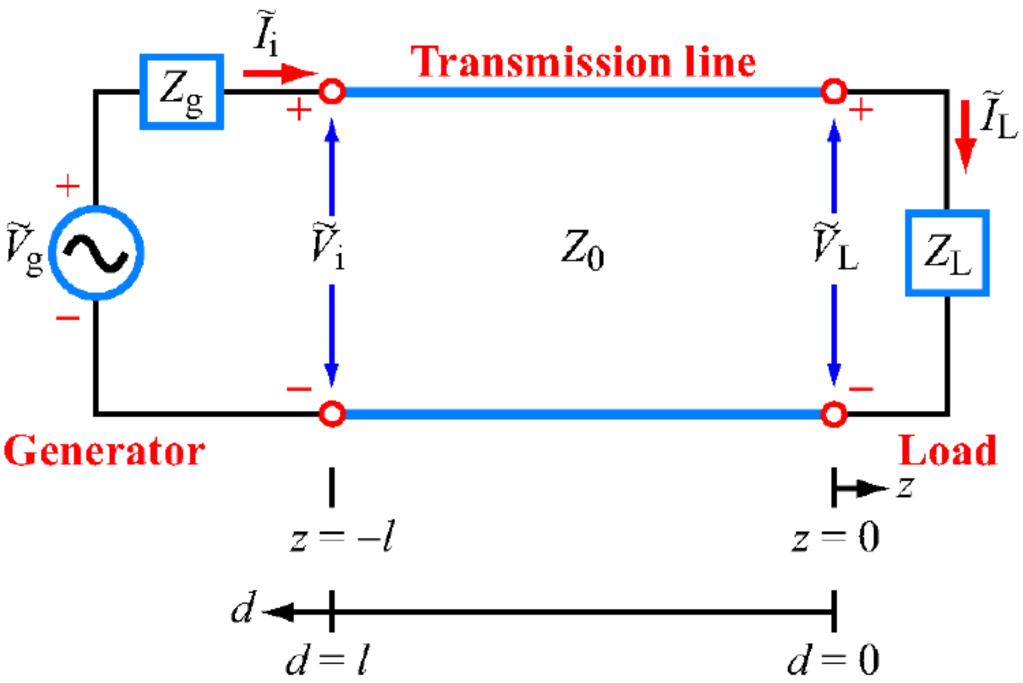
Lecture – 5 **Date: 18.01.2016**

- Voltage Reflection Coefficient
- Examples
- Standing Waves

Voltage Reflection Coefficient

- To determine the unknowns V_0^+ and V_0^-, we need to consider the lossless transmission line in the context of complete circuit including a generator circuit at its input terminals and a load at its output terminals.



At the load side:

$$Z_L = \frac{\tilde{V}_L}{\tilde{I}_L}$$

- Where, $\tilde{V}_L$ and $\tilde{I}_L$ are the total voltage and current at the load.

$$\tilde{V}_L = \tilde{V}(z=0) = V_0^+ + V_0^-$$

$$\tilde{I}_L = \tilde{I}(z=0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

Voltage Reflection Coefficient (contd.)

$$\tilde{V}_L = \tilde{V}(z=0) = V_0^+ + V_0^-$$

$$\tilde{I}_L = \tilde{I}(z=0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} = \frac{V_0^+ - V_0^-}{Z_0}$$

$$Z_L = \frac{\tilde{V}_L}{\tilde{I}_L} \Rightarrow Z_L = \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right) Z_0$$

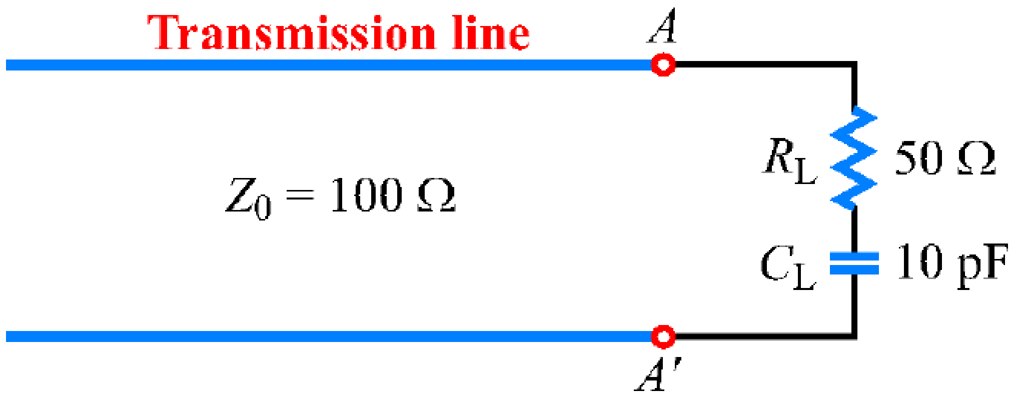
- Solving for V_0^- gives: $V_0^- = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) V_0^+ \Rightarrow \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$

$\frac{V_0^-}{V_0^+} \Rightarrow$ The ratio of the amplitudes of the reflected and the incident voltage waves at the load is called voltage reflection coefficient Γ

$$\therefore \Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Example – 1

- A 100Ω transmission line is connected to a load consisting of a 50Ω resistor in series with a 10pF capacitor. Find the reflection coefficient at the load for a 100MHz wave.



Example – 2

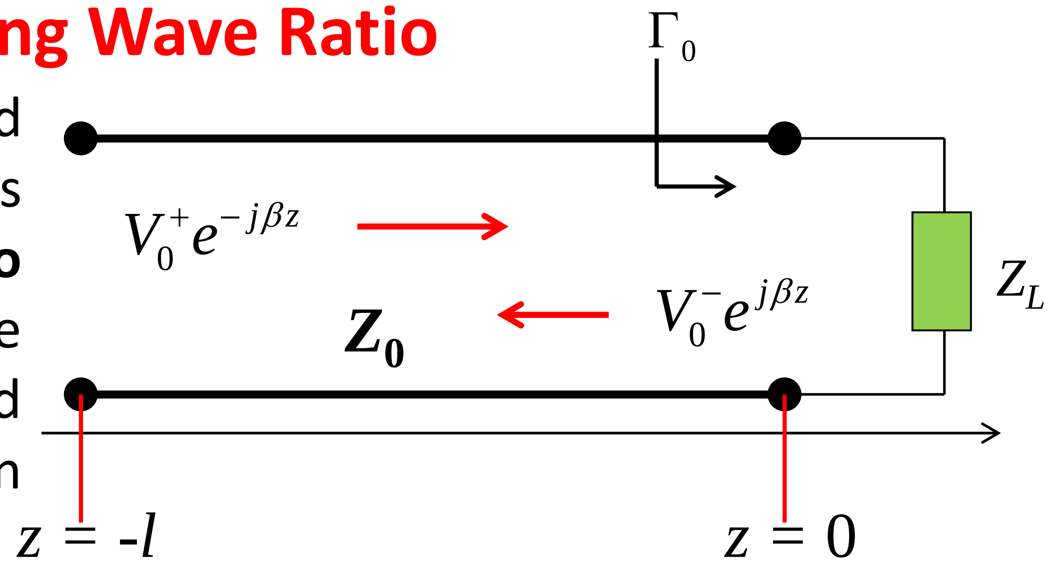
- A 150Ω lossless transmission line is terminated in a capacitor with impedance $Z_L = -j30\Omega$. Calculate Γ.

Example – 3

- Show that $|\Gamma| = 1$ for a lossless line connected to a purely reactive load.

Standing Wave and Standing Wave Ratio

- Another traditional real-valued measure of load match is **Voltage Standing Wave Ratio** (VSWR). Consider again the **voltage** along a terminated transmission line, as a function of **position** z.



$$\tilde{V}(z) = V_0^+ \left[e^{-j\beta z} + \Gamma_0 e^{+j\beta z} \right] \Rightarrow \tilde{V}(-l) = V_0^+ \left[e^{j\beta l} + \Gamma_0 e^{-j\beta l} \right]$$

- For a short circuited line: $\Gamma_0 = -1$ $\Rightarrow$ $\tilde{V}(-l) = V_0^+ \left(e^{+j\beta l} - e^{-j\beta l} \right)$

2*j*sin(β*l*)

$$\therefore \tilde{V}(-l) = 2jV_0^+ \sin \beta l$$

Standing Wave and Standing Wave Ratio (contd.)

$$v(-l,t) = \mathrm{Re}\left(\tilde{V}(-l)e^{j\omega t}\right) = \mathrm{Re}\left(2jV_0^+ \sin(\beta l)e^{j\omega t}\right)$$

$$\therefore v(-l,t) = 2V_0^+ \sin(\beta l)\cos(\omega t + (\pi/2))$$

Definitely not a traveling wave!!

Always zero for **-*l*=0** i.e., the point of short-circuit

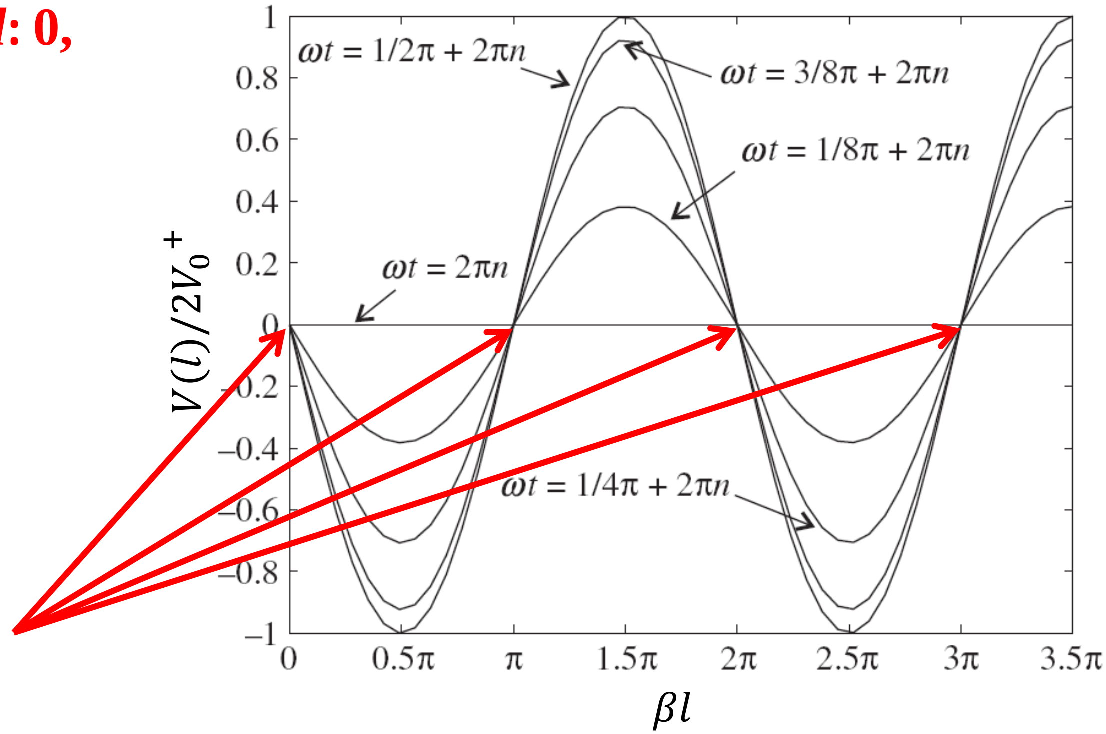
Where has the traveling wave V(z) gone?

- As the time and space are decoupled → No wave propagation takes place
- The incident wave is 180° out of phase with the reflected wave → gives rise to zero crossings of the wave at 0, λ/2, λ, 3λ/2, and so on → standing wave pattern!!!

Standing Wave and Standing Wave Ratio (contd.)

Corresponding β*l*: 0, π, 2π, 3π

Spatial Location: 0, λ/2, λ, 3λ/2



Standing Wave Pattern for Various Instances of Time

- **for arbitrarily terminated line:** $\tilde{V}(-l) = V_0^+\left(e^{+j\beta l} + \Gamma_0 e^{-j\beta l}\right) = V_0^+ e^{+j\beta l}\left(1+\Gamma_0 e^{-j2\beta l}\right)$

$$\Rightarrow \tilde{V}(-l) = A(-l)\left(1+\Gamma(-l)\right)$$

Valid anywhere on the line

Standing Wave and Standing Wave Ratio (contd.)

$$\Rightarrow \tilde{V}(-l) = A(-l)\big(1+\Gamma(-l)\big)$$

Valid anywhere on the line

- Under the matched condition, $\Gamma_0 = 0$ and therefore $\Gamma(-l) = 0$ → as expected, only positive traveling wave exists.
- For other arbitrary impedance loads: Standing Wave Ratio (SWR) or Voltage Standing Wave Ratio (VSWR) is the measure of mismatch.
- SWR is defined as the ratio of maximum voltage (or current) amplitude and the minimum voltage (or current) amplitude along a line → therefore, for an arbitrarily terminated line:

$$VSWR = \left|\frac{\tilde{V}(-l)_{\max}}{\tilde{V}(-l)_{\min}}\right|$$

We have: $\tilde{V}(-l) = V_0^+ e^{+j\beta l}\left(1+\Gamma_0 e^{-j2\beta l}\right)$

- Two possibilities for extreme values: $\Gamma_0 e^{-j\beta l} = 1$ $\quad$ $\Gamma_0 e^{-j\beta l} = -1$

Standing Wave and Standing Wave Ratio (contd.)

Max. voltage: $\left|\tilde{V}(-l)\right|_{max} = \left|V_0^+\right|\left(1+\left|\Gamma_0\right|\right)$ **Min. voltage:** $\left|\tilde{V}(-l)\right|_{min} = \left|V_0^+\right|\left(1-\left|\Gamma_0\right|\right)$

$$\therefore VSWR = \frac{1+\left|\Gamma_0\right|}{1-\left|\Gamma_0\right|}$$

Apparently: $0 \le \Gamma_0 \le 1$ → $\therefore 1 \le VSWR < \infty$

- Note if $|\Gamma_0| = 0$ (i.e., $Z_L = Z_0$), then VSWR = 1. We find for this case: $\left|\tilde{V}(z)\right|_{max} = \left|\tilde{V}(z)\right|_{min} = \left|V_0^+\right|$

In other words, the voltage magnitude is a **constant** with respect to position z.

- Conversely, if $|\Gamma_0| = 1$ (i.e., $Z_L = Z_0$), then VSWR = ∞. We find for **this** case: $\left|\tilde{V}(z)\right|_{max} = 2\left|V_0^+\right|$ $\left|\tilde{V}(z)\right|_{min} = 0$

In other words, the voltage magnitude varies **greatly** with respect to position z.

In practice, SWR can only be defined for lossless line as the SWR equation is not valid for attenuating voltage and current

Example – 4

- A 50Ω lossless transmission line is terminated in a load with impedance $Z_L = (100 + j30)\Omega$. Calculate voltage reflection coefficient and the voltage standing wave ratio.