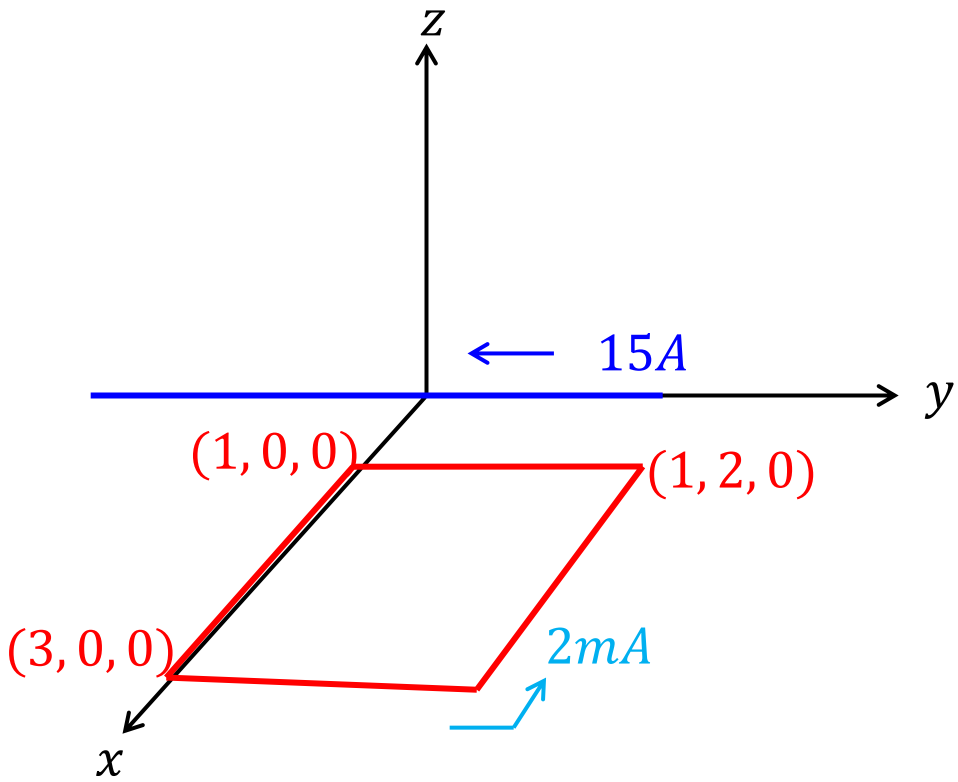
Lecture – 15

Date: 12.03.2015

- Magnetic Torque, Dipole and Moment
- Magnetization in Materials
- Magnetic Field in Materials

Example – 1

- There is a square loop of wire in the $z = 0$ plane carrying $2mA$ in the field of an infinite filament on the $y - axis$ as shown. Find the total force on the loop.



Example – 1 (contd.)

- The field produced by the straight filament in the plane of the loop is:

$$\vec{H} = \frac{I}{2\pi x}\hat{a}_z \; A/m \quad \Rightarrow \quad \vec{B} = \mu_0 \vec{B} = \frac{\mu_0 I}{2\pi x}\hat{a}_z = \frac{3\times 10^{-6}}{x}\hat{a}_z \quad T$$

- Therefore:

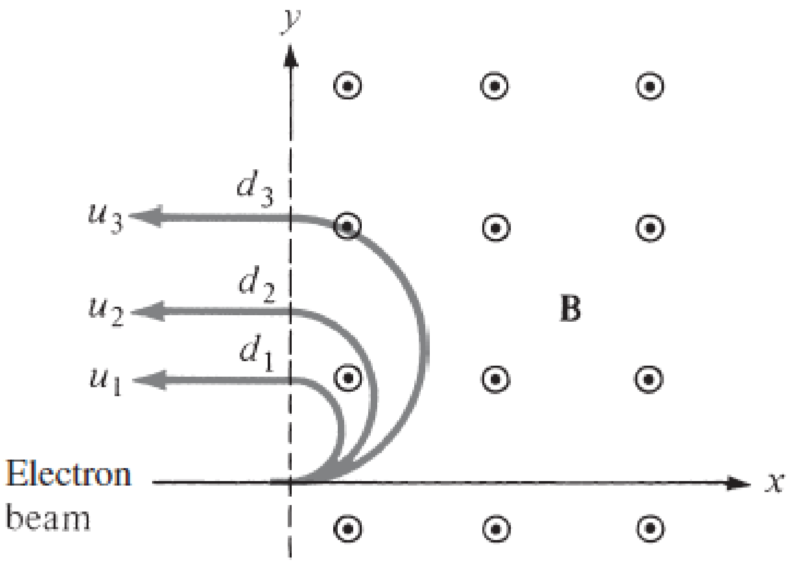
$$\vec{F}_m = I_{loop}\oint \overline{dl}\times\vec{B}$$

$$\Rightarrow \vec{F}_m = 2\times 10^{-3}\times 3\times 10^{-6}\left[\int_{x=1}^{3} dx\hat{a}_x\times\frac{\hat{a}_z}{x} + \int_{y=0}^{2} dy\hat{a}_y\times\frac{\hat{a}_z}{3} + \int_{x=3}^{1} dx\hat{a}_x\times\frac{\hat{a}_z}{x} + \int_{y=2}^{0} dy\hat{a}_y\times\frac{\hat{a}_z}{1}\right]$$

$$\therefore \vec{F}_m = -8\hat{a}_x nN$$

Example – 2

- By injecting an electron beam normally to the plane edge of a uniform field $B_o\hat{a}_z$, electrons can be dispersed according to their velocity as shown in the figure below.



(a) Show that the electrons would be ejected out of the field in path parallel to the input beam as shown.

(b) Derive an expression for the exit distance d above the entry point.

Example – 2 (contd.)

(a) We know: $\vec{F} = m\vec{a} = Q(\vec{u} \times \vec{B})$ → $\vec{F} = m\vec{a} = -e(\vec{u} \times \vec{B})$

$$\Rightarrow -\frac{m}{e}\frac{d\vec{u}}{dt} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ u_x & u_y & u_z \\ 0 & 0 & B_0 \end{vmatrix} \quad \rightarrow \quad \frac{m}{e}\frac{d\vec{u}}{dt} = -u_y B_0 \vec{a}_x + u_x B_0 \vec{a}_y$$

- From the above expression we can deduce:

$$\frac{du_x}{dt} = -u_y \frac{eB_0}{m} = -u_y g \qquad \text{Where: } g = \frac{eB_0}{m}$$

$$\frac{du_y}{dt} = u_x \frac{eB_0}{m} = u_x g$$

$$\frac{du_z}{dt} = 0 \quad \rightarrow \quad u_z = c = 0$$

Example – 2 (contd.)

- In order to determine the terms u_x and u_y, let us combine and simplify the expressions. It results into:

$$\frac{d^2u_x}{dt^2} = -g\frac{d^2u_y}{dt^2} = -g^2u_x \Rightarrow \frac{d^2u_x}{dt^2} + g^2u_x = 0$$

- The solution is: $u_x = A\cos gt + B\sin gt$
- Similarly: $u_y = A\sin gt - B\cos gt$
- Let us assume: at $t = 0 \rightarrow u_x = u_0 \,, u_y = 0$
- Then: $A = u_0 \; and \; B = 0$
- Therefore:

$$u_x = u_0\cos gt \Rightarrow u_x = \frac{dx}{dt} = u_0\cos gt \Rightarrow x = \frac{u_0}{g}\sin gt + c_1$$

$$u_y = u_0\sin gt \Rightarrow u_y = \frac{dy}{dt} = u_0\sin gt \Rightarrow y = -\frac{u_0}{g}\cos gt + c_2$$

Example – 2 (contd.)

- At $t = 0$: $x = 0\ and\ y = 0$
- It gives: $c_1 = 0$ and $c_2 = \frac{u_0}{g}$
- Eventually: $$x^2 + \left(y - \frac{u_0}{g}\right)^2 = \left(\frac{u_0}{g}\right)^2$$
- Therefore:

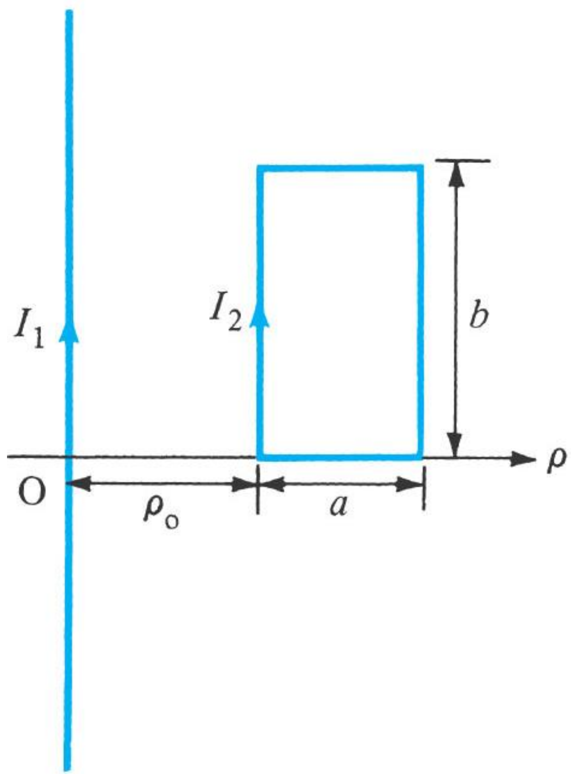
$$x = \frac{u_0}{g}\sin gt \qquad y = \frac{u_0}{g}(1 - \cos gt)$$

It shows that the electron will move in a circle centered at $\left(0, \frac{u_0}{g}\right)$. But since the field does not exist throughout the circular region, the electron passes through a semi-circle and leaves the field horizontally.

(b) Its twice the radius of the semi circle: $d = \frac{2u_0}{g} = \frac{2u_0 m}{B_0 e}$

Example – 3

- A rectangular loop carrying current I_2 is placed parallel to infinitely long filamentary wire carrying current I_1 as shown in figure. Show that the force experienced by the loop is given by:

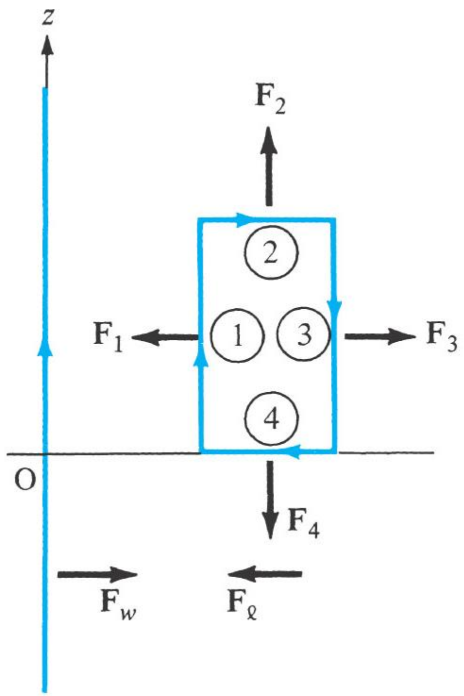


$$\overrightarrow{F_m} = -\frac{\mu_0 I_1 I_2 b}{2\pi}\left[\frac{1}{\rho_0} - \frac{1}{\rho_0 + a}\right]\hat{a}_\rho \text{ N}$$

Example – 3 (contd.)

- Let the force on the loop be: $\vec{F}_m = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$$= I_2 \oint \overline{dl}_2 \times \vec{B}_1$$



For infinitely long wire:

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi\rho_0}\hat{a}_\phi$$

Therefore:

$$\vec{F}_1 = I_2 \int \overline{dl}_2 \times \vec{B}_1 = I_2 \int_{z=0}^{b} dz\hat{a}_z \times \frac{\mu_0 I_1}{2\pi\rho_0}\hat{a}_\phi$$

$$\vec{F}_1 = -\frac{\mu_0 I_1 I_2 b}{2\pi\rho_0}\hat{a}_\rho$$

Example – 3 (contd.)

- Similarly

$$\vec{F}_3 = I_2\int \overline{dl}_2 \times \vec{B}_1 = I_2 \int_{z=b}^{a} dz\hat{a}_z \times \frac{\mu_0 I_1}{2\pi(\rho_0 + a)}\hat{a}_\phi \Rightarrow \vec{F}_3 = \frac{\mu_0 I_1 I_2 b}{2\pi(\rho_0 + a)}\hat{a}_\rho$$

$$\vec{F}_2 = I_2 \int_{\rho=\rho_0}^{\rho_0+a} d\rho\hat{a}_\rho \times \frac{\mu_0 I_1}{2\pi\rho}\hat{a}_\phi \Rightarrow \vec{F}_2 = \hat{a}_z \frac{\mu_0 I_1 I_2}{2\pi}\ln\frac{\rho_0 + a}{\rho_0}$$

$$\vec{F}_4 = I_2 \int_{\rho=\rho_0+a}^{\rho_0} d\rho\hat{a}_\rho \times \frac{\mu_0 I_1}{2\pi\rho}\hat{a}_\phi \Rightarrow \vec{F}_4 = -\hat{a}_z \frac{\mu_0 I_1 I_2}{2\pi}\ln\frac{\rho_0 + a}{\rho_0}$$

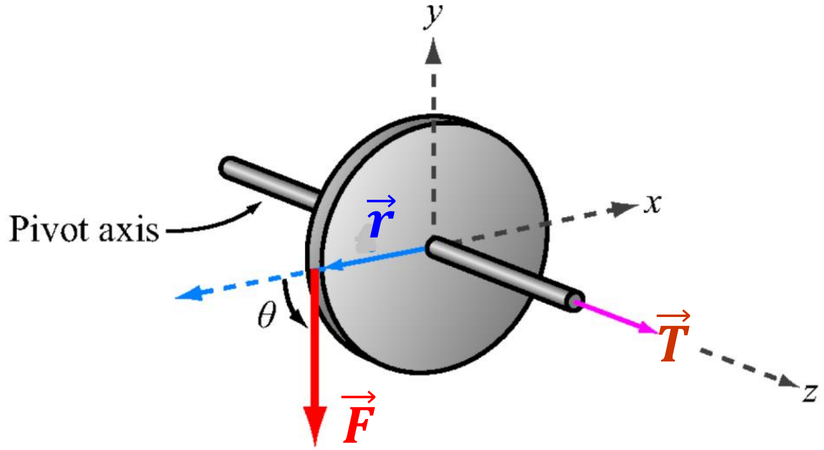
- The summation of all these expressions give the force on the loop:

$$\vec{F}_m = -\frac{\mu_0 I_1 I_2 b}{2\pi}\left[\frac{1}{\rho_0} - \frac{1}{\rho_0 + a}\right]\hat{a}_\rho \ \textbf{N}$$

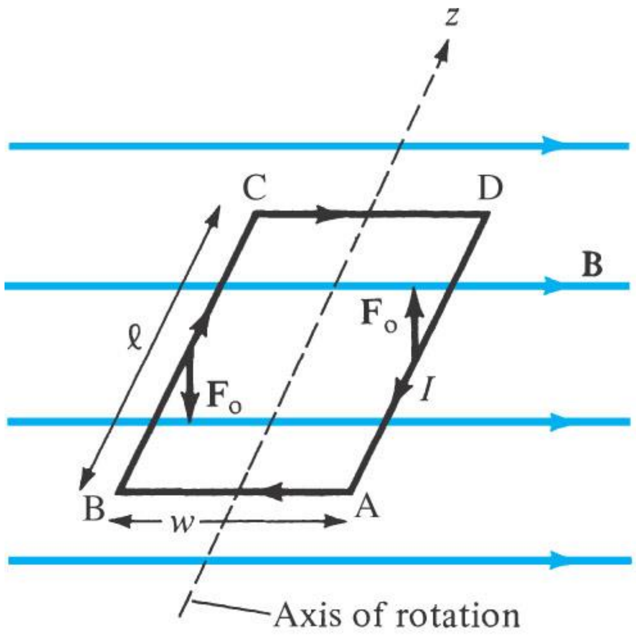
Magnetic Torque

- The torque $\vec{T}$ is the vector product of the force $\vec{F}$ and the moment arm $\bar{r}$.

$$\vec{T} = \vec{r} \times \vec{F}$$



- For the following configuration, the force on the loop is given by:



$$\vec{F} = I\int_B^C \overline{dl} \times \vec{B} + I\int_D^A \overline{dl} \times \vec{B}$$

$$\vec{F} = I\int_0^l dz\hat{a}_z \times \vec{B} + I\int_l^0 dz\hat{a}_z \times \vec{B}$$

$$\therefore \vec{F} = \vec{F}_0 + \left(-\vec{F}\right)_0 = 0$$

Magnetic Torque (contd.)

- Where: $\left|\vec{F}_0\right| = IBl$ ⇐ $\vec{B}$ is considered uniform here

- Apparently no force is exerted on the loop → however, $\vec{F}_0$ and $-\vec{F}_0$ acts on two different points on the loop, thereby creating a couple.
- If normal to the loop plane makes an angle α with $\vec{B}$ then:

$$\left|\vec{T}\right| = BIlw\sin\alpha \Rightarrow \left|\vec{T}\right| = BIS\sin\alpha$$

Let us define a quantity: $\vec{m} = IS\hat{a}_n$ → Magnetic dipole moment

- Therefore: $\vec{T} = \vec{m}\times\vec{B}$

Although this expression is obtained for rectangular loop but is applicable for planar loop of any arbitrary shape.

Example – 4

- A rectangular coil of area $10\ cm^2$ carrying current $50A$ lies on plane $2x + 6y - 3z = 7$ such that the magnetic moment of the coil is directed away from the origin. Calculate its magnetic moment.

$$f(x,y,z) = 2x + 6y - 3z = 0$$

$$\hat{a}_n = \pm \frac{\nabla f}{|\nabla f|} = \frac{2\hat{a}_x + 6\hat{a}_y - 3\hat{a}_z}{\left|\sqrt{49}\right|}$$

$$\vec{m} = IS\hat{a}_n$$

$$\vec{m} = 10\times10^{-4}\times50\times\frac{\left(2\hat{a}_x + 6\hat{a}_y - 3\hat{a}_z\right)}{\left|\sqrt{49}\right|}$$

$$\vec{m} = 7.143\times10^{-3}\times\left(2\hat{a}_x + 6\hat{a}_y - 3\hat{a}_z\right)$$

$$\vec{m} = \left(1.429\hat{a}_x + 4.286\hat{a}_y - 2.143\hat{a}_z\right)\times10^{-2}\quad A.m^2$$

Example – 5

- The coil of last example is surrounded by a uniform field $0.6\hat{a}_x + 0.4\hat{a}_y + 0.5\hat{a}_z \; Wb/m^2$.

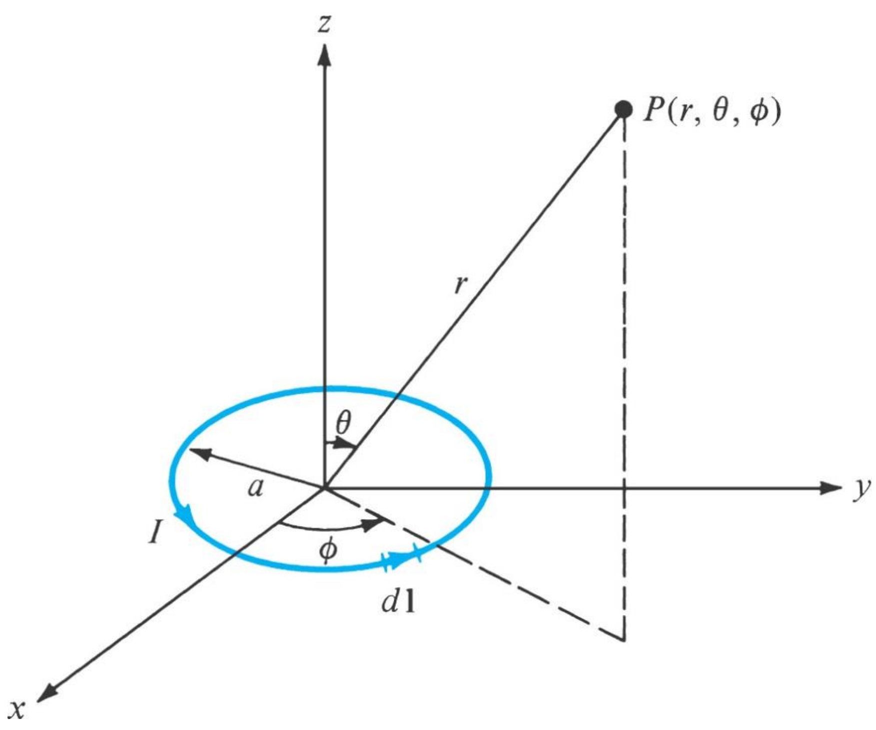
(a) Find the torque on the coil.

(b) Show that the torque on the coil is maximum if placed on plane $2x - 8y + 4z = \sqrt{84}$. Calculate the magnitude of the maximum torque.

$$\vec{T} = \vec{m} \times \vec{B} \quad \Longrightarrow \quad \vec{T} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ m_x & m_y & m_z \\ B_x & B_y & B_z \end{vmatrix}$$

Magnetic Dipole

- A bar magnet or small filamentary current loop is usually referred to as a magnetic dipole.
- The reason will be soon apparent.
- Let us consider the magnetic field $\vec{B}$ at an observation point $P(r, \theta, \phi)$ due to a circular loop carrying current I.



- The magnetic vector potential at P is:

$$\vec{A} = \frac{\mu_0 I}{4\pi} \oint \frac{\overline{dl}}{r}$$

Magnetic Dipole (contd.)

$$\vec{A} = \frac{\mu_0 \vec{m} \times \hat{a}_r}{4\pi r^2}$$

$$\vec{B} = \nabla \times \vec{A} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta \hat{a}_r + \sin\theta \hat{a}_\theta\right)$$

These are similar to the expressions for V and $\vec{E}$ due to an electric dipole

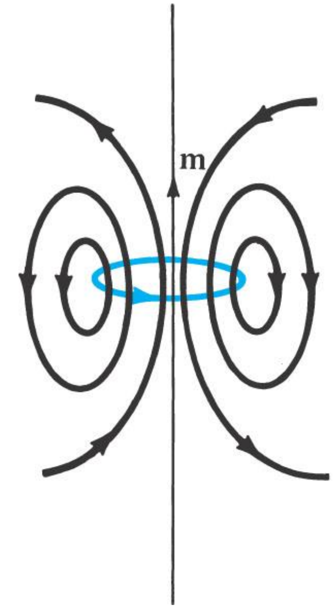
$$V = \frac{\vec{p}.\hat{a}_r}{4\pi\varepsilon_0 r^2}$$

$$\vec{E} = -\nabla V = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta \hat{a}_r + \sin\theta \hat{a}_\theta\right)$$

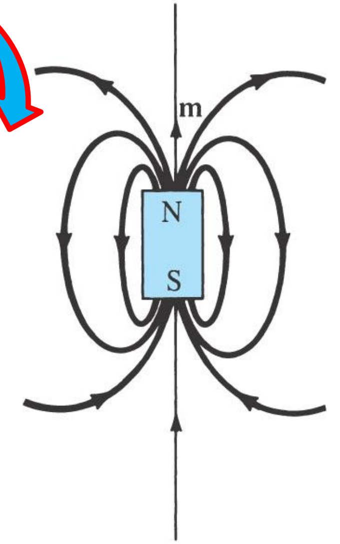
It is therefore reasonable to regard a small current loop as a magnetic dipole

Magnetic Dipole (contd.)

- $\vec{B}$ lines around the magnetic dipole can be illustrated as:



- A short permanent magnet can also be considered as a magnetic dipole.
- The $\vec{B}$ lines due to bar are similar to those due to a small current loop.



Magnetic Materials

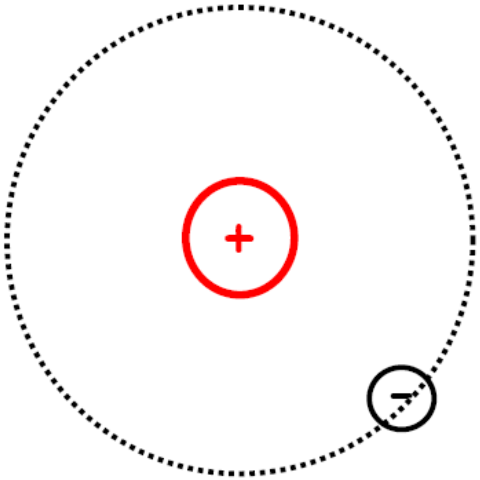
- Recall in dielectrics, electric dipoles were created when an $\vec{E} - field$ was applied.
- Therefore, we defined permittivity ε, electric flux density $\vec{D}$, and a new set of electrostatic equations.
- Recall that **atoms and molecules**, having both positive (i.e., protons) and negative (i.e., electron) charged particles can form **electric dipoles**.
- It will be apparent that that atoms and molecules can also form **magnetic dipoles**!

Q: How??

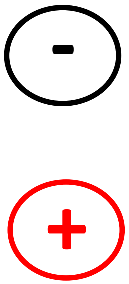
A: Recall a magnetic dipole is formed when current flows in a **small loop**. Current, of course, is **moving charge**, therefore charge moving around a small loop forms a magnetic dipole.

Molecules and atoms **often** exhibit electrons moving around in small loops!

Magnetic Materials (contd.)



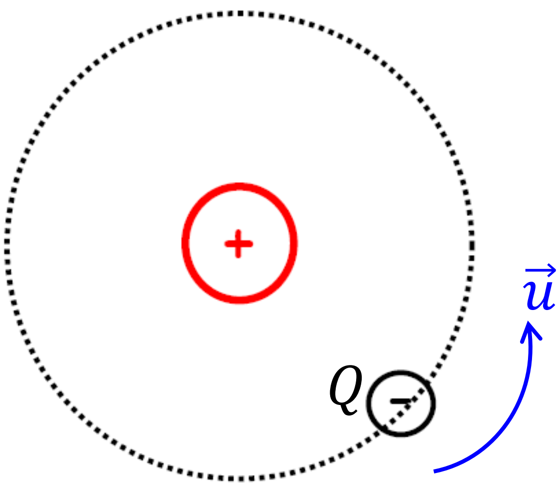
- Again, let us use our **ridiculously** simple model of an atom:

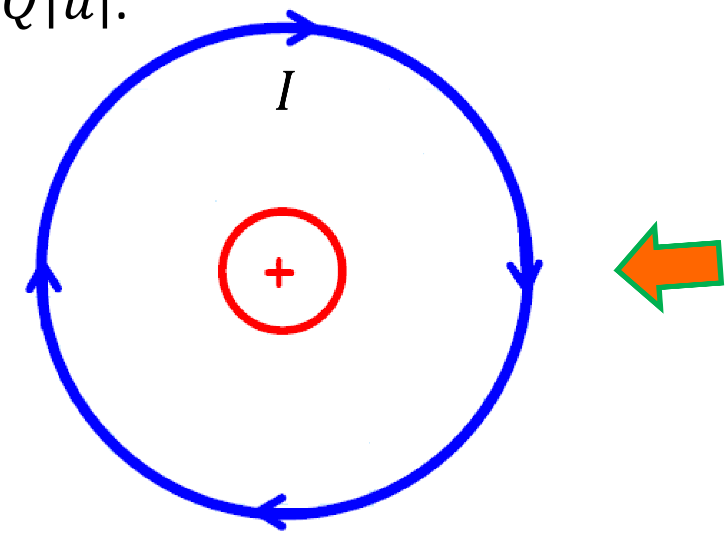


→ electron
(negative charge)

→ nucleus
(positive charge)

- An electron with charge Q orbiting around a nucleus at velocity $\vec{u}$ forms a **small current loop**, where $I = Q|\vec{u}|$.

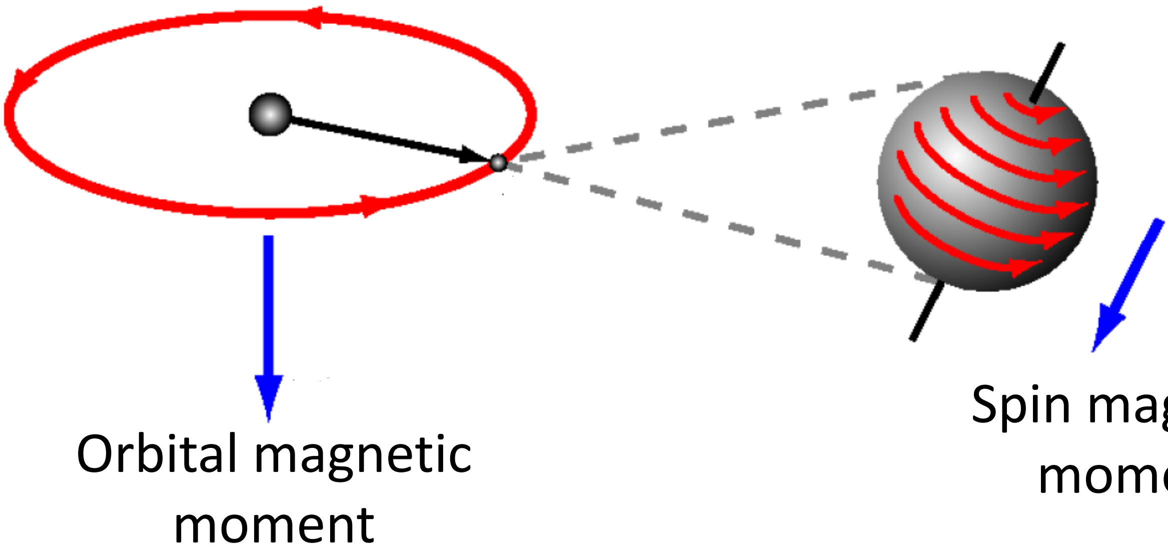




This forms a magnetic dipole

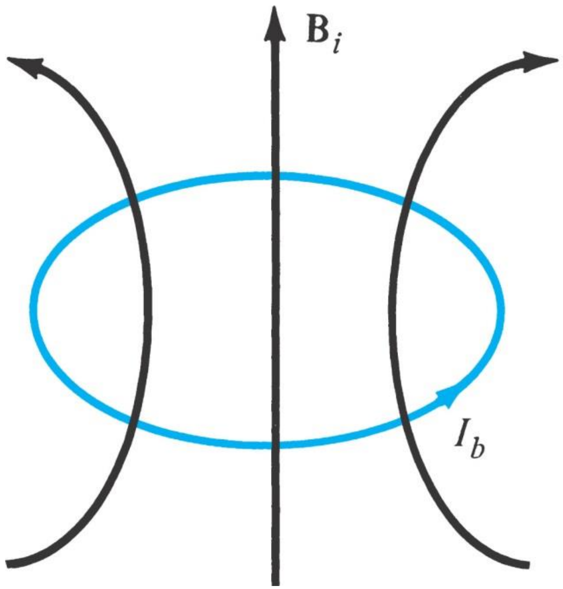
Magnetic Materials (contd.)

- This is a **very simple** atomic explanation of how magnetic dipoles are formed in material.
- In reality, the physical mechanisms that lead to magnetic dipoles can be **far** more complex.
- For example, **electron spin** can also create a magnetic dipole moment.



Magnetic Materials (contd.)

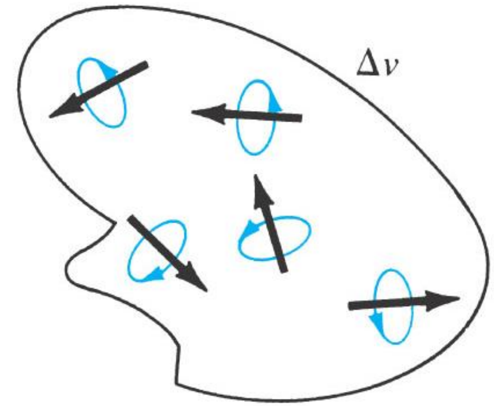
- Both these electronic motions produce internal magnetic fields $\vec{B}_i$ that are similar to the magnetic field produced by a current loop as shown.



This equivalent current loop has a magnetic moment of $\vec{m} = I_b S \hat{a}_n$, where S is the area of the loop and I_b is the bound current (bound to the atom).

Magnetic Materials (contd.)

- Typically, the atoms/molecules of materials exhibit either **no** magnetic dipole moment (i.e., $\vec{m} = 0$), or the dipole moments of each atom/molecule are **randomly oriented**, such that the **net** dipole moment is **zero**.



$$\vec{B} = 0, \vec{M} = 0$$

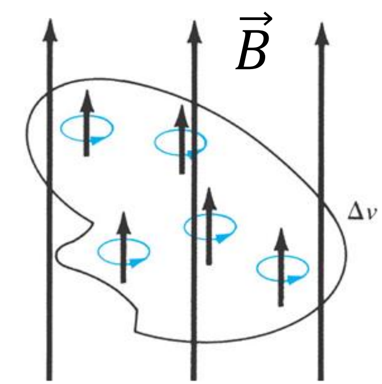
- Therefore, for N randomly oriented magnetic dipoles $\vec{m}_n$, we find:

$$\frac{1}{N}\sum_n \vec{m}_n = 0$$

- Similarly, the **total** magnetic flux density created by these magnetic dipoles is **also zero**:

$$\sum_n \vec{B}_n = 0$$

- However, sometimes the magnetic dipole moment of each atom/molecule is **not** randomly oriented, but in fact are **aligned**!



Magnetic Materials (contd.)

Q: **Why** would these magnetic dipoles be aligned?

A: **Two** possible reasons:

1) the material is a **permanent magnet**.

2) the material is immersed in some **magnetizing field** $\vec{B}$.

The Magnetization Vector

- Recall that we defined the **Polarization vector** of a dielectric material as the **electric dipole density**, i.e.:

$$\vec{P} \doteq \frac{\sum \vec{p}_n}{\Delta v}\left[\frac{dipole\_moment}{unit\_volume} = \frac{C}{m^2}\right]$$

- Similarly, we can define a **Magnetization vector** of a material to be the density of **magnetic** dipole moments:

$$\vec{M} \doteq \frac{\sum \vec{m}_n}{\Delta v \to 0}\left[\frac{magnetic\_dipole\_moment}{unit\_volume} = \frac{A}{m}\right]$$

A medium for which $\vec{M}$ is not zero everywhere is said to be magnetized

The Magnetization Vector (contd.)

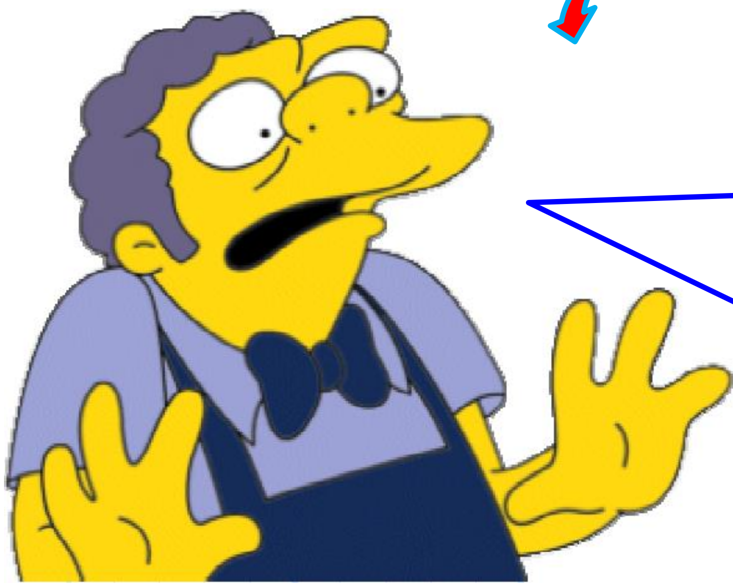
- Note if the dipole moments of atoms/molecules within a material are **completely random**, the Magnetization vector will be **zero** (i.e., $\vec{M} = 0$).
- However, if the dipoles are **aligned**, the Magnetization vector will be **non-zero** (i.e., $\vec{M} \neq 0$).
- Furthermore, for a differential volume dv', the magnetic moment is $\overrightarrow{dm} = \vec{M} dv'$.
- Therefore the vector magnetic potential due to $\overrightarrow{dm}$ can be expressed as:

$$\overrightarrow{dA} = \frac{\mu_0 \vec{M} \times \hat{a}_R}{4\pi R^2} dv' \Rightarrow \overrightarrow{dA} = \frac{\mu_0 \vec{M} \times \vec{R}}{4\pi R^3} dv'$$

$$\therefore \vec{A} = \iiint_v \frac{\mu_0 \vec{M} \times \vec{R}}{4\pi R^3} dv'$$

The Magnetization Vector (contd.)

$$\therefore \vec{A} = \iiint_{v} \frac{\mu_0 \vec{M} \times \vec{R}}{4\pi R^3} dv'$$



Q: This is freaking me out!! I thought that **currents** $\vec{J}$ were responsible for creating magnetic vector potential. In fact, I could have sworn that:

$$\vec{A} = \iiint_{v} \frac{\mu_0 \vec{J}}{4\pi R} dv'$$

A: Relax, **both** expressions are correct!

The Magnetization Currents

- Recall that we could attribute the electric field created by Polarization Vector $\vec{P}$ to **polarization** (i.e., bound) **charges** ρ_{vp} and ρ_{sp}.

$$\rho_{vp} = -\nabla.\vec{P}$$

$$\rho_{sp} = \vec{P}.\hat{a}_n$$

- Similarly, we can **attribute** the magnetic vector potential (and therefore the magnetic flux density) created by Magnetization Vector $\vec{M}$ to **Magnetization Currents** $\vec{J}_b$ and $\vec{K}_b$, the bound volume current density (i.e., magnetization current density) and bound surface current density respectively.

- We have:

$$\vec{A} = \iiint_v \frac{\mu_0 \overrightarrow{M} \times \vec{R}}{4\pi R^3} dv'$$

- Earlier we came across the expression:

$$\frac{\vec{R}}{R^3} = \nabla'\left(\frac{1}{R}\right)$$

- Therefore:

$$\vec{A} = \frac{\mu_0}{4\pi} \iiint_v \overrightarrow{M} \times \nabla'\left(\frac{1}{R}\right) dv'$$

The Magnetization Currents (contd.)

- We can use the identity: $\vec{M} \times \nabla'\left(\frac{1}{R}\right) = \frac{1}{R}\nabla' \times \vec{M} - \nabla' \times \frac{\vec{M}}{R}$
- Therefore we can express: $\vec{A} = \frac{\mu_0}{4\pi}\iiint_v \frac{\nabla' \times \vec{M}}{R} dv' + \frac{\mu_0}{4\pi}\oiint_S \frac{\vec{M} \times \hat{a}_n}{R} ds'$

$$\vec{A} = \frac{\mu_0}{4\pi}\iiint_v \frac{\vec{J}_b}{R} dv' + \frac{\mu_0}{4\pi}\oiint_S \frac{\vec{K}_b ds'}{R}$$

where:

$$\vec{J}_b = \nabla \times \vec{M}$$

$$\vec{K}_b = \vec{M} \times \hat{a}_n$$

Therefore, we find that the magnetization of some material, as described by magnetization vector $\vec{M}$, creates **effective** currents $\vec{J}_b$ and $\vec{K}_b$. We call these effective currents **magnetization currents.**

$\vec{J}_b$ and $\vec{K}_b$ can be derived from $\vec{M}$ and hence are not commonly used

The Magnetic Field

- Now that we have defined **magnetization current**, we find that Ampere's Law for fields **within some material** becomes:

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \vec{J}_b \right)$$

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \nabla \times \vec{M} \right)$$

- This of course is **analogous** to the expression we derived for **Gauss's Law** in a dielectric media:

$$\nabla . \vec{E} = \frac{\rho_v + \rho_{vp}}{\varepsilon_0} = \frac{\rho_v - \nabla . \vec{P}}{\varepsilon_0}$$

Recall that we **removed** the polarization charge from this expression by defining a **new** vector field $\vec{D}$, leaving us with the more **general** expression of Gauss's Law:

$$\nabla . \vec{D} = \rho_v$$

The Magnetic Field (contd.)

Q: Can we similarly define a **new** vector field to "take care" of **magnetization** current ??

A: Yes! We call this vector field the **magnetic field** $\vec{H}$.

- Let's begin by **rewriting** Ampere's Law as:

$$\nabla \times \vec{B} - \mu_0 \vec{J}_b = \mu_0 \vec{J}$$

- Yuck! Now we see clearly the problem. In **free space**, if we know current distribution $\vec{J}$, we can find the resulting magnetic flux density $\vec{B}$ using the **Biot-Savart** Law:

$$\vec{B} = \frac{\mu_0}{4\pi} \iiint_v \frac{\vec{J} \times \vec{R}}{R^3} dv'$$

But this is the solution for current in **free space**! It is **no longer valid** if some **material** is present!

The Magnetic Field (contd.)

Q: Why?

A: Because, the magnetic flux density produced by current $\vec{J}$ may **magnetize** the material (i.e., produce magnetic dipoles), thus producing **magnetization currents** $\vec{J}_b$.

These magnetization currents $\vec{J}_b$ will **also** produce a magnetic flux density—a **modification** of vector field $\vec{B}$ that is **not** accounted for in the Biot-Savart expression shown above!

- To determine the correct solution, we first recall that: $\vec{J}_b = \nabla \times \vec{M}$
- Therefore Ampere's Law is: $\nabla \times \vec{B} - \mu_0 \nabla \times \vec{M} = \mu_0 \vec{J}$

$$\nabla \times \left[\frac{\vec{B}}{\mu_0} - \vec{M} \right] = \vec{J}$$

- Now let's define a **new** vector field $\vec{H}$, called the **magnetic field**: $\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M}$
- Therefore: $\nabla \times \vec{H} = \vec{J}$

The Magnetic Field (contd.)

- For most materials, it has been found that the magnetization vector $\vec{M}$ is directly **proportional** to the magnetic field $\vec{H}$:

$$\vec{M} = \chi_m \vec{H}$$

where the proportionality coefficient χ_m is the **magnetic susceptibility** of the material.

- Note that for a given magnetic field $\vec{H}$, as χ_m **increases**, the magnetization vector $\vec{M}$ **increases**.
- Magnetic susceptibility χ_m therefore indicates how **susceptible** the material is to **magnetization**.
- In other words, χ_m is a measure of how easily (or difficult) it is to create and align **magnetic dipoles** (from atoms/molecules) within the **material**.

Again, note the **analogy** to electrostatics. We defined earlier **electric** susceptibility χ_e, which indicates how susceptible a material is to **polarization** (i.e., the creation of **electric** dipoles).

The Magnetic Field (contd.)

- Therefore: $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ → $\vec{H} = \frac{\vec{B}}{\mu_0} - \chi_m \vec{H}$ → $\mu_0 (1+\chi_m)\vec{H} = \vec{B}$

Hey! We know the magnetic field $\vec{H}$ and magnetic flux density are related by a **simple constant**!

$$\vec{B} = \mu \vec{H}$$

$$\therefore \mu = \mu_0 (1+\chi_m)$$

$$\mu \doteq material\_permeability \left[\frac{Henrys}{meter}\right]$$

- The expression can be **further** simplified by defining a **relative** permeability:

$$\mu_r = 1 + \chi_m$$

The Magnetic Field (contd.)

- Therefore: $\vec{B} = \mu\vec{H} = \mu_0\mu_r\vec{H}$ ← Only valid for linear isotropic materials
- In other words, if the **relative** permeability of some material was, say, $\mu_r = 2$, then the **permeability** of the material is **twice** that of the permeability of **free space** (i.e., $\mu = 2\mu_0$). This perhaps is more readily evident when we write:

$$\mu_r = \frac{\mu}{\mu_0}$$

Note that μ and/or μ_r are **proportional** to magnetic susceptibility χ_m. As a result, permeability is likewise an indication of how **susceptible** a material is to **magnetization**.

- If $\mu_r = 1$, this susceptibility is that of **free space** (i.e., **none**!).
- Alternatively, a **large** μ_r indicates a material that is **easily magnetized**.
- For example, the relative permeability of **iron** is $\mu_r = 4000$!

The Magnetic Field (contd.)

- **Now**, we are **finally** able to determine the **magnetic flux density** in some **material,** produced by current density $\vec{J}$!
- Since $\vec{B} = \mu\vec{H}$ and:

$$\vec{H} = \frac{1}{4\pi}\iiint_v \frac{\vec{J}\times\vec{R}}{R^3}dv'$$

- we find the desired solution:

$$\vec{B} = \frac{\mu}{4\pi}\iiint_v \frac{\vec{J}\times\vec{R}}{R^3}dv'$$

Comparing this result with the Biot-Sarvart Law for **free space**, we see that the only difference is that μ_0 has been replaced with μ.

This last result is therefore a **more general** form of the Biot-Savart Law, giving the correct result for fields within some **material** with permeability μ. Of course, the "material" **could** be free space. However, the expression above will **still** provide the correct answer; because for free space $\mu = \mu_0$, thus returning the equation to its **original** (i.e., free space) form!

The Magnetic Field (contd.)

Summarizing, we can attribute the existence of a **magnetic field** $\vec{H}$ to **conduction** current $\vec{J}$, while we attribute the existence of **magnetic flux density** to the **total** current density, including the magnetization current.