Lecture – 21 Date: 06.04.2015

- Electromagnetic Wave Propagation
- Complex Permittivity, Loss Tangent, Intrinsic Impedance, Skin Depth, Skin Effect etc.
- Electromagnetic Shielding

Introduction

- Let us consider the Maxwell's equations in free space (i.e., $\rho_v = \vec{J} = 0$).

$$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \qquad \nabla . \vec{E} = 0 \qquad \nabla . \vec{H} = 0$$

- **First equation states that:** If $\vec{E}$ is changing with time at some point, then $\vec{H}$ has curl at that point; therefore $\vec{H}$ varies spatially in a direction normal to its orientation direction.
- Also, if $\vec{E}$ is changing with time, then $\vec{H}$ will in general also change with time, although not necessarily in the same way.
- **Next we see from second equation:** a time varying $\vec{H}$ generates $\vec{E}$, which having curl, varies spatially in the direction normal to its orientation.
- We now once more have a changing $\vec{E}$, our original hypothesis, but this field is present at a small distance away from the point of original disturbance.

Clearly demonstrates the propagation of Electric and Magnetic field and in turn transfer of energy.

Introduction (contd.)

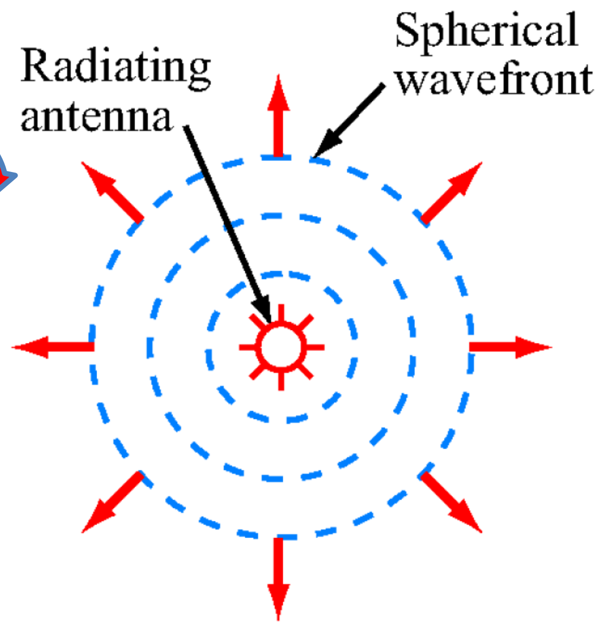
- The velocity with which this effect moves away from the original point is the velocity of light.
- We postulate the existence of *$uniform\ plane\ wave$*, in which both fields $\vec{E}$ and $\vec{H}$, lie in the transverse plane → that is, the plane whose normal is the direction of propagation.

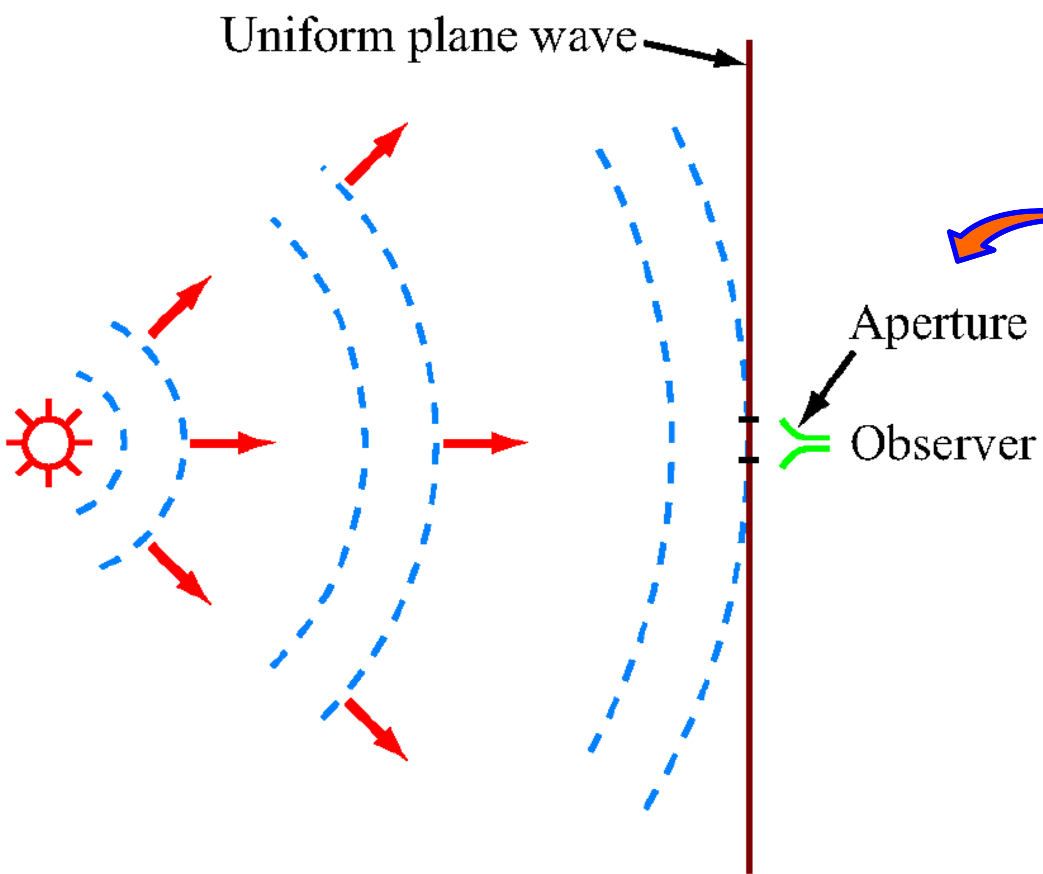
A *$uniform\ plane\ wave$* is characterized by electric and magnetic fields that have uniform properties at all points across an infinite plane.

A *$plane\ wave$* has no electric or magnetic field components along its direction of propagation

Introduction (contd.)

For example, a wave produced by a localized source, such as an antenna, expands outwardly in the form of spherical wave.

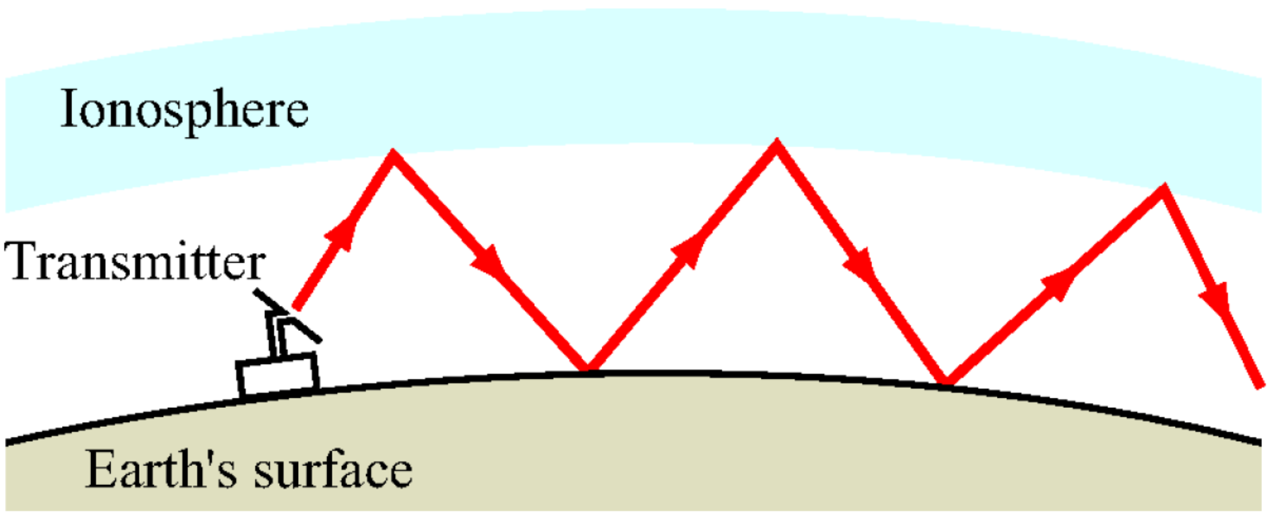




However, it looks a part of a uniform plane wave, with an identical properties at all points in the plane tangent to the wavefront, to an observer very far.

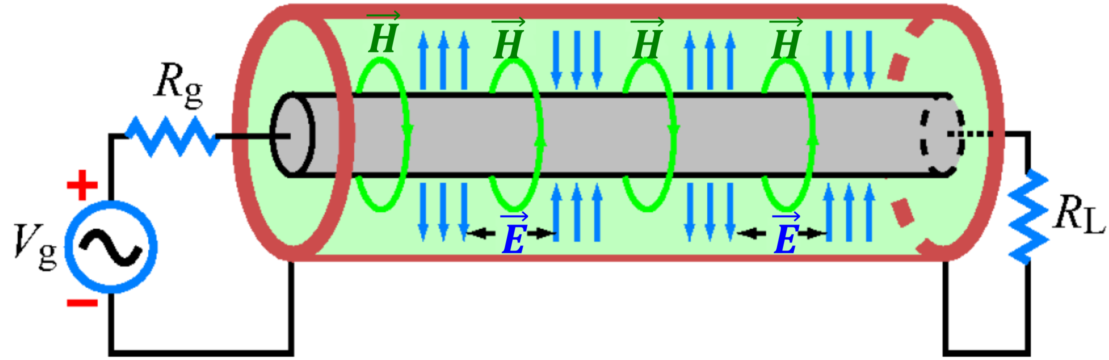
Introduction (contd.)

- When a wave propagates through a homogeneous medium without interacting with obstacles or material interfaces, it is called *unbounded* and when a wave propagates along a material structure, it is called guided.
- Earth's surface and ionosphere constitute parallel boundaries of a natural structure capable of guiding short-wave radio transmission in the HF band (3 to 30MHz).



Indeed, the ionosphere is a good reflector at HF band.

Introduction (contd.)



- Similarly, a transmission line such as coaxial can guide a wave. For example, when an ac source excites an incident wave that travels down the coaxial line toward the load.
- Unless the load is matched to the line, part (or all) of the incident wave is reflected back toward the source.
- At any point on the line, the instantaneous total voltage $v(z,t)$ is the sum of the reflected and incident waves, both of which vary sinusoidally with time.
- Associated with the voltage difference between the inner and outer conductors is a radial electric field $\vec{E}(z,t)$ that exists in the dielectric material. $\vec{E}(z,t)$ is also sinusoidal as $v(z,t)$ varies sinusoidally.
- Furthermore, the current flowing through the inner conductor induces an azimuthal magnetic field $\vec{H}(z,t)$.
- The coupled $\vec{E}(z,t)$ & $\vec{H}(z,t)$ constitute an EM field and models the wave propagation on a transmission line.
- So, propagation can be talked in terms of $v(z,t)$ & $i(z,t)$ or $\vec{E}(z,t)$ & $\vec{H}(z,t)$.

Wave Propagation in Lossy Dielectrics

- Let us develop formulations for wave propagation in lossy dielectrics – it provides the general case of wave propagation.
- A lossy dielectric is a medium in which an EM wave, as it propagates, loses power owing to imperfect dielectric.
- In other words, a lossy dielectric is partially conducting medium (imperfect dielectric or imperfect conductor) with $\sigma \neq 0$, as distinct from perfect dielectric in which $\sigma = 0$.
- The Maxwell's equations in a linear, isotropic, homogeneous, lossy dielectric medium that is charge free is given by:

$$\nabla \times \vec{H}_s = (\sigma + j\omega\varepsilon)\vec{E}_s$$

$$\nabla \times \vec{E}_s = -j\omega\mu\vec{H}_s$$

$$\nabla . \vec{E}_s = 0$$

$$\nabla . \vec{H}_s = 0$$

The time factor $e^{j\omega t}$ has been suppressed in above expressions.

Wave Propagation in Lossy Dielectrics (contd.)

$$\nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s$$

Take curl on both sides

$$\nabla \times \nabla \times \vec{E}_s = -j\omega\mu\left(\nabla \times \vec{H}_s\right)$$

$$\nabla\left(\nabla . \vec{E}_s\right) - \nabla^2 \vec{E}_s = -j\omega\mu(\sigma + j\omega\varepsilon)\vec{E}_s$$

=0

$$\Rightarrow \nabla^2 \vec{E}_s - j\omega\mu(\sigma + j\omega\varepsilon)\vec{E}_s = 0$$

$$\nabla^2 \vec{E}_s - \gamma^2 \vec{E}_s = 0$$

Where, $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

γ is called the propagation constant

- We can similarly find expression for magnetic field: $\nabla^2 \vec{H}_s - \gamma^2 \vec{H}_s = 0$
- These expressions are called vector Helmholtz's equations.
- In cartesian coordinates, for example, **each of these two vector equations** are equivalent to three scalar wave equations → one for each components of $\vec{E}_s$ or $\vec{H}_s$ along $\hat{a}_x$, $\hat{a}_y$, and $\hat{a}_z$.

Wave Propagation in Lossy Dielectrics (contd.)

$$\frac{\partial^2 E_{sx}}{\partial x^2}+\frac{\partial^2 E_{sx}}{\partial y^2}+\frac{\partial^2 E_{sx}}{\partial z^2}=\gamma^2 E_{sx}$$

$$\frac{\partial^2 H_{sx}}{\partial x^2}+\frac{\partial^2 H_{sx}}{\partial y^2}+\frac{\partial^2 H_{sx}}{\partial z^2}=\gamma^2 H_{sx}$$

$$\frac{\partial^2 E_{sy}}{\partial x^2}+\frac{\partial^2 E_{sy}}{\partial y^2}+\frac{\partial^2 E_{sy}}{\partial z^2}=\gamma^2 E_{sy}$$

$$\frac{\partial^2 H_{sy}}{\partial x^2}+\frac{\partial^2 H_{sy}}{\partial y^2}+\frac{\partial^2 H_{sy}}{\partial z^2}=\gamma^2 H_{sy}$$

$$\frac{\partial^2 E_{sz}}{\partial x^2}+\frac{\partial^2 E_{sz}}{\partial y^2}+\frac{\partial^2 E_{sz}}{\partial z^2}=\gamma^2 E_{sz}$$

$$\frac{\partial^2 H_{sz}}{\partial x^2}+\frac{\partial^2 H_{sz}}{\partial y^2}+\frac{\partial^2 H_{sz}}{\partial z^2}=\gamma^2 H_{sz}$$

The component fields of any time-harmonic EM wave must individually satisfy these six partial differential equations. In many cases, the EM wave will not contain all six components. An example of this is the $plane\ wave$.

Wave Propagation in Lossy Dielectrics (contd.)

- If we assume that the wave propagates along $+\hat{a}_z$ and that $\vec{E}_s$ has only an x-component, then:

$$\vec{E}_s = E_{xs}(z)\hat{a}_x$$

- Substitution of this into Helmholtz equation results in:

$$\left(\nabla^2 - \gamma^2\right)E_{xs}(z) = 0$$

- Therefore:

$$\frac{\partial^2 E_{xs}(z)}{\partial x^2} + \frac{\partial^2 E_{xs}(z)}{\partial y^2} + \frac{\partial^2 E_{xs}(z)}{\partial z^2} - \gamma^2 E_{xs}(z) = 0$$

=0

- Hence:

$$\left[\frac{d^2}{dz^2} - \gamma^2\right]E_{xs}(z) = 0$$

Scalar wave equation

It is a linear homogeneous differential equation whose solution is:

$$E_{xs}(z) = E_0^+ e^{-\gamma z} + E_0^- e^{\gamma z}$$

- Where, the first component is the wave propagating in $+z$ *direction* and the second term is the wave propagating in $-z$ *direction*.
- We assumed, wave only propagating in $+z$ *direction*. Therefore, $E_0^- = 0$.

Wave Propagation in Lossy Dielectrics (contd.)

- Since γ is a complex quantity, we can express it as:

$$\gamma = \alpha + j\beta \Rightarrow \gamma^2 = \alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu(\sigma + j\omega\varepsilon)$$

- Simplification gives:

$$\operatorname{Re}\gamma^2 = \alpha^2 - \beta^2 = -\omega^2\mu\varepsilon \Rightarrow \beta^2 - \alpha^2 = \omega^2\mu\varepsilon$$

- Furthermore:

$$\left|\gamma^2\right| = \beta^2 + \alpha^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2}$$

- From the above two expressions we can obtain:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

- Therefore the simplified solution of wave equation is:

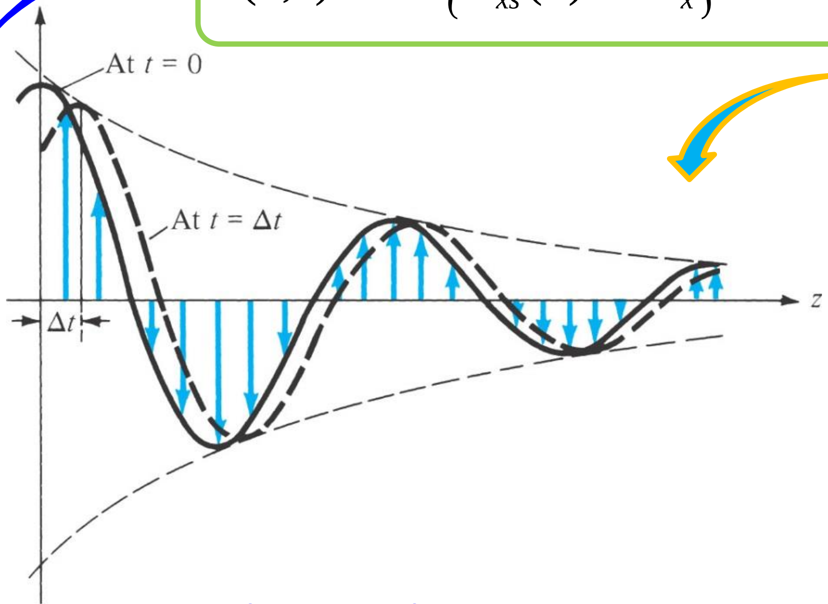
$$E_{xs}(z) = E_0^+ e^{-\gamma z} = E_0^+ e^{-(\alpha + j\beta)z}$$

- Inserting the time factor in the solution yields:

$$\vec{E}(z,t) = \operatorname{Re}\left\{E_{xs}(z)e^{j\omega t}\hat{a}_x\right\} = \operatorname{Re}\left\{E_0^+ e^{-\alpha z} e^{j(\omega t - \beta z)}\hat{a}_x\right\}$$

Wave Propagation in Lossy Dielectrics (contd.)

$$\vec{E}(z,t) = \mathrm{Re}\left\{E_{xs}(z)e^{j\omega t}\hat{a}_x\right\} = \mathrm{Re}\left\{E_0^+ e^{-\alpha z} e^{j(\omega t - \beta z)}\hat{a}_x\right\}$$



An electric field with an x-component traveling in +z direction at $t = 0$ and $t = \Delta t$; arrows indicate instantaneous values of Electric Field.

- It is apparent that as the wave propagates along $+\hat{a}_z$, it decreases or attenuates in amplitude by a factor $e^{-\alpha z}$, and therefore α is known as the attenuation constant or attenuation coefficient of the medium → It is a measure of the spatial rate of decay of the wave in the medium, measured in nepers per meter → For free space, $\sigma = 0$ and therefore $\alpha = 0$ → the wave doesn't attenuate in free space.
- The quantity β is a measure of phase shift per unit length in radians per meter and is called the phase constant or wave number.

Wave Propagation in Lossy Dielectrics (contd.)

- The solution for magnetic field is: $\vec{H}(z,t) = \mathrm{Re}\left\{H_0^+ e^{-\alpha z} e^{j(\omega t - \beta z)} \hat{a}_y\right\}$

- Where: $H_0^+ = \dfrac{E_0^+}{\eta}$

η is a complex quantity known as the *intrinsic impedance* of the medium.

$$\eta = \eta = |\eta| e^{j\theta_\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

Derive it !

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\dfrac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$$

$$0 \le \theta_\eta \le 45^\circ$$

- Therefore the magnetic field expression is:

$$\vec{H}(z,t) = \mathrm{Re}\left\{\frac{E_0}{|\eta| e^{j\theta_\eta}} e^{-\alpha z} e^{j(\omega t - \beta z)} \hat{a}_y\right\} \Rightarrow \vec{H}(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\hat{a}_y$$

It is evident that $\vec{E}$ and $\vec{H}$ are out of phase by θ_η.

Wave Propagation in Lossy Dielectrics (contd.)

- In terms of β, the wave velocity u and wavelength λ are:

$$u = \frac{\omega}{\beta}$$

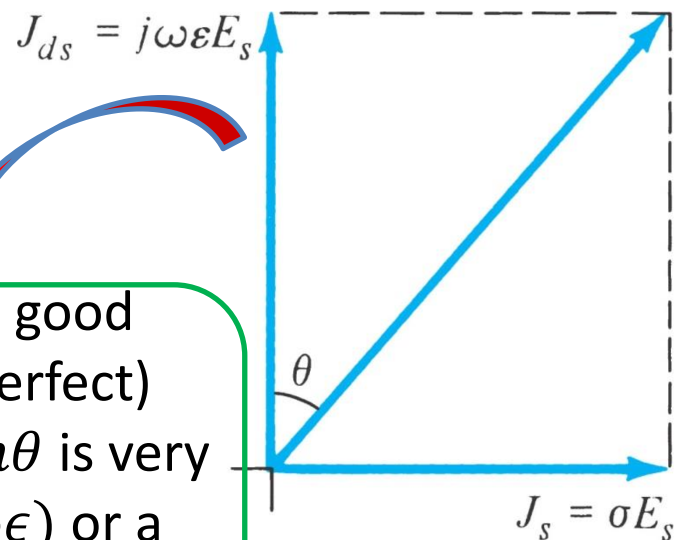
$$\lambda = \frac{2\pi}{\beta}$$

- Furthermore, the ratio of the magnitude of conduction current density $\vec{J}_c$ to that of the displacement current density $\vec{J}_d$ is:

$$\frac{|\vec{J}_{cs}|}{|\vec{J}_{ds}|} = \frac{|\sigma \vec{E}_s|}{|j\omega\varepsilon \vec{E}_s|} = \frac{\sigma}{\omega\varepsilon} = \tan\theta$$

$tan\theta$ is known as the *loss tangent* and θ is the *loss angle* of the medium.

A medium is good (lossless or perfect) dielectric if $tan\theta$ is very small ($\sigma \ll \omega\epsilon$) or a good conductor if $tan\theta$ is large ($\sigma \gg \omega\epsilon$)



Wave Propagation in Lossy Dielectrics (contd.)

- In general, for propagation of wave, characteristics of any medium doesn't only depend on the parameters $\sigma, \epsilon, and\ \mu$ but also on frequency of operation.
- A medium that is regarded as good conductor at low frequency may be a good dielectric at high frequencies.
- We have:
  - From definition of intrinsic impedance: $\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$
  - From definition of loss tangent: $\frac{\sigma}{\omega\varepsilon} = \tan\theta$
  - Therefore: $\theta = 2\theta_\eta$
- Furthermore: $\nabla \times \vec{H}_s = (\sigma + j\omega\varepsilon)\vec{E}_s$ → $\nabla \times \vec{H}_s = j\omega\varepsilon\left[1 - \frac{j\sigma}{\omega\varepsilon}\right]\vec{E}_s$

$$\varepsilon_c = \varepsilon\left[1 - \frac{j\sigma}{\omega\varepsilon}\right]$$

$$\Rightarrow \nabla \times \vec{H}_s = j\omega\varepsilon_c \vec{E}_s$$

Wave Propagation in Lossy Dielectrics (contd.)

$$\varepsilon_c = \varepsilon\left[1 - \frac{j\sigma}{\omega\varepsilon}\right]$$

$$\varepsilon_c = \varepsilon' - j\varepsilon''$$

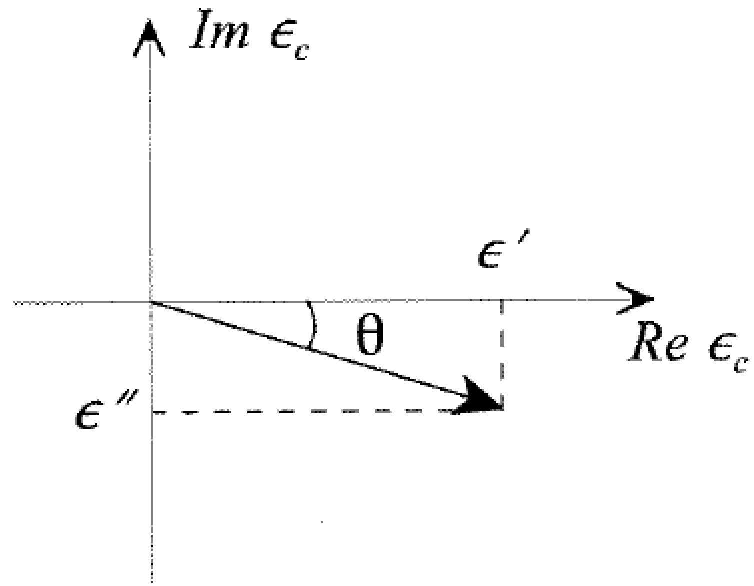
$$\varepsilon' = \varepsilon$$

$$\varepsilon'' = \frac{\sigma}{\omega}$$

ε_c is called the complex permittivity of the medium.

- The loss tangent is:

$$\tan\theta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon}$$



Example – 1

- If the magnetic field phasor of a plane wave traveling in a medium with intrinsic impedance $\eta = 100\Omega$ is given by $\vec{H}_s = \left(10\hat{a}_y + 20\hat{a}_z\right)e^{-j4x}\,\frac{mA}{m}$. Find the associated electric field phasor.

- It is clear that the wave travels in $x - direction$.
- Therefore:

$$\vec{E}_s = -\eta(\hat{a}_x \times \vec{H}_s)$$

$$\vec{E}_s = -100\left[\hat{a}_x \times (10\hat{a}_y + 20\hat{a}_z)\right]e^{-j4x} \times 10^{-3}$$

$$\therefore\ \vec{E}_s = \left(-\hat{a}_z + 2\hat{a}_y\right)e^{-j4x}\,\frac{V}{m}$$

Example – 2

- In the previous example, determine the electric field if the magnetic field is given by $\vec{H}_s = \hat{a}_y\left(10e^{-j3x} - 20e^{j3x}\right)\frac{mA}{m}$.

- This magnetic field is composed of two components, one with amplitude of $10\ mA/m$ belonging to a wave traveling along $+\hat{a}_x$ and another with amplitude of $20\ mA/m$ belonging to a separate wave traveling in the opposite direction $-\hat{a}_x$. Hence, we need to treat these two components separately.

$$\vec{H}_s = \vec{H}_{1s} + \vec{H}_{2s} = \hat{a}_y 10e^{-j3x}\frac{mA}{m} - \hat{a}_y\ 20e^{j3x}\frac{mA}{m}$$

- Then use: $\vec{E}_s = -\eta(\hat{a}_x \times \vec{H}_s)$

$$\therefore \vec{E}_s = \hat{a}_z\left(e^{-j3x} + 2e^{j3x}\right)\frac{V}{m}$$

Plane Waves in Lossless Dielectrics

- In a lossless dielectrics, $\sigma \ll \omega\varepsilon$.
- In such a scenario: $\sigma \approx 0,\ \varepsilon = \varepsilon_0\varepsilon_r,\quad \mu = \mu_0\mu_r$.
- Therefore:

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$u = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\angle 0^\circ$$

Thus $\vec{E}$ and $\vec{H}$ are in time phase with each other.

Plane Waves in Free Space

- In this case: $\sigma = 0,\ \varepsilon = \varepsilon_0,\ \ \mu = \mu_0.$
- Therefore: $\alpha = 0$ $\beta = \omega\sqrt{\mu_0\varepsilon_0} = \dfrac{\omega}{c}$ $u = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}} = c$ $\lambda = \dfrac{2\pi}{\omega c}$ $\eta = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}\angle 0°$
- The fact that EM waves travel in free space with the speed of light is significant.
- It provides evidence that light is the manifestation of an EM wave.
- We have:

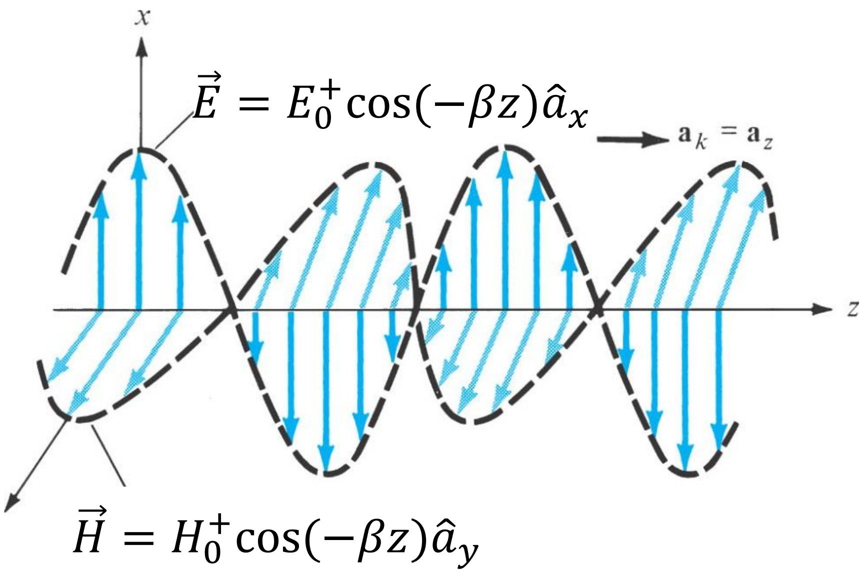
$$\eta = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \simeq 377\Omega$$

- Furthermore:

$$\vec{E} = E_0^+ \cos(\omega t - \beta z)\hat{a}_x$$

$$\vec{H} = \frac{E_0^+}{\eta_0}\cos(\omega t - \beta z)\hat{a}_y$$

- The plots of $\vec{E}$ and $\vec{H}$ are shown below.



Plane Waves in Free Space (contd.)

- In general, if $\hat{a}_E$, $\hat{a}_H$ and $\hat{a}_k$ are unit vectors along $\vec{E}$, $\vec{H}$ and the direction of propagation, then:

$$\hat{a}_k \times \hat{a}_E = \hat{a}_H$$

$$\hat{a}_k \times \hat{a}_H = -\hat{a}_E$$

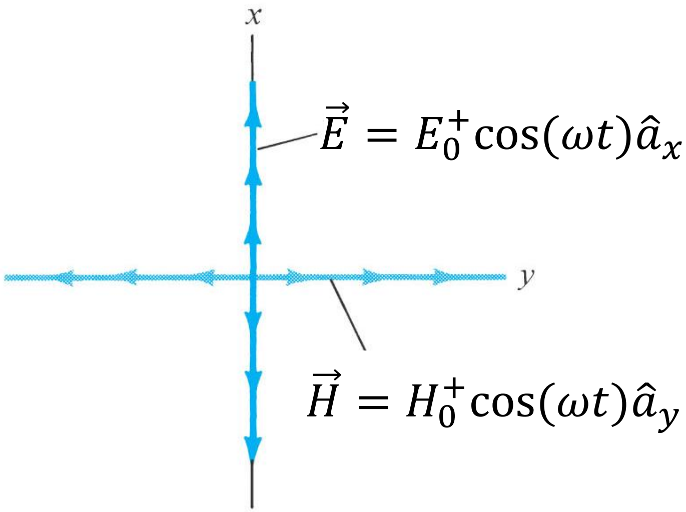
$$\hat{a}_E \times \hat{a}_H = \hat{a}_k$$

- Both $\vec{E}$ and $\vec{H}$ fields are everywhere normal to the direction of wave propagation.
- It means that the fields lie in a plane that is transverse or orthogonal to the direction of propagation.
- They form an EM wave that has no electric or magnetic field components along the direction of propagation → such a wave is called transverse electromagnetic (TEM) wave.
- A combination of $\vec{E}$ and $\vec{H}$ is called a uniform plane wave because fields have same magnitude throughout any transverse plane.
- The direction in which the electric field points is the **polarization** of a TEM wave → Essentially, polarization of a uniform plane wave describes the locus traced by the tip of the $\vec{E}$ vector (in the plane orthogonal to the direction of propagation) at a given point in space as a function of time.

Plane Waves in Free Space (contd.)

$$\vec{E}(z,t) = E_0^+ e^{-\alpha z}\cos(\omega t - \beta z)\hat{a}_x$$

It is polarized in x-direction



It illustrates a uniform plane wave

- In practice, a uniform plane wave can't exist because it stretches to infinity and would represent an infinite energy → however these waves are characteristically simple and fundamentally important.
- These serve as approximations for practical waves such as those from radio antenna at distances sufficiently far from radiating sources.
- The on-going discussion are applicable for any other isotropic medium.

Plane Waves in Good Conductors

- In a good conductor, displacement current is negligible in comparison to conduction current $(J_{conduction} \gg J_{displacement}) \leftrightarrow$ Because, for a perfect or good conductor, $\sigma \gg \omega\varepsilon$.
- **Although this inequality is frequency dependent,** most good conductors (such as copper and aluminum) have conductivities on the order of $10^7\ mho/m$ and negligible polarization such that we never encounter the frequencies at which the displacement current becomes comparable to the conduction current.
- For a good conductor: $\sigma = \infty,\ \varepsilon = \varepsilon_0,\quad \mu = \mu_0\mu_r.$
- Therefore:

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \Rightarrow \alpha = \beta = \sqrt{\pi f \mu\sigma}$$

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}} \qquad \lambda = \frac{2\pi}{\beta}$$

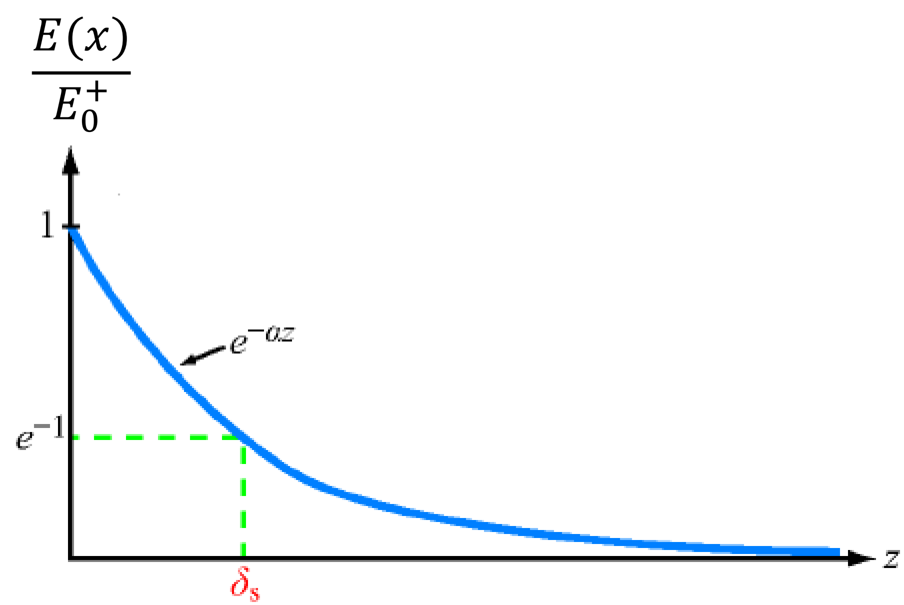
Plane Waves in Good Conductors (contd.)

- Furthermore: $\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45°$

Thus $\vec{E}$ leads $\vec{H}$ by 45°

- If: $\vec{E} = E_0^+ e^{-\alpha z}\cos(\omega t - \beta z)\hat{a}_x$
- Then: $\vec{H} = \frac{E_0^+}{\sqrt{\omega\mu/\sigma}} e^{-\alpha z}\cos(\omega t - \beta z - 45°)\hat{a}_y$

- The amplitude of $\vec{E}$ or $\vec{H}$ is attenuated by the factor $e^{-\alpha z}$ as it travels along the medium.
- The rate of attenuation in a good conductor is characterized by distance called $skin\ depth\ (\delta)$ $\leftrightarrow$ a distance over which plane wave is attenuated by a factor e^{-1} (about 37% of the original value) in a good conductor.



Plane Waves in Good Conductors (contd.)

- *skin depth* is a measure of the depth to which an EM wave can penetrate the medium.

$$E_0^+ e^{-\alpha\delta} = E_0^+ e^{-1} \Rightarrow \therefore \delta = \frac{1}{\alpha}$$

Valid for any material medium

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

For a partially conducting medium, the skin depth can be considerably large.

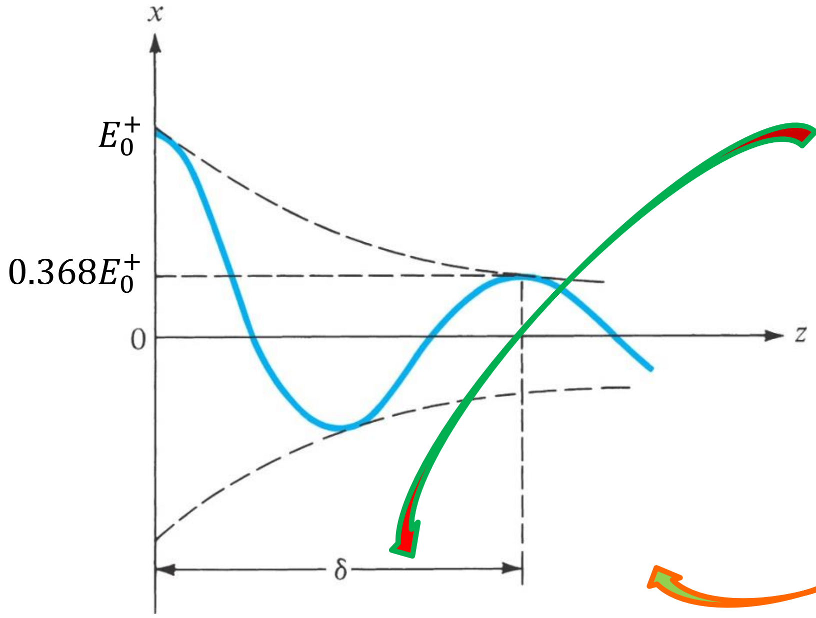
- For a good conductor: $\eta = \frac{1}{\sigma\delta}\sqrt{2}e^{j\pi/4} = \frac{1+j}{\sigma\delta}$

- For good conductors, $\alpha = \beta = \frac{1}{\delta}$, therefore: $\vec{E} = E_0^+ e^{-z/\delta}\cos(\omega t - \frac{z}{\delta})\hat{a}_x$

Plane Waves in Good Conductors (contd.)

$$\vec{E} = E_0^+ e^{-z/\delta} \cos(\omega t - \frac{z}{\delta})\hat{a}_x$$

It shows that *skin depth* (δ) is the measure of exponential damping the wave experiences as it travels through the conductor.



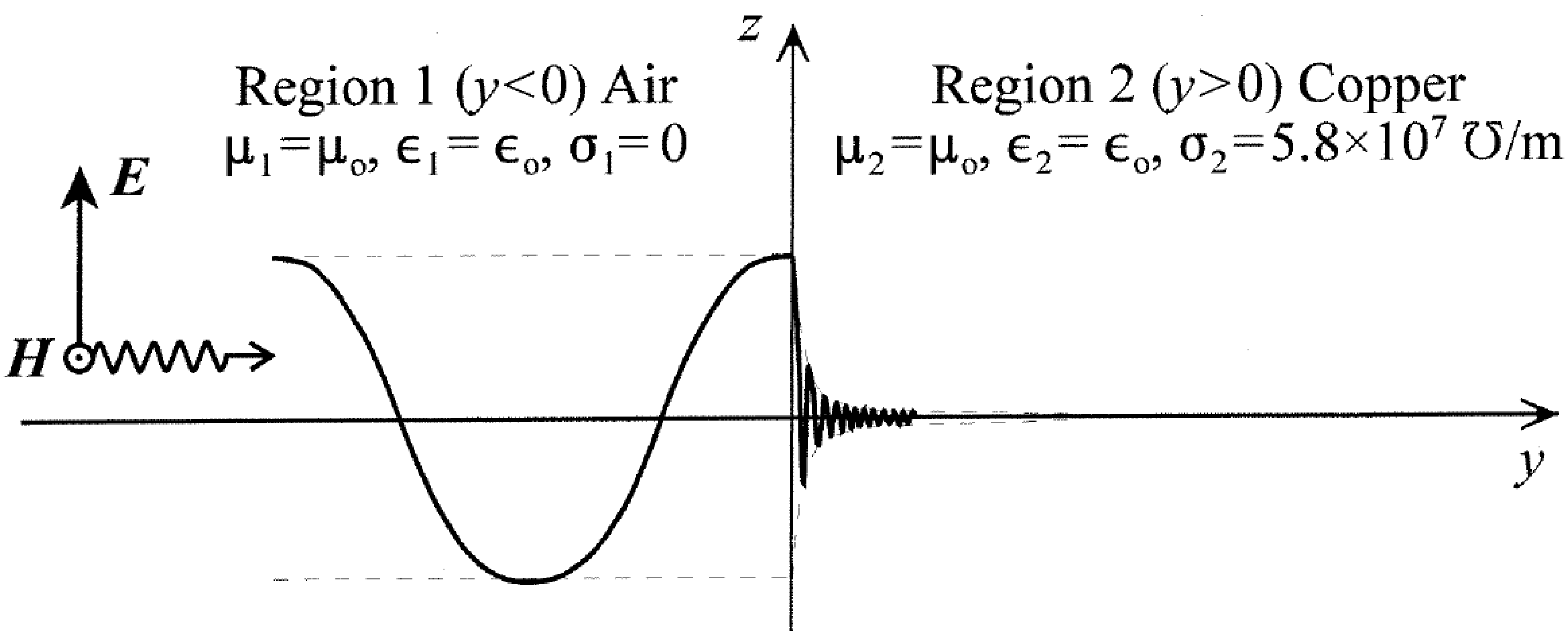
$$\delta = 66.1/\sqrt{f}\,(\text{mm})$$

For Copper !

It demonstrates that the fields dampen and will hardly propagate through good conductors

Example – 3

- Uniform plane wave ($f = 1MHz$) at an air/copper interface.



Determine $\alpha_1, \alpha_2, \beta_1, \beta_2, u_1, u_2, \lambda_1, and\ \lambda_2$.

$$\alpha_1 = 0, \quad \beta_1 = \frac{\omega}{c}$$

$$\alpha_2 = \beta_2 = \frac{1}{\delta}$$

Example – 3 (contd.)

- In the air, $c = 3\times 10^8 m/s$

$$\beta_1 = \frac{\omega}{c} = \frac{2\pi\times 10^6}{3\times 10^8} = 0.0209 rad/m$$

$$\lambda_1 = \frac{c}{f} = \frac{3\times 10^8}{10^6} = 300m$$

- In the copper,

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}} = \frac{1}{\sqrt{\pi f \left(4\pi\times 10^{-7}\right)\left(5.8\times 10^7\right)}} = \frac{0.066}{\sqrt{f}}$$

at 1 MHZ: $\delta = 0.066mm$

$$\alpha_2 = \beta_2 = \frac{1}{\delta} = 15.2\times 10^3 Np/m$$

$$\lambda_2 = \frac{2\pi}{\beta_2} = 2\pi\delta = 0.415mm$$

$$u_2 = \lambda_2 f = 415m/s$$

Plane Waves in Good Conductors (contd.)

Electromagnetic Shielding

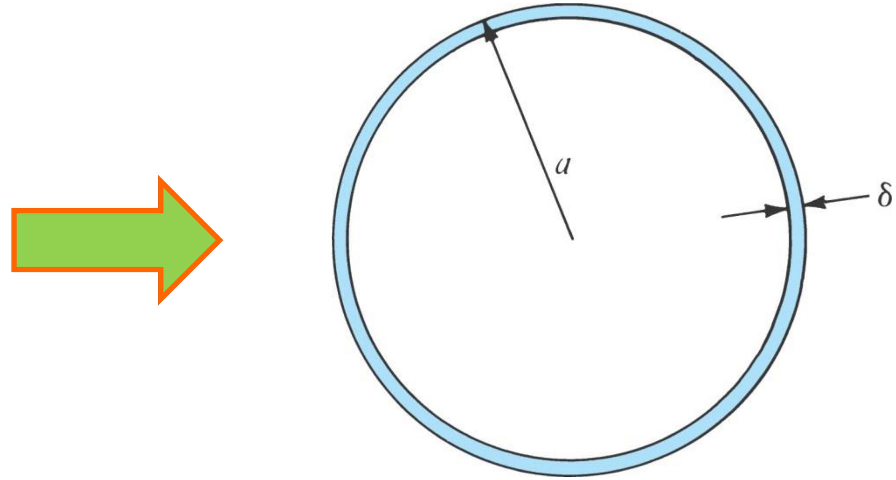
The previous example shows that we may enclose a volume with a thin layer of good conductor to act as an electromagnetic shield. Depending on the application, the electromagnetic shield may be necessary to prevent waves radiating out of the shielded volume or to prevent waves from penetrating into the shielded volume.

Plane Waves in Good Conductors (contd.)

- Given a plane wave incident on a highly-conducting surface, the electric field (and thus the current density) is found to be concentrated at the surface of the conductor.
- The same phenomenon occurs for a current carrying conductor such as a wire.
- The effect is frequency dependent, just as it is in the incident plane wave example.
- This phenomenon is known as the $skin\ effect$.
- Therefore, one can say, The process whereby the field intensity in a conductor rapidly decreases is called $skin\ effect$.
- $skin\ effect$ is the tendency of the charges to migrate from the bulk of the conducting material to the surface, resulting in higher resistances (for ac!)
- The fields and associated currents are confined to a very thin layer $(the\ skin)$ of the conductor surface.

Plane Waves in Good Conductors (contd.)

- For a wire of radius a, it is a good approximation at high frequencies to assume that all of the current flows in the circular ring of thickness δ.



- $skin\ effect$ is used to advantage in many applications.
- For example, because the $skin\ depth$ in silver is very small, the difference in performance between a pure silver and silver-plated brass component is negligible, so silver plating is often used to reduce the material cost of waveguide components.
- Furthermore, hollow tubular conductors are used instead of solid conductors in outdoor television antennas.

Plane Waves in Good Conductors (contd.)

- The *skin depth* is useful in calculating the ac resistance.
- The resistance $\left(R = \frac{l}{\sigma S}\right)$ is called the dc resistance R_{dc}.
- The *skin resistance* R_s is the real part of η.

$$R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\pi f \mu}{\sigma}}$$

Resistance of a unit width and unit length of the conductor having cross-sectional area $1 \times \delta$.

- Therefore, for a given width w and length l, the ac resistance is:

$$R_{ac} = \frac{l}{w\sigma\delta} = \frac{R_s l}{w}$$

- For a conductor wire of radius a:

$$\frac{R_{ac}}{R_{dc}} = \frac{\dfrac{l}{w\sigma\delta}}{\dfrac{l}{\sigma S}} = \frac{\dfrac{l}{\sigma(2\pi a)\delta}}{\dfrac{l}{\sigma(\pi a^2)}} = \frac{a}{2\delta}$$

Since, $\delta \ll a$ at high frequencies, R_{ac} is far greater than R_{dc}. In general, the ratio of the ac and dc resistance starts at 1.0 for dc and very low frequencies and increases as the frequency increases.

General Relations Between $\vec{E}$ and $\vec{H}$

- We learn earlier that if $\hat{a}_E$, $\hat{a}_H$ and $\hat{a}_k$ are unit vectors along $\vec{E}$, $\vec{H}$ and the direction of propagation, then:

$$\hat{a}_k \times \hat{a}_E = \hat{a}_H \qquad \hat{a}_k \times \hat{a}_H = -\hat{a}_E \qquad \hat{a}_E \times \hat{a}_H = \hat{a}_k$$

- In general it can be deduced that:

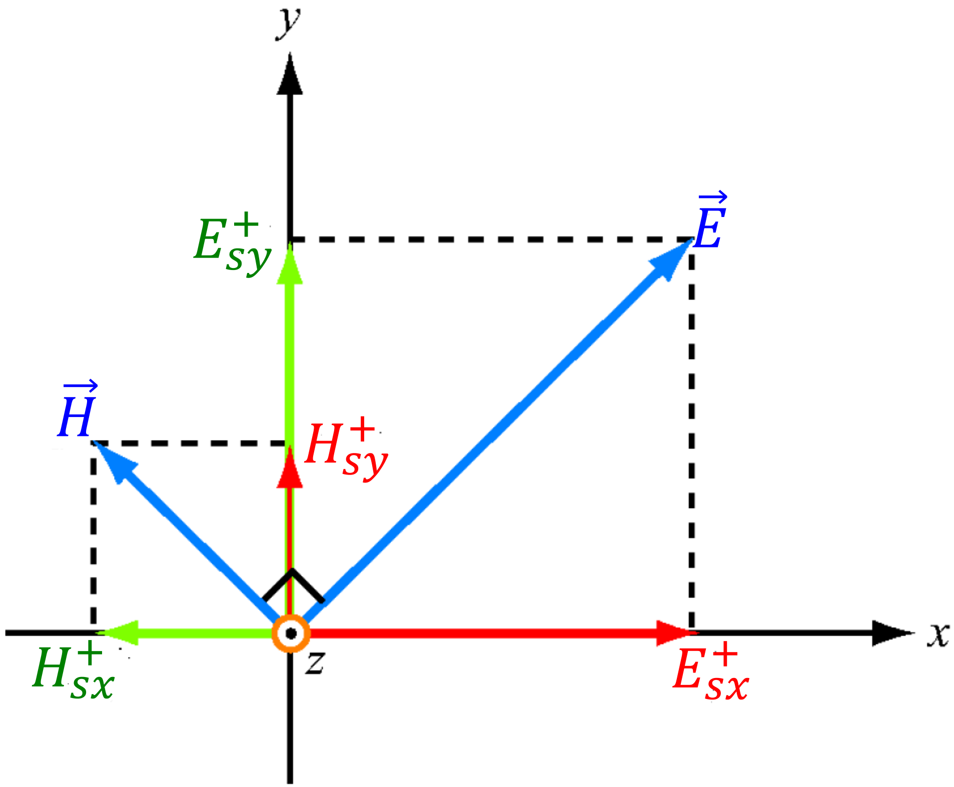
$$\vec{H}_s = \frac{1}{\eta}\hat{a}_k \times \vec{E}_s \qquad \vec{E}_s = -\eta\hat{a}_k \times \vec{H}_s$$

- Furthermore, a uniform plane wave travelling in the $+\hat{a}_z$ direction may have both $x-$ and $y-components$.
- In such a scenario: $\vec{E}_s = \hat{a}_x\vec{E}_{sx}^+(z) + \hat{a}_y\vec{E}_{sy}^+(z)$
- The associated magnetic field will be: $\vec{H}_s = \hat{a}_x\vec{H}_{sx}^+(z) + \hat{a}_y\vec{H}_{sy}^+(z)$
- The exact expression of magnetic field in terms of electric field will be:

$$\vec{H}_s = \frac{1}{\eta}\hat{a}_z \times \vec{E}_s = -\hat{a}_x\frac{\vec{E}_{sy}^+(z)}{\eta} + \hat{a}_y\frac{\vec{E}_{sx}^+(z)}{\eta}$$

- Thus: $\vec{H}_{xs}^+ = -\frac{\vec{E}_{sy}^+(z)}{\eta}$ $\qquad \vec{H}_{ys}^+(z) = \frac{\vec{E}_{sx}^+(z)}{\eta}$

General Relations Between $\vec{E}$ and $\vec{H}$ (contd.)



In general, a TEM wave may have an electric field in any direction in the plane orthogonal to the direction of wave travel, and the associated magnetic field is also in the same plane with appropriate magnitude and direction.