Lecture – 11

Date: 12.02.2015

- Magnetostatics
- Biot-Savart Law
- Ampere’s Circuital Law
- Applications of Ampere’s Law

Test – 3

1. If $V = \rho^2 z sin\phi$, calculate the energy within the region defined by $1 < \rho < 4, -2 < z < 2, 0 < \phi < \frac{\pi}{3}$.

Start:

$$\vec{E} = -\nabla V \Rightarrow \vec{E} = -\left(\frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z\right)$$

$$\therefore \vec{E} = -\left(2\rho z \sin\phi \hat{a}_\rho + \rho z \cos\phi \hat{a}_\phi + \rho^2 \sin\phi \hat{a}_z\right)$$

Therefore:

$$W_E = \frac{1}{2}\varepsilon_0 \int_v \left|\vec{E}\right|^2 dv$$

$$\frac{2W_E}{\varepsilon_0} = \iiint_v \left(4\rho^2 z^2 \sin^2\phi \hat{a}_\rho + \rho^2 z^2 \cos^2\phi \hat{a}_\phi + \rho^4 \sin^2\phi \hat{a}_z\right)\rho d\phi dz d\rho$$

$$\therefore W_E = \frac{1507.67}{2}\left(\frac{10^{-9}}{36\pi}\right)$$

Test – 3

1. For the current density $\vec{J} = 10zsin^2\phi\hat{a}_\rho \ A/m^2$, find the current through the cylindrical surface $\rho = 2, 1 \le z \le 5 \ m$.

$$\overrightarrow{dS} = \rho d\phi dz\hat{a}_\rho \quad \Rightarrow \quad I = \int_S \vec{J}.\overrightarrow{dS} \quad \Rightarrow \quad I = \int_{\phi=0}^{2\pi}\int_{z=1}^{5} 10z\sin^2\phi\rho dzd\phi|_{\rho=2}$$

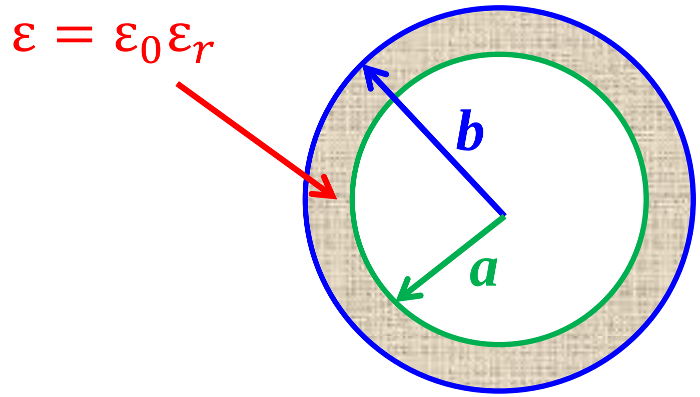
$$\Rightarrow I = \int_{\phi=0}^{2\pi}\int_{z=1}^{5} 10z\sin^2\phi\rho dzd\phi|_{\rho=2} = 10(2)\left[\frac{z^2}{2}\right]_1^5 \int_0^{2\pi}\frac{1}{2}(1-\cos 2\phi)d\phi \quad \Rightarrow \quad \therefore I = 240\pi = 754 \text{ A}$$

Test – 3

2. Show that: $\vec{P} = \left(\varepsilon - \varepsilon_0\right)\vec{E}$ and $\vec{D} = \dfrac{\varepsilon_r}{\varepsilon_r - 1}\vec{P}$

Test – 3

2. At the center of a hollow dielectric sphere ($\varepsilon = \varepsilon_0 \varepsilon_r$) is placed a point charge Q. If the sphere has inner radius a and outer radius b, calculate $\vec{D}$, $\vec{E}$ and $\vec{P}$.



For $0 < r < a$

Gauss's law gives: $$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r$$

$$\vec{P} = \vec{D} - \varepsilon_0\vec{E} = 0$$

For $a < r < b$

Gauss's law gives: $$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}\hat{a}_r$$

$$\vec{P} = \chi_e\varepsilon_0\vec{E} = \frac{\varepsilon_r - 1}{\varepsilon_r}\frac{Q}{4\pi r^2}\hat{a}_r$$

Test – 3

For $r > b$

Gauss's law gives: $\vec{D} = \dfrac{Q}{4\pi r^2}\hat{a}_r$ ⇒ $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r$ ⇒ $\vec{P} = \vec{D} - \varepsilon_0\vec{E} = 0$

Therefore:

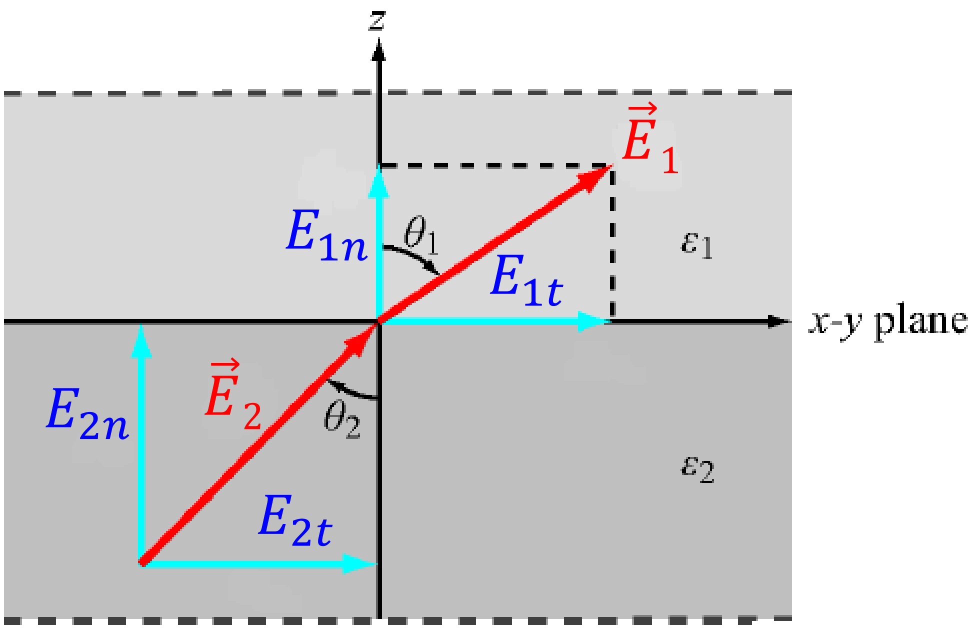
$$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r \qquad r > 0$$

$$\vec{E} = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}\hat{a}_r & a < r < b \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r & \text{otherwise} \end{cases}$$

$$\vec{P} = \begin{cases} \dfrac{\varepsilon_r - 1}{\varepsilon_r}\dfrac{Q}{4\pi r^2}\hat{a}_r & a < r < b \\ 0 & \text{otherwise} \end{cases}$$

Test – 3

3. Find $\vec{E}_1$ in the following figure, if $\vec{E}_2 = 2\hat{a}_x - 3\hat{a}_y + 3\hat{a}_z$ (V/m), $\varepsilon_1 = 2\varepsilon_0$, $\varepsilon_2 = 8\varepsilon_0$ and the boundary is charge free.



- Given that the x–y plane is the boundary between the two media, the x- and y-components of $\vec{E}_2$ are parallel to the boundary, and therefore are the same across the two sides of the boundary. Thus,

$$E_{1x} = E_{2x} = 2 \qquad E_{1y} = E_{2y} = -3$$

For the z-component

$$\varepsilon_1 E_{1z} = \varepsilon_2 E_{2z} \Rightarrow E_{1z} = \frac{8\varepsilon_0}{2\varepsilon_0} E_{2z} = 12$$

- Therefore: $\vec{E}_1 = E_{1x}\hat{a}_x + E_{1y}\hat{a}_y + E_{1z}\hat{a}_z \Rightarrow \vec{E}_1 = 2\hat{a}_x - 3\hat{a}_y + 12\hat{a}_z$ V/m

Test – 3

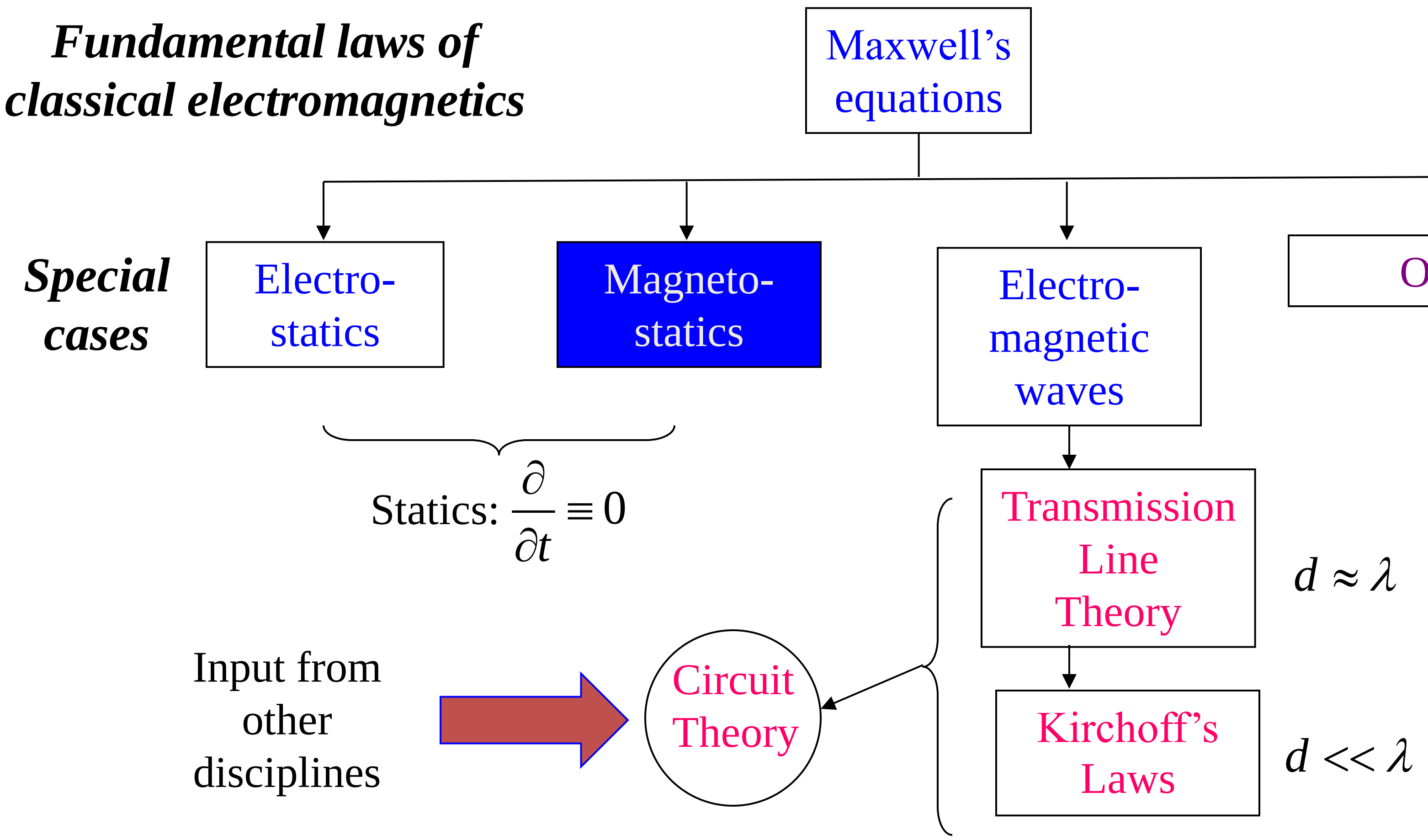
4. Let us assume there is a parallel plate capacitor along x-axis. The upper plate is maintained at –100V, and the lower plate is maintained at 0V. The plates are 10cm apart, and they are infinitely large. Find $V(\bar{r})$ and $\vec{E}(\bar{r})$ in the parallel-plate region.

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$V(\bar{r}) = -1000x$$

$$\vec{E}(\bar{r}) = 1000\hat{a}_x$$

Overview of Electromagnetics



Magnetostatics

- **Magnetostatics** is the branch of electromagnetics dealing with the effects of electric charges in steady motion (i.e, steady current or DC).
- The fundamental law of magnetostatics is **Ampere's law of force**.
- Ampere's law of force is analogous to Coulomb's law in electrostatics.
- In magnetostatics, the magnetic field is produced by steady currents.
- The magnetostatic field does not allow for
    - inductive coupling between circuits
    - coupling between electric and magnetic fields

Magnetostatic Fields

- Static magnetic fields are characterized by $\vec{H}$ or $\vec{B}$.
- These are **analogous** to $\vec{E}$ or $\vec{D}$
- A **definite link** between electric and magnetic field was established by a Danish professor **Hans Christian Oersted.**
- We know, an electrostatic field is produced by static or stationary charges. If the charges are moving with constant velocity, **a static magnetic field is produced.**
- A magnetostatic field is produced by a constant current flow (or direct current).
- These currents could be due to **magnetization currents** as in permanent magnets, electron beam currents as in vacuum tubes, or conduction currents as in current-carrying wires.
- First lets consider magnetostatic in free space.

Magnetostatic Fields (contd.)

- **Foremost**, study of magnetostatics is not a dispensable luxury.
- Its indispensable necessity.
- Motors, Transformers, Microphones, Compasses, Telephone Bell Ringers, Television Focusing Controls, Advertising Displays, Magnetically Levitated High Speed Trains, Volatile and Non-Volatile Memories, Magnetic Separators etc could not have been developed without an understanding of magnetostatic phenomena.

Maxwell's Equations for Magnetostatics

$$\nabla . \vec{B}(\bar{r}) = 0$$

$$\nabla \times \vec{B}(\bar{r}) = \mu_0 \vec{J}(\bar{r})$$

- From the **point form** of Maxwell's equations, we find that the **static** case reduces to another (in addition to electrostatics) pair of **decoupled differential equations** involving magnetic flux density $\vec{B}(\bar{r})$ and current density $\vec{J}(\bar{r})$:
- We know from the **Lorentz force equation** that the magnetic flux density $\vec{B}(\bar{r})$ will apply a **force** on current density $\vec{J}(\bar{r})$ flowing in volume dv equal to:

$$d\vec{F} = \left(\vec{J}(\bar{r}) \times \vec{B}(\bar{r})\right) dv$$

- Current density $\vec{J}(\bar{r})$ is of course expressed in units of **Amps/meter²**. The units of magnetic flux density $\vec{B}(\bar{r})$ are:

$$\frac{Newton.seconds}{Coulomb.meter} \equiv \frac{Weber}{meter^2} \equiv Tesla$$

- Recall the units for **electric** flux density $\vec{D}(\bar{r})$ are **Coulombs/m²**. **Compare** this to the units for **magnetic** flux density—Webers/m².
- We can say therefore that the units of **electric** flux are **Coulombs**, whereas the units of **magnetic** flux are **Webers**.

Maxwell's Equations for Magnetostatics (contd.)

- The concept of **magnetic flux** is much more important and useful than the concept of electric flux, as there is **no** such thing as **magnetic charge**.
- We will talk much more about the concept of **magnetic flux** later!
- Now, let us consider specifically the **two** magnetostatic equations.

First, we note that they specify both the **divergence** and **curl** of magnetic flux density $\vec{B}(\bar{r})$, thus **completely** specifying this vector field.

Second, it is apparent that the magnetic flux density $\vec{B}(\bar{r})$ is **not conservative** (i.e, $\nabla \times \vec{B}(\bar{r}) = \mu_0 \vec{J}(\bar{r}) \neq 0$).

Finally, we note that the magnetic flux density **is** a **solenoidal** vector field (i.e, $\nabla . \vec{B}(\bar{r}) = 0$).

Maxwell's Equations for Magnetostatics (contd.)

- Consider the **first** of the magnetostatic equations:

$$\nabla.\vec{B}(\bar{r})=0$$

This equation is sometimes referred to as **Gauss's Law for magnetics**, for its obvious **similarity** to Gauss's Law of electrostatics.

This equation essentially states that the magnetic flux density does **not diverge** nor converge from any point. In other words, it states that there is no such thing as **magnetic charge** !

Maxwell's Equations for Magnetostatics (contd.)

- This of course is **consistent** with our understanding of **solenoidal** vector fields. The vector field will **rotate** about a point, but not diverge from it.

Q: Just what **does** the magnetic flux density $\vec{B}(\bar{r})$ rotate around ?

A: Look at the **second** magnetostatic equation!

Maxwell's Equations for Magnetostatics (contd.)

- The **second** magnetostatic equation is referred to as **Ampere's Circuital Law**:

$$\nabla \times \vec{B}(\bar{r}) = \mu_0 \vec{J}(\bar{r})$$

Ampere's Circuital Law

This equation indicates that the magnetic flux density $\vec{B}(\bar{r})$ **rotates around** current density $\vec{J}(\bar{r})$ --the **source** of magnetic flux density is current!.