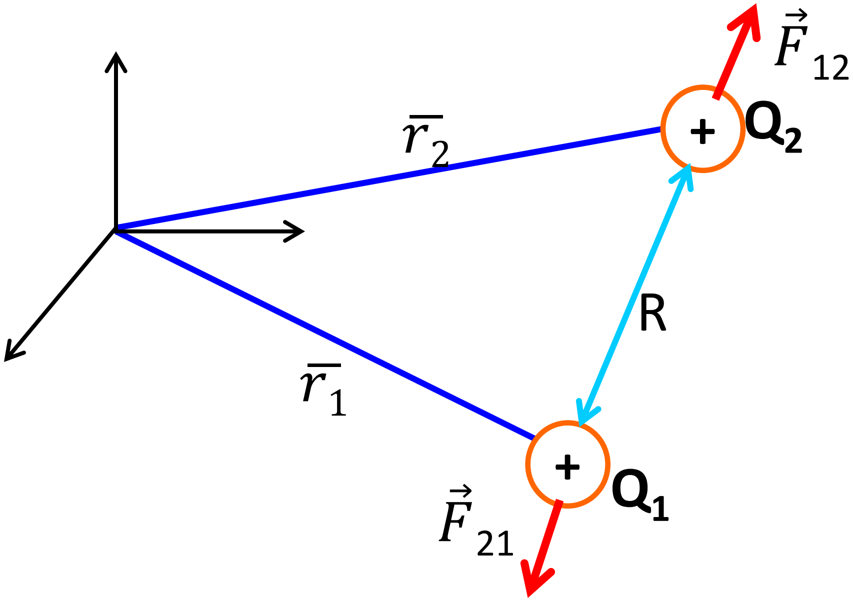


Goal of next few lectures

- Develop dexterity in applying the expressions for the electric field intensity $\vec{E}$ induced by specified distribution of charge.
- For now, our discussion will be limited to electrostatic fields generated by stationary charges.
- We will begin by considering the expression for the electric field developed by **Coulomb**.

Coulomb's Law

- Let us Consider **two positive** point charges, Q_1 and Q_2, in free space located at positions $\bar{r}_1$ and $\bar{r}_2$, respectively.
- Clarification: by point charge it is assumed that the charge is located on a body whose dimensions are much smaller than other relevant dimensions.



- **Each** charge exert a **force** $\vec{F}$ (with magnitude and direction) on the other.
- This force is **dependent** on both the **sign** (+ or −) and the **magnitude** of charges Q_1 and Q_2, as well as the **distance** R between the charges.

- **Charles Coulomb** determined this relationship in the 18th century! We call his result **Coulomb's Law**:

$$\overrightarrow{F_{21}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}\hat{a}_{21}$$

This force $\vec{F}_{21}$ is the force exerted on charge Q_1 by Q_2.

Coulomb's Law (contd.)

- Likewise, the force exerted **by** charge Q_1 **on** charge Q_2 is equal to:

$$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{R^2}\hat{a}_{12}$$

- In these formula, the value ε_0 is a **constant** that describes the **permittivity of free space** (i.e., a vacuum) given by:

$$\varepsilon_0 = 8.854\times10^{-12}\ \text{C}^2/\text{Nm}^2 \ \text{ or F/m} \Rightarrow = \frac{1}{36\pi}\times10^{-9}\ \text{C}^2/\text{Nm}^2 \ \text{ or F/m}$$

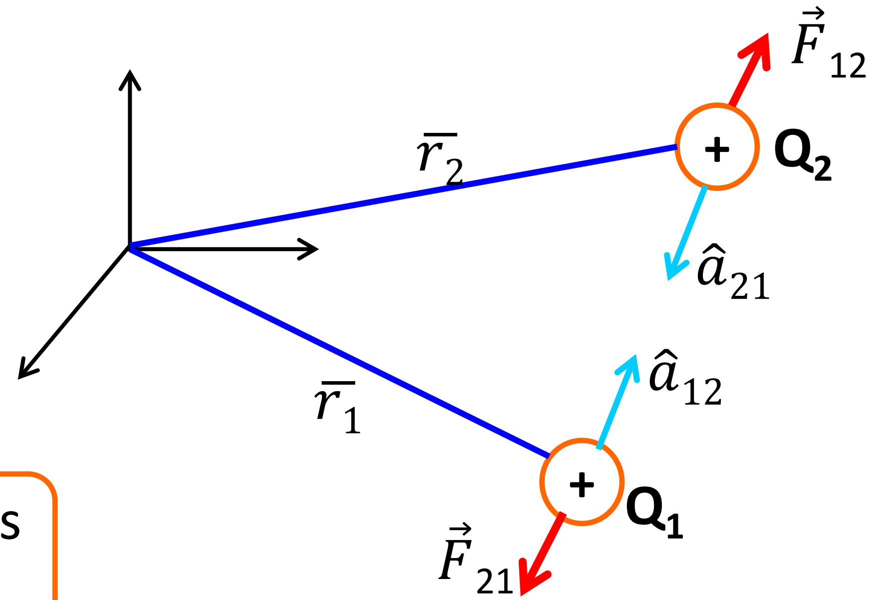
- Note the **only difference** between the equations for forces $\vec{F}_{21}$ and $\vec{F}_{12}$ are the **unit vectors** $\hat{a}_{21}$ and $\hat{a}_{12}$.
- Unit vector $\hat{a}_{21}$ points **from** the location of Q_2 (i.e., $\overline{r_2}$) **to** the location of charge Q_1 (i.e., $\overline{r_1}$).
- Likewise, unit vector $\hat{a}_{12}$ points **from** the location of Q_1 (i.e., $\overline{r_1}$) **to** the location of charge Q_2 (i.e., $\overline{r_2}$).

Coulomb's Law (contd.)

- Note therefore, that these unit vectors point in **opposite** directions, a result we express mathematically as $\hat{a}_{21} = -\hat{a}_{12}$.
- Therefore :

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{R^2}\hat{a}_{21} \Rightarrow = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{R^2}(-\hat{a}_{12}) \Rightarrow = -\left(\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{R^2}\hat{a}_{12}\right) \Rightarrow = -\vec{F}_{12}$$

- **Look!** Forces $\vec{F}_{21}$ and $\vec{F}_{12}$ have **equal magnitude**, but point in **opposite directions**!



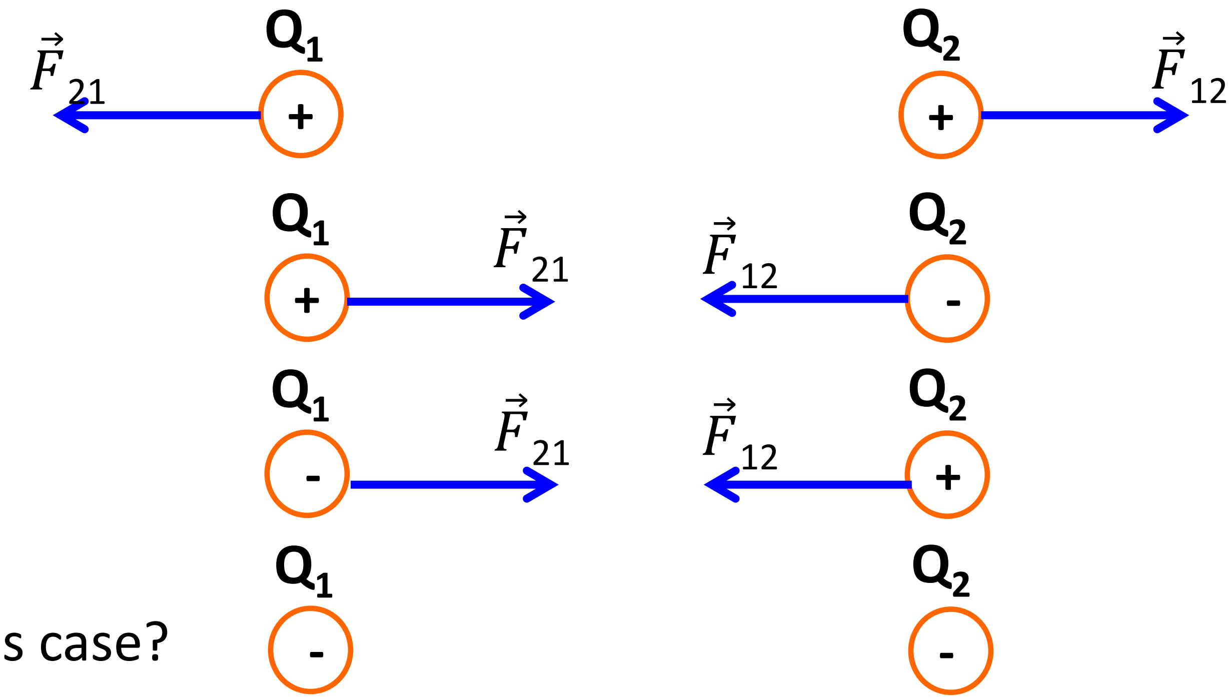
Note in this case **both** charges were **positive**.

Coulomb's Law (contd.)

Q: What happens when **one** of the charges is **negative**?

A: Look at Coulomb's Law! If one charge is positive, and the other is negative, then the **product** Q_1Q_2 is **negative**. The resulting force vectors are therefore negative—they point in the **opposite** direction of the previous (i.e., both positive) case!

- Therefore:
- What about this case?



Coulomb's Law (contd.)

We come to the **important** conclusion that:

1. charges of **opposite** sign **attract**.
2. charges with the **same** sign **repel**.

Charles-Augustin de Coulomb (1736-1806)

a military civil engineer, retired from the French army because of ill health after years in the West Indies. Forced from Paris by the disturbances of the revolution, he began working at his family estate and discovered that the torsion characteristics of long fibers made them ideal for the sensitive measurement of **magnetic** and **electric** forces. He was familiar with Newton's **inverse-square law** and in the period 1785-1791 he succeeded in showing that **electrostatic** forces obey the **same** rule.

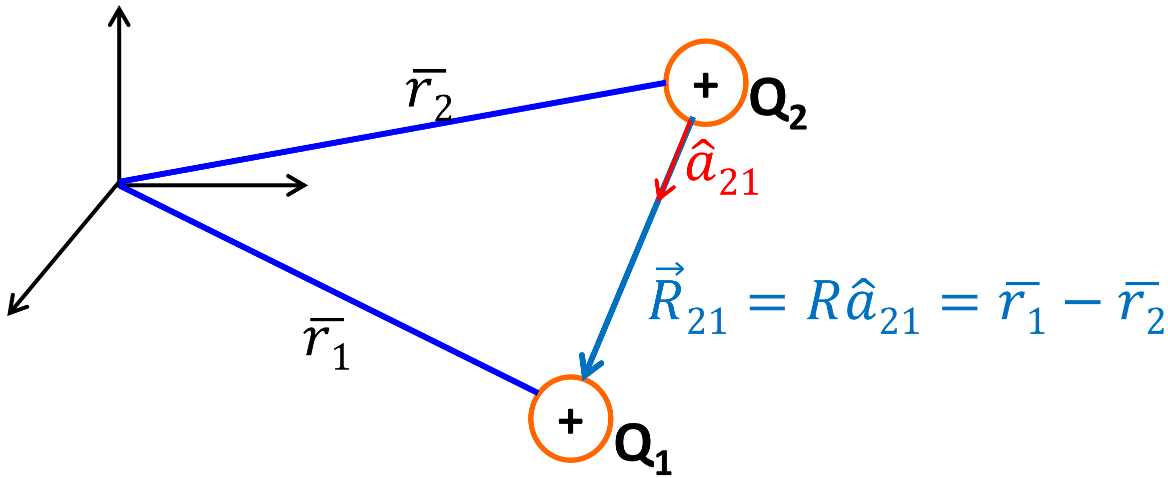
The Vector Form of Coulomb's Law of Force

- The **position vector** can be used to make the **calculations** of Coulomb's Law of Force more **explicit**.
- Recall: $$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}\hat{a}_{21}$$

Specifically, we ask ourselves the question: **how** do we determine the **unit vector** $\hat{a}_{21}$ and **distance** R ??

- Recall the **unit vector** $\hat{a}_{21}$ is a unit vector directed **from** Q_2 **toward** Q_1, and R is the **distance** between the two charges.
- The **directed distance** vector $\vec{R}_{21} = R\hat{a}_{21}$ can be determined from the **difference** of position vectors $\overline{r}_1$ and $\overline{r}_2$.

The Vector Form of Coulomb's Law of Force (contd.)



- This directed distance $\vec{R}_{21} = \bar{r}_1 - \bar{r}_2$ is **all** we need to determine **both** unit vector $\hat{a}_{21}$ and distance R (i.e., $\vec{R}_{21} = R\hat{a}_{21}$)!
- For example, since the **direction** of directed distance $\vec{R}_{21}$ is $\hat{a}_{21}$, we can **explicitly** find this unit vector by **dividing** $\vec{R}_{21}$ by its **magnitude**:

$$\hat{a}_{21} = \frac{\vec{R}_{21}}{\left|\bar{r}_1 - \bar{r}_2\right|}$$

- Likewise, the **distance** R between the two charges is simply the magnitude of directed distance $\vec{R}_{21}$!

$$\hat{a}_{21} = \frac{\vec{R}_{21}}{\left|\bar{r}_1 - \bar{r}_2\right|}$$

The Vector Form of Coulomb's Law of Force (contd.)

- We can therefore express **Coulomb's Law** entirely in terms of $\vec{R}_{21}$, the **directed distance** relating the location of Q_1 with respect to Q_2:

$$\overrightarrow{F_{21}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{R^2}\hat{a}_{21} \Rightarrow = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{\left|\overrightarrow{R_{21}}\right|^2}\frac{\overrightarrow{R_{21}}}{\left|\overrightarrow{R_{21}}\right|} \Rightarrow = \frac{Q_1Q_2}{4\pi\varepsilon_0}\frac{\overrightarrow{R_{21}}}{\left|\overrightarrow{R_{21}}\right|^3}$$

- Explicitly using the relation $\vec{R}_{21} = \overline{r}_1 - \overline{r}_2$, we can express:

$$\overrightarrow{F_{21}} = \frac{Q_1Q_2}{4\pi\varepsilon_0}\frac{\overline{r}_1 - \overline{r}_2}{\left|\overline{r}_1 - \overline{r}_2\right|^3}$$

The Vector Form of Coulomb's Law of Force (contd.)

- We could likewise define a directed distance:

$$\overrightarrow{R_{12}} = \left|\overline{r_2} - \overline{r_1}\right|$$

relates the location of Q_2 with respect to Q_1.

- We can then describe the force on charge Q_2 as:

$$\overrightarrow{F_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0} \frac{\overline{r_2} - \overline{r_1}}{\left|\overline{r_2} - \overline{r_1}\right|^3}$$

- Note since $\vec{R}_{12} = -\vec{R}_{21}$, (thus $|\vec{R}_{12}| = |\vec{R}_{21}|$), it is apparent that:

$$\vec{F}_{12} = -\vec{F}_{21}$$

The forces on each charge have **equal** magnitude but **opposite** direction.

Confirmation of Newton's 3rd Law

Example – 3

- Point charge 5nC is located at (2, 0, 4). Determine the force on a 1nC point charge located at (1, –3, 7)

Solution:

Apply Coulomb's Law: $\vec{F} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{Q_1 Q_2 \left(\bar{r}_1 - \bar{r}_2\right)}{\left|\bar{r}_1 - \bar{r}_2\right|^3}$

Here: $Q_1 = 1nC$ $\quad Q_2 = 5nC$ $\quad \bar{r}_1 = \hat{a}_x - 3\hat{a}_y + 7\hat{a}_z$ $\quad \bar{r}_2 = 2\hat{a}_x + 4\hat{a}_z$

Therefore: $\vec{F} = \dfrac{1\times10^{-9} \times 5\times10^{-9}}{4\pi\left(\dfrac{10^{-9}}{36\pi}\right)} \dfrac{\left(-a_x - 3\hat{a}_y + 3\hat{a}_z\right)}{19^{3/2}} N$

$$\therefore \vec{F} = \frac{45\left(-a_x - 3\hat{a}_y + 3\hat{a}_z\right)}{19^{3/2}} nN$$

Electric Field

- An electric field is an *invisible entity* which exists in the region around a charged particle. It is caused to exist by the charged particle.
- **The effect of an electric field** is to **exert a force on any charged particle** **(other than the charged particle causing the electric field to exist)** that finds itself at a point in space at which the electric field exists.
- **The electric field at an empty point in space** is the **force-per-unit-charge-of-would-be-victim** **at that empty point in space.**
- The charged particle that is causing the electric field to exist is called a **source charge.**
- The electric field exists in the region around the **source charge** whether or not **there is a victim** charged particle for the electric field to exert a force upon.
- At every point in space where the electric field exists, it has both magnitude and direction. Hence, the electric field is a vector at each point in space at which it exists.

Electric Field (contd.)

- We call the force-per-unit-charge-of-would-be-victim **vector** at a particular point in space the **"electric field"** at that point.
- We also call the infinite set of all such vectors, **in the region around the source charge**, the electric field of the source charge.
- **We use the symbol** $\vec{E}$ to represent the electric field. I am using the word "victim" for any particle upon which an electric field is exerting a force.
- The electric field will **only** exert a force on a particle if that particle has charge. So all "victims" of an electric field have charge.
- If there does happen to be a charged particle in an electric field, then that charged particle (the victim) will experience a force:

$$\vec{F} = Q_v \vec{E}$$

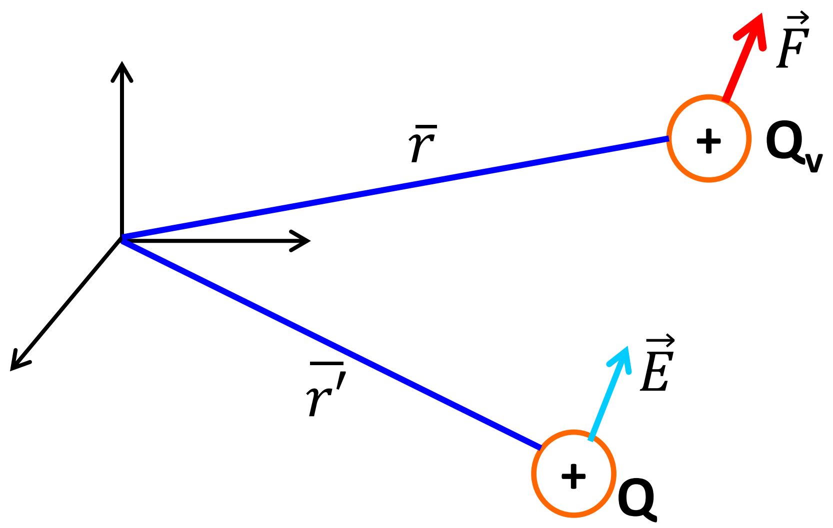
where Q_v is the charge of the victim and $\vec{E}$ is the electric field vector at the location of the victim.

Electric Field (contd.)

We can think of the electric field as a characteristic of space. The force experienced by the victim charged particle is the product of a characteristic of the victim (its charge) and a characteristic of the point in space (the electric field) at which the victim happens to be.

Electric Field due to a Point Charge

- **For Q > 0**, the electric field $\vec{E}$ is in the direction of $\vec{F}$.

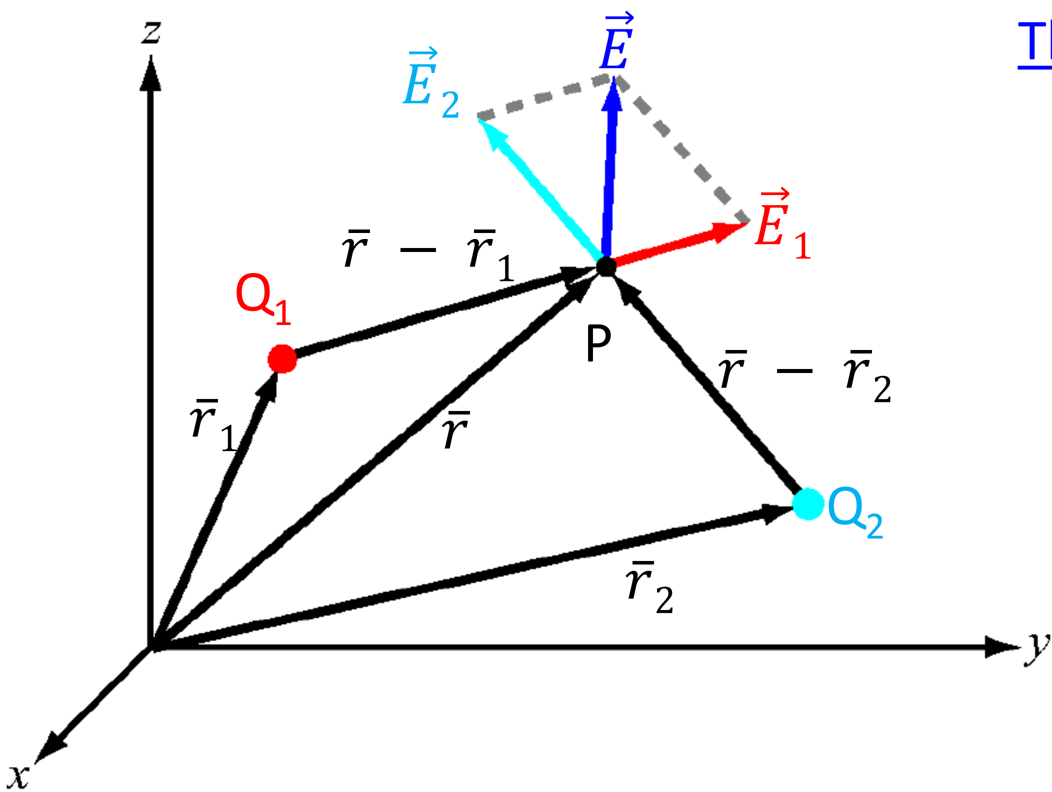


- Therefore, the electric field at point $\bar{r}$ due to a point charge located at $\bar{r}'$ is:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R = \frac{Q}{4\pi\varepsilon_0}\frac{(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}$$

Electric Field due to a Point Charge (contd.)

- The electric field $\vec{E}$ expression due to a single point charge can be extended to multiple point charges.
- Let us consider two **positive point charges** Q_1 and Q_2 located at position vectors $\bar{r}_1$ and $\bar{r}_2$. Then evaluate the electric field $\vec{E}$ at point P located at $\bar{r}$.



Therefore:

$$\vec{E}_1 = \frac{Q_1}{4\pi\varepsilon_0}\frac{(\bar{r} - \bar{r}_1)}{|\bar{r} - \bar{r}_1|^3}$$

⇐ Due to Q_1

$$\vec{E}_2 = \frac{Q_2}{4\pi\varepsilon_0}\frac{(\bar{r} - \bar{r}_2)}{|\bar{r} - \bar{r}_2|^3}$$

⇐ Due to Q_2

Electric Field of Point Charge (contd.)

- The total electric field $\vec{E}$ due to both point charges is the vector sum of the individual electric fields $\vec{E}_1$ and $\vec{E}_2$.

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = \frac{Q_1}{4\pi\varepsilon_0}\frac{(\bar{r}-\bar{r}_1)}{|\bar{r}-\bar{r}_1|^3} + \frac{Q_2}{4\pi\varepsilon_0}\frac{(\bar{r}-\bar{r}_2)}{|\bar{r}-\bar{r}_2|^3}$$

$$\therefore \vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1(\bar{r}-\bar{r}_1)}{|\bar{r}-\bar{r}_1|^3} + \frac{Q_2(\bar{r}-\bar{r}_2)}{|\bar{r}-\bar{r}_2|^3}\right]$$

Obeys principles of superposition

- For N point charges Q_1, Q_2, …., Q_N located at $\bar{r}_1, \bar{r}_2, \ldots \bar{r}_N$, the electric field at point $\bar{r}$ is given by:

$$\vec{E} = \frac{Q_1}{4\pi\varepsilon_0}\frac{(\bar{r}-\bar{r}_1)}{|\bar{r}-\bar{r}_1|^3} + \frac{Q_2}{4\pi\varepsilon_0}\frac{(\bar{r}-\bar{r}_2)}{|\bar{r}-\bar{r}_2|^3} + \ldots + \frac{Q_N}{4\pi\varepsilon_0}\frac{(\bar{r}-\bar{r}_N)}{|\bar{r}-\bar{r}_N|^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\bar{r}-\bar{r}_k)}{|\bar{r}-\bar{r}_k|^3}$$

Example – 4

- Two point charges with $Q_1 = 2 \times 10^{-5}\ C$ and $Q_2 = -4 \times 10^{-5}\ C$ are located in free space at points with Cartesian coordinates (1, 3, -1) and (-3, 1, -2) respectively. Find (a) the electric field $\vec{E}$ at (3, 1, -2) and (b) the force on $8 \times 10^{-5}\ C$ charge located at that point. All distances are in meters.

Solution

(a)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1(\bar{r}-\bar{r}_1)}{|\bar{r}-\bar{r}_1|^3} + \frac{Q_2(\bar{r}-\bar{r}_2)}{|\bar{r}-\bar{r}_2|^3}\right]$$

Here:

$$\bar{r} = 3\hat{a}_x + \hat{a}_y - 2\hat{a}_z \qquad \bar{r}_1 = \hat{a}_x + 3\hat{a}_y - \hat{a}_z \qquad \bar{r}_2 = -3\hat{a}_x + \hat{a}_y - 2\hat{a}_z$$

$$\therefore \bar{r} - \bar{r}_1 = (3\hat{a}_x + \hat{a}_y - 2\hat{a}_z) - (\hat{a}_x + 3\hat{a}_y - \hat{a}_z) = 2\hat{a}_x - 2\hat{a}_y - \hat{a}_z$$

$$\therefore \bar{r} - \bar{r}_2 = (3\hat{a}_x + \hat{a}_y - 2\hat{a}_z) - (-3\hat{a}_x + \hat{a}_y - 2\hat{a}_z) = 6\hat{a}_x$$

Solution – 4 (contd.)

Therefore:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2\left(2\hat{a}_x - 2\hat{a}_y - \hat{a}_z\right)}{\left|\left(2\hat{a}_x - 2\hat{a}_y - \hat{a}_z\right)\right|^3} - \frac{4\left(6\hat{a}_x\right)}{\left|\left(6\hat{a}_x\right)\right|^3}\right]\times 10^{-5}$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2\left(2\hat{a}_x - 2\hat{a}_y - \hat{a}_z\right)}{27} - \frac{4\left(6\hat{a}_x\right)}{216}\right]\times 10^{-5}$$

$$\therefore \vec{E} = \frac{\hat{a}_x - 4\hat{a}_y - 2\hat{a}_z}{108\pi\varepsilon_0}\times 10^{-5} \quad \text{V/m}$$

(b)

$$\vec{F} = Q_v\vec{E} \qquad \Rightarrow \vec{F} = 8\times 10^{-5}\times\frac{\hat{a}_x - 4\hat{a}_y - 2\hat{a}_z}{108\pi\varepsilon_0}\times 10^{-5}$$

$$\therefore \vec{F} = \frac{2\hat{a}_x - 8\hat{a}_y - 4\hat{a}_z}{27\pi\varepsilon_0}\times 10^{-10} \quad \text{N}$$

Example – 5

- Point charges 5nC and –2nC are located at (2, 0, 4) and (–3, 0, 5) respectively.
  (i) Determine the force on a 1nC point charge located at (1, –3, 7).
  (ii) Find the electric field $\vec{E}$ at (1, –3, 7).

Solution:

Apply Coulomb's Law:

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{Q_1(\bar{r} - \bar{r}_2)}{|\bar{r} - \bar{r}_1|^3} + \frac{Q_2(\bar{r} - \bar{r}_2)}{|\bar{r} - \bar{r}_2|^3}\right]$$

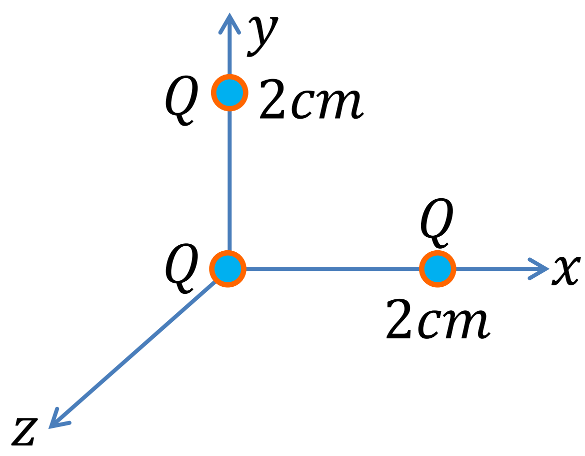
Here: $Q_1 = 5nC \qquad Q_2 = -2nC \qquad Q = 1nC$

$\bar{r}_1 = 2\hat{a}_x + 4\hat{a}_z \qquad \bar{r}_2 = -3\hat{a}_x + 5\hat{a}_z \qquad \bar{r} = \hat{a}_x - 3\hat{a}_y + 7\hat{a}_z$ Then find $\vec{F}$

Then: $\vec{E} = \dfrac{\vec{F}}{Q}$

Example – 6

Three point charges, each with $Q = 3\,nC$ are located at the corners of a triangle in the xy-plane, with one corner at the origin, another at $(2cm, 0, 0)$, and the third at $(0, 2cm, 0)$. Find the force acting on the charge located at the origin.



Example – 7

- Two identical charges are located on the x-axis at $x = 3$ and $x = 7$. At what point in space is the **net electric field** zero.
- **Since both charges are on x-axis**, the point at which the fields due to the two charges can cancel **has to lie on the x-axis also**. Intuitively, since the two charges are identical, that point is midway between them at $(5, 0, 0)$.

Example – 8

- Using Coulomb's law, determine the units of permittivity of free space.

Example – 9

- Find the magnitude of the Coulomb force that exists between an electron and proton in a hydrogen atom. Compare the Coulomb force and the gravitational force between the two particles. The two particles are separated approximately by 1×10^{-10} m.

$$F_{Coulomb} = \frac{Q^2}{4\pi\varepsilon_0 R^2} \approx \frac{\left(1.602\times10^{-19}\right)^2}{4\pi\left(\frac{10^{-9}}{36\pi}\right)\left(10^{-10}\right)^2} \approx 2.3\times10^{-8}\,N$$

$$F_{gravitational} = G\frac{m_{electron}M_{proton}}{R^2} = \left(6.67\times10^{-11}\right)\left(\frac{\left(9.11\times10^{-31}\right)\left(1836\times9.11\times10^{-31}\right)}{\left(1\times10^{-10}\right)^2}\right) = 1.02\times10^{-47}\,N$$

$$\therefore \frac{F_{Coulomb}}{F_{gravitational}} = 2.27\times10^{39}$$

Such a large Coulomb force helps explain why chemical bonds that hold atoms, molecules, and compounds together can be very strong.

Electric Field due to a Line Charge

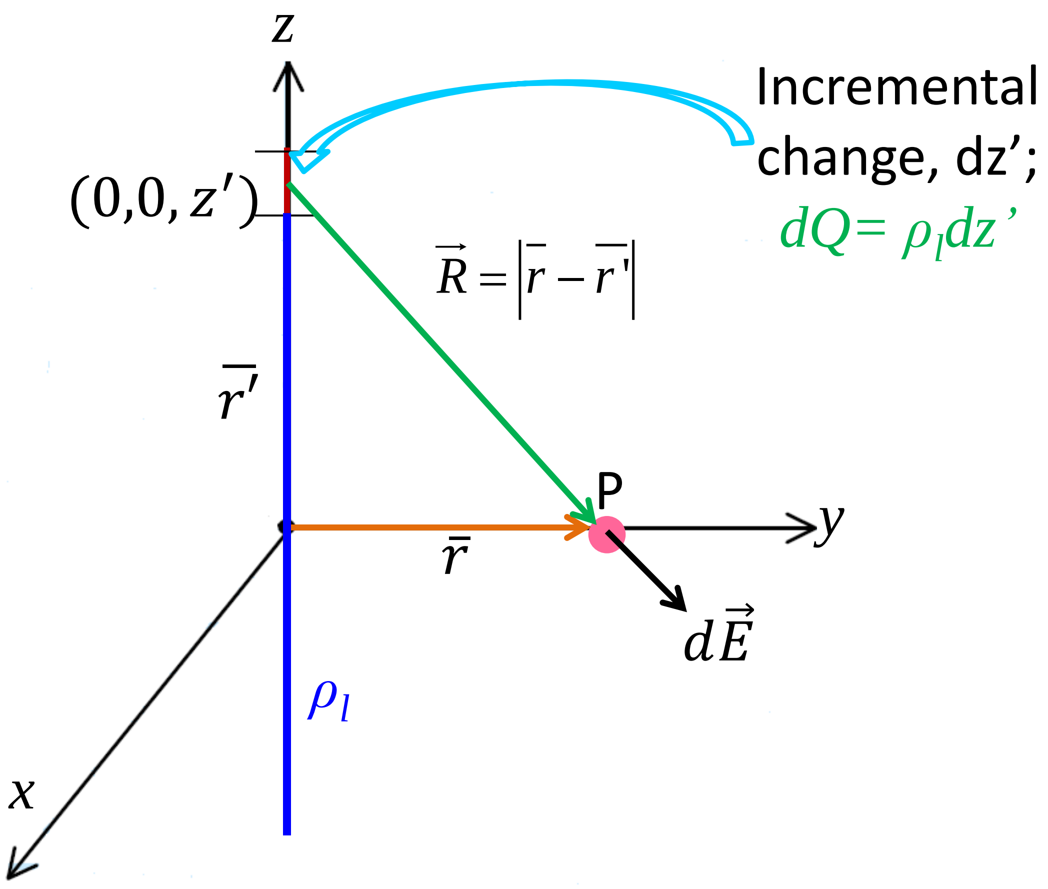
- Filament like distribution of charge density.
- For example, sharp beam in a cathode-ray tube or charged conductor of a very small radius.
- Let us assume an infinite straight-line charge, with charge density ρ_l C/m, lying along the z-axis.

Q: What electric field $\vec{E}(\bar{r})$ is produced by this line charge?

A: Apply Coulomb's Law.

Electric Field due to a Line Charge (contd.)

- For the calculation of electric field $\vec{E}$ at $P(0, y, 0)$, the first step is to determine the incremental field at P due to the incremental charge $dQ = \rho_l dz'$



We have:

$$\overrightarrow{dE} = \frac{dQ}{4\pi\varepsilon_0}\frac{(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3} = \frac{\rho_l dz'}{4\pi\varepsilon_0}\frac{(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}$$

$$\bar{r} = y\hat{a}_y = \rho\hat{a}_\rho$$

$$\bar{r}' = z'\hat{a}_z$$

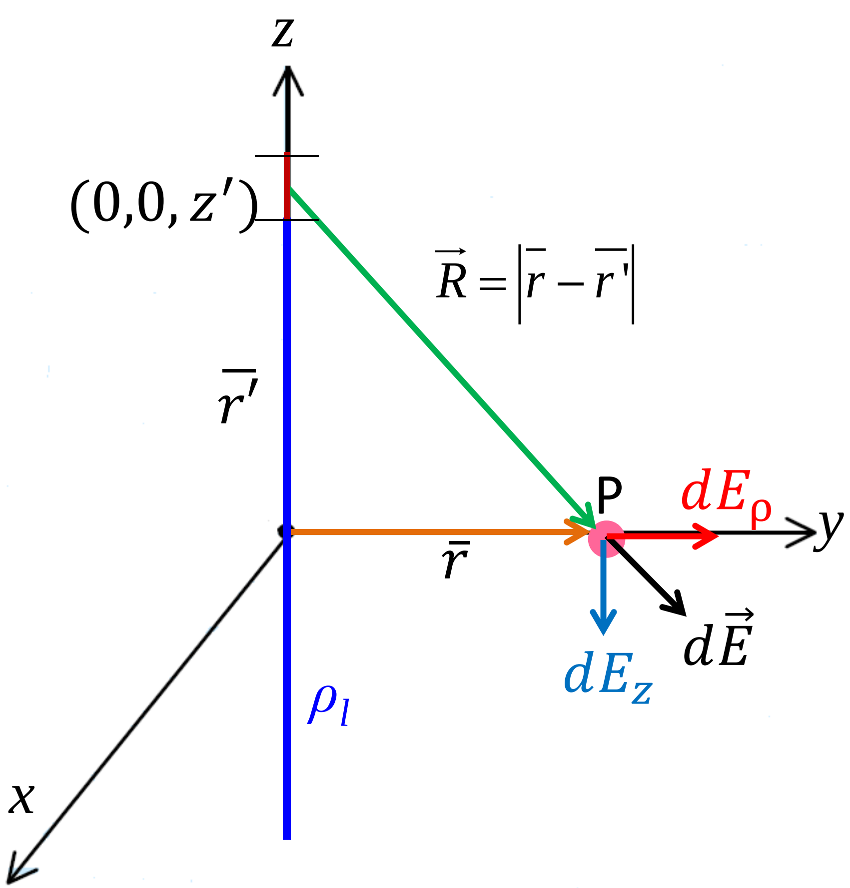
$$\therefore \vec{R} = \bar{r} - \bar{r}' = \rho\hat{a}_\rho - z'\hat{a}_z$$

Electric Field due to a Line Charge (contd.)

$$\Rightarrow \overrightarrow{dE} = \frac{\rho_l dz'}{4\pi\varepsilon_0}\frac{\left(\rho\hat{a}_\rho - z'\hat{a}_z\right)}{\left(\rho^2 + z'^2\right)^{3/2}}$$

$$\therefore \overrightarrow{dE} = \frac{\rho_l \rho dz'}{4\pi\varepsilon_0\left(\rho^2 + z'^2\right)^{3/2}}\hat{a}_\rho - \frac{\rho_l z' dz'}{4\pi\varepsilon_0\left(\rho^2 + z'^2\right)^{3/2}}\hat{a}_z$$

$d\vec{E}_\rho$ $d\vec{E}_z$



$$\therefore \overrightarrow{dE} = \hat{a}_\rho dE_\rho - \hat{a}_z dE_z$$

Electric Field due to a Line Charge (contd.)

Now:

$$dE_\rho = \int_{z'=-\infty}^{z'=\infty} \frac{\rho_l \rho}{4\pi\varepsilon_0 \left(\rho^2 + z'^2\right)^{3/2}} dz' = \frac{\rho_l \rho}{4\pi\varepsilon_0} \int_{z'=-\infty}^{z'=\infty} \frac{dz'}{\left(\rho^2 + z'^2\right)^{3/2}}$$

$$\Rightarrow E_\rho = \frac{\rho_l \rho}{4\pi\varepsilon_0} \left[\frac{1}{\rho^2} \frac{z'}{\left(\rho^2 + z'^2\right)^{1/2}} \right]_{z'=-\infty}^{z'=\infty}$$

$$\therefore E_\rho = \frac{\rho_l}{2\pi\varepsilon_0 \rho}$$

AND:

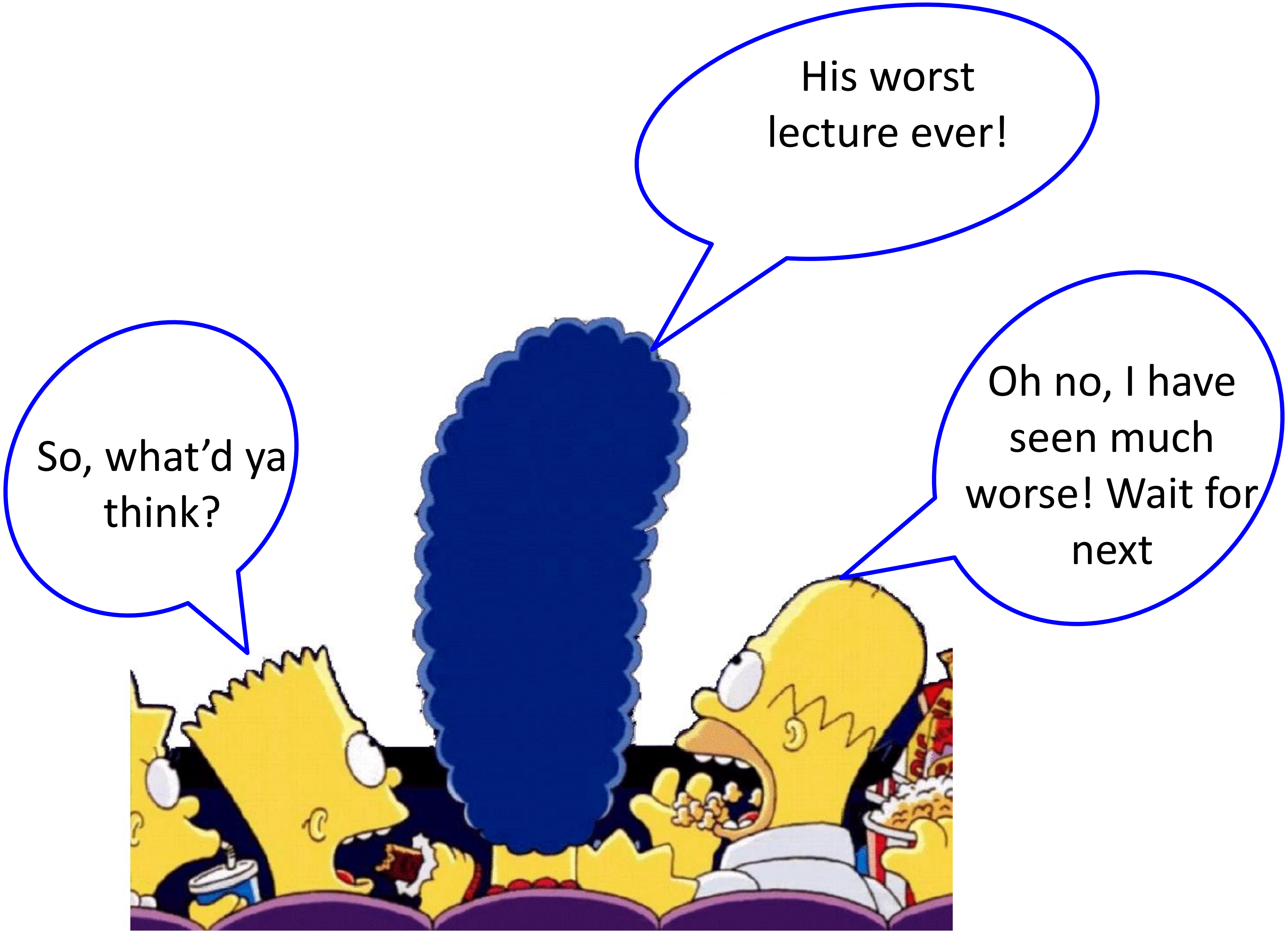
$$dE_z = \frac{\rho_l}{4\pi\varepsilon_0} \int_{z'=-\infty}^{z'=\infty} \frac{z' dz'}{\left(\rho^2 + z'^2\right)^{3/2}}$$

$$\therefore E_z = \frac{\rho_l}{4\pi\varepsilon_0} \times (0) = 0$$

Therefore:

$$\vec{E}\left(\bar{r}\right) = E_\rho \hat{a}_\rho - E_z \hat{a}_z = \frac{\rho_l}{2\pi\varepsilon_0 \rho} \hat{a}_\rho$$

BTW, there are multiple ways of solving this problem. You can master this art through practice!


His worst lecture ever!
So, what’d ya think?
Oh no, I have seen much worse! Wait for next