Lecture – 16

Date: 16.03.2015

- Magnetic Materials
- Permanent Magnets
- Magnetic Boundary Conditions

Example – 1

- Consider an **infinite cylinder** made of **magnetic** material. This cylinder is centered along the z-axis, has a **radius of $2m$**, and a **permeability** of $4\mu_0$.

Inside the cylinder there exists a **magnetic flux density**:

$$\vec{B} = \frac{8\mu_0}{\rho}\hat{a}_\phi \qquad (\rho \leq 1)$$

Determine the **magnetization current** $\vec{K}_b$ flowing **on the surface** of this cylinder, as well as the magnetization current $\vec{J}_b$ flowing **within the volume** of this cylinder.

Example – 1 (contd.)

- First, we note that we must know the **magnetization vector** $\vec{M}$ in order to find the magnetization currents:

$$\vec{J}_b = \nabla \times \vec{M}$$

$$\vec{K}_b = \vec{M} \times \hat{a}_n$$

- We must know the **magnetic susceptibility** χ_m and the magnetic field $\vec{H}$ to determine magnetization vector.

$$\vec{M} = \chi_m \vec{H}$$

- Likewise, we need to know the **relative permeability** $\boldsymbol{\mu_r}$ to determine magnetic susceptibility:

$$\vec{J}_b = \nabla \times \vec{M}$$

- and we need to know the **magnetic flux density** $\vec{B}$ to determine the magnetic field:

$$\vec{H} = \frac{\vec{B}}{\mu}$$

Example – 1 (contd.)

- But guess what! We **know** the relative permeability $\boldsymbol{\mu_r}$ of the material, as well as the magnetic flux density within it!

$$\mu = 4\mu_0 \quad \Rightarrow \quad \mu_r = 4$$

$$\vec{B} = \frac{8\mu_0}{\rho}\hat{a}_\phi \qquad (\rho \le 1)$$

- Therefore, the **magnetic field** is: $\vec{H} = \frac{\vec{B}}{\mu} = \frac{1}{4\mu_0}\frac{8\mu_0}{\rho}\hat{a}_\phi = \frac{2}{\rho}\hat{a}_\phi$

- and the **magnetic susceptibility** is: $\chi_m = \mu_r - 1 = 4 - 1 = 3$

- So the **magnetization vector** is: $\vec{M} = \chi_m \vec{H} = (3)\frac{2}{\rho}\hat{a}_\phi = \frac{6}{\rho}\hat{a}_\phi$

Example – 1 (contd.)

- Now (**finally!**) we can determine the **magnetization currents**:

$$\vec{J}_b = \nabla \times \vec{M} = \nabla \times \left(\frac{6}{\rho} \hat{a}_\phi \right) = 0$$

The volume magnetization current density is **zero**—there is no magnetization current flowing **within** the cylinder!

Q: No magnetization currents! So we're **done** right? This problem is **solved**?

A: hardly!

Example – 1 (contd.)

- Although there are no magnetization currents flowing **within** the cylinder, there might be magnetization currents flowing on the cylinder **surface** $\vec{K}_b$!

$$\vec{K}_b = \vec{M} \times \hat{a}_n$$

Note for this problem, the unit vector normal to the surface of the cylinder is $\hat{a}_n = \hat{a}_\rho$.

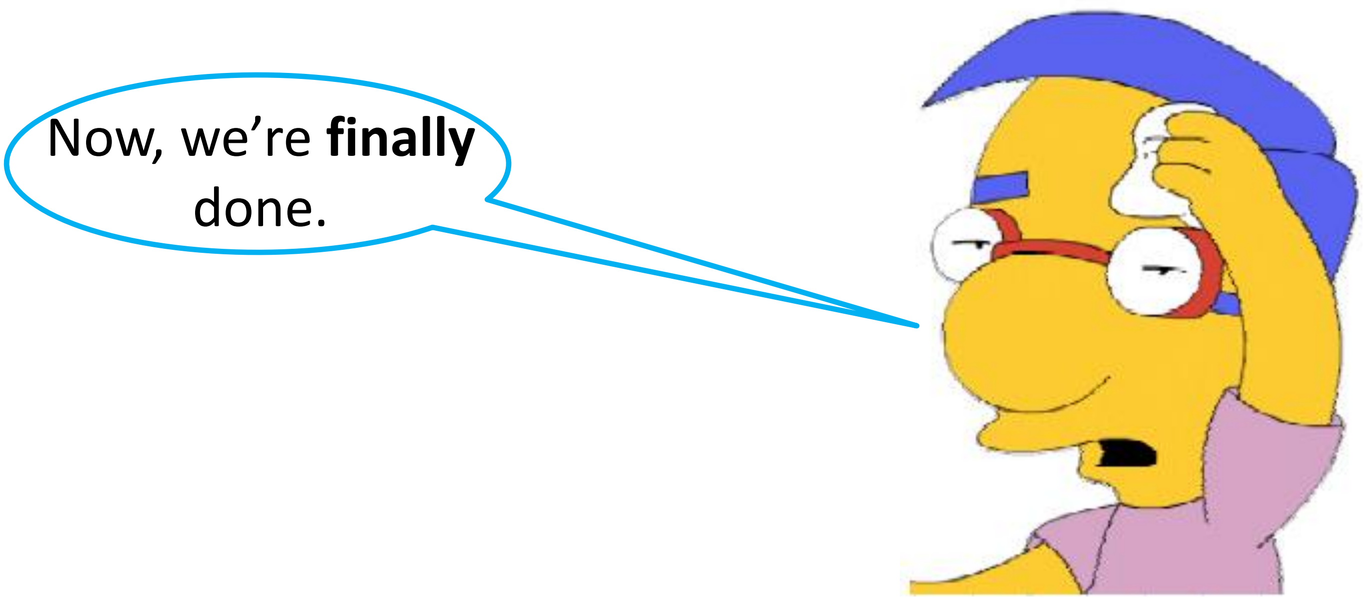
Likewise, the magnetization vector **evaluated at the cylinder surface** (i.e., at ρ = 2) is:

$$\vec{M} = \vec{M}(\rho = 2) = \frac{6}{\rho} \hat{a}_\phi \,|_{\rho=2} = 3\hat{a}_\phi$$

Example – 1 (contd.)

- Therefore, the **magnetization current density** on the cylinder surface is:

$$\vec{K}_b(\rho=2)=\vec{M}(\rho=2)\times\hat{a}_n=3\hat{a}_\phi\times\hat{a}_\rho=-3\hat{a}_z \quad A/m$$



Magnetic Materials

- A material usually is classified as diamagnetic, paramagnetic, or ferromagnetic on the basis of the value of its χ_m.
- **Diamagnetic** materials have negative susceptibilities **whereas paramagnetic** materials have positive susceptibilities.
- The absolute **magnitude of χ_m** is of the **order of 10^{-5}** for both classes of materials, which for most applications allows us to ignore χ_m.
- **Therefore, $\mu_r \cong 1$ or $\mu = \mu_0$ for diamagnetic and paramagnetic substances**, **which include dielectric materials and most metals.**
- In contrast, $|\mu_r| \gg 1$ **for ferromagnetic materials.**

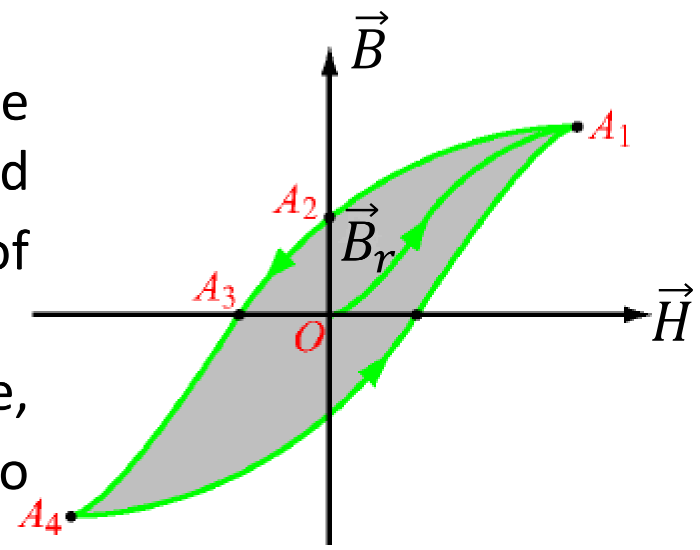
Properties of Magnetic Materials

	Diamagnetism	Paramagnetism	Ferromagnetism
Permanent magnetic Dipole Moment	No	Yes, but weak	Yes, and strong
Primary Magnetization Mechanism	Electron orbital magnetic moment	Electron spin magnetic moment	Magnetized domains
Direction of induced magnetic field (relative to external magnetic field)	Opposite	Same	Hysteresis
Common Substances	Bismuth, copper, diamond, gold, lead, mercury, silver, silicon	Aluminium, calcium, chromium, magnesium, niobium, platinum, tungsten	Iron, nickel, cobalt
Typical value of χ_m	$\cong -10^{-5}$	$\cong 10^{-5}$	$\gg 1$ *and hysteretic*
Typical value of μ_r	≈ 1	≈ 1	$\gg 1$ *and hysteretic*

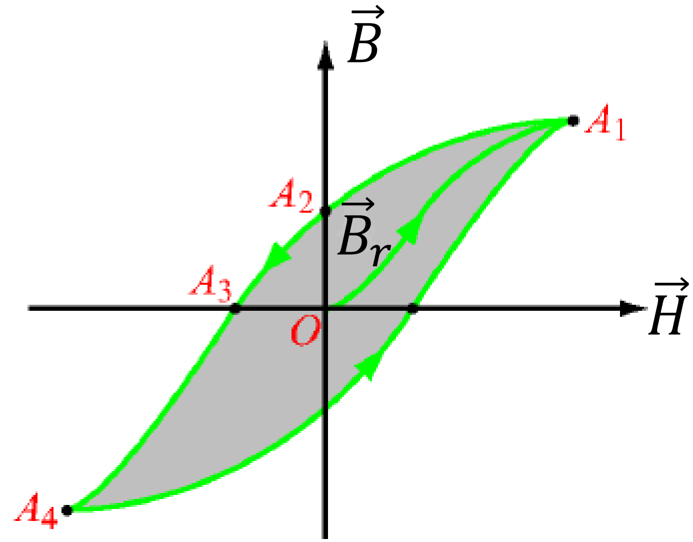
Magnetic Materials (contd.)

- Ferromagnetic materials exhibit unique magnetic properties due to the fact that their magnetic moments tend to readily align along the direction of external magnetic field.
- The magnetization behavior of a ferromagnetic material can be understood in terms of its $\vec{B} - \vec{H}$ magnetization curve.

- Suppose, we start with an unmagnetized sample of iron, denoted by point O.
- Increase in $\vec{H}$, for example by increasing the current passing through a wire wound around the sample, increases $\vec{B}$ up to point of saturation A_1.
- Removal of current through the wire (i.e, decrease in $\vec{H}$ to zero) doesn't bring back $\vec{B}$ to zero.
- Instead there remains a residual flux density $\vec{B}_r$



Magnetic Materials (contd.)

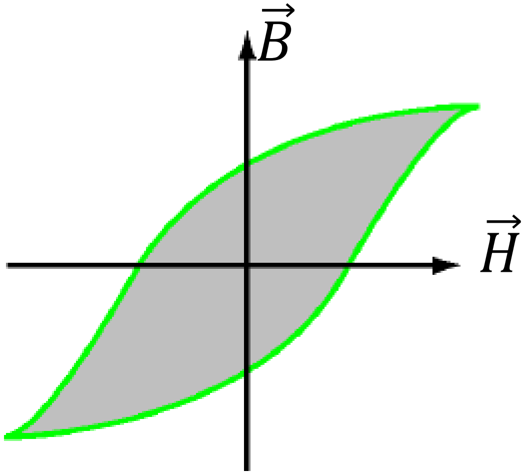


- The presence of $\vec{B}_r$ enables the iron sample to be used as permanent magnet.
- Reversing the direction of $\vec{H}$ and increasing its intensity causes decrease in $\vec{B}$ → further increase in $\vec{H}$ while maintaining the direction, the magnetization moves to saturation point A_4.
- Finally as $\vec{H}$ is made to return to zero and then increased again then the curve follows the path from A_4 to A_1.
- This overall process is called magnetic hysteresis.
- The existence of ***hysteresis loop*** implies that the magnetization process in ferromagnetic materials depends not only on the magnetic field $\vec{H}$, but also on the magnetic history of the material.
- The shape and extent of the hysteresis loop depend on the properties of the ferromagnetic material and the peak-to-peak range over which $\vec{H}$ is made to vary.

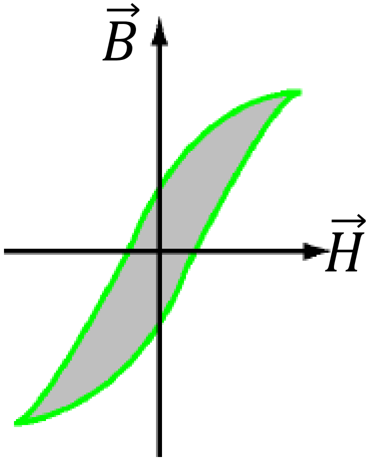
Magnetic Materials (contd.)

- Hard ferromagnetic materials are characterized by wide hysteresis loops.
- They can't be easily demagnetized by an external magnetic field because they have large residual magnetization $\vec{B}_r$.
- Hard ferromagnetic materials are used in the fabrication of permanent magnets for motors and generators.
- Soft ferromagnetic materials have narrow hysteresis loops and can be easily magnetized and demagnetized.

To demagnetize any ferromagnetic material, the material is subjected to several hysteresis cycles while gradually decreasing the peak-to-peak range of applied field



Hard material

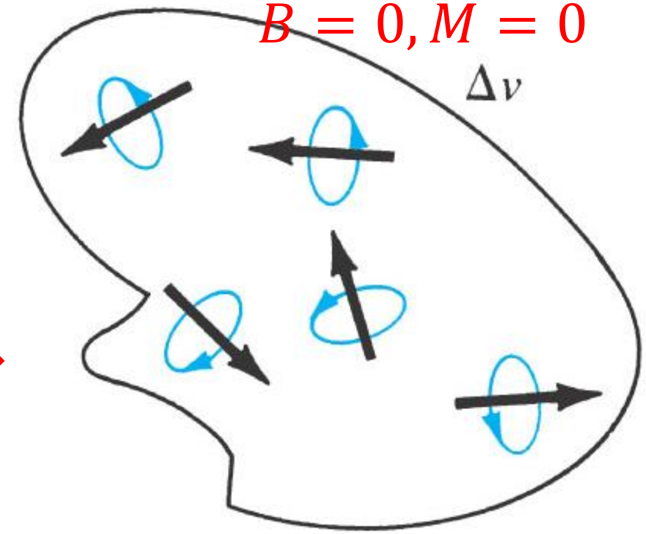


Soft material

Permanent Magnets

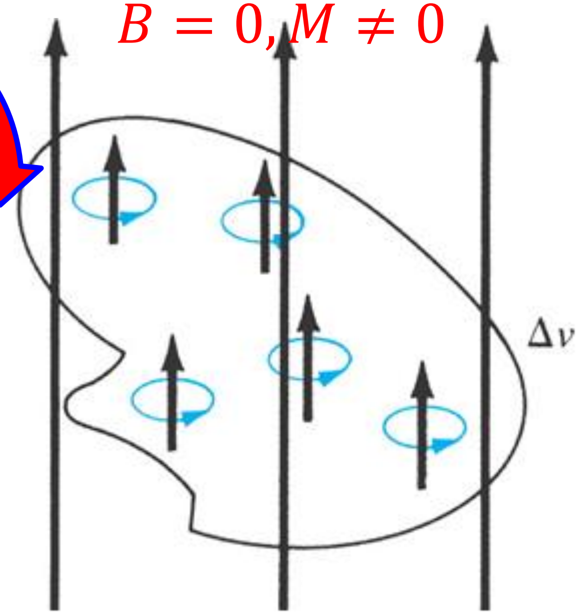
- For **most** magnetic material (i.e., where $\mu = \mu_0$), we find that the magnetization vector $\vec{M}$ will return to **zero** when a magnetization field $\vec{B}$ is removed. In other words, the **magnetic dipoles** will vanish, or at least return to their random state.

$\vec{B} = 0, \vec{M} = 0$



- However, some magnetic material, called **ferromagnetic** material, **retain** its dipole orientation, even when the magnetizing field is removed !

$\vec{B} = 0, \vec{M} \neq 0$



In this case, a **permanent magnet** is formed (just like the ones you stick on your fridge)!

Permanent Magnets

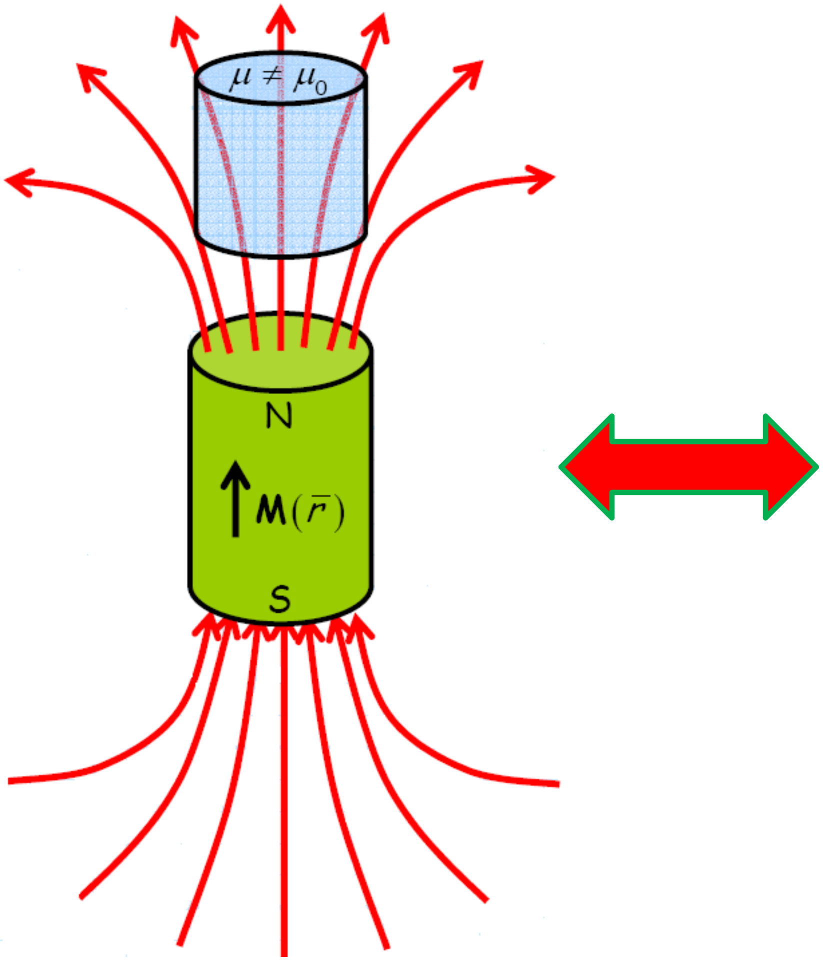
- Ferromagnetic materials have **numerous applications**.
- For example, they will **attract** magnetic material.

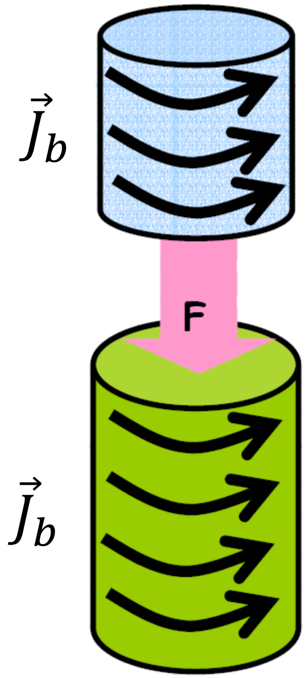
Q: How?

A: A permanent magnet will produce **everywhere** a magnetic flux density $\vec{B}$, which we can **either** attribute to the magnetic **dipoles** within the material, **or** to the equivalent magnetic **current** $\vec{J}_b$.

The magnetic flux density produced by the magnet will act as a **magnetizing** field for some **other** magnetic material nearby, thus creating a **second** magnetization **current** $\vec{J}_b$ within the nearby material. The magnetization currents of the material and the magnet will **attract**!

Permanent Magnets





Permanent Magnets

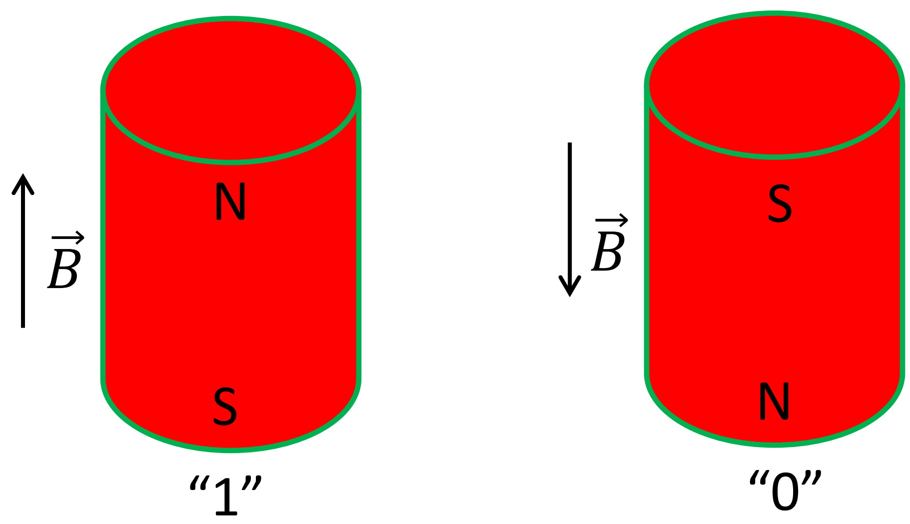
- Another interesting application of ferromagnetic material is in non-volatile **data storage** (e.g., tape or disk). Ferromagnetics can be used as **binary memory** !

Q: How?

A: Recall that the magnetization vector in ferromagnetic material retains its direction after the magnetizing field $\vec{B}$ has been removed. In other words, it "**remembers**" the direction of the magnetizing field.

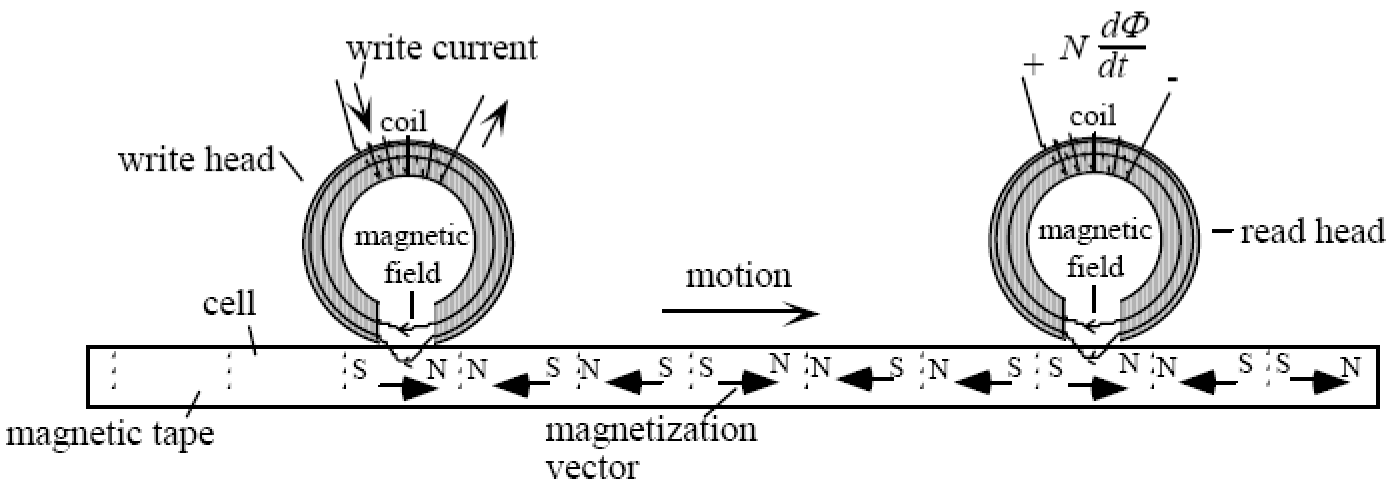
Permanent Magnets

- We can assign each of **two** different magnetizing directions, therefore, a **binary** state:



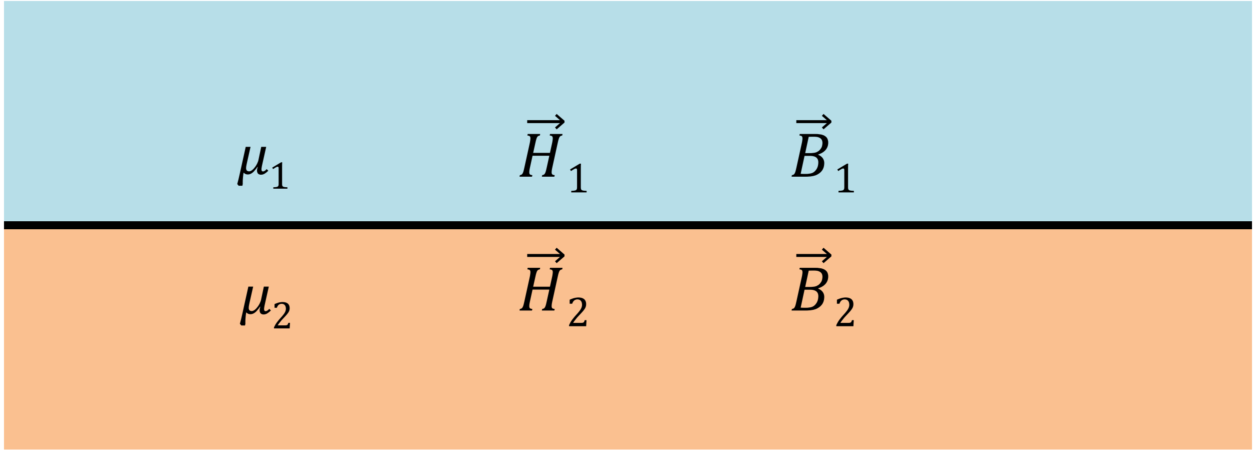
Permanent Magnets

- If ferromagnetic material is **embedded** in a tape or disk, we can magnetize (e.g., **write**) small sections of the media, or detect the magnetization (e.g., **read**) small sections of the media.



Magnetic Boundary Conditions

- Consider the **interface** between two **different materials** with dissimilar **permeabilities**:



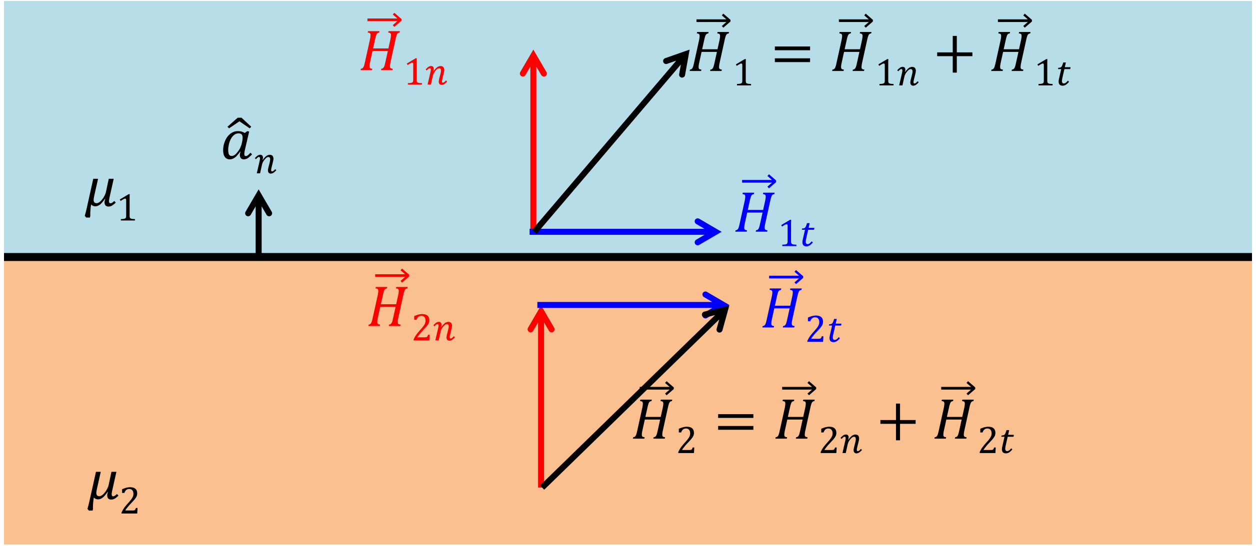
- Say that a magnetic field and a magnetic flux density is present in **both** regions.

Q: How are the fields in dielectric **region 1** (i.e., $\vec{H}_1$ and $\vec{B}_1$) related to the fields in **region 2** (i.e., $\vec{H}_2$ and $\vec{B}_2$)

A: They must satisfy the **magnetic boundary conditions** !

Magnetic Boundary Conditions (contd.)

- First, let's write the fields **at the interface** in terms of their **normal** $\vec{H}_n$ and **tangential** $\vec{H}_t$ vector components:



- Our first boundary condition states that the **tangential** component of the magnetic field is **continuous** across a boundary. In other words:

$$\vec{H}_{1t} = \vec{H}_{2t}$$

Magnetic Boundary Conditions (contd.)

The **tangential** component of the magnetic field on **one** side of the material boundary is **equal** to the tangential component on the **other** side !

Furthermore:

$$\frac{\vec{B}_{1t}}{\mu_1} = \frac{\vec{B}_{2t}}{\mu_2}$$

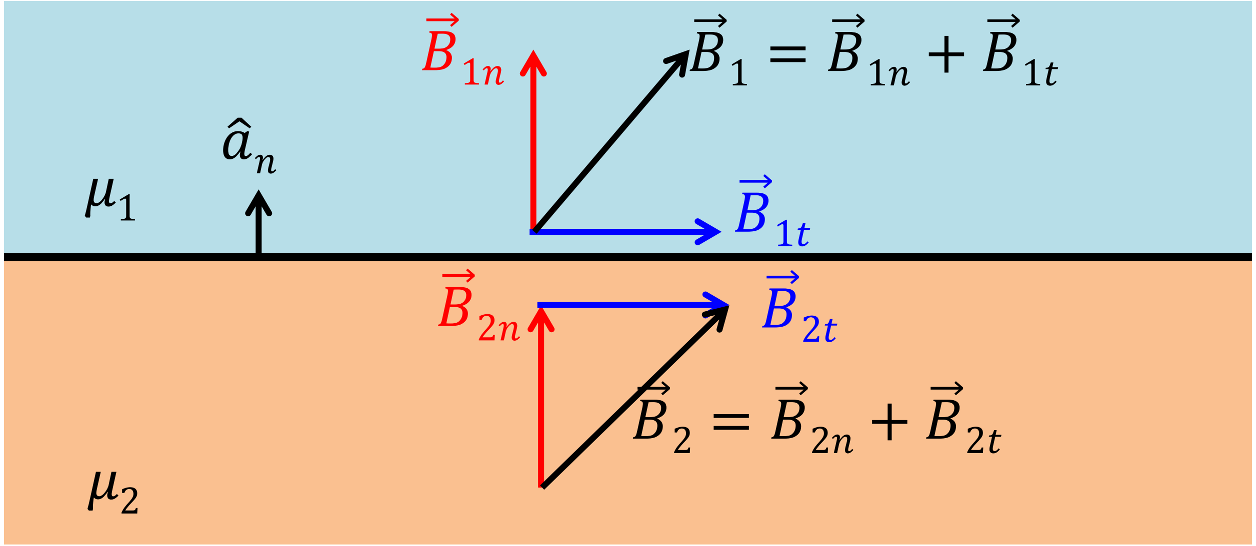
Tangential component of magnetic flux density is discontinuous

- Interface having bound surface charge density $\vec{K}_b$ will have the modified relationship:

$$\vec{H}_{1t} - \vec{H}_{2t} = \vec{K}_b$$

Magnetic Boundary Conditions

- We can likewise consider the **magnetic flux densities** on the material interface in terms of their **normal** and **tangential** components:



- The second magnetic boundary condition states that the **normal** vector component of the **magnetic flux density** is **continuous** across the material boundary. In other words:

$$\vec{B}_{1n} = \vec{B}_{2n}$$

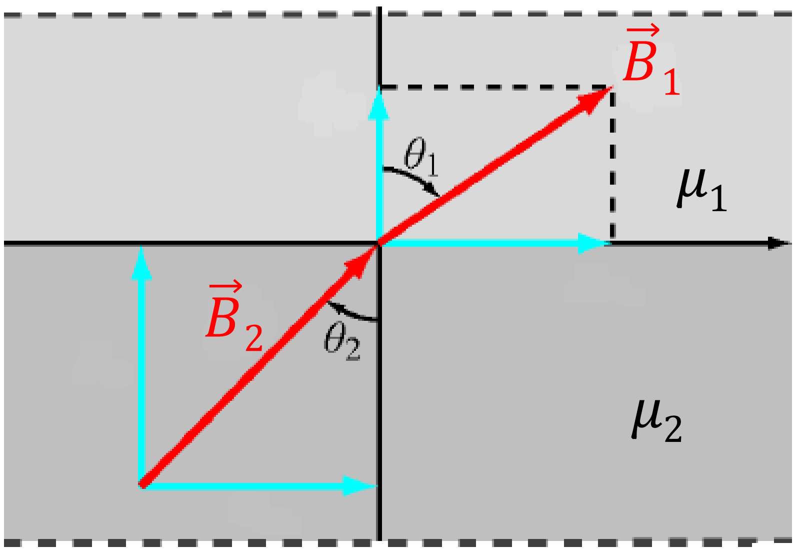
Magnetic Boundary Conditions (contd.)

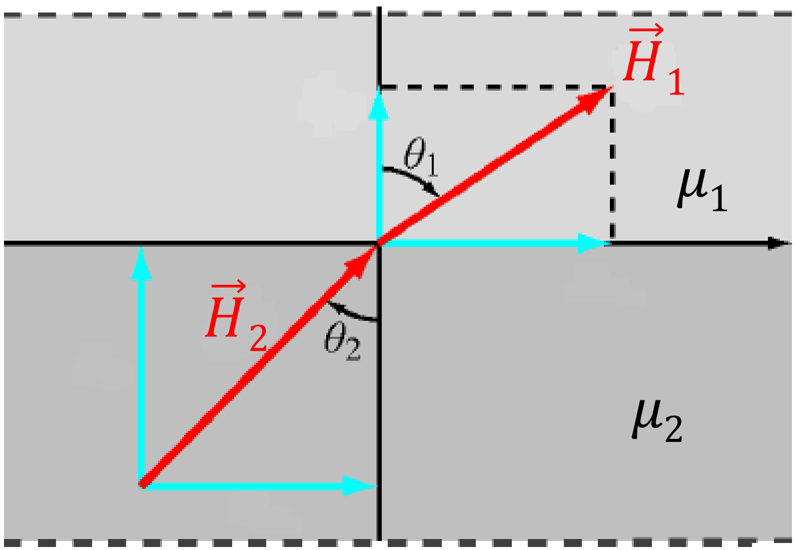
Furthermore: $\mu_1 \overrightarrow{H}_{1n} = \mu_2 \overrightarrow{H}_{2n}$

Normal component of magnetic field is discontinuous

Magnetic Boundary Conditions (contd.)

- If the fields make angle θ with the normal to the interface then:





$$B_1 \cos\theta_1 = \vec{B}_{1n} = \vec{B}_{2n} = B_2 \cos\theta_2$$

$$\frac{B_1}{\mu_1}\sin\theta_1 = \vec{H}_{1t} = \vec{H}_{2t} = \frac{B_2}{\mu_2}\sin\theta_2$$

- Simplification gives: $\dfrac{\tan\theta_2}{\tan\theta_1} = \dfrac{\mu_1}{\mu_2}$ → Law of refraction for magnetic flux lines at a boundary with no surface current

Example – 2

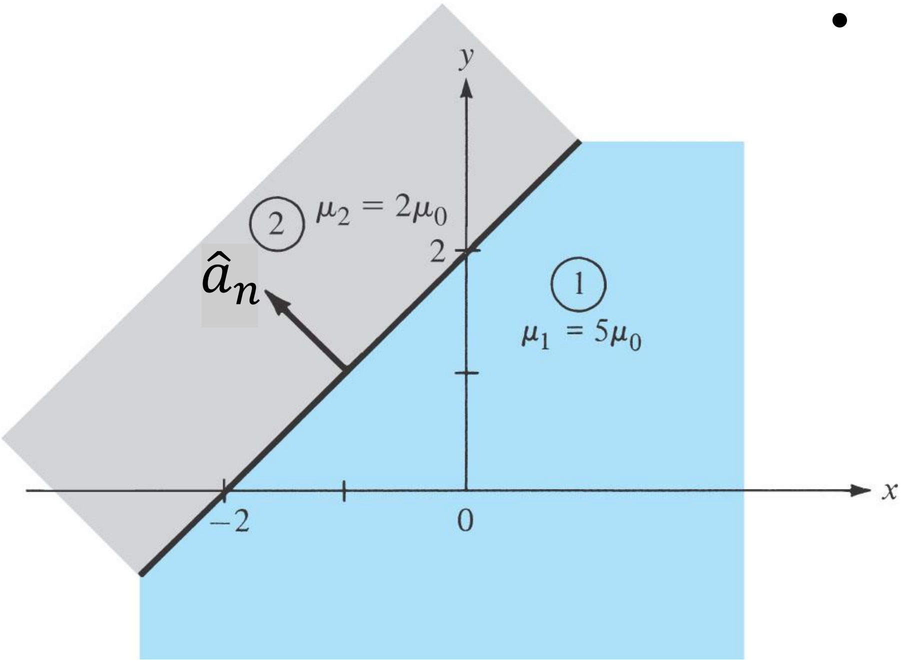
- Given that $\vec{H}_1 = -2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z\ A/m$ in region $y - x - 2 \leq 0$, where $\mu_1 = 5\mu_0$, calculate:

  (a) $\vec{M}_1$ and $\vec{B}_1$

  (b) $\vec{H}_2$ and $\vec{B}_2$ in region $y - x - 2 \geq 0$, where $\mu_2 = 2\mu_0$

Example – 2 (contd.)

- Since $y - x - 2 = 0$ is a plane, $y - x \leq 2$ or $y \leq x + 2$ is region-1 as shown in figure.



- If we let the surface of the plane be described by $f(x,y) = y - x - 2$, then a unit vector normal to the plane will be:

$$\hat{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{\hat{a}_y - \hat{a}_x}{\sqrt{2}}$$

(a) $\vec{M}_1 = \chi_{m1}\vec{H}_1 = (\mu_{r1} - 1)\vec{H}_1$

$$= (5-1)(-2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z)$$

$$\therefore \vec{M}_1 = -8\hat{a}_x + 24\hat{a}_y + 16\hat{a}_z \quad A/m$$

Example – 2 (contd.)

$$\vec{B}_1 = \mu_1 \vec{H}_1 = \mu_{r1}\mu_0 \vec{H}_1 \qquad \Rightarrow \vec{B}_1 = 4\pi \times 10^{-7}(5)\left(-2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z\right)$$

$$\therefore \vec{B}_1 = -12.57\hat{a}_x + 37.7\hat{a}_y + 25.13\hat{a}_z \;\; \mu Wb/m^2$$

(b) $$\vec{H}_{1n} = \left(\vec{H}_1.\hat{a}_n\right)\hat{a}_n \qquad \Rightarrow \vec{H}_{1n} = \left[\left(-2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z\right).\left(\frac{\hat{a}_y - \hat{a}_x}{\sqrt{2}}\right)\right]\left(\frac{\hat{a}_y - \hat{a}_x}{\sqrt{2}}\right)$$

$$\therefore \vec{H}_{1n} = -4\hat{a}_x + 4\hat{a}_y$$

- We can find the tangential component as: $\vec{H}_{1t} = \vec{H}_1 - \vec{H}_{1n}$

$$\therefore \vec{H}_{1t} = 2\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z$$

Example – 2 (contd.)

- Use of boundary conditions give:

$$\vec{H}_{2t} = \vec{H}_{1t} = 2\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z$$

$$\vec{B}_{2n} = \vec{B}_{1n} \rightarrow \mu_2 \vec{H}_{2n} = \mu_1 \vec{H}_{1n}$$

$$\vec{H}_{2n} = \frac{\mu_1}{\mu_2}\vec{H}_{1n} = \frac{5}{2}\left(-4\hat{a}_x + 4\hat{a}_y\right) = -10\hat{a}_x + 10\hat{a}_y$$

- Thus: $$\vec{H}_2 = \vec{H}_{2t} + \vec{H}_{2n} = -8\hat{a}_x + 12\hat{a}_y + 4\hat{a}_z \quad A/m$$

$$\vec{B}_2 = \mu_2 \vec{H}_2 = \mu_{r2}\mu_0 \vec{H}_2 = -20.11\hat{a}_x + 30.16\hat{a}_y + 10.05\hat{a}_z \quad \mu Wb/m^2$$

Example – 3

Region -1, described by $3x + 4y \geq 10$, is free space, whereas region-2, described by $3x + 4y \leq 10$, is a magnetic material for which $\mu = 10\mu_0$. Assuming that the boundary between the material and free space is current free, find $\vec{B}_2$ if $\vec{B}_1 = 0.1\hat{a}_x + 0.4\hat{a}_y + 0.2\hat{a}_z \, Wb/m^2$.

- If we let the surface of the plane be described by $f(x,y) = 3x + 4y - 10$, then a unit vector normal to the plane will be:

$$\hat{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{3\hat{a}_x + 4\hat{a}_y}{5}$$

$$\Rightarrow \vec{B}_{1n} = \left(\vec{B}_1 . \hat{a}_n\right)\hat{a}_n = \left(\frac{0.3 + 1.6}{5}\right)\left(\frac{3\hat{a}_x + 4\hat{a}_y}{5}\right) \Rightarrow \vec{B}_{1n} = \frac{57\hat{a}_x + 76\hat{a}_y}{250}$$

$$\therefore \vec{B}_{1n} = 0.228\hat{a}_x + 0.304\hat{a}_y \Rightarrow \vec{B}_{2n} = 0.228\hat{a}_x + 0.304\hat{a}_y$$

From Boundary Condition

Example – 3 (contd.)

$$\vec{B}_{1t} = \vec{B}_1 - \vec{B}_{1n} = -0.128\hat{a}_x + 0.096\hat{a}_y + 0.2\hat{a}_z$$

$$\vec{B}_{2t} = \frac{\mu_2}{\mu_1}\vec{B}_{1t} = \frac{10\mu_0}{\mu_0}\vec{B}_{1t} = -1.28\hat{a}_x + 0.96\hat{a}_y + 2\hat{a}_z$$

$$\therefore \vec{B}_2 = \vec{B}_{2n} + \vec{B}_{2t} = -1.052\hat{a}_x + 1.264\hat{a}_y + 2\hat{a}_z \quad Wb/m^2$$

Example – 4

The interface $4x - 5z = 0$ between two magnetic media carries current $35\hat{a}_y\ A/m$. If $\vec{H}_1 = 25\hat{a}_x - 30\hat{a}_y + 45\hat{a}_z\ A/m$ in region $4x - 5z \leq 0$ where $\mu_{r1} = 5$, calculate $\vec{H}_1$ in the region $4x - 5z \geq 0$ where $\mu_{r2} = 10$.