Lecture – 24

Date: 13.04.2015

- Reflection of Plane Wave at Oblique Incidence (Snells' Law, Brewster's Angle, Parallel Polarization, Perpendicular Polarization etc.)
- Introduction to RF/Microwave
- Introduction to Transmission Lines

Introduction

- One can't expect plane waves to be incident normally on a plane in all types of applications.
- Therefore one must consider the general problem of a plane wave propagating along a specified axis that is arbitrarily located relative to a rectangular coordinate system.
- The most general form of a plane wave in a lossless media is given by:

$$\vec{E}(r,t) = \vec{E}_o \cos(\vec{\beta}.\bar{r} - \omega t)$$

Where: $\vec{\beta} = \beta_x \hat{a}_x + \beta_y \hat{a}_y + \beta_z \hat{a}_z$ $\quad$ $\bar{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ $\quad$ $\beta^2 = \beta_x^2 + \beta_y^2 + \beta_z^2$

One can deduce Maxwell's equations in the following form:

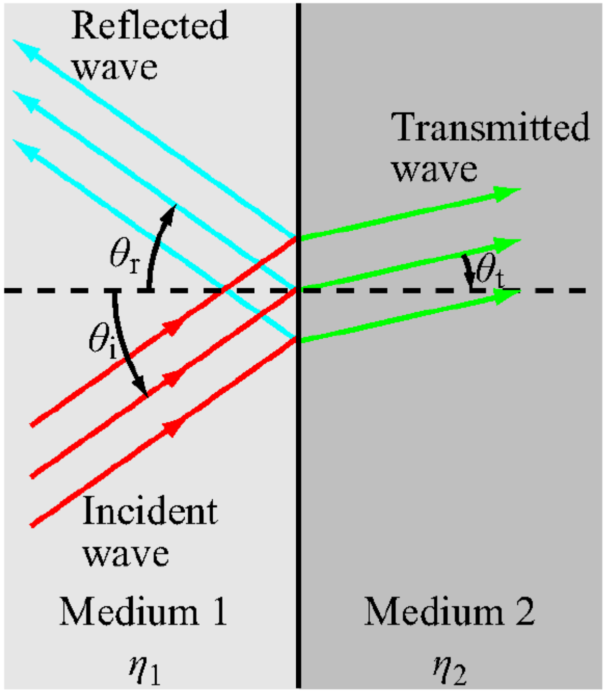
$$\vec{\beta} \times \vec{E} = \omega\mu\vec{H} \qquad \vec{\beta} \times \vec{H} = -\omega\varepsilon\vec{E} \qquad \vec{\beta}.\vec{H} = 0 \qquad \vec{\beta}.\vec{E} = 0$$

They show two things: (i) $\vec{E}, \vec{H}$ and $\vec{\beta}$ are orthogonal, (ii) $\vec{E}$ and $\vec{H}$ lie on the same plane

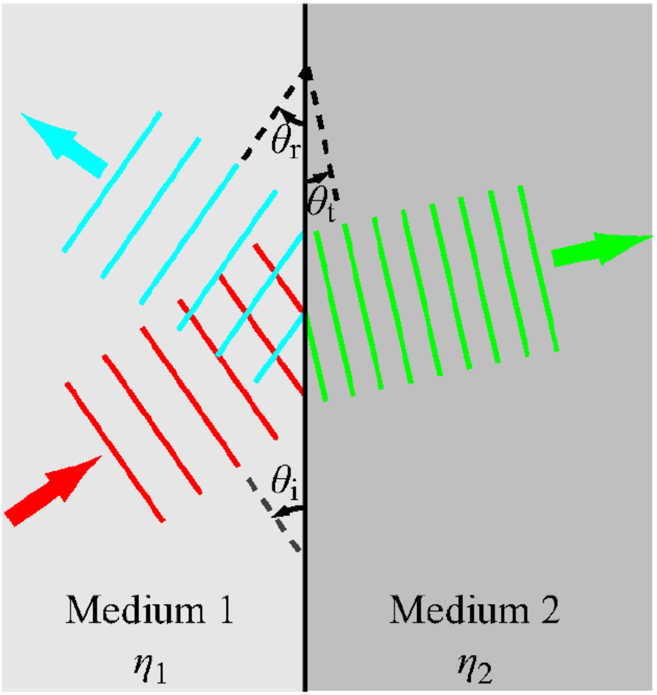
Introduction (contd.)

- Furthermore: $\vec{\beta}.\bar{r} = \beta_x x + \beta_y y + \beta_z z = constant$

- The corresponding magnetic field is: $\vec{H} = \frac{1}{\omega\mu}\vec{\beta}\times\vec{E} = \frac{\hat{a}_\beta \times \vec{E}}{\eta}$



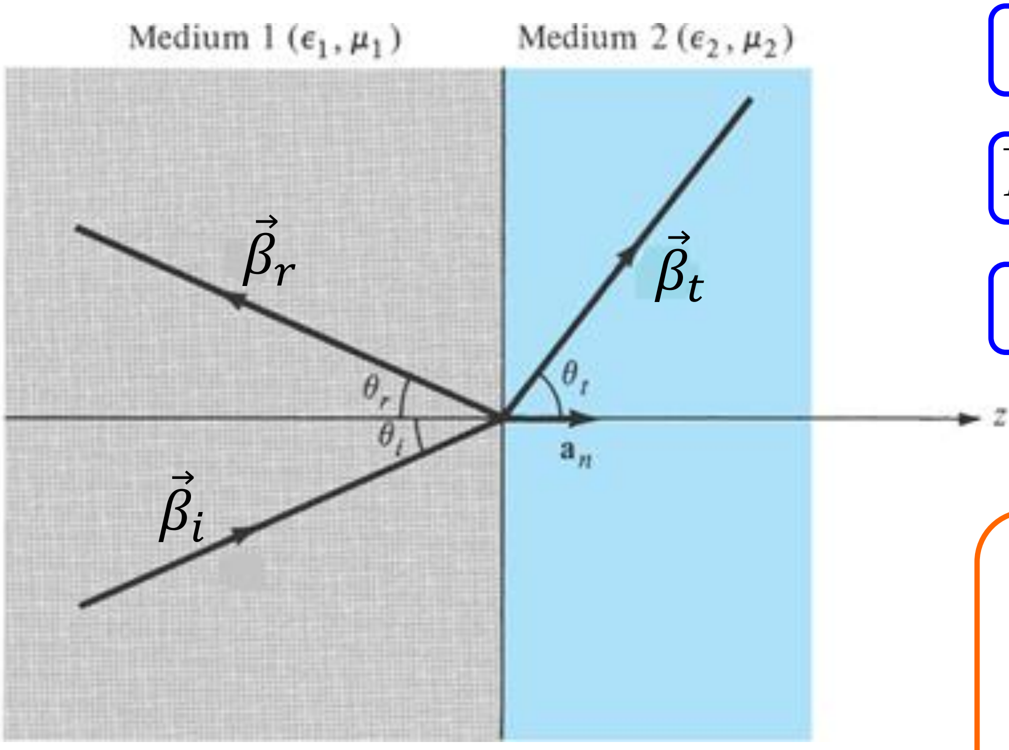
Ray representation of oblique incidence



Wavefront representation of oblique incidence

Reflection at Oblique Incidence

- The plane defined by the propagation vector $\vec{\beta}$ and a unit normal vector $\hat{a}_n$ to the boundary is called the plane of incidence.
- The angle between $\vec{\beta}$ and $\hat{a}_n$ is the *angle of incidence*.



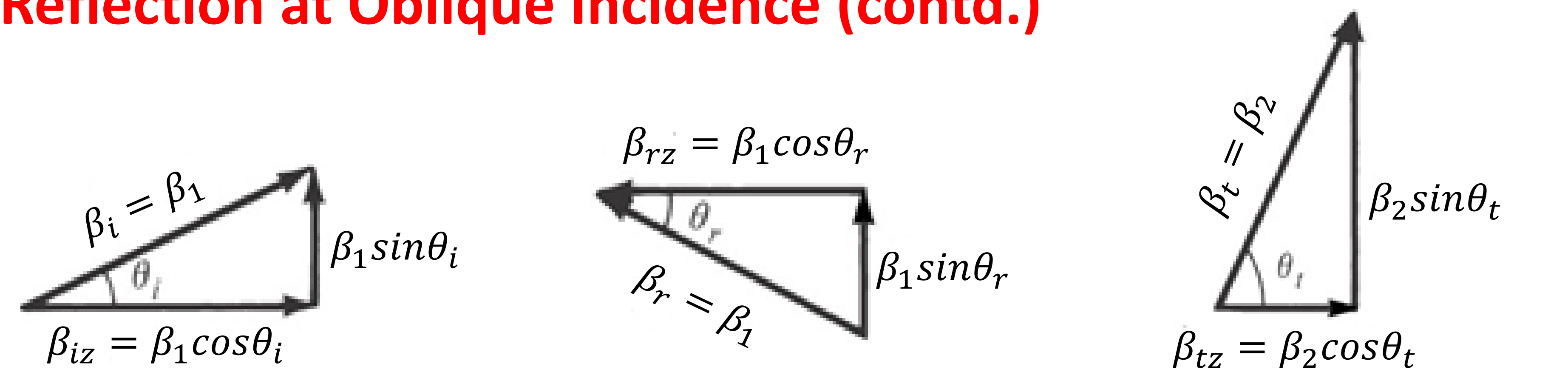
$$\vec{E}_i = \vec{E}_{io}\cos(\beta_{ix}x + \beta_{iy}y + \beta_{iz}z - \omega_i t)$$

$$\vec{E}_r = \vec{E}_{ro}\cos(\beta_{rx}x + \beta_{ry}y + \beta_{rz}z - \omega_r t)$$

$$\vec{E}_t = \vec{E}_{to}\cos(\beta_{tx}x + \beta_{ty}y + \beta_{tz}z - \omega_t t)$$

Where: β_i, β_r and β_t will have normal and tangential components to the plane of incidence.

Reflection at Oblique Incidence (contd.)



- From boundary condition we can write: the tangential component of $\vec{E}$ must be continuous at $z = 0$.

$$\vec{E}_{i\tan}(z=0) + \vec{E}_{r\tan}(z=0) = \vec{E}_{t\tan}(z=0)$$

This boundary condition can be satisfied if:

$$\omega_i = \omega_r = \omega_t = \omega$$

$$\beta_{ix} = \beta_{rx} = \beta_{tx} = \beta_x$$

$$\beta_{iy} = \beta_{ry} = \beta_{ty} = \beta_y$$

First condition implies that the frequency remains unchanged.

Reflection at Oblique Incidence (contd.)

- From second and third conditions we can write:

$$\beta_1 sin\theta_i = \beta_1 sin\theta_r$$

$$\beta_1 sin\theta_i = \beta_2 sin\theta_t$$

Where, θ_r is the *angle of reflection* and θ_t is the *angle of transmission*.

$$\theta_i = \theta_r$$

- We know for lossless media: $\beta_1 = \omega\sqrt{\mu_1\varepsilon_1}$ $\beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\beta_1}{\beta_2} = \frac{u_2}{u_1} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}$$

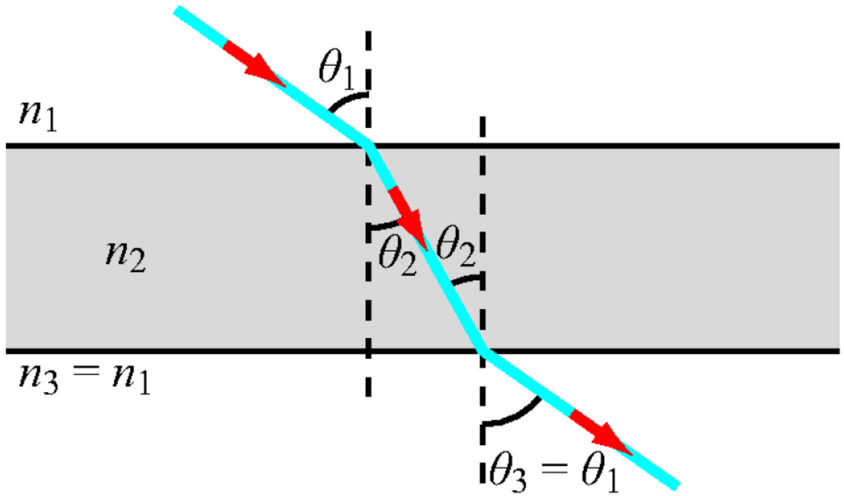
Snell's Law

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

n_1 and n_2 are the refractive indices of the media

Example – 1

A dielectric slab with index of refraction n_2 is surrounded by a medium with index of refraction n_1 as shown. If $\theta_i < \theta_c$, show that the emerging beam is parallel to the incident beam.



At the upper surface:

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

Similarly at the lower surface:

$$\sin\theta_3 = \frac{n_2}{n_3}\sin\theta_2 \quad \Rightarrow \quad \sin\theta_3 = \frac{n_2}{n_1}\sin\theta_2$$

$$\Rightarrow \sin\theta_3 = \left(\frac{n_2}{n_1}\right)\left(\frac{n_1}{n_2}\right)\sin\theta_1 = \sin\theta_1$$

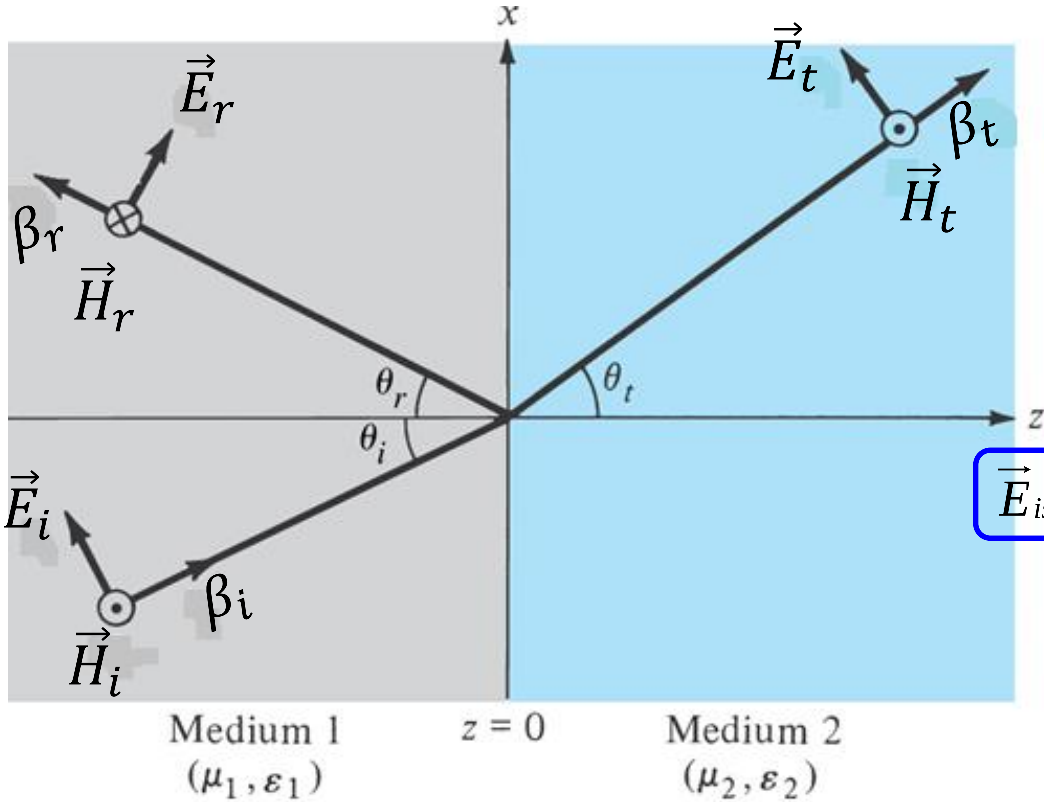
The slab displaces the beam's position but the beam's direction remains unchanged.

Reflection at Oblique Incidence (contd.)

- For normal incidence, the reflection and transmission coefficients Γ and τ at a boundary between two media are independent of the polarization of the incident wave, as both the $\vec{E}$ and $\vec{H}$ of a normally incident plane wave are tangential to the boundary regardless to the wave polarization.
- This is not the case for wave travelling at an angle $\theta_i \neq 0$ with respect to the normal to the interface.
- A wave of arbitrary polarization may be described as the superposition of two orthogonally polarized waves, one with its $\vec{E}$ parallel to the plane of incidence (parallel polarization or transverse magnetic (TM) polarization) and the other with $\vec{E}$ perpendicular to the plane of incidence (perpendicular polarization or transverse electric (TE) polarization).

Parallel Polarization

- Consider this figure: $\vec{E}$ field lies in the xz-plane, the plane of incidence.
- It illustrates the case of "Parallel Polarization".



- In medium 1 the incident waves are:

$$\vec{E}_{is} = \vec{E}_{io}(\hat{a}_x \cos\theta_i - \hat{a}_z \sin\theta_i)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{H}_{is} = \frac{\vec{E}_{io}}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \hat{a}_y$$

Parallel Polarization (contd.)

- In medium 1 the reflected waves are:

$$\vec{E}_{rs} = \vec{E}_{ro}(\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r) e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{H}_{rs} = -\frac{\vec{E}_{io}}{\eta_1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \hat{a}_y$$

- The transmitted fields in medium 2 are given by:

$$\vec{E}_{ts} = \vec{E}_{to}(\hat{a}_x \cos\theta_t - \hat{a}_z \sin\theta_t) e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\vec{H}_{ts} = \frac{\vec{E}_{to}}{\eta_2} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \hat{a}_y$$

- We know: $\theta_i = \theta_r$ and tangential components of electric and magnetic fields are continuous at the boundary z=0.
- Therefore:

$$(E_{io} + E_{ro})\cos\theta_i = E_{to}\cos\theta_t$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{1}{\eta_2} E_{to}$$

Parallel Polarization (contd.)

- Simplification gives:

$$\Gamma_{\parallel} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\tau_{\parallel} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

Fresnel's Equations for parallel polarization

- For $\theta_i = \theta_t = 0$, we get:

$$\Gamma_{\parallel} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma$$

$$\tau_{\parallel} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1} = \tau$$

- Furthermore, the expressions for reflection coefficient and transmission coefficient can be written in terms of *angle of incidence*.

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - \left(\frac{u_1^2}{u_2^2}\right)\sin^2\theta_i}$$

- In addition:

$$1 + \Gamma_{\parallel} = \tau_{\parallel}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$$

Parallel Polarization (contd.)

- The reflection coefficient $\Gamma_{||}$ equals zero when there is no reflection (only the parallel component is not reflected), and the *incident angle* at which this happens is called *Brewster's Angle* $\theta_{B||}$.
- The *Brewster's Angle* is also known as *polarizing angle*.
- At this angle, the perpendicular component of $\vec{E}$ will be reflected.
- Brewster's concept is utilized in *laser tube* used in surgical procedures.

- For Brewster's Angle, set $\Gamma_{||} = 0$:

$$\eta_2 \cos\theta_t = \eta_1 \cos\theta_{B||}$$

$$\Rightarrow \eta_2^2\left(1-\sin^2\theta_t\right) = \eta_1^2\left(1-\sin^2\theta_{B||}\right)$$

$$\sin^2\theta_{B||} = \frac{1-\dfrac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}}{1-\left(\dfrac{\varepsilon_1}{\varepsilon_2}\right)^2}$$

For a lossless and nonmagnetic medium:

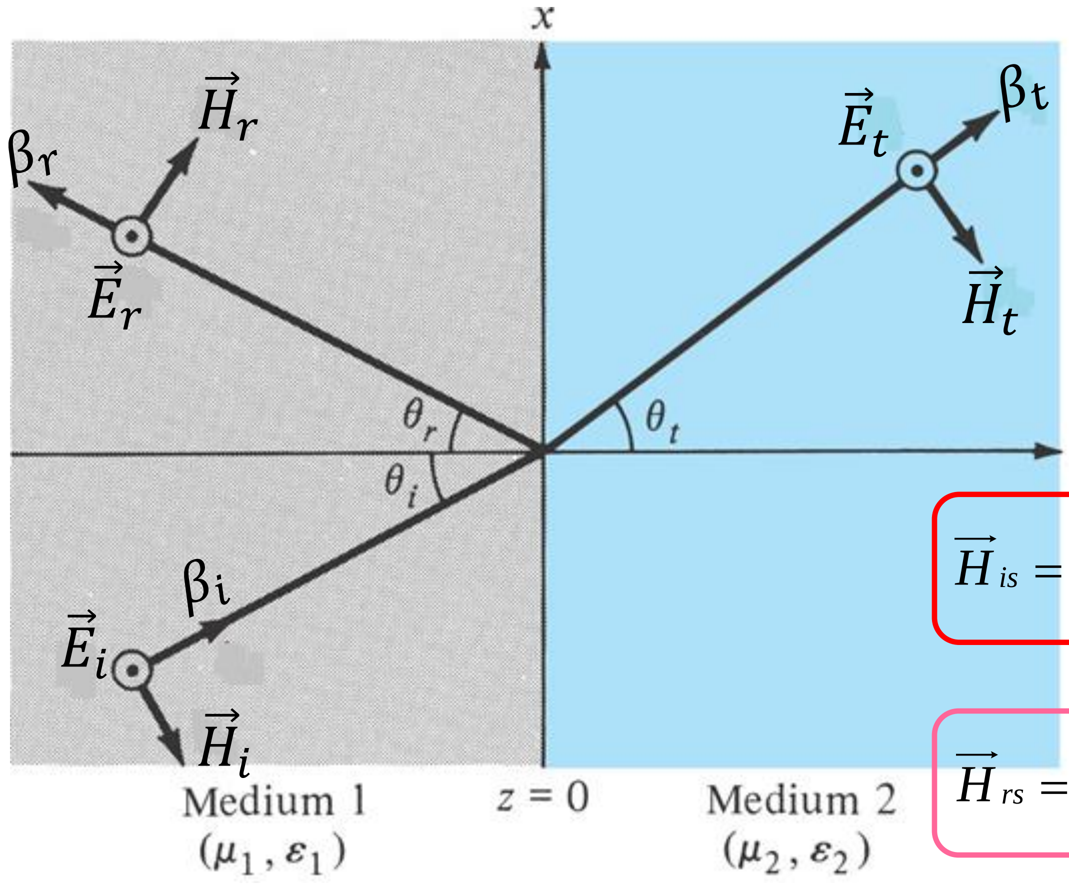
$$\sin^2\theta_{B||} = \frac{1}{1+\dfrac{\varepsilon_1}{\varepsilon_2}}$$

$$\sin\theta_{B||} = \sqrt{\frac{\varepsilon_2}{\varepsilon_2+\varepsilon_1}}$$

There is a Brewster Angle for any combination of ε_1 and ε_2.

Perpendicular Polarization

- The $\vec{E}$ field is perpendicular to the plane of incidence (the xz-plane).
- In this situation we get "Perpendicular Polarization".
- Here, $\vec{H}$ field is parallel to the plane of incidence.



$$\vec{E}_{is} = \vec{E}_{io} e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)} \hat{a}_y$$

$$\vec{E}_{rs} = \vec{E}_{io} e^{-j\beta_1 (x\sin\theta_r - z\cos\theta_r)} \hat{a}_y$$

$$\vec{H}_{is} = \frac{\vec{E}_{io}}{\eta_1} (-\hat{a}_x \cos\theta_i + \hat{a}_z \sin\theta_i) e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{H}_{rs} = \frac{\vec{E}_{ro}}{\eta_2} (\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r) e^{-j\beta_1 (x\sin\theta_r - z\cos\theta_r)}$$

Perpendicular Polarization (contd.)

- The transmitted fields in medium 2 are given by:

$$\vec{E}_{ts} = \vec{E}_{to} e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)} \hat{a}_y$$

$$\vec{H}_{ts} = \frac{\vec{E}_{to}}{\eta_2} (-\hat{a}_x \cos\theta_t + \hat{a}_z \sin\theta_t) e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)}$$

- Again, $\theta_i = \theta_r$ and tangential components of electric and magnetic fields are continuous at the boundary z=0.
- Therefore:

$$(E_{io} + E_{ro}) = E_{to}$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro})\cos\theta_i = \frac{1}{\eta_2} E_{to} \cos\theta_t$$

- Simplification gives:

$$\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro})\cos\theta_i = \frac{1}{\eta_2} E_{to} \cos\theta_t$$

Fresnel's Equations for perpendicular polarization

- For $\theta_i = \theta_t = 0$, we get:

$$\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma$$

$$\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1} = \tau$$

Perpendicular Polarization (contd.)

- Simplification for Brewster's Angle in Perpendicular Polarization gives:

$$\eta_2 \cos\theta_{B\perp} = \eta_1 \cos\theta_t \rightarrow \eta_2^2\left(1-\sin^2\theta_{B\perp}\right) = \eta_1^2\left(1-\sin^2\theta_t\right)$$

$$\sin^2\theta_{B\perp} = \frac{1-\dfrac{\mu_1\varepsilon_2}{\mu_2\varepsilon_1}}{1-\left(\dfrac{\mu_1}{\mu_2}\right)^2}$$

For nonmagnetic media, $\mu_1 = \mu_2 = \mu_0$ and therefore:

$$\sin^2\theta_{B\perp} \rightarrow \infty$$

Brewster's Angle doesn't exist as sine of an angle is never greater than unity

- If $\mu_1 \neq \mu_2$ and $\varepsilon_1 = \varepsilon_2$ then:

$$\sin^2\theta_{B\perp} = \frac{1}{1+\dfrac{\mu_1}{\mu_2}} \rightarrow \sin\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_2+\mu_1}}$$

Theoretically possible but rarely occurs in practice

Reflection at Oblique Incidence (contd.)

- The Brewster's Angle is also called Polarizing Angle.
- This is because if a wave composed of both the perpendicular and parallel polarization components is incident on a nonmagnetic surface at the Brewster angle $\theta_{B||}$, the parallel polarized component totally transmitted into the second medium and only the perpendicularly polarized component is reflected by the surface.
- Natural light, including sunlight and light generated by most manufactured sources, is $unpolarized$ because it consists of equal parallel and perpendicular rays. When they are incident upon a surface at the Brewster angle, the reflected wave is strictly perpendicularly polarized. Hence the surface acts as a polarizer.

Example – 2

- A wave in air is incident upon a soil surface at $\theta_i = 50°$. If soil has $\varepsilon_r = 4$ and $\mu_r = 1$, determine the following:

$\Gamma_\perp$ $\tau_\perp$ $\Gamma_\parallel$ $\tau_\parallel$ The Brewster angle

Applications of RF/Microwaves

- The use of RF/microwaves has greatly expanded.
- Examples include telecommunications, radio astronomy, land surveying, radar, meteorology, UHF television, terrestrial microwave links, solid-state devices, heating, medicine, and identification systems.
- Features that make microwaves attractive for communications include wide available bandwidths (capacities to carry information) and directive properties for short wavelengths.
- Currently, there are three main techniques to carry energy over long distances: (a) microwave links, (b) coaxial cables, and (c) fibre optics.
- A microwave system normally consists of a transmitter (including a microwave oscillator, waveguides, amplifiers, and transmitting antenna) and a receiver subsystem (including a receiver antenna, transmission line or waveguide, and amplifiers).
- A microwave network is usually an interconnection of various microwave components and devices.

Applications of RF/Microwaves (contd.)

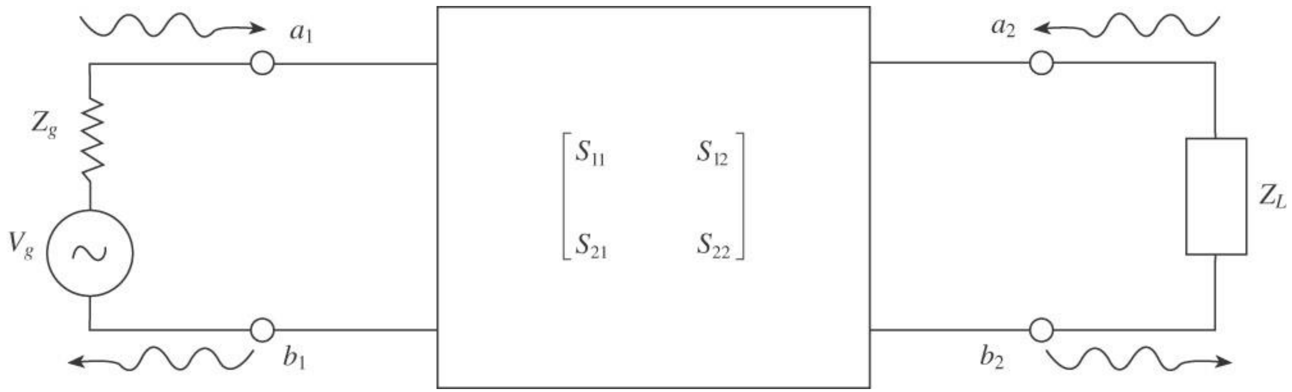
- Common microwave components include:
- Coaxial cables, which are transmission lines for interconnecting microwave components.
- Waveguide sections, which may be straight, curved, or twisted.
- Antenna, which transmit or receive EM waves efficiently.
- Terminators, which are designed to absorb the input power and therefore acts as one port network.
- Attenuators, which are designed to absorb some of the EM power passing through the device, thereby decreasing the power level of the microwave signal.
- Directional couplers, with a mechanism to couple between different ports.
- Isolators, which allow energy flow in only one direction.
- Circulators, which are designed to establish various entry/exit points where power can be either fed or extracted.
- Filters, which suppress unwanted signals and/or separate signals of different frequencies.

RF/Microwave Circuit

- A microwave circuit consists of microwave components such as sources, transmission lines, waveguides, attenuators, circulators, and filters.
- One way of analyzing, such circuits, are to relate the input and output variables of each component.
- At RF/microwave frequencies, where current and voltage are not well defined, it is a common practice to use S-parameters for analysis.
- S-parameters are defined in terms of wave variables which are more easily measurable at high frequencies than voltage and current.

RF/Microwave Circuit (contd.)

- Let us consider following 2-port network:



- The traveling waves are related to the S-parameters as:

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

where, a_1 and a_2 are incident waves at port 1 and 2 respectively; while b_1 and b_2 represent the reflected waves.

RF/Microwave Circuit (contd.)

- In matrix form:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

The off-diagonal terms represent transmission coefficients, while the diagonal terms represent reflection coefficients.

- If the network is reciprocal, it will have the same transmission characteristic in either direction.

$$S_{12} = S_{21}$$

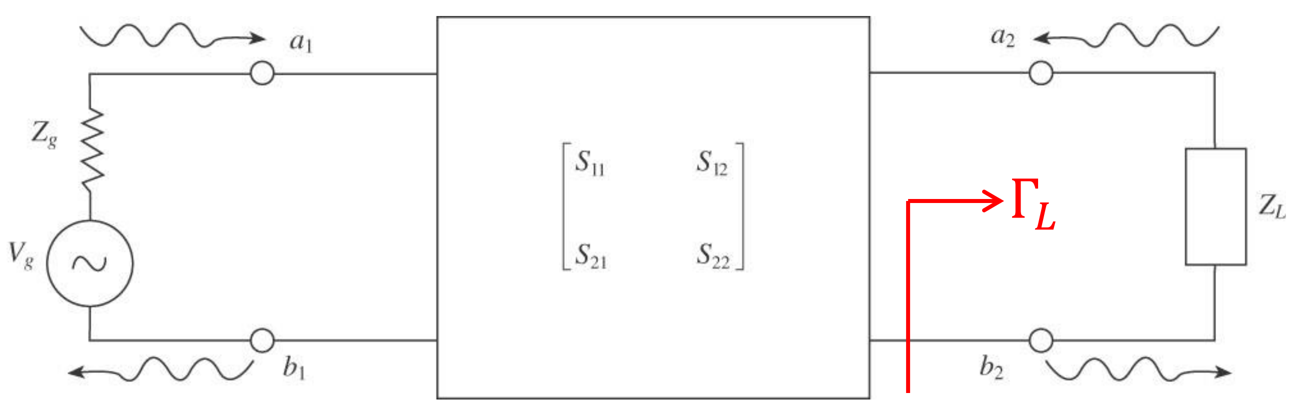
- If the network is symmetric, then:

$$S_{11} = S_{22}$$

- For matched two port network:

$$S_{11} = S_{22} = 0$$

RF/Microwave Circuit (contd.)



- The input reflection coefficient in terms of the S-parameters and the load Z_L:

$$\Gamma_i = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

RF/Microwave Circuit (contd.)

- Similarly, the output reflection coefficient (with $V_g = 0$) can be expressed in terms of the generator impedance Z_g:

$$\Gamma_o = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}\Gamma_g}{1 - S_{11}\Gamma_g}$$

$$\Gamma_g = \frac{Z_g - Z_{in}}{Z_g + Z_{in}}$$

Example – 3

- S-parameters are obtained for a microwave transistor operating at 2.5 GHz: $S_{11} = 0.85 < -30°, S_{12} = 0.07 < 56°, S_{21} = 1.68 < 120°, S_{22} = 0.85 < -40°$. Determine the input reflection coefficient when $Z_L = Z_o = 75\Omega$.

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = 0$$

$$\Gamma_i = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = S_{11} = 0.85\angle -30°$$

Waveguiding Structure

- A waveguiding structure is one that carries a wave (or power) from one point to another.
- There are three common types:

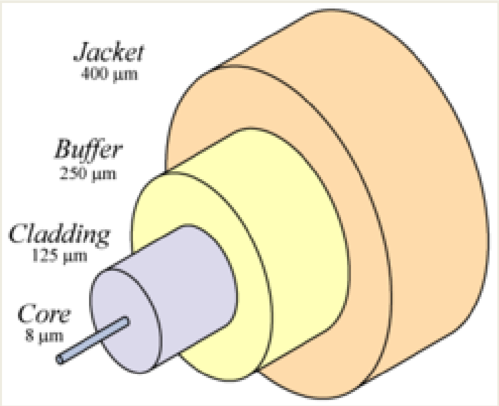
  - Fiber-optic guides
  - Waveguides
  - Transmission lines

Note: An alternative to waveguiding structures is wireless transmission using antennas.

Fiber-Optic Guides

Properties

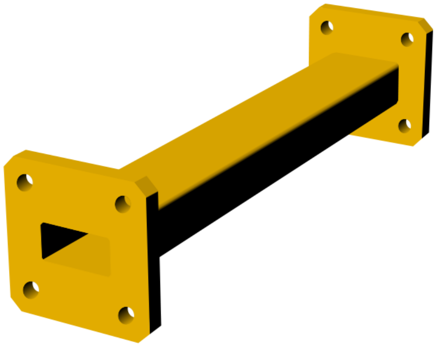
- Can propagate a signal at any frequency (in theory)
- Can be made very low loss
- Has minimal signal distortion
- Very immune to interference
- Not suitable for high power
- Has both E_z and H_z components of the fields



Waveguides

Properties

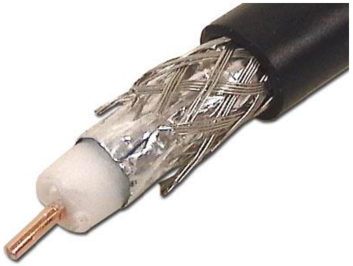
- Has a single hollow metal pipe
- Can propagate a signal only at high frequency: $\omega > \omega_c$
- The width must be at least one-half of a wavelength
- Has signal distortion, even in the lossless case
- Immune to interference
- Can handle large amounts of power
- Has low loss (compared with a transmission line)
- Has either E_z or H_z component of the fields (TM_z or TE_z)



Transmission Line

Properties

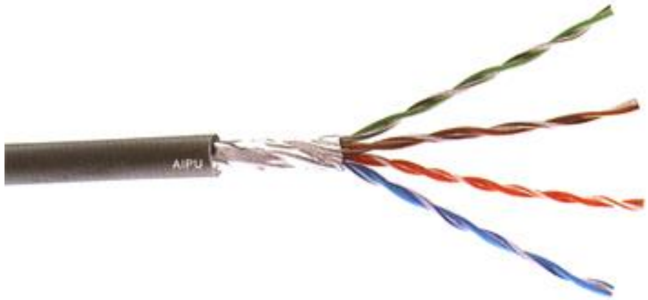
- Has two conductors running in parallel
- Can propagate a signal at any frequency (in theory)
- Becomes lossy at high frequency
- Can handle low or moderate amounts of power
- Does not have signal distortion, unless there is loss
- May or may not be immune to interference
- Does not have E_z or H_z components of the fields (TEM_z)



Coaxial cable (coax)

Twin lead
(shown connected to a impedance-transforming balun)

Transmission Line (contd.)

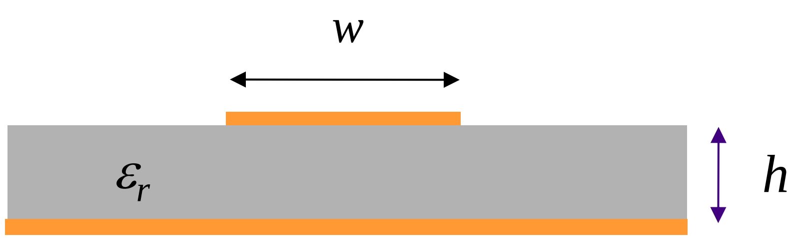


CAT 5 cable
(twisted pair)

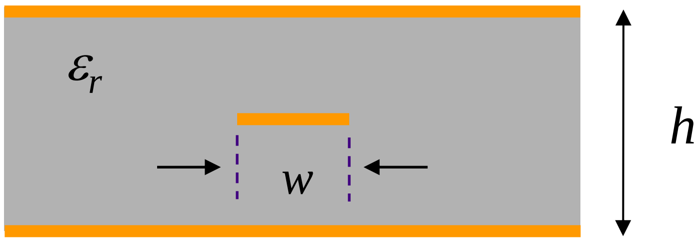
The two wires of the transmission line are twisted to reduce interference and radiation from discontinuities.

Transmission Line (contd.)

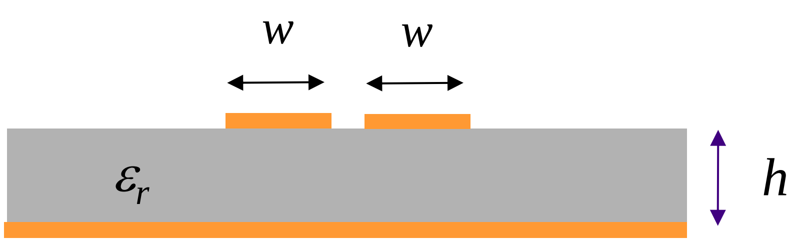
Transmission lines commonly used on printed-circuit boards



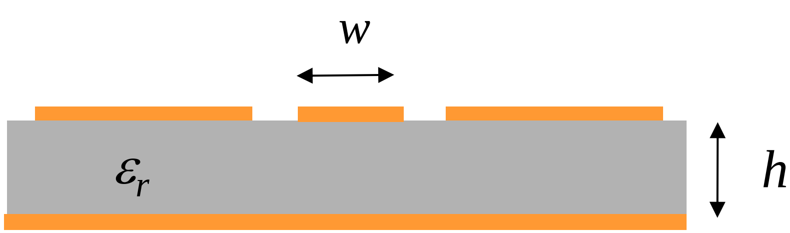
Microstrip



Stripline



Coplanar strips

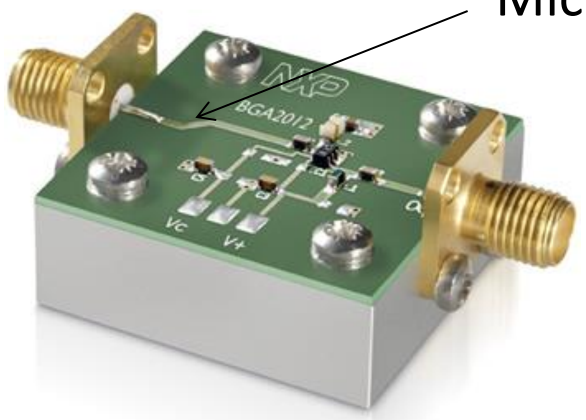


Coplanar waveguide (CPW)

Transmission Line (contd.)

Transmission lines commonly used on printed-circuit boards

Microstrip line



A microwave integrated circuit

Transmission Line Theory

- Lumped circuits: resistors, capacitors, inductors

  neglect time delays (phase)

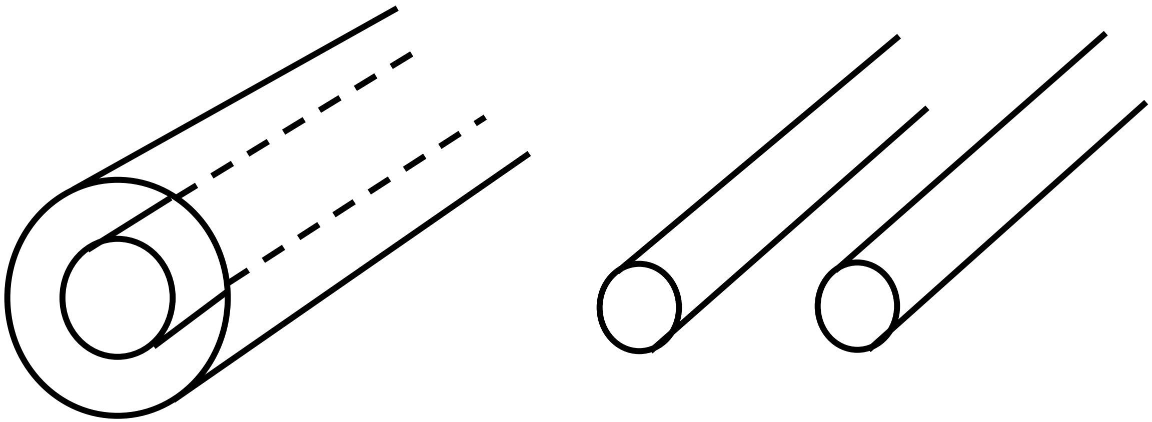
- Distributed circuit elements: transmission lines

  account for propagation and time delays (phase change)

We need transmission-line theory whenever the length of a line is significant compared with a wavelength.

Transmission Line Theory (contd.)

2 conductors



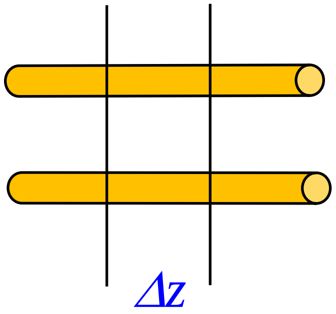
4 per-unit-length parameters:

C = capacitance/length [F/m]

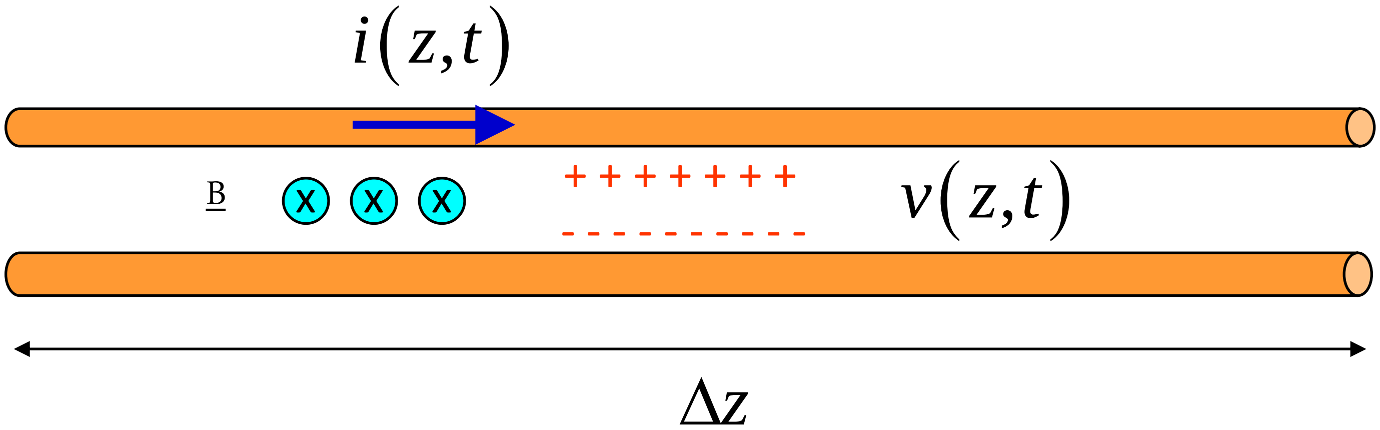
L = inductance/length [H/m]

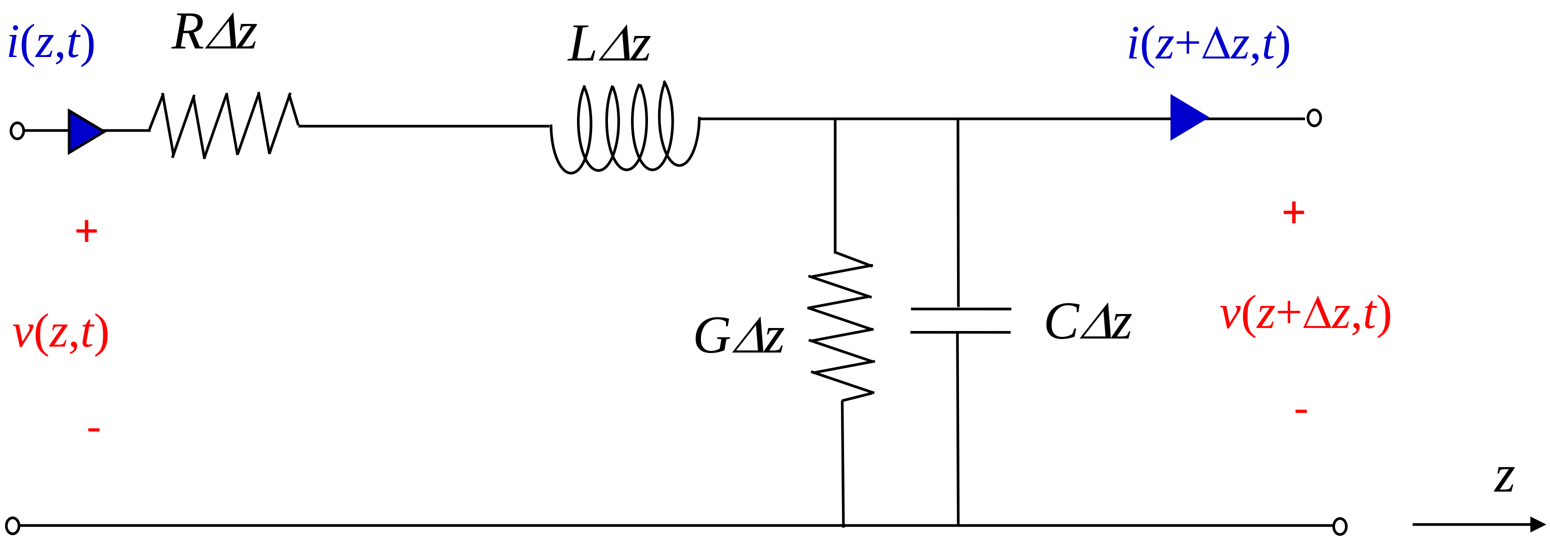
R = resistance/length [Ω/m]

G = conductance/length [S/m]



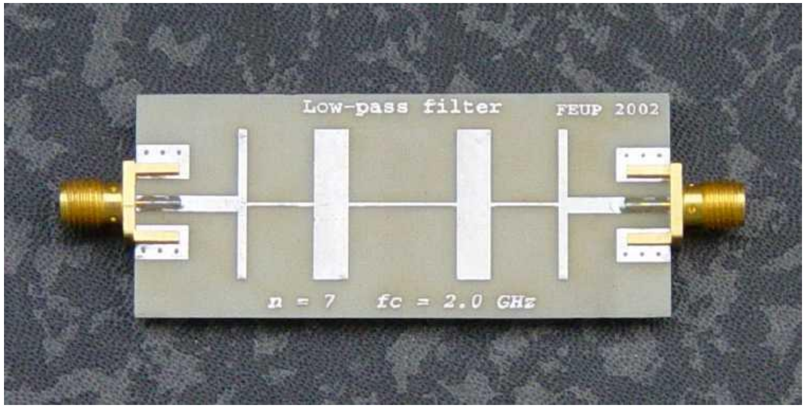
Transmission Line Theory (contd.)





Using Transmission Lines to Synthesize Impedances

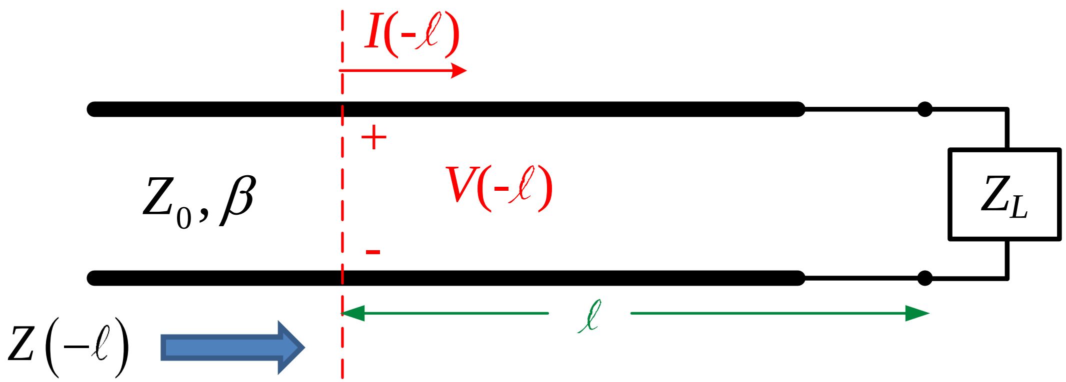
- This is very useful is RF/microwave engineering.



A microwave filter constructed from microstrip.

Using Transmission Lines to Synthesize Impedances (contd.)

- A lossless transmission line terminated in load impedance Z_L



(A) Matched load: ($Z_L = Z_0$)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

← No reflection from the load

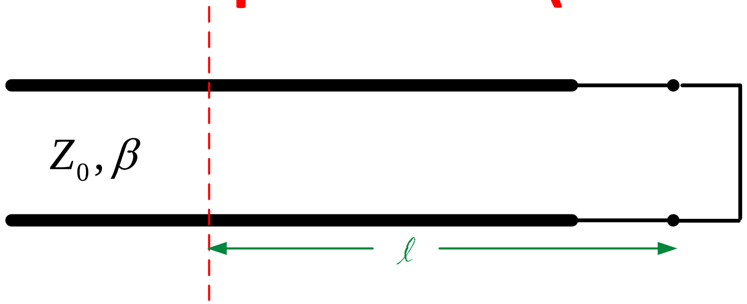
$$\Rightarrow V(-\ell) = V_0^+ e^{+j\beta\ell}$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{+j\beta\ell}$$

$$\Rightarrow Z(-\ell) = Z_0$$

For any ℓ

Using Transmission Lines to Synthesize Impedances (contd.)



(B) Short circuit load: $(Z_L = 0)$

$$\Gamma_L = \frac{0 - Z_0}{0 + Z_0} = -1$$

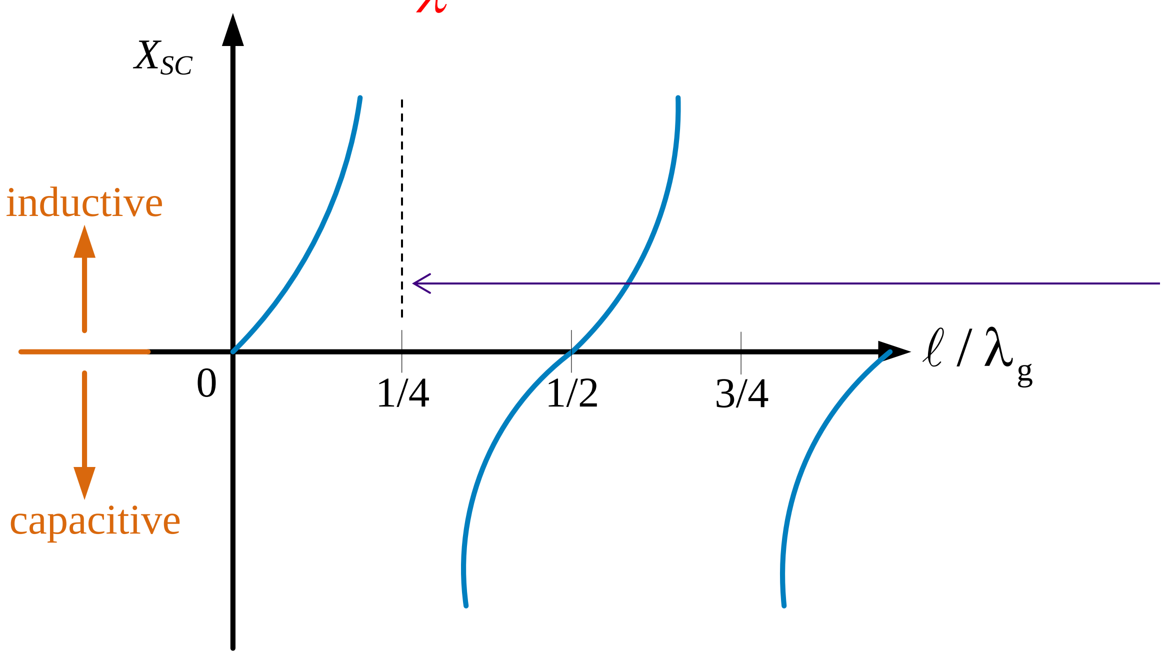
$$\Rightarrow Z(-\ell) = jZ_0 \tan(\beta\ell)$$

Always imaginary!

$$\Rightarrow Z(-\ell) = jX_{sc}$$

$$X_{sc} = Z_0 \tan(\beta\ell)$$

Note: $\beta\ell = 2\pi \frac{\ell}{\lambda}$



S.C. can become an O.C. with a $\lambda/4$ trans. line