Lecture – 10 Date: 09.02.2015

- Electrostatic Boundary Value Problems (contd.)
- Capacitances
- Energy Storage in a Capacitor

Example – 1

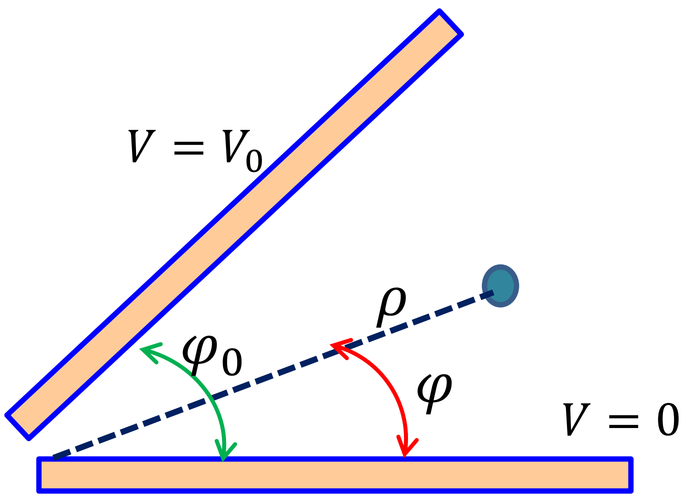
- Let us assume there is a parallel plate capacitor along x-axis. The upper plate is maintained at –100V, and the lower plate is maintained at 0V. The plates are 10cm apart, and they are infinitely large. Find $V(\bar{r})$ and $\vec{E}(\bar{r})$ in the parallel-plate region.

$$V(\bar{r}) = -1000x$$

$$\vec{E}(\bar{r}) = 1000\hat{a}_x$$

Example – 2

- Two semi-infinite plates are arranged at an angle φ_0, as shown in following figure. One plate is charged to V_0 volts and the other to 0V. A gap at the tip insulates one plate from the other. Find $V(\bar{r})$ and the corresponding $\vec{E}(\bar{r})$ in the region $0 < \varphi < \varphi_0$.

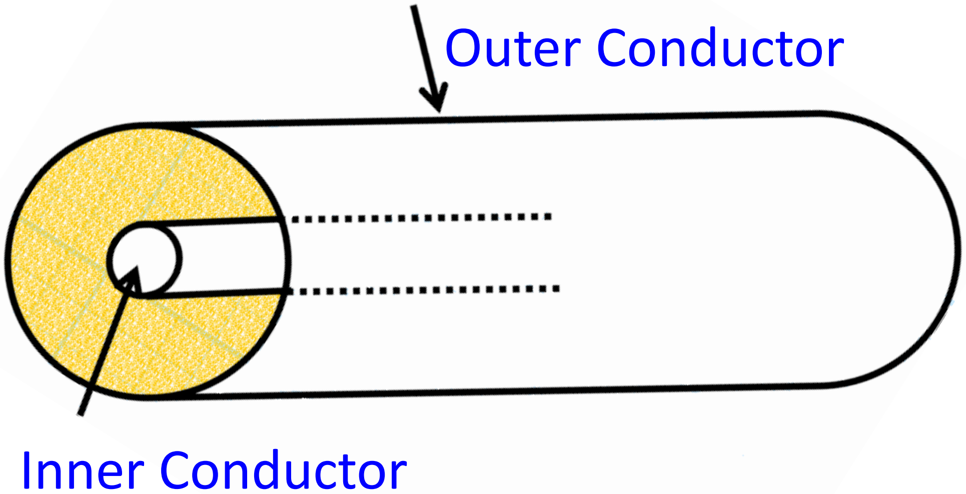


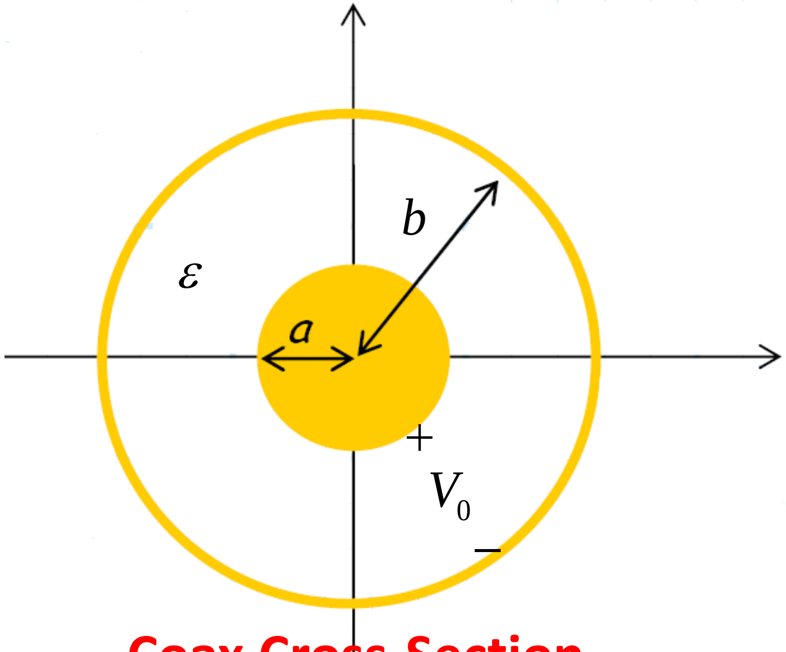
$$V(\bar{r}) = \left(\frac{V_0}{\varphi_0}\right)\varphi$$

$$\vec{E}(\bar{r}) = -\frac{1}{\rho}\left(\frac{V_0}{\varphi_0}\right)\hat{a}_\varphi$$

Example – 3: The Electrostatic Fields of a Coaxial Line

- A common form of a transmission line is the **coaxial** cable.





Coax Cross-Section

The coax has an **outer** radius b, and an **inner** radius a. The space between the conductors is filled with **dielectric** material of permittivity ε .

Say a voltage V_0 is placed across the conductors, such that the electric potential of the **outer** conductor is **zero**, and the electric potential of the **inner** conductor is V_0.

Example – 3 (contd.)

- The potential **difference** between the inner and outer conductor is therefore $V_0 - 0 = V_0$ volts.

Q: What electric potential field $V(\bar{r})$, electric field $\vec{E}(\bar{r})$, and charge density $\rho_s(\bar{r})$ is produced by this situation?

A: We must solve a **boundary-value** problem! We must find solutions that:

a) Satisfy the **differential equations** of electrostatics (e.g., Poisson's, Laplace's, Gauss's).

b) Satisfy the electrostatic **boundary conditions**.

Yikes! Where do we start ?

Example – 3 (contd.)

We might start with the electric potential field $V(\bar{r})$, since it is a **scalar** field.

a) The electric potential function must satisfy **Poisson's** equation:

$$\nabla^2 V(\bar{r}) = \frac{-\rho_v(\bar{r})}{\varepsilon_0}$$

b) It must also satisfy the **boundary conditions**:

$$V(\rho = a) = V_0 \qquad V(\rho = b) = 0$$

- Consider first the **dielectric** region $(a < \rho < b)$. Since the region is a dielectric, there is **no** free charge, and:

$$\rho_v(\bar{r}) = 0$$

Example – 3 (contd.)

- Therefore, Poisson's equation reduces to **Laplace's** equation:

$$\nabla^2 V(\bar{r}) = 0$$

- This particular problem (i.e., coaxial line) is directly solvable because the structure is **cylindrically symmetric**. Rotating the coax around the z-axis (i.e., in the $\hat{a}_\phi$ direction) does not change the geometry at all. As a result, we know that the electric potential field is a function of ρ **only**! i.e.,:

$$V(\bar{r}) = V(\rho)$$

- This make the problem much **easier**. Laplace's equation becomes:

$$\nabla^2 V(\bar{r}) = 0 \Rightarrow \nabla^2 V(\rho) = 0$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V(\rho)}{\partial \rho}\right) + 0 + 0 = 0 \Rightarrow \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V(\rho)}{\partial \rho}\right) = 0$$

Example – 3 (contd.)

$$\therefore \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V(\rho)}{\partial \rho}\right) = 0$$

Be **very** careful during **this** step! Make sure you implement the **Laplacian** operator correctly.

Example – 3 (contd.)

- Integrating **both sides** of the resulting equation, we find:

$$\int \frac{\partial}{\partial \rho}\left(\rho \frac{\partial V(\rho)}{\partial \rho}\right) d\rho = \int (0)\, d\rho \quad \Longrightarrow \quad \rho \frac{\partial V(\rho)}{\partial \rho} = C_1$$

where C_1 is some constant.

- Rearranging the above equation, we find:

$$\frac{\partial V(\rho)}{\partial \rho} = \frac{C_1}{\rho}$$

- Integrating both sides **again**, we get:

$$\int \frac{\partial V(\rho)}{\partial \rho} d\rho = \int \frac{C_1}{\rho} d\rho \quad \Longrightarrow \quad V(\rho) = C_1 \ln[\rho] + C_2$$

We find that this final equation $V(\rho) = C_1 \ln[\rho] + C_2$ will satisfy Laplace's equation (try it!).

We must now apply the **boundary conditions** to determine the value of constants C_1 and C_2.

Example – 3 (contd.)

- We know that on the outer surface of the inner conductor (i.e., $\rho = a$), the electric potential is equal to V_0 (i.e., $V(\rho = a) = V_0$).
- And, we know that on the inner surface of the outer conductor (i.e., $\rho = b$) the electric potential is equal to zero (i.e., $V(\rho = b) = 0$).

Therefore, we can write:

$$V(\rho = a) = C_1 \ln[a] + C_2 = V_0$$

$$V(\rho = b) = C_1 \ln[b] + C_2 = 0$$

- Solving for C_1 and C_2 we get:

$$C_1 = \frac{-V_0}{\ln[b] - \ln[a]} = \frac{-V_0}{\ln[b/a]}$$

$$C_2 = \frac{V_0 \ln[b]}{\ln[b/a]}$$

- Therefore, the **electric potential** field within the dielectric is found to be:

$$V(\bar{r}) = \frac{-V_0 \ln[\rho]}{\ln[b/a]} + \frac{V_0 \ln[b]}{\ln[b/a]} \qquad (b > \rho > a)$$

Example – 3 (contd.)

- Before we move on, we should do a **sanity check** to make sure we have done everything correctly. Evaluating our result at *ρ = a*, we get:

$$V(\rho=a)=\frac{-V_0\ln[a]}{\ln[b/a]}+\frac{V_0\ln[b]}{\ln[b/a]} \Rightarrow =\frac{V_0(\ln[b]-\ln[a])}{\ln[b/a]} \Rightarrow =\frac{V_0\ln[b/a]}{\ln[b/a]}=V_0 \quad \checkmark$$

- Likewise, we evaluate our result at *ρ = b*:

$$V(\rho=b)=\frac{-V_0\ln[b]}{\ln[b/a]}+\frac{V_0\ln[b]}{\ln[b/a]} \Rightarrow =\frac{V_0(\ln[b]-\ln[b])}{\ln[b/a]} \Rightarrow \therefore V(\rho=b)=0 \quad \checkmark$$

Our result **is** correct!

Example – 3 (contd.)

- Now, we can determine the **electric field** within the dielectric by taking the gradient of the electric potential field:

$$\vec{E}(\bar{r}) = -\nabla V(\bar{r}) = \frac{V_0}{\ln[b/a]}\frac{1}{\rho}\hat{a}_\rho \qquad (b > \rho > a)$$

- Note that **electric flux density** is therefore:

$$\vec{D}(\bar{r}) = \varepsilon\vec{E}(\bar{r}) == \frac{\varepsilon V_0}{\ln[b/a]}\frac{1}{\rho}\hat{a}_\rho \qquad (b > \rho > a)$$

Finally, we need to determine the **charge density** that actually created these fields!

Example – 3 (contd.)

Q1: Just where **is** this charge? After all, the dielectric (if it is perfect) will contain **no** free charge.

A1: The free charge, as we might expect, is in the **conductors**. Specifically, the charge is located at the surface of the conductor.

Q2: Just how do we **determine** this surface charge $\rho_s(\bar{r})$?

A2: Apply the boundary conditions!

- Recall that we found that **at** a conductor/dielectric **interface**, the **surface charge density** on the conductor is related to the **electric flux density** in the dielectric as:

$$D_n = \vec{D}(\bar{r}).\hat{a}_n = \rho_s(\bar{r})$$

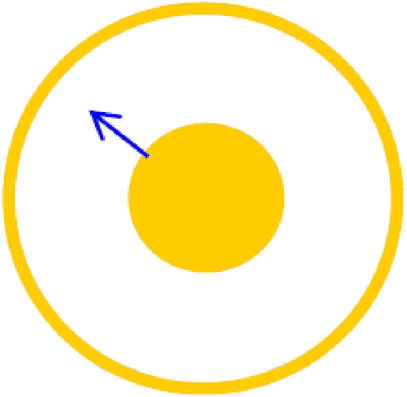
Example – 3 (contd.)

- First, we find that the electric flux density **on the surface** of the inner conductor (i.e., at $\rho = a$) is:

$$\vec{D}(\bar{r})|_{\rho=a} = \hat{a}_\rho \frac{\varepsilon V_0}{\ln[b/a]} \frac{1}{\rho}|_{\rho=a} = \frac{\varepsilon V_0}{\ln[b/a]} \frac{1}{a} \hat{a}_\rho$$

- For **every** point on **outer** surface of the **inner** conductor, we find that the unit vector **normal** to the conductor is:

$$\hat{a}_n = \hat{a}_\rho$$

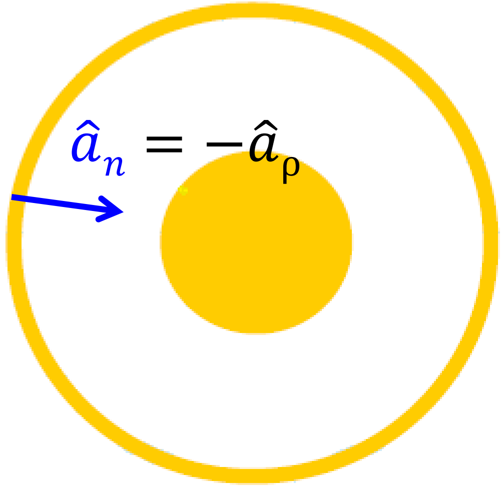


- Therefore, we find that the **surface charge density** on the outer surface of the inner conductor is:

$$\rho_{sa}(\bar{r}) = \hat{a}_n.\vec{D}(\bar{r})|_{\rho=a} \Rightarrow \rho_{sa}(\bar{r}) = \hat{a}_\rho.\hat{a}_\rho \frac{\varepsilon V_0}{\ln[b/a]} \frac{1}{a} \Rightarrow \rho_{sa}(\bar{r}) = \frac{\varepsilon V_0}{\ln[b/a]} \frac{1}{a} \quad \rho = a$$

Example – 3 (contd.)

- Likewise, we find the unit vector **normal** to the **inner** surface of the **outer** conductor is (do you see why?):

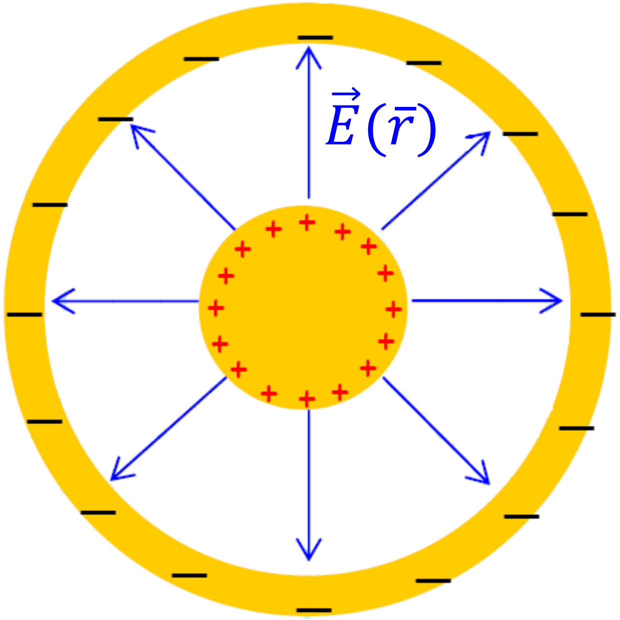


- Therefore, evaluating the electric flux density on the inner surface of the outer conductor (i.e., ρ = b), we find:

$$\rho_{sb}(\overline{r}) = \hat{a}_n.\vec{D}(\overline{r})|_{\rho=b} \Rightarrow \rho_{sb}(\overline{r}) = -\hat{a}_\rho.\hat{a}_\rho \frac{\varepsilon V_0}{\ln[b/a]}\frac{1}{b} \Rightarrow \rho_{sb}(\overline{r}) = \frac{-\varepsilon V_0}{\ln[b/a]}\frac{1}{b} \quad \rho = b$$

Example – 3 (contd.)

- Note the charge on the outer conductor is **negative**, while that of the inner conductor is **positive**. Hence, the electric field points from the inner conductor to the outer.



Example – 3 (contd.)

- We should **note** several things about these solutions:

1) $\nabla \times \vec{E}(\bar{r}) = 0$

2) $\nabla . \vec{D}(\bar{r}) = 0$ and $\nabla^2 V(\bar{r}) = 0$

3) $\vec{D}(\bar{r})$ and $\vec{E}(\bar{r})$ are normal to the surface of the conductor (i.e., their tangential components equal zero!

In other words, the **fields** $\vec{E}(\bar{r})$, $\vec{D}(\bar{r})$, and $V(\bar{r})$ are attributable to **free charge densities** $\rho_{sa}(\bar{r})$ and $\rho_{sb}(\bar{r})$.

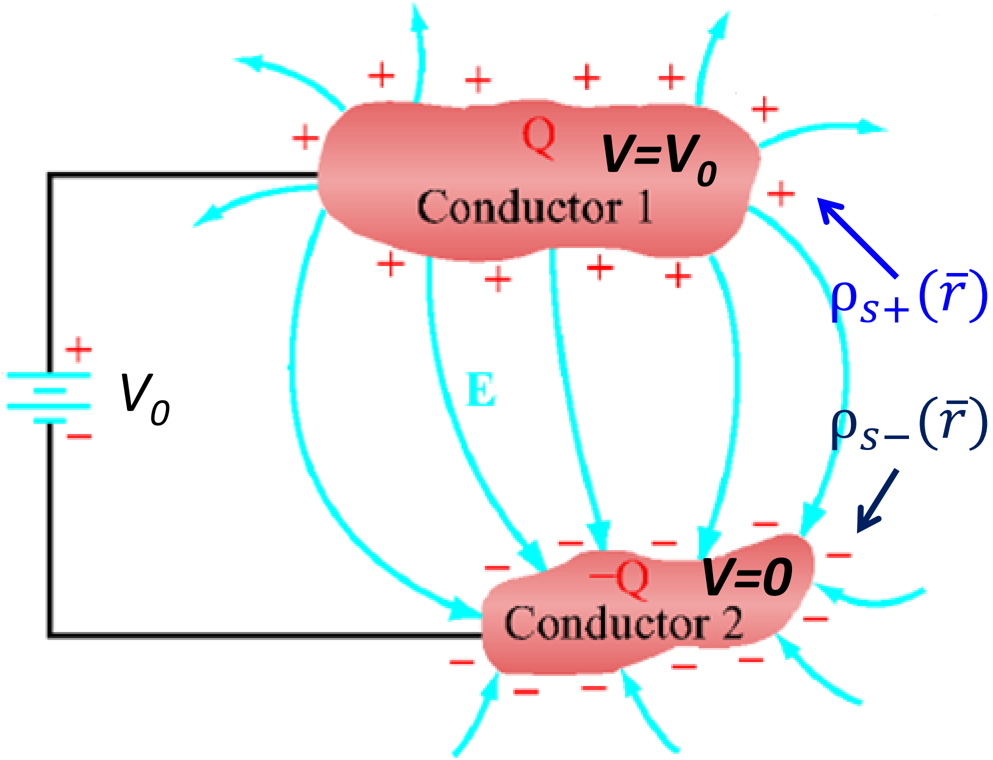
Capacitance

- Any two conducting bodies, when separated by an insulating (dielectric) medium, regardless of their shapes and sizes form a capacitor.
- If a dc voltage is connected across them, the surfaces of conductors connected to the positive and negative source terminals will accumulate charges +Q and –Q respectively.
- If a conductor has excess charge, it distributes the charge on its surface in such a manner as to maintain a zero electric field everywhere within the conductor → to ensure that electric potential is same at every point in the conductor.

Capacitance (contd.)

- Consider two **conductors**, with a **potential difference** of V volts.

- Since there is a potential difference between the conductors, there must be an **electric potential field** $V(\bar{r})$, and therefore an **electric field** $\vec{E}(\bar{r})$ in the region between the conductors.



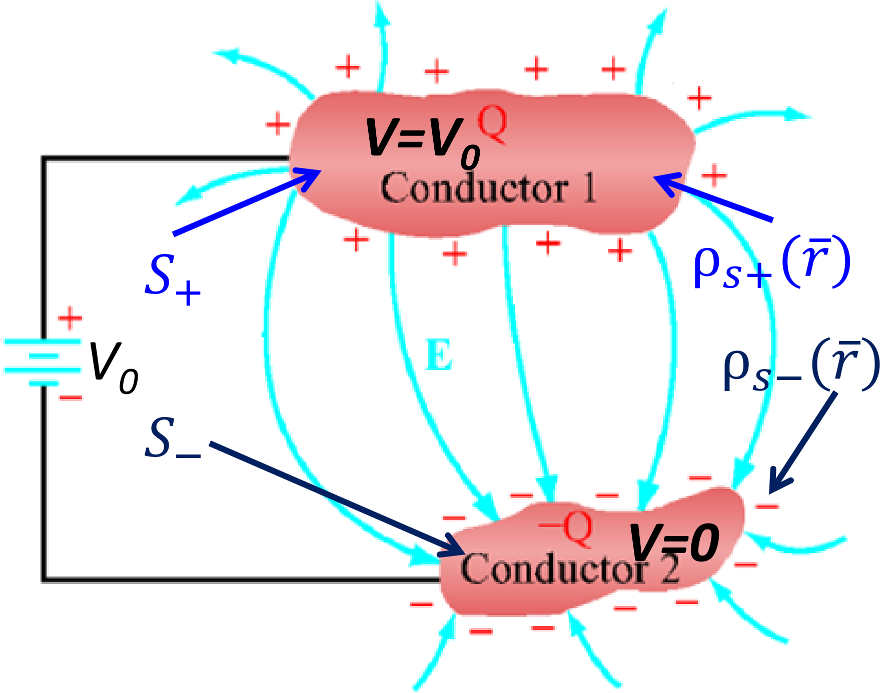
- Likewise, if there is an electric field, then we can specify an **electric flux density** $\vec{D}(\bar{r})$, which we can use to determine the **surface charge density** $\rho_s(\bar{r})$ on each of the conductors.

Capacitance (contd.)

- We find that if the total net charge on **one** conductor is Q then the charge on the **other** will be equal to $-Q$.
- In other words, the total net charge on each conductor will be **equal** but **opposite**!
- Note that this does **not** mean that the surface charge densities on each conductor are equal (i.e., $\rho_{s+}(\bar{r}) \neq \rho_{s-}(\bar{r})$). Rather, it means that:

$$\oiint_{S_+} \rho_{s+}(\bar{r})dS = -\oiint_{S_-} \rho_{s-}(\bar{r})dS = Q$$

where surface S_+ is the surface surrounding the conductor with the positive charge (and the higher electric potential), while the surface S_- surrounds the conductor with the negative charge.



Capacitance (contd.)

Q: How much free **charge** Q is there on each conductor, and how does this charge relate to the **voltage** V_0?

A: We can determine this from the mutual **capacitance** C of these conductors!

- The mutual **capacitance** between two conductors is **defined** as:

$$C = \frac{Q}{V} \qquad \left[\frac{Coulombs}{Volts} \equiv Farad\right]$$

where Q is the **total charge** on **each conductor**, and V is the **potential difference** between each conductor (for our example, $V = V_0$).

- Recall that the total charge on a conductor can be determined by **integrating** the surface charge density $\rho_S(\bar{r})$ across the **entire surface** S of a conductor:

$$Q = \oiint_{S_+} \rho_{s+}(\bar{r})dS = -\oiint_{S_-} \rho_{s-}(\bar{r})dS$$

Capacitance (contd.)

- But recall also that the surface charge density on the surface of a conductor can be determined from the **electric flux density** $\vec{D}(\bar{r})$:

$$\rho_s(\bar{r}) = \vec{D}(\bar{r}).\hat{a}_n$$

where $\hat{a}_n$ is a unit vector **normal** to the conductor.

- **Combining** the two equations, we get:

$$Q = \oiint_{S_+} \vec{D}(\bar{r}).\hat{a}_n dS = -\oiint_{S_-} \vec{D}(\bar{r}).\hat{a}_n dS \Rightarrow Q = \oiint_{S_+} \vec{D}(\bar{r}).\overline{dS} = -\oiint_{S_-} \vec{D}(\bar{r}).\overline{dS}$$

where we remember that $\overline{dS} = \hat{a}_n dS$.

- Hey! This is **no surprise**! We **already** knew that:

$$Q = \oiint_{S} \vec{D}(\bar{r}).\overline{dS}$$

This expression is also known as____________________!!

Capacitance (contd.)

- Note since $\vec{D}(\bar{r}) = \varepsilon\vec{E}(\bar{r})$ we can also say:

$$Q = \oiint_S \varepsilon \vec{E}(\bar{r}).\overline{dS}$$

- The **potential difference** V between two conductors can likewise be determined as:

$$V = \int_C \vec{E}(\bar{r}).\overline{dl}$$

where C is **any contour** that leads from one conductor to the other.

Q: Why **any** contour?

A:

- We can therefore determine the **capacitance** between two conductors as:

$$C = \frac{Q = \oiint_S \varepsilon \vec{E}(\bar{r}).\overline{dS}}{V = \int_C \vec{E}(\bar{r}).\overline{dl}} \qquad [Farad]$$

- Where the contour C must start at **some** point on surface S_+ and end at **some** point on surface S_-.
- $S_+ = S_-$

Capacitance (contd.)

- Note this expression can be written as: $Q = CV$

In other words, the charge **stored** by two conductors is equal to the product of their mutual capacitance and the potential difference between them.

Therefore, the **greater** capacitance, the **greater** the amount of **charge** that is stored.

- Furthermore, try taking the **time derivative** of the above equation:

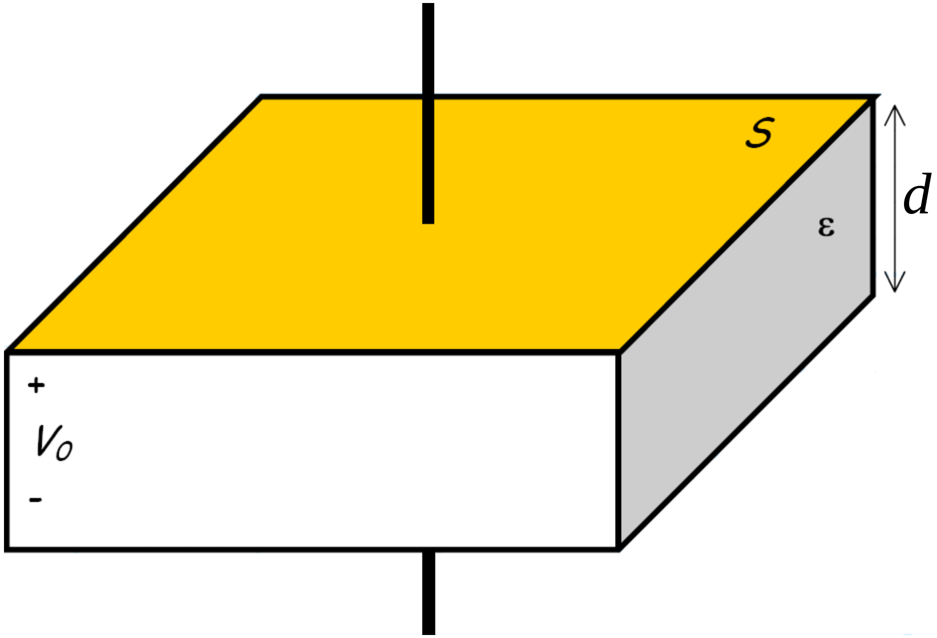
$$\frac{dQ}{dt} = C\frac{dV}{dt} \Rightarrow I = C\frac{dV}{dt}$$

Look familiar ?

By the way, the current I in this equation is **displacement current**.

The Parallel Plate Capacitor

- Consider the geometry of a **parallel plate capacitor**:



Recall that we determined the fields and surface charge density of an **infinite** pair of parallel plates. We can use those results to approximate the fields and charge densities of this **finite** structure, where the **area** of each plate is S.

Where:

V_0 = the **potential difference** between the plates

S = **surface area** of each conducting plate

d = **distance** between plates

ε = **permittivity** of the dielectric between the plates

The Parallel Plate Capacitor (contd.)

- For example, we determined that the **surface charge density** on the upper plate is:

$$\rho_{s+}(\overline{r}) = \frac{\varepsilon V_0}{d}$$

- The **total charge** on the upper plate is therefore:

$$Q = \iint_{S_+} \rho_{s+}(\overline{r})ds \Rightarrow Q = \iint_{S_+} \frac{\varepsilon V_0}{d} ds \Rightarrow Q = \frac{\varepsilon V_0}{d} \iint_{S_+} ds \Rightarrow Q = \frac{\varepsilon V_0 S}{d}$$

- The **capacitance** of this structure is therefore:

$$C = \frac{Q}{V} \Rightarrow C = \left(\frac{\varepsilon V_0 S}{d}\right)\left(\frac{1}{V_0}\right) \Rightarrow \therefore C = \frac{\varepsilon S}{d}$$

The Parallel Plate Capacitor (contd.)

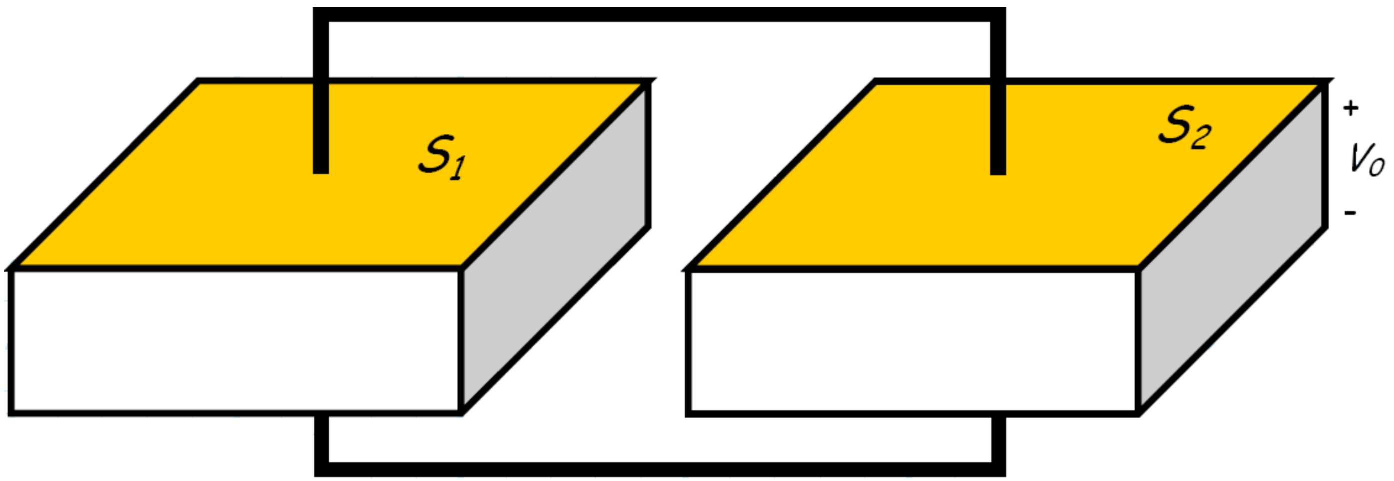
$$\therefore C = \frac{\varepsilon S}{d}$$

Therefore, we can **increase** the capacitance of a parallel plate capacitor by:

1) Increasing surface area S.
2) Decreasing separation distance d.
3) Increasing the dielectric permittivity ε.

The Parallel Plate Capacitor (contd.)

- Consider now the structure:



Note the **two** upper plates form **one** conducting structure, and the **two** bottom plates form **another**.

Q: What is the **capacitance** between these two conducting structures?

A: The potential difference between them is V_0. The **total charge** on one conducting structure is simply the **sum** of the charges on **each plate**:

$$Q = Q_1 + Q_2 = \frac{\varepsilon V_0 S_1}{d} + \frac{\varepsilon V_0 S_2}{d}$$

The Parallel Plate Capacitor (contd.)

- Therefore, the **capacitance** of this structure is:

$$C = \frac{Q}{V} = \left(\frac{\varepsilon V_0 \left(S_1 + S_2\right)}{d}\right)\left(\frac{1}{V_0}\right)$$

$$\Rightarrow \quad C = \frac{\varepsilon \left(S_1 + S_2\right)}{d}$$

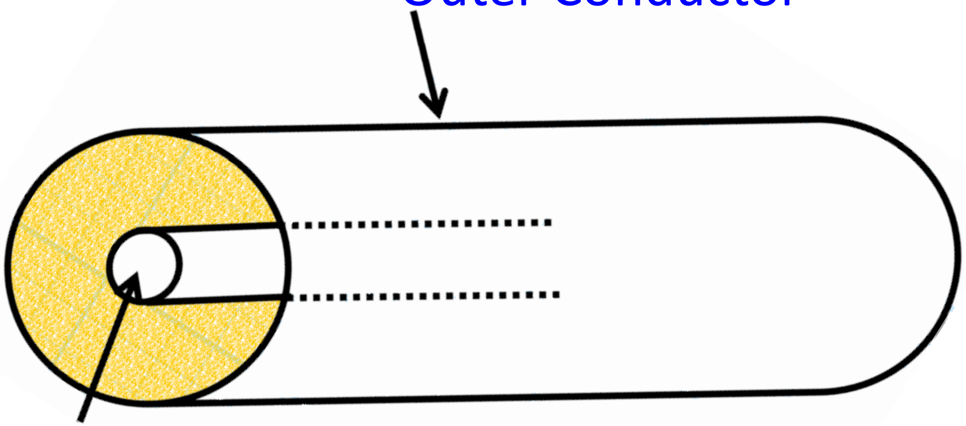
$$\Rightarrow \quad C = \frac{\varepsilon S_1}{d} + \frac{\varepsilon S_2}{d}$$

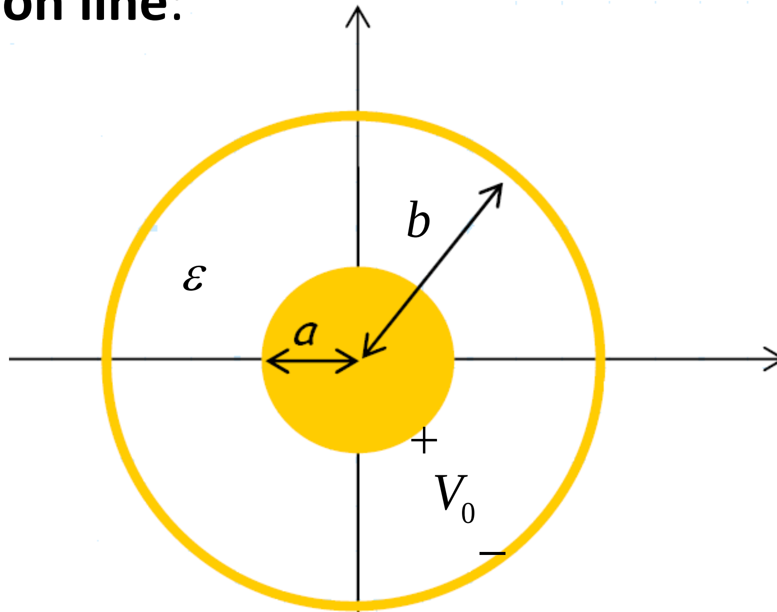
$$\Rightarrow \quad C = C_1 + C_2$$

But **you** knew this! The total capacitance of two capacitors in **parallel** is equal to the **sum** of **each** capacitance.

Capacitance of a Coaxial Transmission Line

- Recall the geometry of a **coaxial transmission line**:





Coax Cross-Section

- We earlier determined that if a **potential difference** of V_0 volts is placed across the conductors, the **surface charge density** on the **inner** conductor is:

$$\rho_{sa}(\bar{r}) = \frac{\varepsilon V_0}{\ln[b/a]}\frac{1}{a} \qquad \rho = a$$

Capacitance of a Coaxial Transmission Line (contd.)

- The **total charge** Q on the **inner** conductor of a coax of length l is determined by **integrating** the surface charge density across the **conductor surface**:

$$Q = \iint_{S_+} \rho_{s+}(\bar{r})ds$$

$$Q = \int_0^l \int_0^{2\pi} \frac{\varepsilon V_0}{\ln[b/a]} \frac{1}{a} \rho d\phi dz$$

$$Q = \frac{\varepsilon V_0}{\ln[b/a]} \frac{1}{a} \rho \int_0^l \int_0^{2\pi} d\phi dz$$

$$Q = \left[\frac{\varepsilon V_0}{\ln[b/a]} \frac{1}{a} \rho \right]_{\rho=a} \int_0^l \int_0^{2\pi} d\phi dz$$

$$\therefore Q = \frac{\varepsilon V_0}{\ln[b/a]} 2\pi l$$

Capacitance of a Coaxial Transmission Line (contd.)

- We can now determine the **capacitance** of this coaxial line!
- Since $C = Q/V$, and since the **potential difference** between the conductors is $V = V_0$, we find:

$$C = \frac{Q}{V} = \left(\frac{\varepsilon V_0}{\ln[b/a]} 2\pi l\right)\left(\frac{1}{V_0}\right) \quad \Rightarrow \quad C = \frac{2\pi\varepsilon}{\ln[b/a]} l$$

- This value represents the capacitance of a coaxial line of length l. A more useful expression is the capacitance of a coaxial line **per unit length** (e.g. farads/meter). We find this simply by **dividing** it by length l:

$$\frac{C}{l} = \frac{2\pi\varepsilon}{\ln[b/a]} \qquad \left[\frac{Farads}{metre}\right]$$

Capacitance of a Coaxial Transmission Line (contd.)

$$C = \frac{2\pi\varepsilon}{\ln\left[b/a\right]} l$$

Note the **longer** the transmission line, the **greater** the capacitance!

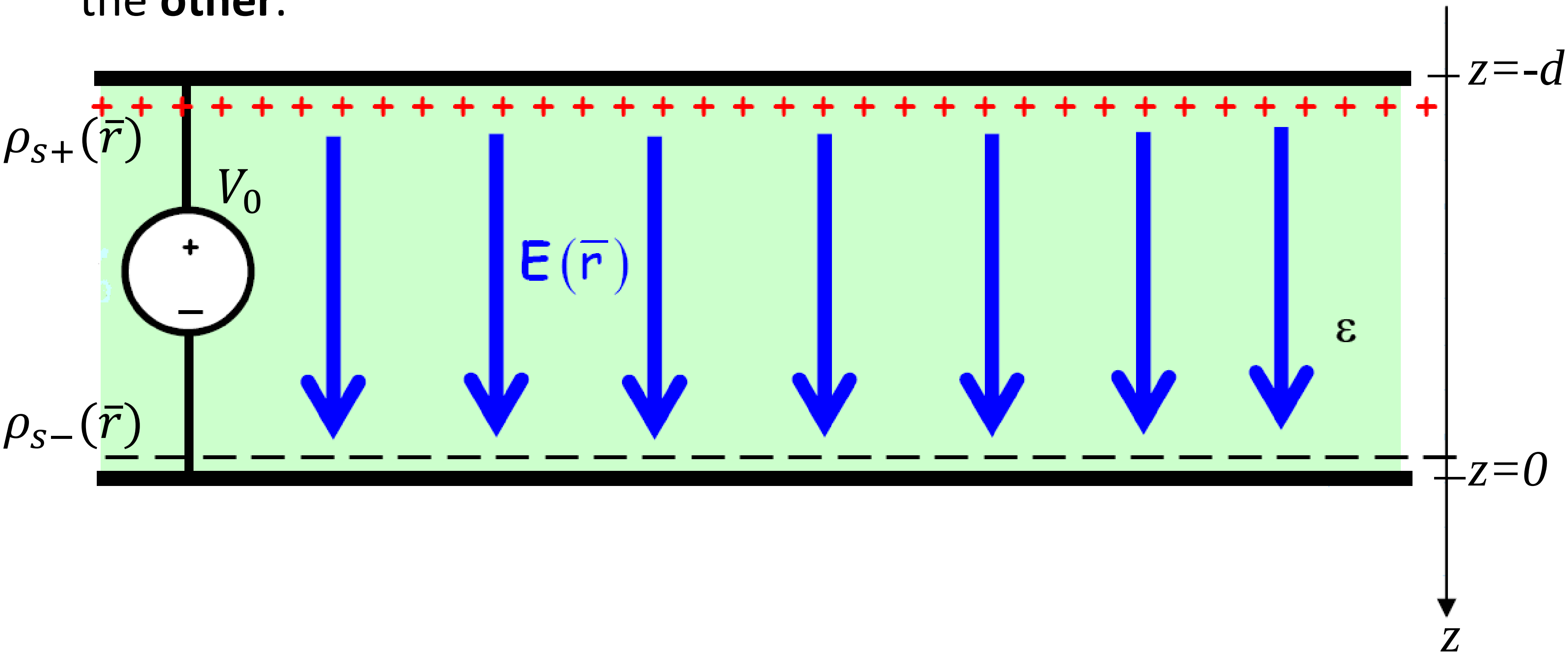
This can cause **great difficulty** if the voltage across the transmission line conductors is **time varying** (as it almost certainly will be!).

For **long** transmission lines, engineers cannot consider a transmission line simply as a "**wire**" conductor that connects circuit elements together. Instead, capacitance (and inductance) make the transmission line **itself** a **circuit element**!

In this case, engineers must use **transmission line theory** to analyze circuits!

Energy Storage in Capacitors

- Recall in a **parallel plate capacitor**, a surface charge distribution $\rho_{s+}(\bar{r})$ is created on **one** conductor, while charge distribution $\rho_{s-}(\bar{r})$ is created on the **other**.



Q: How much **energy** is stored by these charges?

Energy Storage in Capacitors (contd.)

- We learnt that the energy **stored** by a **charge distribution** is:

$$W_e = \frac{1}{2}\iiint_v \rho_v(\bar{r})V(\bar{r})dv$$

- The **equivalent** equation for **surface** charge distributions is:

$$W_e = \frac{1}{2}\iint_S \rho_s(\bar{r})V(\bar{r})dS$$

- For the parallel plate capacitor, we must integrate over **both** plates:

$$W_e = \frac{1}{2}\iint_{S_+} \rho_{s+}(\bar{r})V(\bar{r})dS + \frac{1}{2}\iint_{S_-} \rho_{s-}(\bar{r})V(\bar{r})dS$$

- But on the **top** plate (i.e., S_+), we know that: $V(z=-d)=V_0$

- While on the **bottom** plate (i.e., S_-): $V(z=0)=0$

Energy Storage in Capacitors (contd.)

- Therefore:

$$W_e = \frac{V_0}{2}\iint_{S_+} \rho_{s+}(\bar{r})dS + \frac{0}{2}\iint_{S_-} \rho_{s-}(\bar{r})dS$$

$$W_e = \frac{V_0}{2}\iint_{S_+} \rho_{s+}(\bar{r})dS$$

here

$$\iint_{S_+} \rho_{s+}(\bar{r})dS = Q$$

$$\therefore W_e = \frac{1}{2}QV$$

here

- $Q = CV$
- and V is the **potential difference** between the two conductors

- Combining these **two** equations, we find: $$W_e = \frac{1}{2}CV^2$$

Energy Storage in Capacitors (contd.)

$$W_e = \frac{1}{2}CV^2$$

It shows that the **energy stored** within a capacitor is proportional to the product of its capacitance and the squared value of the voltage across the capacitor.

- Recall that we also can determine the stored energy from the **fields** within the dielectric:

$$W_e = \frac{1}{2}\iiint_v \vec{D}(\bar{r}).\vec{E}(\bar{r})dv \;\Rightarrow\; W_e = \frac{1}{2}\frac{\varepsilon V^2}{d^2}(volume)$$

- Here $volume = Sd$, therefore:

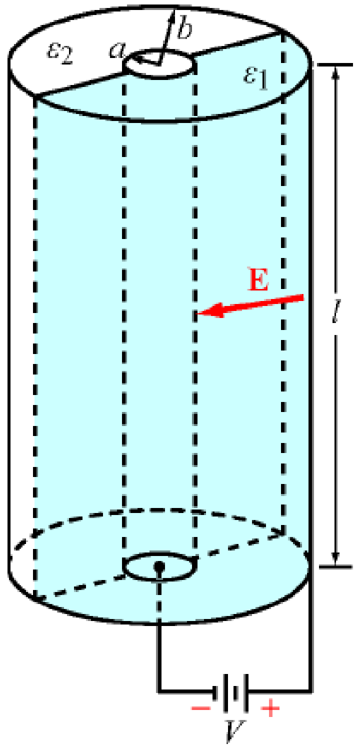
$$W_e = \frac{1}{2}\frac{\varepsilon S}{d}V^2 \;\Rightarrow\; W_e = \frac{1}{2}CV^2$$

Example – 4

- A coaxial capacitor consists of two concentric, conducting, cylindrical surfaces, one of radius a and another of radius b. The insulating layer separating the two conducting surfaces is divided equally into two semi-cylindrical sections, one filled with dielectric ε_1 and the other filled with dielectric ε_2.

(a) Develop an expression for C in terms of the length l and the given quantities.

(b) Evaluate the value of C for a = 2 mm, b = 6 mm, ε_{r1} = 2, ε_{r2} = 4, and l = 4 cm.



Example – 4 (contd.)

(a) For the indicated voltage polarity, the **electric field** inside the capacitor exists in only the dielectric materials and **points radially inward**. Let $\vec{E}_1$ be the field in dielectric ε_1 and $\vec{E}_2$ be the field in dielectric ε_2.

(b) At the interface between the two dielectric sections, $\vec{E}_1$ is parallel to $\vec{E}_2$ and both are tangential to the interface.

(c) Since boundary conditions require that the tangential components of $\vec{E}_1$ and $\vec{E}_2$ be the same, it follows that:

$$\vec{E}_1 = \vec{E}_2 = -E\hat{a}_\rho$$

- At r = a (surface of inner conductor), in medium 1, the boundary condition on $\vec{D}$, leads to:

$$\vec{D}_1 = \varepsilon_1 \vec{E}_1 = \rho_{s1}\hat{a}_n \;\Rightarrow\; -\varepsilon_1 E\hat{a}_\rho = \rho_{s1}\hat{a}_\rho \;\Rightarrow\; \rho_{s1} = -\varepsilon_1 E$$

Example – 4 (contd.)

- Similarly, in medium 2: $\rho_{s2} = -\varepsilon_2 E$

- Thus, the electric fields will be the same in the two dielectrics, but the charge densities will be different along the two sides of the inner conducting cylinder.
- Since the same voltage applies for the two sections of the capacitor, we can treat them as two capacitors in parallel. For first half of the cylinder that includes dielectric ε_1, we can express:

$$C_1 = \frac{\pi \varepsilon_1}{\ln[b/a]} l$$

Only half cylinder

- Similarly:

$$C_2 = \frac{\pi \varepsilon_2}{\ln[b/a]} l$$

Therefore:

$$C = C_1 + C_2 = \frac{\pi l (\varepsilon_1 + \varepsilon_2)}{\ln[b/a]}$$