Lecture-1 Date: 05.01.2015

- Introduction
- Vector Arithmetic (Review)
- Coordinate System and Transformations
- Examples

Fields and Waves (ECE230)

Instructor: Dr. Mohammad S. Hashmi

TAs: D. Sharma, A. Maktoomi, P. Singhal, S. Sharma

Class Timings: Monday and Thursday (11:30 – 13:00) in C03

Tutorial: Tuesday (15:00-17:00) in LR1

Lab Session: Wednesday (14:00-17:00) in RF & Propagation Lab

Office Hours: Friday (9:00 – 11:00)

Pre-requisites: Vector Calculus, Calculus, Complex Variables

Course URL:
Available at: http://www.iiitd.edu.in/~mshashmi/Teaching.html

Course Focus:
Foundations of Electromagnetic Field Theory

Course Objectives:
On the completion of this course students should

- Gain thorough understanding of Time Varying Fields, Maxwell's Equations, Time Harmonic Fields, and Plane Electromagnetic Waves
- Understand the operating principles of resistors, capacitors and inductors
- Be able to analyse simple electronic equipments utilizing electromagnetics concept

Lab Component:

- MATLAB based sessions and possibly some demo

Evaluation:

- Assignments + Lab – 20% (6 assignments)
- Surprise Quizzes – 20% (5 quizzes)
- Exams – mid-sem (25%) & end-sem (25%)
- Project – 10%

Attendance and Classroom Behavior:

- Attendance not mandatory – however skip classes at your own risk
- Students will be responsible for any notes, announcements etc. made during the class
- Prompt arrival to the class is requested – please do not enter the class if you are late by 5 minutes!
- No eating, drinking, smoking allowed in the class – definitely no facebooking will be entertained

Text Book:

- Principles of electromagnetics *4th Edition*, Mathew N. O. Sadiku

Syllabus:

- Chapters 1-10 from Mathew N. O. Sadiku

Other Recommended Books:

- Electromagnetics Fields and Waves *2nd Edition*, David K. Cheng
- Fundamentals of Applied Electromagnetics *6th Edition*, Fawwaz T. Ulaby
- Engineering Electromagnetics *2nd Edition*, Karl E. Lonngrenn et al.

Course Website:

http://www.iiitd.edu.in/~mshashmi/Teaching.html

Info related to ECE230 can be found here

Why Study Electromagnetics?

This course is about electromagnetics (EM), the electrical foundation of Electrical and Computer Engineering, or, how electricity *really* works.

-- Look into the black boxes.

- Linear Circuit is a simple part of EM, so it was taught first.
- However there are an increasing number of cases in ECE where circuit theory fails (e.g. faster computers, higher communications frequencies, power electronics, power system transients,), and therefore EM must supplement circuit theory. *But, don't worry…*
- Also EM is the basis for many devices (machinery, antennas, etc.), and one of the physical foundations of any active electronic device.

Why Study Electromagnetics?

Electrical Engineering is Applied Electromagnetics

- As devices get smaller and smaller, and frequencies get higher and higher, circuit theory is less able to adequately describe the performance or to predict the operation of circuits.
- At very high frequencies, transmission line and guided wave theory must be used in applications such as high speed electronics, micro/nano electronics, integrated circuits.
- Other applications include: Fiber Optics, Microwave Communication Systems, Antennas and Wave Propagation, Optical Computing, Electromagnetic Interference, Electromagnetic Compatibility, Biology and Medicine/Biomedical Imaging.

Why Study Electromagnetics?

- As use of the electromagnetic frequency spectrum increases, the demand for engineers who have practical working knowledge in the area of electromagnetics continues to grow.
- Electromagnetic engineers design: high frequency or optoelectronic circuits, antennas and waveguides; electrical circuits that function properly in the presence of external interference while not interfering with other equipment.
- The electromagnetics technical specialty prepares future engineers for employment in industry in the areas of radar, antennas, fiber optics, high frequency circuits, electromagnetic compatibility and microwave communication.

Electromagnetics is Everywhere

Electromagnetics is fundamental to the advancement of electrical and computer technology!

Why Study Electromagnetics?

Objective

Introduce the basic principles of the electromagnetic phenomena in terms of a few relatively simple laws

Outcome

Students are well equipped :

- to handle important practical problems in electrical & computer engineering
- to gain physical intuition about nature around themselves

What is Electromagnetics?

- Electromagnetics is the study of Charges:

  (i) at rest (ii) in motion

- The subject of electromagnetics may be divided into 3 branches:
  - Electrostatics: charges are at rest (no time-variation)
  - Magnetostatics: charges are in steady-motion (no time-variation)
  - Electrodynamics: charges are in time-varying motion

    (give rise to waves that propagate and carry energy and information)

Steps in Studying Electromagnetics

- Define basic quantities (e.g., E-field, H-field)
- Define the rules of operation (mathematics) of these quantities (e.g., Vector Algebra, PDEs)
- Postulate fundamental laws

Why is Electromagnetics Difficult?

Electric and Magnetic Field:

- are 3-dimensional !
- are vectors !
- vary in space and as well as in time !
- are governed by PDEs (partial differential equations)

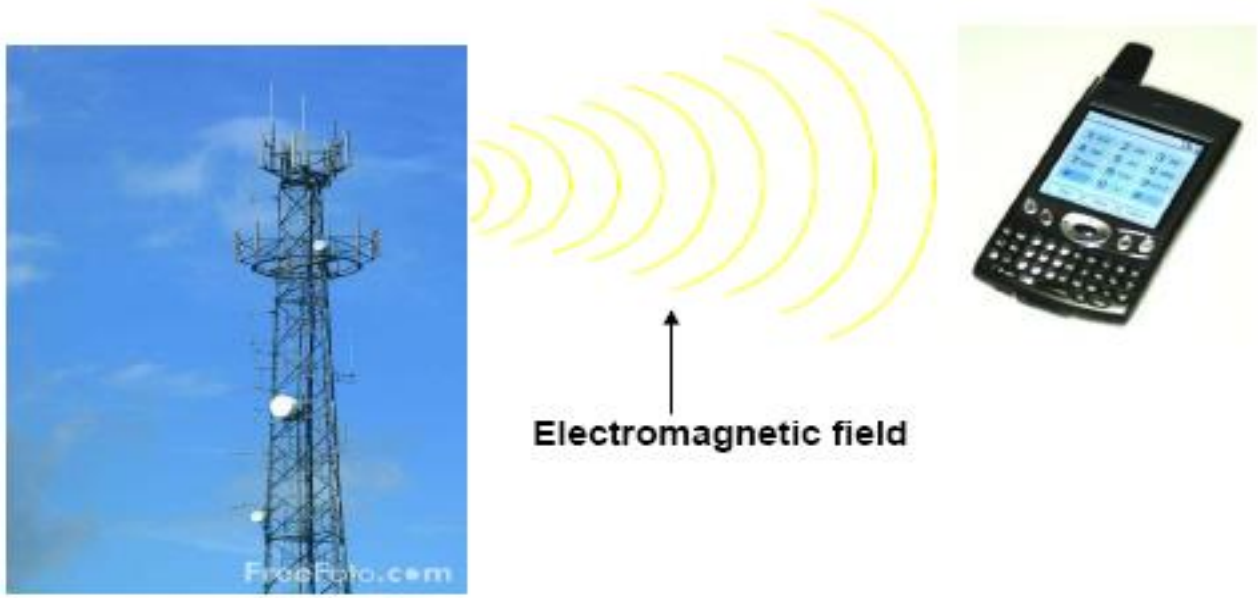
Therefore →

- Solution of electromagnetic problems requires a high level of abstract thinking !
- Students must develop a deep physical understanding !

Math is just a powerful tool !

Examples of Electromagnetic Applications

Communication Technology



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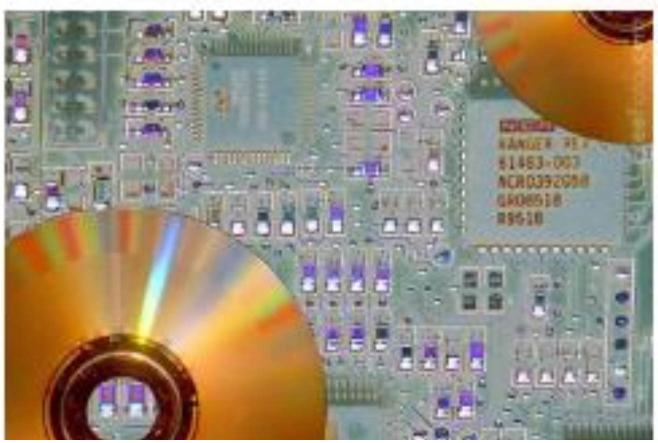
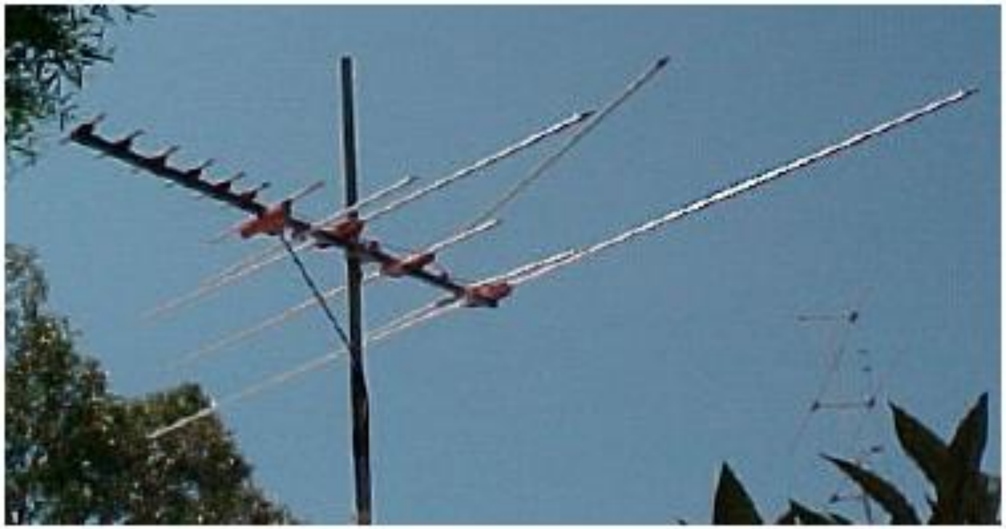
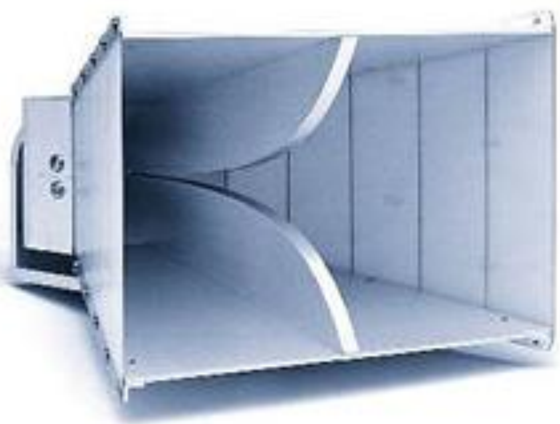
Computer Technology

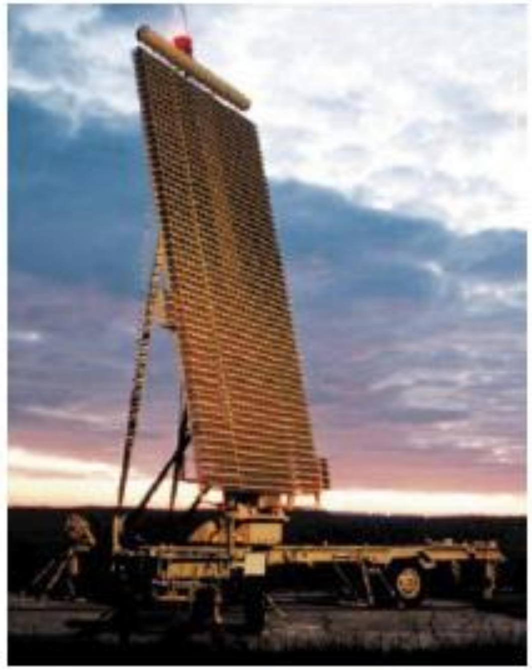


Antenna Technology

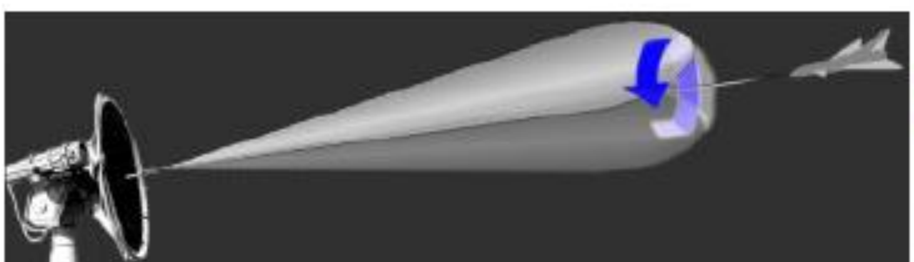


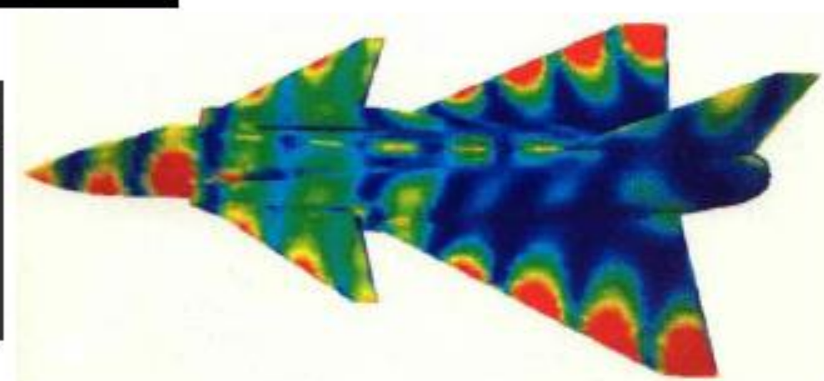


Military and Defense Applications

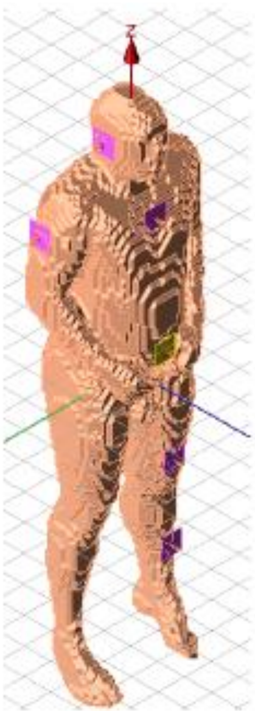


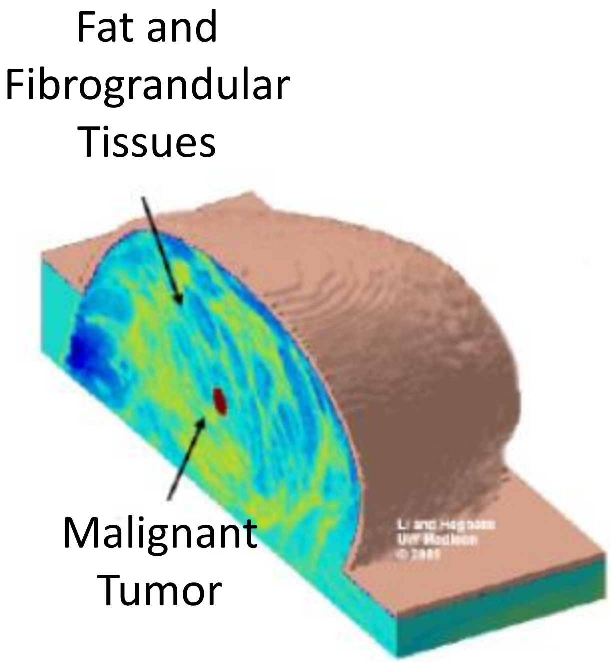
Radars

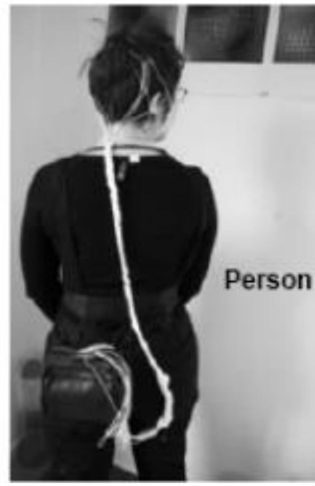




Biomedical Applications







Person wearing electrodes for EEG

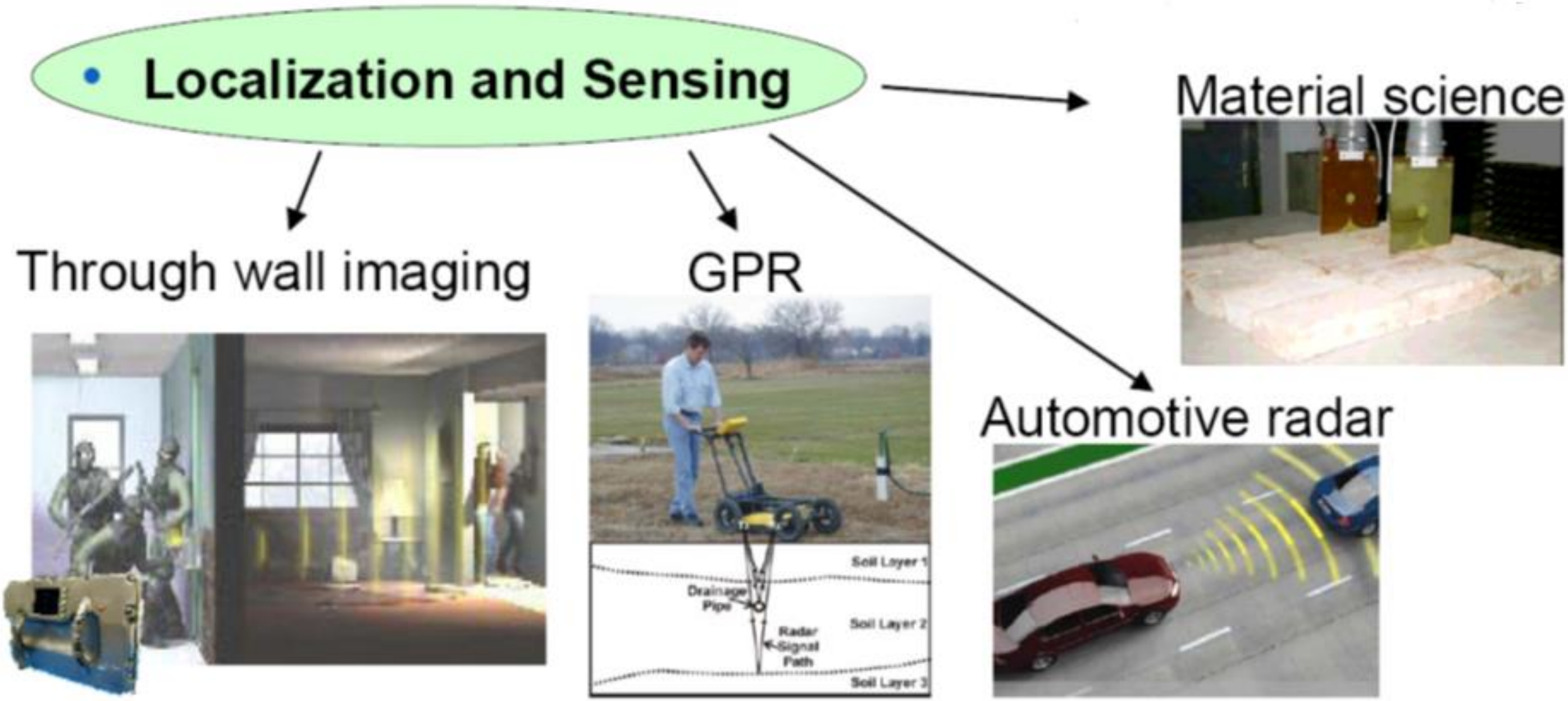
EEG (Electroencephalography) measures the electrical activity produced by the brain as recorded from electrodes placed on the scalp.

ECG (Electrocardiogram) records the electrical activity of heart over time.

Transportation

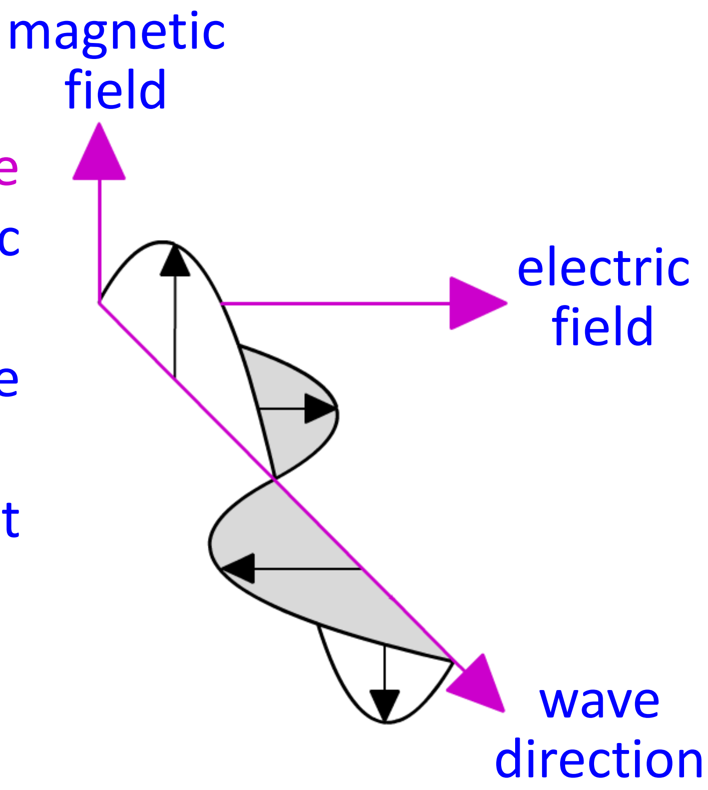
- Levitated trains: Maglev prototype

Localization and Sensing



What are electromagnetic waves?

- Electromagnetic waves are transverse waves made up of electric and magnetic fields.
- All electromagnetic waves travel at the same speed.
- In a vacuum (space), they travel at 300,000,000 m/s!



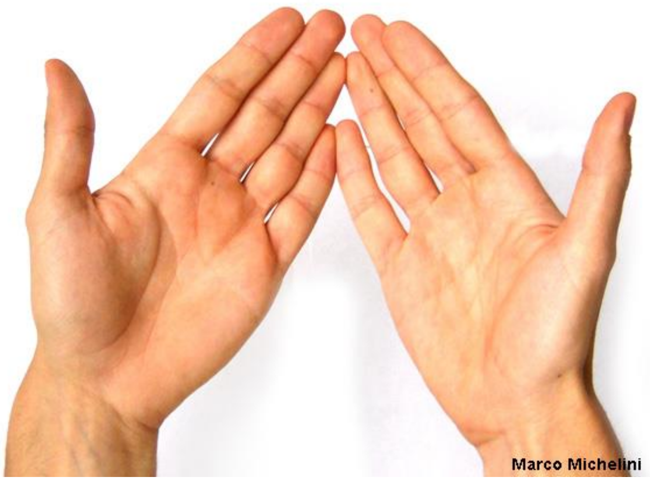
How do electromagnetic waves differ?

- Different electromagnetic waves carry different amounts of energy.
- For example, microwaves carry less energy than X-rays.
- The amount of energy carried by an electromagnetic wave depends on the wavelength:
    - the shorter the wavelength, the higher its energy.
- Wavelength and frequency are linked properties of a wave: the shorter the wavelength, the higher its frequency.
- So, frequency also tells you about the energy of a wave:
    - the higher its frequency, the higher is energy.

Do microwaves have a shorter wavelength than X-rays?

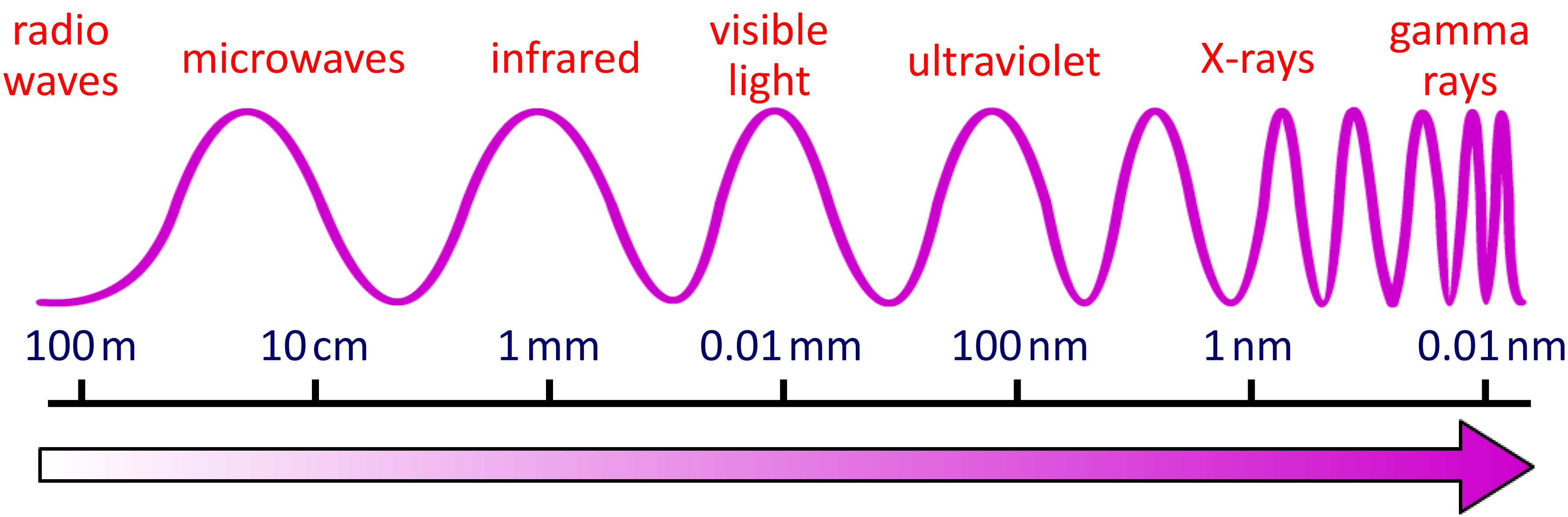
What happens when waves hit a surface?

- When electromagnetic waves hit a surface, they can be reflected, absorbed or transmitted.
- How the waves behave, depends on their energy and the type of material.
- For example, light waves are reflected by skin but X-rays pass straight through.
- If electromagnetic waves are absorbed, some of their energy is absorbed by the material. This usually increases the temperature of the material.



What is the electromagnetic spectrum?

- The electromagnetic waves are grouped into types that have similar wavelengths and so have similar properties.



smaller wavelength → higher frequency → energy and hazard

Electromagnetic waves form a continuous series in order of changing wavelength, frequency and energy. This series is called the electromagnetic spectrum.

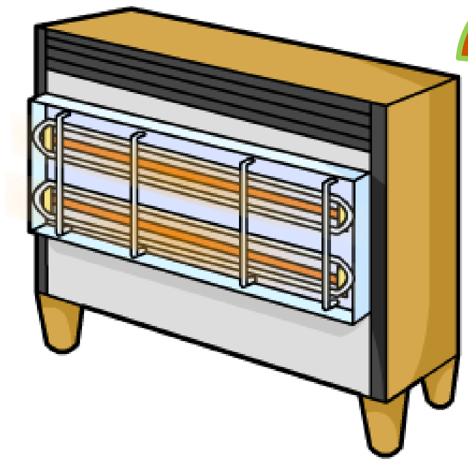
How do radio waves affect humans?

- Radio waves are the longest-wavelength electromagnetic waves and mostly pass through the body.
- They are not strongly absorbed and are thought to have no effect on the health of living tissue.

- Microwaves are radio waves with short wavelengths. They are very slightly absorbed by the body and can cause a minor heating effect.
- However, the microwaves produced by mobile phones have not yet been proved to cause health problems.

How do infrared waves affect humans?

- Infrared waves are absorbed by skin to a limited depth. They transfer their energy to the skin tissue warming it up.
- This heating effect is detected by temperature-sensitive nerve endings in the skin.



Infrared waves from this grill heat the surface of the meat. If the meat absorbs too much energy, it will be burnt.

If skin is exposed to too many high-energy infrared waves, it will be burnt.

How do visible light affect humans?

- Your eyes detect visible light, which does not normally pose any health risk.
- However, very bright light can damage your eyes and may even make you blind. This is why you should not look at the Sun through a telescope or binoculars.

- Lasers are very intense sources of visible light. The lasers used in light shows are not powerful enough to cause harm but must be used safely.
- Some very powerful lasers can cut through materials such as metal.
- These would also be able to burn through living tissue.

Courses @ IIITD

1. Antenna Theory and Design
2. Radar Engineering
3. RF Circuit Design
4. RF Laboratory
5. System Design for Wireless

Project Themes

1. Global Positioning System (GPS)
2. Microwave Imaging
3. Plasma
4. Mobile Phone Technology
5. Microwave and Satellite Comm

Minimal requirement: thorough reading and understanding the concept by going through various papers, book chapters, white papers etc.. You are expected to explore (such as simulation or test setup development) as much as possible if you want to get the optimum marks and bonus marks.

Vector Arithmetic

Vector Addition

Q: Say we **add** two vectors $\vec{A}$ and $\vec{B}$ together; what is the **result**?

A: The addition of two vectors results in **another vector**, which we will denote as $\vec{C}$. Therefore, we can say:

$$\boxed{\vec{A}+\vec{B}=\vec{C}}$$

The **magnitude** and **direction** of $\vec{C}$ is determined by the **head-to-tail rule**.

This is not a **provable** result, rather the head-to-tail rule is the **definition** of vector addition. This definition is used because it has many **applications** in physics.

Some important properties of vector addition:

1. Vector addition is **commutative:** $\vec{A}+\vec{B}=\vec{B}+\vec{A}$
2. Vector addition is **associative:** $(\vec{X}+\vec{Y})+\vec{Z}=\vec{X}+(\vec{Y}+\vec{Z})=\vec{K}$

From these two properties, we can conclude that the addition of **several** vectors can be executed in **any order**

Vector Subtraction

- We consider the addition of a negative vector as a **subtraction**.

Q: Is $\vec{A} + \vec{B} = \vec{B} - \vec{A}$**?**

A: What do **you** think ?

Multiplication

- Consider a scalar quantity a and a vector quantity $\vec{B}$. We express the multiplication of these two values as:

$$\boxed{a\vec{B} = \vec{C}}$$

In other words, the product of a scalar and a vector is a **vector**!

Q: OK, but what **is** vector $\vec{C}$**?** What is the **meaning** of $\vec{C}$**?**

A: The resulting vector $\vec{C}$ has a **magnitude** that is equal to a times the **magnitude** of $\vec{B}$. In other words:

$$\boxed{\left|\vec{C}\right| = a\left|\vec{B}\right|}$$

The **direction** of vector $\vec{C}$ is **exactly** that of $\vec{B}$**.**

→ Jut to reiterate, multiplying a vector by a scalar changes the **magnitude** of the vector, but **not** its direction.

Multiplication (contd.)

Some important properties of vector multiplication:

1. The scalar-vector multiplication is **distributive**: $\boxed{a\vec{B}+b\vec{B}=(a+b)\vec{B}}$
2. also **distributive** as: $\boxed{a\vec{B}+a\vec{C}=a\left(\vec{B}+\vec{C}\right)}$
3. Scalar-Vector multiplication is also **commutative**: $\boxed{a\vec{B}=\vec{B}a}$
4. Multiplication of a vector by a **negative** scalar is interpreted as: $\boxed{-a\vec{B}=a\left(-\vec{B}\right)}$
5. **Division** of a vector by a scalar is the same as multiplying the vector by the **inverse** of the scalar: $\boxed{\frac{\vec{B}}{a}=\left(\frac{1}{a}\right)\vec{B}}$

Unit Vector

- Lets begin with vector $\vec{A}$. Say we **divide** this vector by its **magnitude** (a scalar value). We create a new vector, which we will denote as $\hat{a}_A$:

$$\hat{a}_A = \frac{\vec{A}}{|\vec{A}|}$$

Q: How is vector $\hat{a}_A$ related to vector $\vec{A}$**?**

A: Since we divided $\vec{A}$ by a scalar value, the vector $\hat{a}_A$ has the **same direction** as vector $\vec{A}$.

- But, the **magnitude** of $\hat{a}_A$ is:

$$|\hat{a}_A| = \frac{|\vec{A}|}{|\vec{A}|} = 1$$

The vector $\hat{a}_A$ has a magnitude equal to **one**! We call such a vector a **unit vector**.

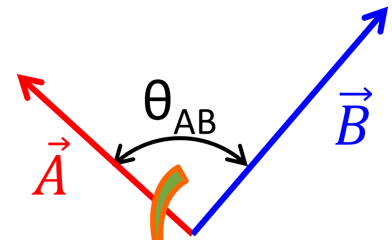
- A unit vector is essentially a **description of direction** only, as its magnitude is always **unit valued** (i.e., equal to one). Therefore:
  - $|\vec{A}|$ is a scalar value that describes the **magnitude** of vector $\vec{A}$.
  - $\hat{a}_A$ is a vector that describes the **direction** of $\vec{A}$**.**

The Dot Product

- The **dot product** of two vectors, $\vec{A}$ and $\vec{B}$, is **denoted** as $\vec{A}\,.\,\vec{B}$

$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta_{AB}$$

angle θ_{AB} is the angle formed **between** the vectors $\vec{A}$ and $\vec{B}$.



$$0 \le \theta_{AB} \le \pi$$

IMPORTANT NOTE: The dot product is an operation involving **two vectors**, but the result is a **scalar** !! e.g.,:

$$\vec{A}.\vec{B} = c$$

The dot product is also called the **scalar product** of two vectors.

- Note also that the dot product is **commutative**, i.e.,:

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

The Dot Product (contd.)

1. The dot product of a vector **with itself** is equal to the **magnitude** of the vector **squared.**

$$\boxed{\vec{A}.\vec{A} = |\vec{A}|.|\vec{A}|\cos 0^\circ = |\vec{A}|^2} \Rightarrow \boxed{|\vec{A}| = \sqrt{\vec{A}.\vec{A}}}$$

2. If $\vec{A}\,.\,\vec{B} = 0$ (and $\vec{A} \neq 0$, $\vec{B} \neq 0$), then it must be true that: $\boxed{\theta_{AB} = 90^\circ}$

3. If $\vec{A}\,.\,\vec{B} = |\vec{A}||\vec{B}|$, then it must be true that: $\boxed{\theta_{AB} = 0}$

4. If $\vec{A}\,.\,\vec{B} = -|\vec{A}||\vec{B}|$, then it must be true that: $\boxed{\theta_{AB} = 180^\circ}$

5. The dot product is **distributive** with addition, such that:

$$\boxed{\vec{A}.\left(\vec{B}+\vec{C}\right) = \vec{A}.\vec{B} + \vec{A}.\vec{C}}$$

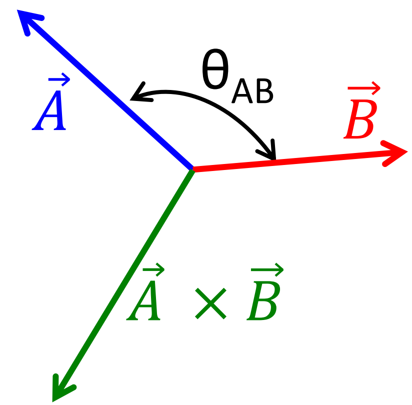
The Cross Product

- The **cross product** of two vectors, $\vec{A}$ and $\vec{B}$**,** is **denoted** as $\vec{A} \times \vec{B}$.

$$\vec{A} \times \vec{B} = \hat{a}_n |\vec{A}||\vec{B}| \sin\theta_{AB}$$

$$0 \le \theta_{AB} \le \pi$$

Just as with the dot product, the angle θ_{AB} is the angle between the vectors $\vec{A}$ and $\vec{B}$. The unit vector $\hat{a}_n$ is **orthogonal** to both $\vec{A}$ and $\vec{B}$ (i.e., $\hat{a}_n . \vec{A}$ =0 and $\hat{a}_n . \vec{B}$ =0.)



IMPORTANT NOTE: The cross product is an operation involving **two vectors**, and the result is also a **vector**. e.g.,:

$$\vec{A} \times \vec{B} = \vec{C}$$

- The **magnitude** of vector $\vec{A} \times \vec{B}$ is therefore: $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta_{AB}$

While the **direction** of vector $\vec{A} \times \vec{B}$ is described by unit vector $\hat{a}_n$.

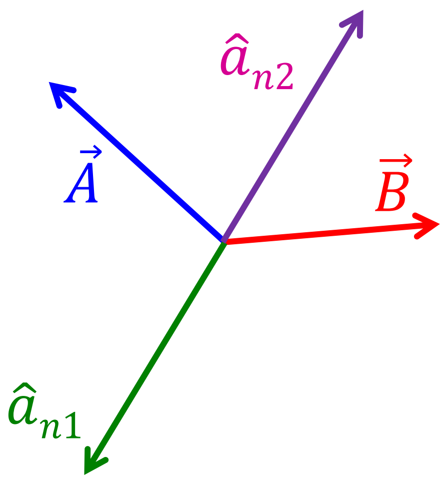
The Cross Product (contd.)

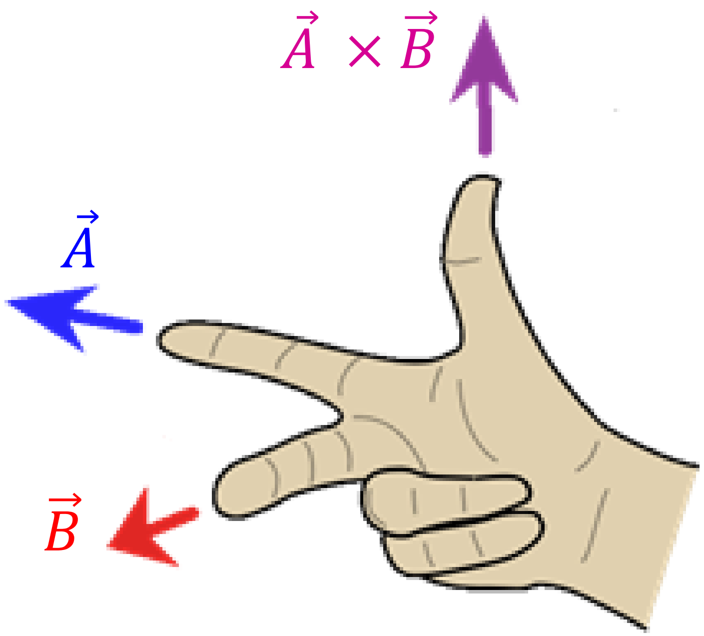
Problem! There are **two** unit vectors that satisfy the equations $\hat{a}_n \cdot \vec{A}$ =0 and $\hat{a}_n \cdot \vec{B}$ =0!! These two vectors are **antiparallel**.

$$\boxed{\vec{A}.\hat{a}_{n1} = \vec{A}.\hat{a}_{n2} = 0}$$

$$\boxed{\vec{B}.\hat{a}_{n1} = \vec{B}.\hat{a}_{n2} = 0}$$

$$\boxed{\hat{a}_{n1} = -\hat{a}_{n2}}$$





Q: Which unit vector is correct?

A: Use the **right-hand rule**

The Cross Product (contd.)

1. If θ_{AB} = 90° (i.e., **orthogonal**), then: $\vec{A}\times\vec{B}=\hat{a}_n|\vec{A}||\vec{B}|\sin 90^\circ=\hat{a}_n|\vec{A}||\vec{B}|$

2. If θ_{AB} = 0° (i.e., **parallel**), then: $\vec{A}\times\vec{B}=\hat{a}_n|\vec{A}||\vec{B}|\sin 0^\circ=0$

   Note that $\vec{A}\times\vec{B}=0$ also if θ$_{AB}$ = 180°.

3. The cross product is **not** commutative! In other words, $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$.

   While evaluating the cross product of two vectors, the **order** is important !

   $\vec{A}\times\vec{B}\neq-(\vec{B}\times\vec{A})$

4. The **negative** of the cross product is: $-(\vec{A}\times\vec{B})=\vec{A}\times(-\vec{B})$

5. The cross product is also **not** associative: $\vec{A}\times\vec{B}\times\vec{C}\neq\vec{A}\times\left(\vec{B}\times\vec{C}\right)$

6. But, the cross product is **distributive**, in that: $\vec{A}\times\left(\vec{B}+\vec{C}\right)=\left(\vec{A}\times\vec{B}\right)+\left(\vec{A}\times\vec{C}\right)$

The Triple Product

- The **triple product** is not a “new” operation, as it is simply a combination of the **dot** and **cross** products.
- For example, the triple product of vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ is **denoted** as:

$$\boxed{\vec{A}.\vec{B}\times\vec{C}}$$

Q: Yikes! Does this mean: $\boxed{\left(\vec{A}.\vec{B}\right)\times\vec{C}}$ **OR** $\boxed{\vec{A}.\left(\vec{B}\times\vec{C}\right)}$

A: The answer is **easy**! Only one of these two interpretations makes sense:

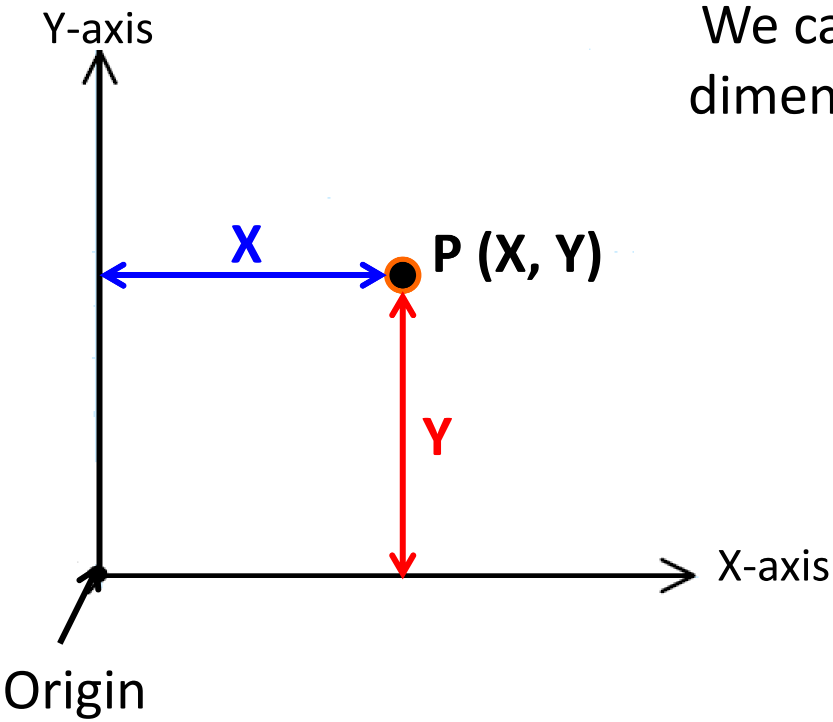
$\left(\vec{A}.\vec{B}\right)\times\vec{C} =$ **Scalar X Vector** ← **makes no sense**

$\vec{A}.\left(\vec{B}\times\vec{C}\right) =$ **Vector . Vector** ← **dot product**

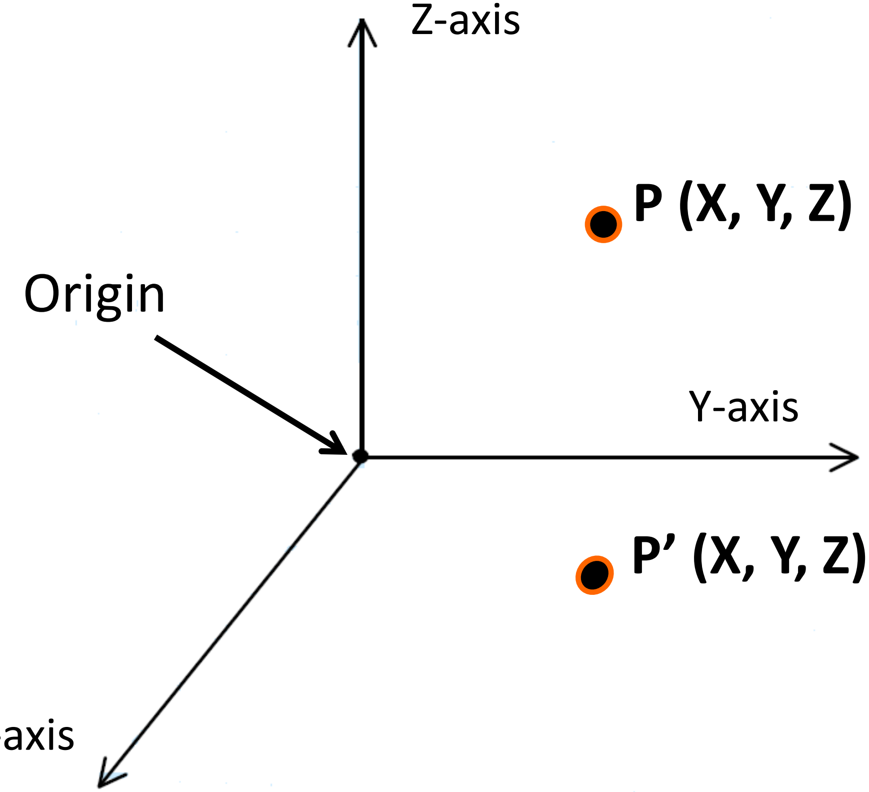
Coordinate System

Cartesian Coordinates

- In **two** dimensions, we can specify a point on a plane using **two** scalar values, generally called X and Y.

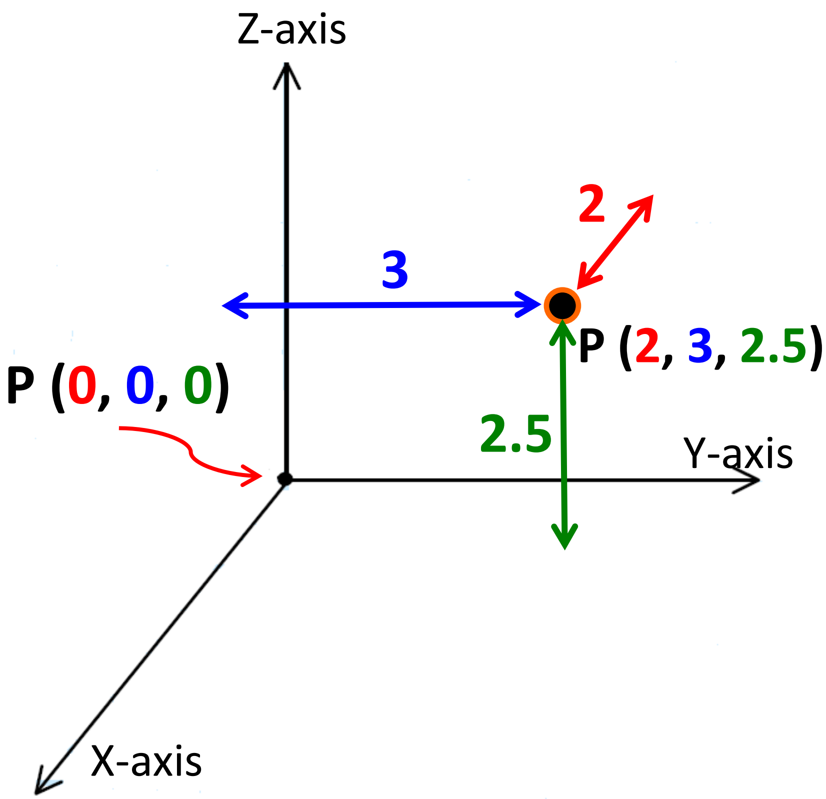


We can extend this to **three-** dimensions, by adding a **third** scalar value Z.



Cartesian Coordinates

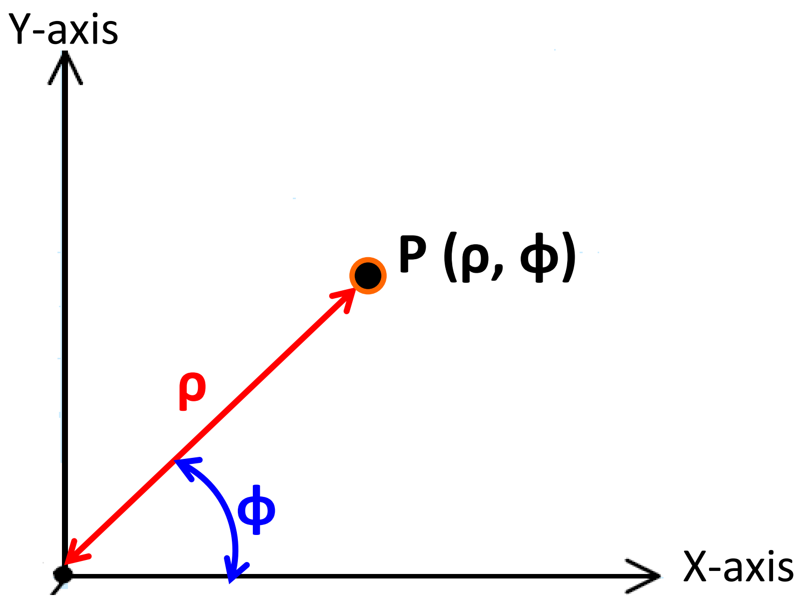
- Note the coordinate values in the Cartesian system effectively represent the **distance** from a **plane** intersecting the origin.
- For **example**, x =3 means that the point is **3 units** from the y-z **plane** (i.e., the x = 0 plane).
- Likewise, the y coordinate provides the **distance** from the x-z (y=0) plane, and the z coordinate provides the **distance** from the x-y (z =0) plane.
- Once **all three** distances are specified, the **position** of a point is **uniquely** identified.

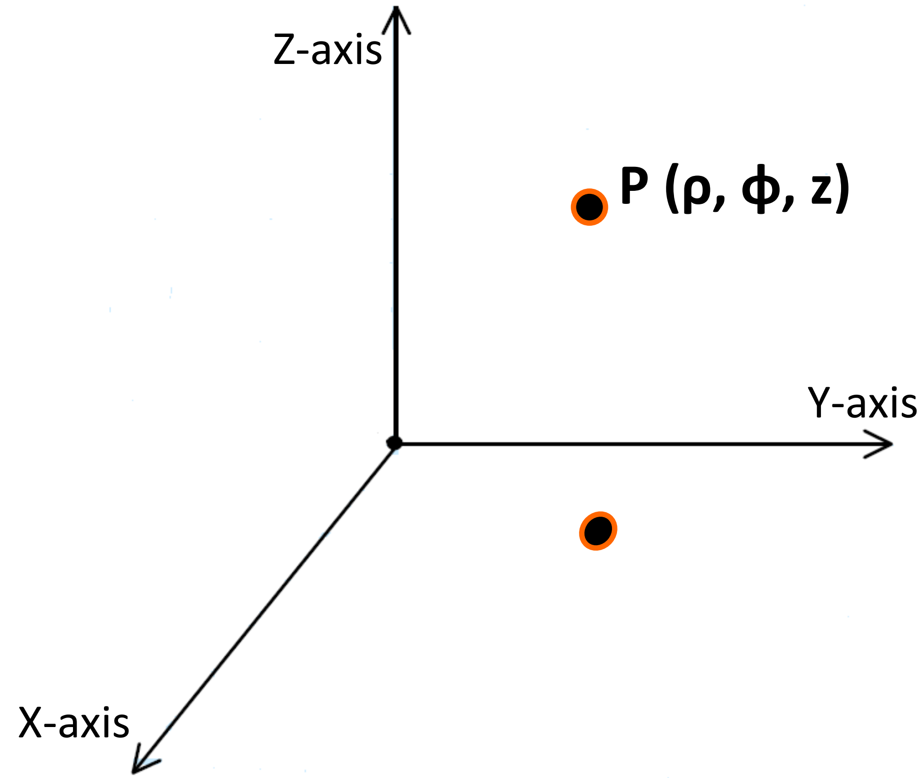


Cylindrical Coordinates

- You're also familiar with **polar coordinates**. In **two** dimensions, we specify a point with **two** scalar values, generally called ρ and ϕ.

We can extend this to **3**-dimensions, by adding a **third** scalar value z. This method for identifying the position of a point is referred to as **cylindrical coordinates**.

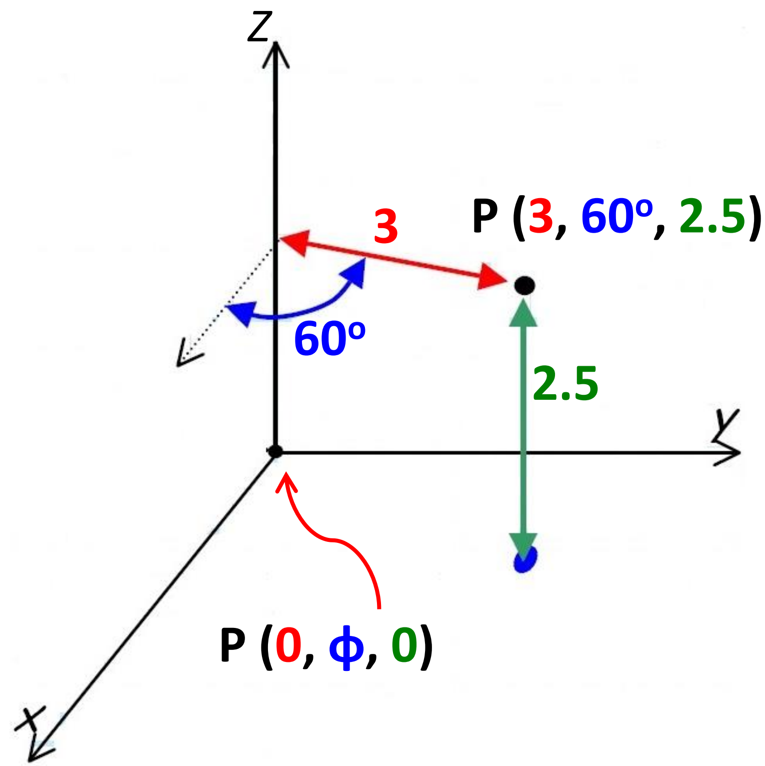




Cylindrical Coordinates

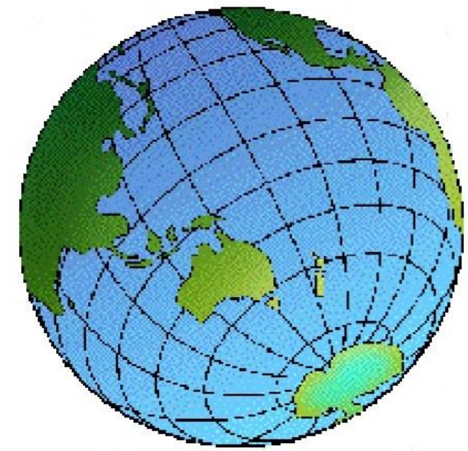
Note the **physical** significance of each parameter of **cylindrical** coordinates:

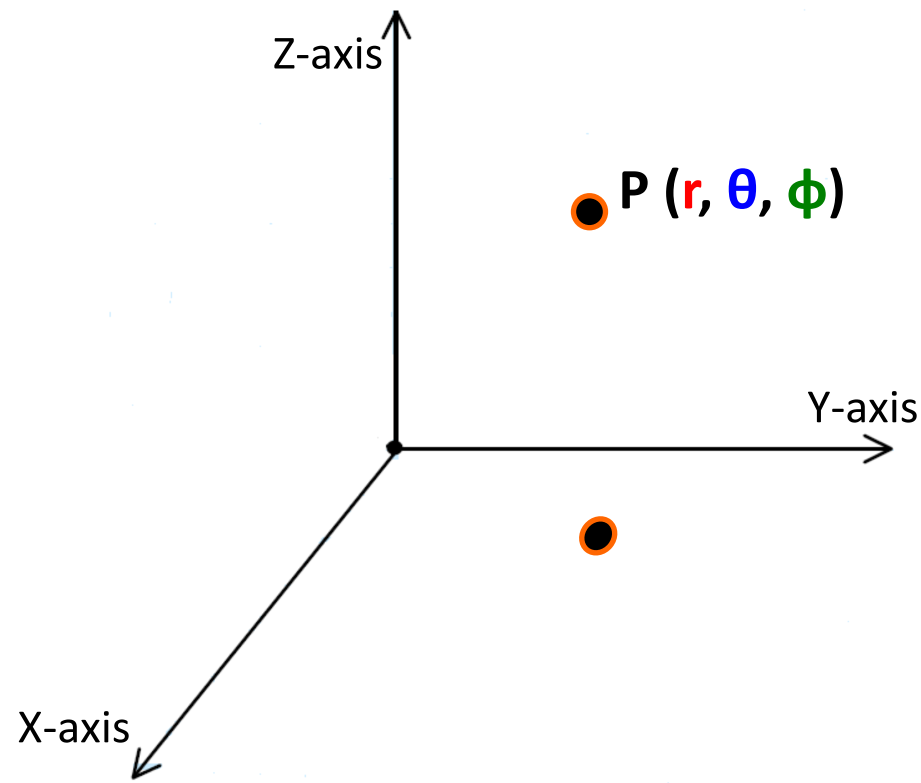
1. The value **ρ** indicates the **distance** of the point from the **z-axis** ($0 \leq \rho < \infty$).
2. The value **ϕ** indicates the **rotation angle** around the **z-axis** ($0 \leq \phi < 2\pi$), **precisely** the same as the angle **ϕ** used in **spherical** coordinates.
3. The value **z** indicates the **distance** of the point from the x-y (z = 0) plane ($-\infty < z < \infty$), **precisely** the same as the same coordinate **z** used in **Cartesian** coordinates.
4. Once **all three** values are specified, the **position** of a point is **uniquely** identified.



Spherical Coordinates

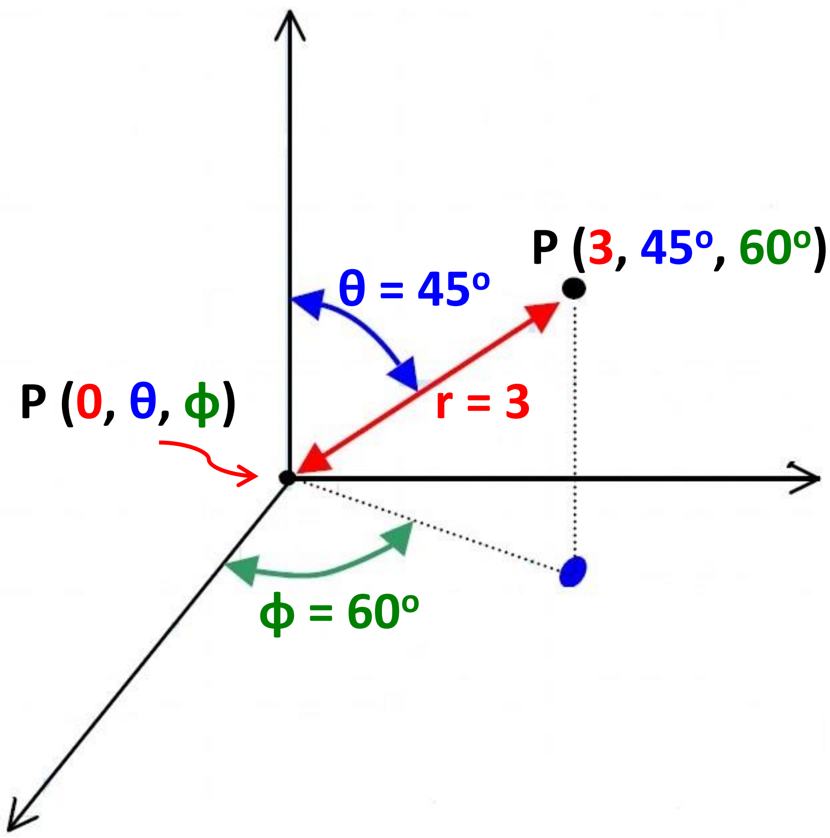
- **Geographers** specify a location on the Earth's surface using **three** scalar values: **longitude**, **latitude**, and **altitude**.
- Both longitude and latitude are **angular** measures, while altitude is a measure of **distance.**
- Latitude, longitude, and altitude are similar to **spherical coordinates.**
- Spherical coordinates consist of one scalar value (r), with units of **distance**, while the other two scalar values (θ, ϕ) have **angular** units (degrees or radians).





Spherical Coordinates

- For spherical coordinates, **r** (0≤r<∞) expresses the **distance** of the point from the **origin** (i.e., similar to **altitude**).
- Angle **θ** (0 ≤θ ≤π) represents the angle formed **with the z-axis** (i.e., similar to **latitude**).
- Angle **ϕ** (0≤ϕ<2π) represents the rotation angle around the z-axis, **precisely** the same as the **cylindrical** coordinate ϕ (i.e., similar to **longitude**).



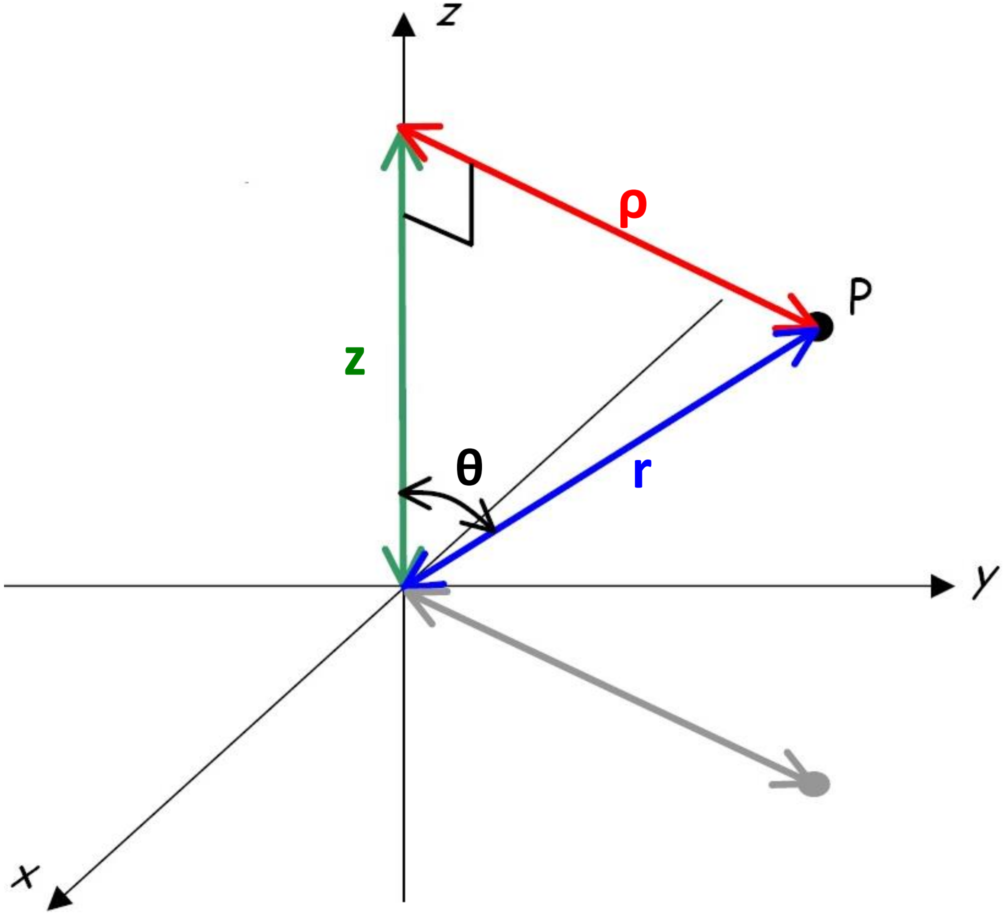
Thus, using **spherical** coordinates, a point in space can be unambiguously defined by **one distance** and **two angles**.

Coordinate Transformations

- Say we **know** the location of a point, or the description of some scalar field in terms of **Cartesian** coordinates (e.g., T (x, y, z)).
- What if we decide to express this point or this scalar field in terms of **cylindrical** or **spherical** coordinates **instead**?
- We see that the coordinate values ***z, ρ, r,*** and θ are all variables of a **right triangle**! We can use our knowledge of trigonometry to relate them to each other.
- In fact, we can **completely derive** the relationship between **all six** independent coordinate values by considering just **two very important right triangles**!
  - Hint: **Memorize these 2 triangles!!!**

Coordinate Transformations (contd.)

Right Triangle #1



$$z = r \times \cos\theta = \rho \times \cot\theta = \sqrt{r^2 - \rho^2}$$

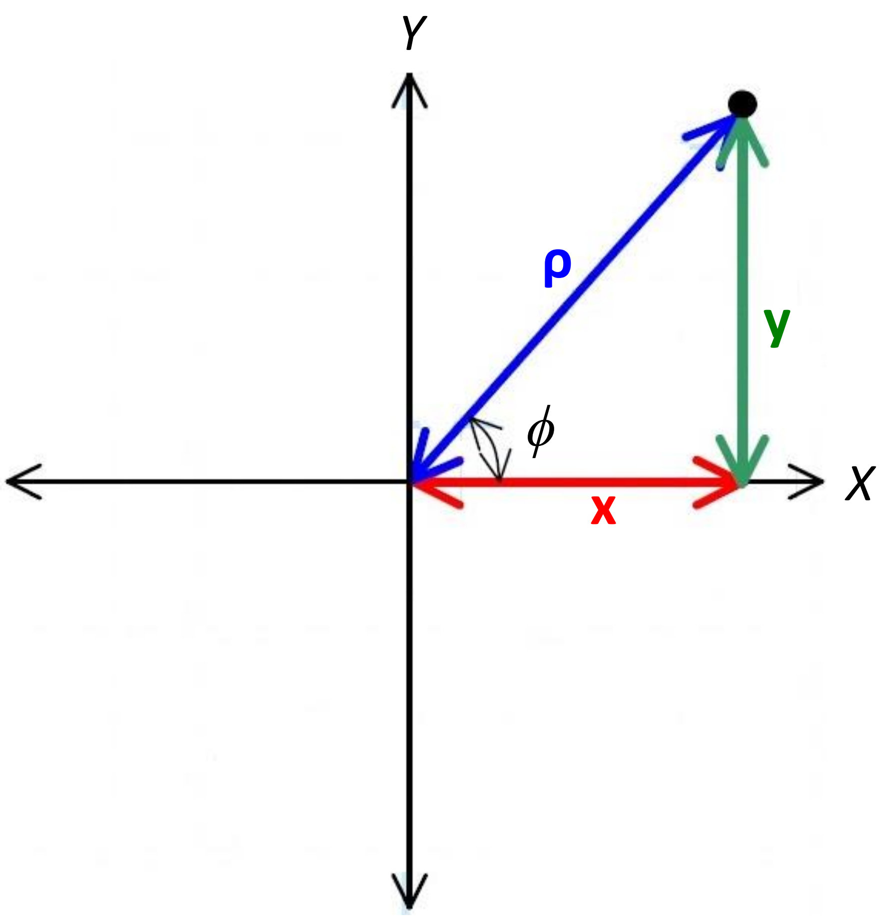
$$\rho = r \times \sin\theta = z \times \tan\theta = \sqrt{r^2 - z^2}$$

$$r = \sqrt{\rho^2 + z^2} = \rho \times \operatorname{cosec}\theta = z \times \sec\theta$$

$$\theta = \tan^{-1}\left[\frac{\rho}{z}\right] = \sin^{-1}\left[\frac{\rho}{r}\right] = \cos^{-1}\left[\frac{z}{r}\right]$$

Coordinate Transformations (contd.)

Right Triangle #2



$$x = \rho \times \cos\phi = y \times \cot\phi = \sqrt{\rho^2 - y^2}$$

$$y = \rho \times \sin\phi = x \times \tan\phi = \sqrt{\rho^2 - x^2}$$

$$\rho = \sqrt{x^2 + y^2} = x \times \sec\phi = y \times \operatorname{cosec}\phi$$

$$\phi = \tan^{-1}\left[\frac{y}{x}\right] = \sin^{-1}\left[\frac{y}{\rho}\right] = \cos^{-1}\left[\frac{x}{\rho}\right]$$

Coordinate Transformations (contd.)

Combining the results of the two triangles allows us to write each coordinate set in terms of each other

- **Cartesian and Cylindrical**

$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}\left[\frac{y}{x}\right]$$

$$z = z$$

$$x = \rho \times \cos\phi$$

$$y = \rho \times \sin\phi$$

$$z = z$$

- **Cartesian and Spherical**

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}\left[\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right]$$

$$\phi = \tan^{-1}\left[\frac{y}{x}\right]$$

$$x = r \times \sin\theta \times \cos\phi$$

$$y = r \times \sin\theta \times \sin\phi$$

$$z = r \times \cos\theta$$

Coordinate Transformations

- **Cylindrical and Spherical**

$$\rho = r \times \sin\theta$$
$$\phi = \phi$$
$$z = r \times \cos\theta$$

$$\Leftrightarrow$$

$$r = \sqrt{\rho^2 + z^2}$$
$$\theta = \tan^{-1}\left[\frac{\rho}{z}\right]$$
$$\phi = \phi$$

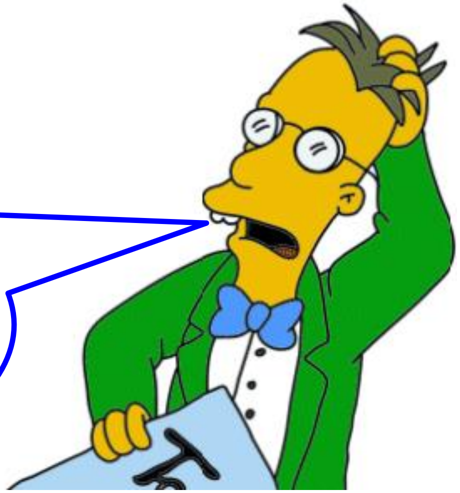
Example – 1

- Say we have denoted a **point** in space (using **Cartesian** Coordinates) as $P(x = -3, y = -3, z = 2)$.
- Let's **instead** define this **same** point using **cylindrical** coordinates ρ, ϕ, z.

$$\boxed{\rho = \sqrt{(-3)^2 + (-3)^2} = 3\sqrt{2}} \quad \boxed{\phi = \tan^{-1}\left[\frac{-3}{-3}\right] = 45^o} \quad \boxed{z = 2}$$

Therefore, the location of this point can **perhaps** be defined **also** as $P\left(\rho = 3\sqrt{2}, \phi = 45^o, z = 2\right)$.

Q: **Wait!** Something has gone **horribly wrong**. Coordinate ϕ = 45° indicates that point P is located in **quadrant-I**, whereas the coordinates x =-3, y =-3 tell us it is in fact in **quadrant-III**!

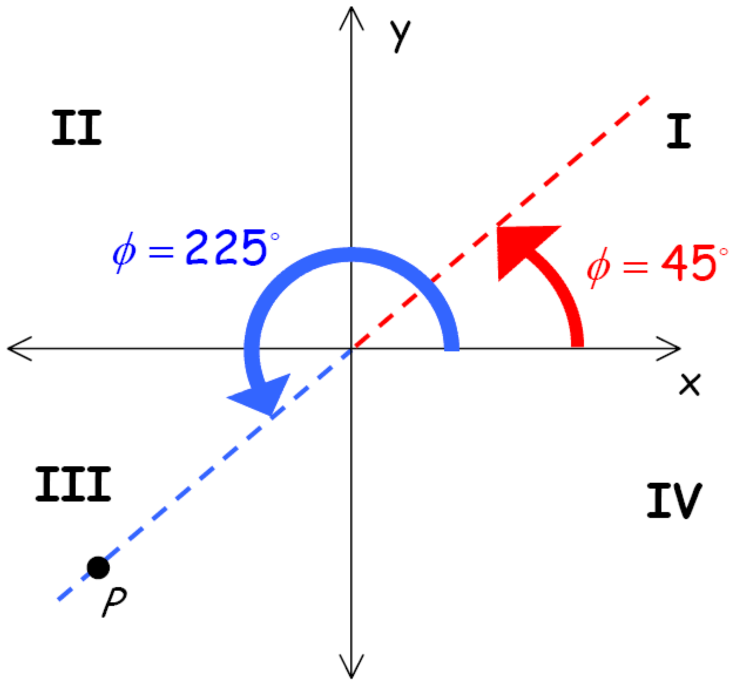


Example – 1 (contd.)

A: The problem is in the interpretation of the **inverse tangent!**

Remember that $0 \le \phi < 360^o$, so that we must do a **four quadrant** inverse tangent. Your calculator likely only does a **two quadrant** inverse tangent (i.e., $90^o \le \phi \le -90^o$), so **be careful**!

Therefore, if we **correctly** find the coordinate ϕ: $\phi = \tan^{-1}\left[\frac{-3}{-3}\right] = 225^o$



The location of point P can be expressed as **either** $P(x = -3, y = -3, z = 2)$ or $P\left(\rho = 3\sqrt{2}, \phi = 225^o, z = 2\right)$.

Example – 2

Coordinate transformation on a Scalar field

- Consider the scalar field (i.e., scalar function): $g(\rho,\phi,z)=\rho^3 z\sin\phi$

rewrite this function in terms of **Cartesian** coordinates.

- Note that since $\rho=\sqrt{x^2+y^2}$ → $\boxed{\rho^3=\left(x^2+y^2\right)^{3/2}}$

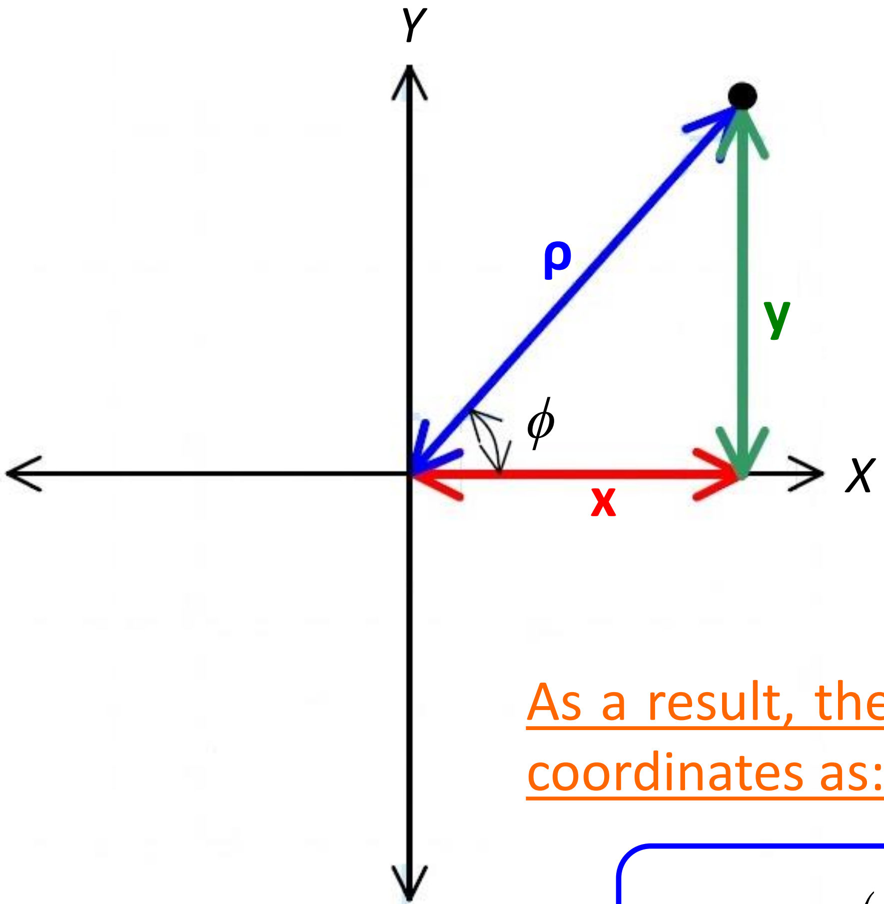
- Now, what about **sin**ϕ?

We know that $\phi=\tan^{-1}\left[\frac{y}{x}\right]$, We might be tempted to write:

$$\boxed{\sin\phi=\sin\left[\tan^{-1}\left[\frac{y}{x}\right]\right]}$$

Technically correct, this is one **ugly** expression. We can instead turn to one of the **very important right triangles** that we discussed earlier

Example – 2 (contd.)



From **this** triangle, it is apparent that:

$$\sin\phi = \frac{y}{\sqrt{x^2 + y^2}}$$

As a result, the scalar field can be written in **Cartesian** coordinates as:

$$g(x,y,z) = \left(x^2 + y^2\right)^{3/2} \frac{y}{\sqrt{x^2 + y^2}} z = \left(x^2 + y^2\right) yz$$

Example – 2 (contd.)

Although the scalar fields: $g(\rho,\phi,z)=\rho^3 z\sin\phi$ **and** $g(x,y,z)=\left(x^2+y^2\right)yz$

look very different, they are in fact **exactly** the same functions—only expressed using different **coordinate variables**.

- For **example**, if you **evaluate** each of the scalar fields at the **point** described earlier, you will get **exactly the same** result!

$$g(x=-3,y=-3,z=2)=-108$$

$$g(\rho=3\sqrt{2},\phi=225^o,z=2)=-108$$