Lecture – 14 **Date: 09.03.2015**

- Force Due to Magnetic Field

Example – 1

- Given the magnetic vector potential $\vec{A} = -\frac{\rho^2}{4}\hat{a}_z \frac{\text{Wb}}{\text{m}}$, calculate the total magnetic flux crossing the surface $\phi = \frac{\pi}{2},\ 1 \le \rho \le 2\ m,\ 0 \le z \le 5\ m.$

Method-1: $\vec{B} = \nabla \times \vec{A} = -\frac{\partial A_z}{\partial \rho}\hat{a}_\phi$ $\qquad \overline{dS} = d\rho dz \hat{a}_\phi$

- Therefore: $\psi = \int_S \vec{B}.\overline{ds}$ → $\psi = \frac{1}{2}\int_{z=0}^{5}\int_{\rho=1}^{2} \rho d\rho dz$ → $\psi = 3.75Wb$

Example – 2

- A current distribution gives rise to the vector magnetic potential $\vec{A} = x^2 y\hat{a}_x + y^2 x\hat{a}_y - 4xyz\hat{a}_z \frac{Wb}{m}$. Calculate the following:
  (a) $\vec{B}$ at $(-1,\ 2,\ 5)$
  (b) The flux through the surface defined by $\mathrm{z} = 1, 0 \le \mathrm{x} \le 1, -1 \le \mathrm{y} \le 4$

(a) $\vec{B} = \nabla \times \vec{A} \quad = (-4xz - 0)\hat{a}_x + (0 + 4yz)\hat{a}_y + (y^2 - x^2)\hat{a}_z$

$$\therefore \vec{B}(-1, 2, 5) = 20\hat{a}_x + 40\hat{a}_y + 3\hat{a}_z$$

(b) The flux through the given surface:

$$\psi = \int_S \vec{B}.\overline{ds} \Rightarrow \psi = \int_{y=-1}^{4} \int_{x=0}^{1} \left(y^2 - x^2\right) \partial x \partial y \Rightarrow \psi = 20Wb$$

Forces Due to Magnetic Fields

- Three possible ways for forces due to magnetic fields –
  - Due to moving charged particle
  - On a current element in another magnetic field $\vec{B}$
  - Between two current elements

Force on Charged Particle

- We know, the electric force $\vec{F}_e$ on a stationary or moving electric charge Q in an electric field is given by:

$$\vec{F}_e = Q\vec{E}$$

Therefore for positive Q: $\vec{F}_e$ and $\vec{E}$ have same directions

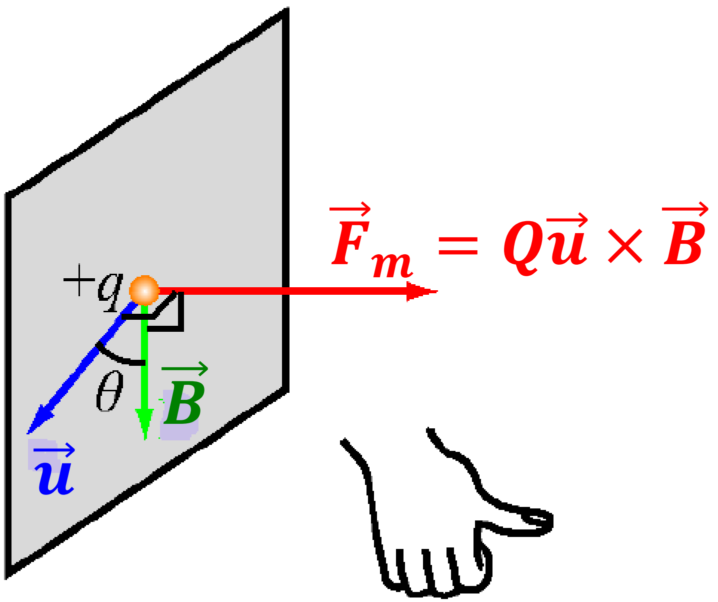
- A magnetic field can exert force only on moving charge. Experimentally it was observed that:

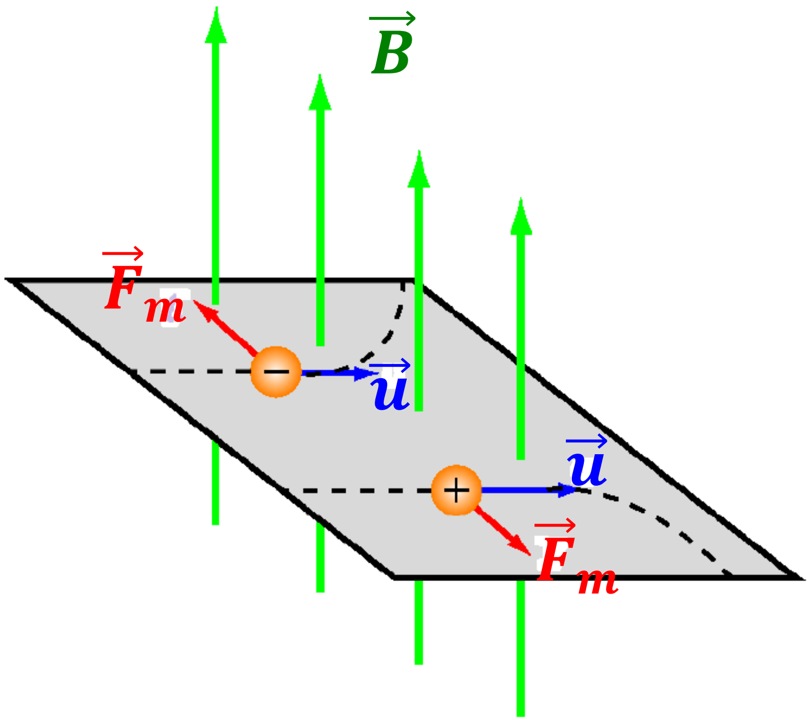
$$\vec{F}_m = Q\vec{u} \times \vec{B}$$

Force on Charged Particle (contd.)

$$\vec{F}_m = Q\vec{u} \times \vec{B}$$

Obviously, $\vec{F}_m$ is perpendicular to both $\vec{u}$ and $\vec{B}$





Force on Charged Particle (contd.)

$$\vec{F}_e = Q\vec{E}$$

$$\vec{F}_m = Q\vec{u} \times \vec{B}$$

- $\vec{F}_e$ is independent of the velocity of charge and can perform work on the charge and change its kinetic energy.
- $\vec{F}_m$ depends on the velocity of charge and is normal to it → as a consequence $\vec{F}_m$ can't perform work because it is normal to the direction of motion of the charge ($\vec{F}_m . \vec{u} = 0$).
- The work performed when a particle is displaced by a differential distance $\overline{dl} = \vec{u}dt$ is: $dW = \vec{F}_m . \overline{dl} = (\vec{F}_m . \vec{u})dt = 0.$
- Since no work is done, $\vec{F}_m$ doesn't cause any increase in the kinetic energy of the charge.
- The magnetic field can change the direction of motion of a charged particle, but not its speed.

The magnitude of $\overrightarrow{F}_m$ is generally small as compared to $\overrightarrow{F}_e$ except at high velocities.

Force on Charged Particle (contd.)

- A charge Q moving in presence of both electric and magnetic fields experiences a force:

$$\vec{F} = \vec{F}_e + \vec{F}_m = Q\left(\vec{E} + \vec{u} \times \vec{B}\right)$$

Lorentz Force Equation

- If the mass of moving charge Q is m then:

$$\vec{F} = m\frac{d\vec{u}}{dt} = Q\left(\vec{E} + \vec{u} \times \vec{B}\right)$$

- This equation gives the velocity of charge
- It is important to note that the energy transfer in the process is only due to electric field

Example – 3

- An electron moving in the positive x-direction perpendicular to the magnetic field is deflected in the negative z-direction. What is the direction of the magnetic field?

Solution:

- The magnetic force acting on a moving charged particle is: $\vec{F}_m = Q\vec{u} \times \vec{B}$

$$Q = -e, \vec{u} = u\hat{a}_x, \vec{F}_m = -F_m\hat{a}_z$$

- Therefore: $-F_m\hat{a}_z = -eu\hat{a}_x \times \vec{B}$

For the cross product to apply, $\vec{B}$ has to be in the positive y-direction.

Example – 4

- A proton moving with a speed of $2 \times 10^6\ m/s$ through a magnetic field with magnetic flux density of 2.5T experiences a magnetic force of magnitude $4 \times 10^{-13} N$. What is the angle between the magnetic field and the proton's velocity?

$$\vec{F}_m = Q\vec{u} \times \vec{B} \Rightarrow F_m = QuB\sin\theta \Rightarrow \theta = 30° \text{ or } 150°$$

Example – 5

A charged particle with velocity $\vec{u}$ is moving in a medium containing uniform fields $\vec{E} = E\hat{a}_x$ and $\vec{B} = B\hat{a}_y$. What should $\vec{u}$ be so that the particle experiences no net force on it?

$$\vec{F}_e = Q\vec{E} = QE\hat{a}_x \qquad \vec{F}_m = Q\vec{u} \times \vec{B} = Q(\vec{u} \times \hat{a}_y B)$$

- For net force to be zero, $\vec{F}_m$ has to be along $-\hat{a}_x$, which requires $\vec{u}$ to be along $+\hat{a}_z$. Thus,

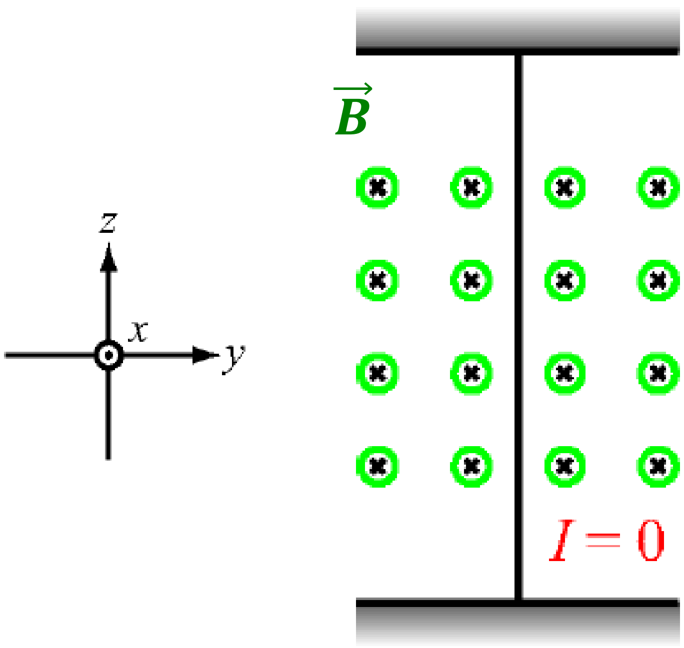
$$QE = QuB$$

$$u = \frac{E}{B} \Longrightarrow \vec{u} = \frac{E}{B}\hat{a}_z$$

- If $\vec{u}$ also has a y-component, that component will exercise no force on the particle.

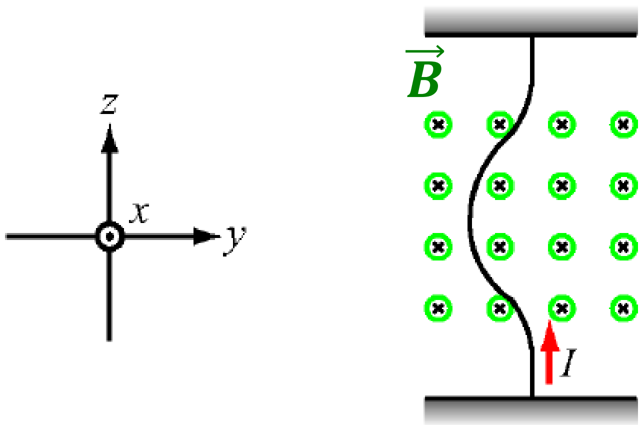
Force on a Current Element

- A current flowing through a conducting wire consists of charged particles drifting through the material of the wire.
- As a consequence, when a current carrying wire is a placed in a magnetic field, it will experience a force equal to the sum of the magnetic forces acting on the charged particles moving within it.

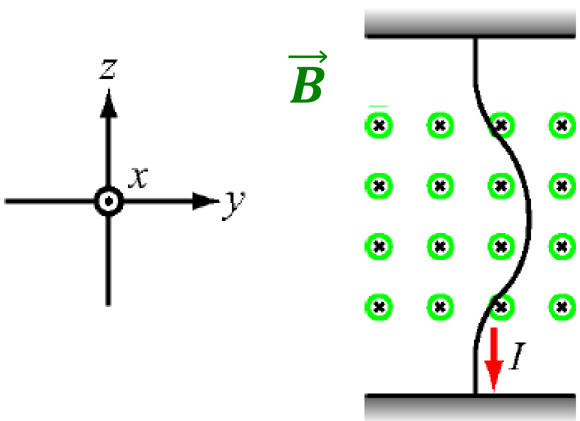


A vertical wire (carrying no current) oriented along the z-direction placed in a magnetic field $\vec{B}$ (in the $-\hat{a}_x$ direction) will experience no force.

Force on a Current Element (contd.)



The wire will deflect to $-\hat{a}_y$ direction if the direction of current flow is upward ($+\hat{a}_z$) direction.



The wire will deflect to $+\hat{a}_y$ direction if the direction of current flow is downward ($-\hat{a}_z$) direction.

Force on a Current Element (contd.)

- Mathematically, this phenomenon can be expressed as:

$$\overrightarrow{dF_m} = I\,\overline{dl} \times \vec{B}$$

force on a current element

- If the current is through a closed path C then: $\overrightarrow{F_m} = \oint_C I\,\overline{dl} \times \vec{B}$

It should be noted that the magnetic field in this expression is due to another source i.e., it is external to the current element → Just for clarity, the magnetic field produced by the current element doesn't exert a force on itself.

- **If the magnetic field is uniform then:**

$$\overrightarrow{F_m} = I\left(\oint_C \overline{dl}\right) \times \vec{B} = 0$$

Conveys that the total magnetic force on any closed current loop in a uniform magnetic field is zero.

Example – 6

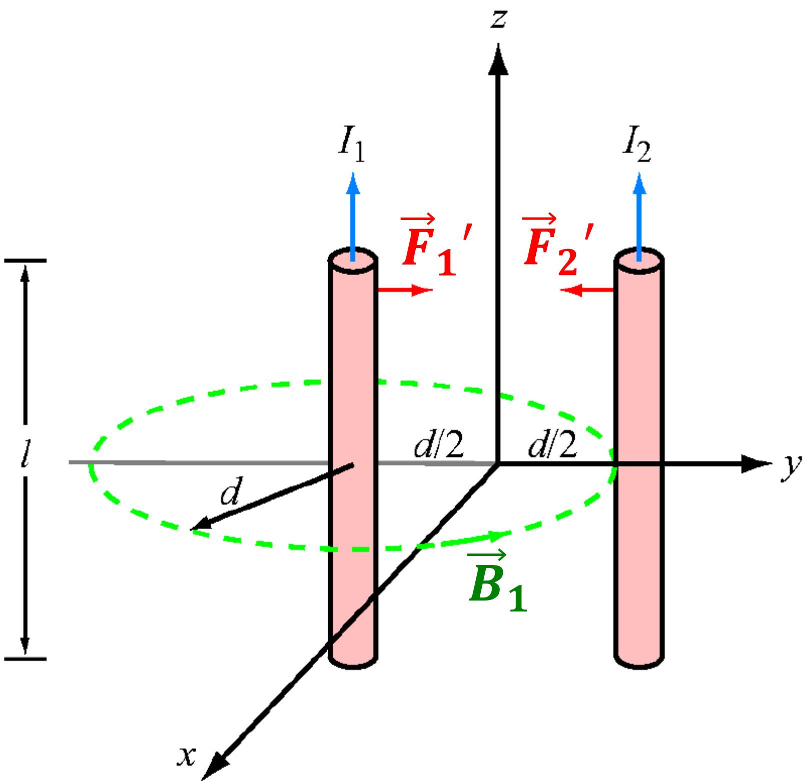
- A charged particle moves with a uniform velocity $4\hat{a}_x \; m/s$ in a region where $\vec{E} = 20\hat{a}_y \; V/m$ and $\vec{B} = B_0\hat{a}_z \; \frac{Wb}{m^2}$. Determine B_0 such that the velocity of the particle remains constant.

If the particle moves with a constant velocity, it is implied that its acceleration is zero. In other words, particle experiences no net force.

$$\vec{F} = m\vec{a} = Q\left(\vec{E} + \vec{u} \times \vec{B}\right) = 0 \quad \Rightarrow \quad B_0 = 5$$

This example illustrates an important concept employed in velocity filter

Force Between Two Current Elements



Let us consider two very long (or infinitely long) and straight parallel wires separated by a distance d and carrying currents I_1 and I_2 in the z-direction at $y = -\frac{d}{2}$ and $y = \frac{d}{2}$ respectively.

- Let us denote by $\vec{B}_1$ the magnetic field due to current I_1 at the location of the wire carrying current I_2, and conversely $\vec{B}_2$ the magnetic field due to I_2 at the location of the wire carrying current I_1.

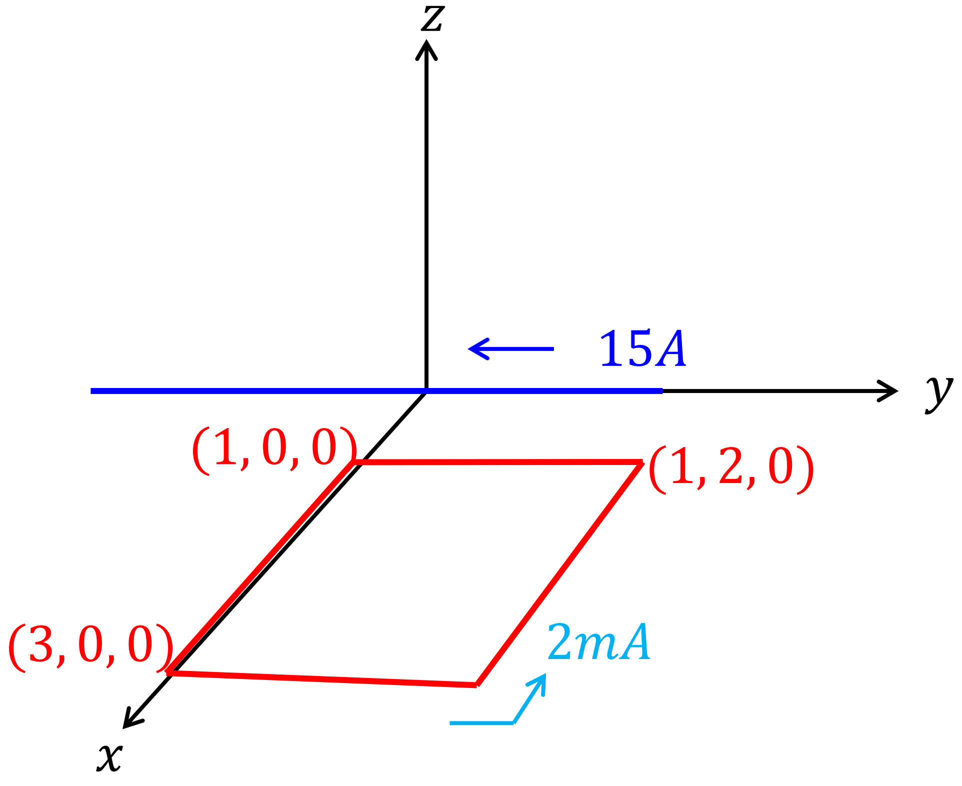
Force Between Two Current Elements (contd.)

- We know: $$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi d}\hat{a}_x$$
- Therefore the force $\vec{F}_2$ exerted on a length l of wire I_2 due to its presence in field $\vec{B}_1$ is: $$\vec{F}_2 = I_2\left(l\hat{a}_z \times \vec{B}_1\right) = I_2\left(l\hat{a}_z \times \left(-\hat{a}_x\right)\frac{\mu_0 I_1}{2\pi d}\right) = -\hat{a}_y \frac{\mu_0 I_1 I_2 l}{2\pi d}$$
- The corresponding force per unit length: $$\vec{F}_2' = -\hat{a}_y \frac{\mu_0 I_1 I_2}{2\pi d}$$
- Similarly, force per unit length exerted on wire carrying I_1 is: $$\vec{F}_1' = \hat{a}_y \frac{\mu_0 I_1 I_2}{2\pi d}$$

Thus two parallel wires carrying currents in the same direction attract each other with equal force. If the currents are in opposite directions, the wires will repel one another with equal force.

Example – 7

- There is a square loop of wire in the $z = 0$ plane carrying $2mA$ in the field of an infinite filament on the $y - axis$ as shown. Find the total force on the loop.



Example – 7 (contd.)

- The field produced by the straight filament in the plane of the loop is:

$$\vec{H} = \frac{I}{2\pi x}\hat{a}_z \ A/m$$

$$\vec{B} = \mu_0 \vec{B} = \frac{\mu_0 I}{2\pi x}\hat{a}_z = \frac{3\times10^{-6}}{x}\hat{a}_z \quad T$$

- Therefore:

$$\vec{F_m} = I_{loop}\oint \overline{dl}\times\vec{B}$$

$$\Rightarrow \vec{F_m} = 2\times10^{-3}\times3\times10^{-6}\left[\int_{x=1}^{3} dx\hat{a}_x\times\frac{\hat{a}_z}{x} + \int_{y=0}^{2} dy\hat{a}_y\times\frac{\hat{a}_z}{3} + \int_{x=3}^{1} dx\hat{a}_x\times\frac{\hat{a}_z}{x} + \int_{y=2}^{0} dy\hat{a}_y\times\frac{\hat{a}_z}{1}\right]$$

$$\therefore \vec{F_m} = -8\hat{a}_x nN$$