Lecture – 3

Date: 11.01.2016

- Examples
- Transmission Lines
- Transmission Line Equations

Example – 1

- A laser beam traveling through fog was observed to have an intensity of $1\,(\frac{\mu W}{m^2})$ at a distance of $2m$ from the laser gun and an intensity of $0.2\,(\frac{\mu W}{m^2})$ at a distance of $3m$. Given that the intensity of an electromagnetic wave is proportional to the square of its electric field amplitude, find the attenuation constant α of fog.

Example – 2

- If $z = 3e^{j\pi/6}$, find the value of e^{z}.

Example – 3

- Find the instantaneous time sinusoidal functions corresponding to the following phasors:

$$(a)\ \tilde{V} = -5e^{\frac{j\pi}{3}} \qquad V$$

$$(b)\ \tilde{V} = j6e^{-j\pi/4} \qquad V$$

$$(c)\ \tilde{I} = (6 + j8) \qquad A$$

$$(d)\ \tilde{I} = (-3 + j2) \qquad A$$

$$(e)\ \tilde{I} = (j) \qquad A$$

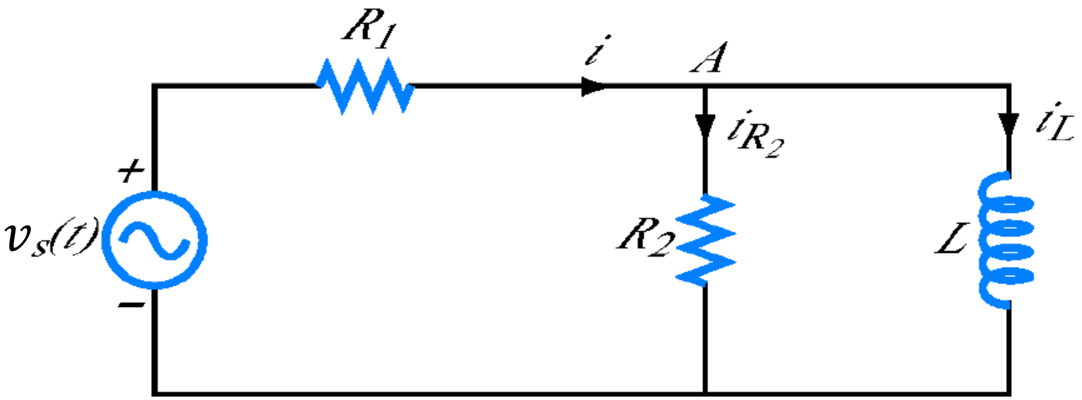
$$(f)\ \tilde{I} = 2e^{j\pi/6} \qquad A$$

Example – 4

- The voltage source of the circuit shown below is given by:

$$v_s(t) = 25cos(4 \times 10^4 t - 45°) \qquad V$$

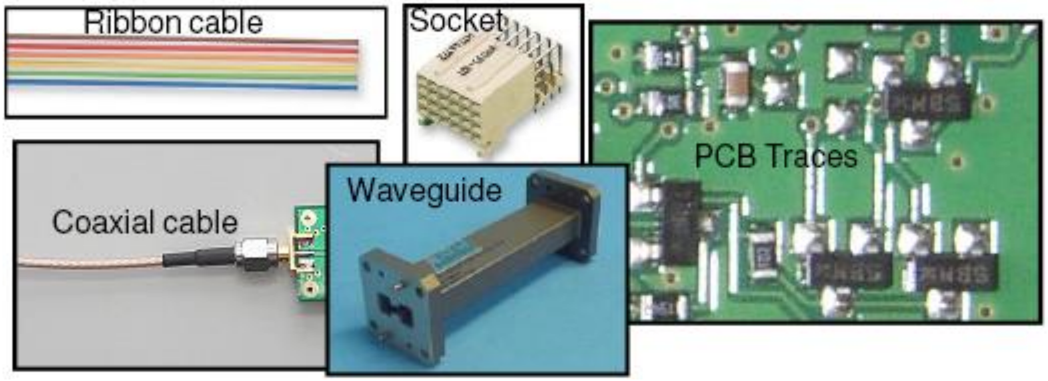
Obtain an expression for $i_L(t)$, the current flowing through the inductor.



$R_1 = 20$, $R_2 = 30$, $L = 0.4\ mH$

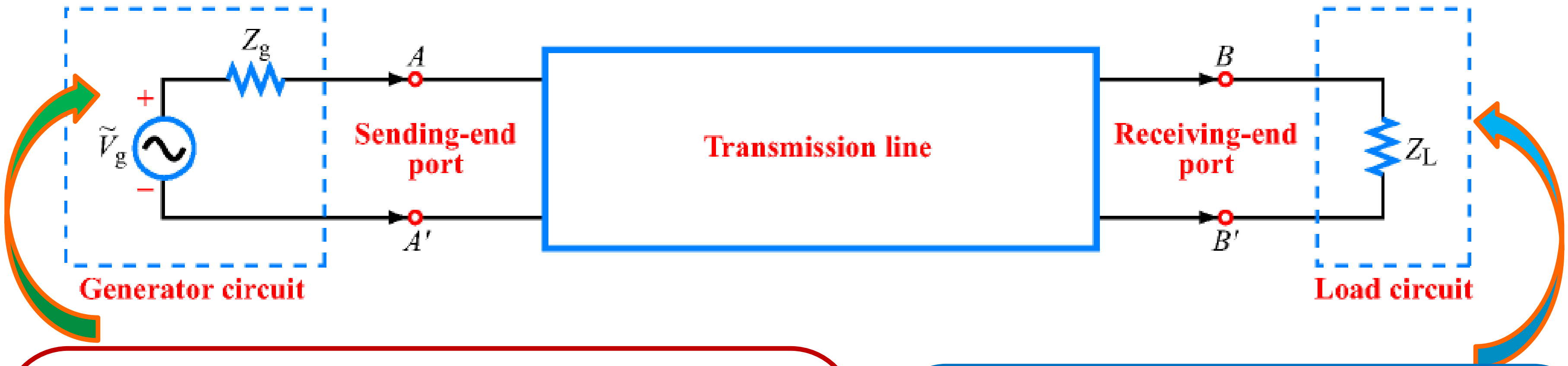
Transmission Line (TL)

- It encompasses all structures and media that serve to transfer energy or information between two points.
- In this course we talk about TL that guide EM waves.
- Such TLs include telephone wires, coaxial cable carrying audio and video information to TV sets or digital data to computers, microstrips printed on microwave circuit boards, and optical fibers carrying light waves.



Fundamentally, TL is a two port network with each port consisting of two terminals.

Transmission Line (TL) (contd.)



Source end may be any circuit generating an output voltage e.g.,

- Radar transmitter
- Amplifier
- Computer terminal in transmitting mode

Load end may be:

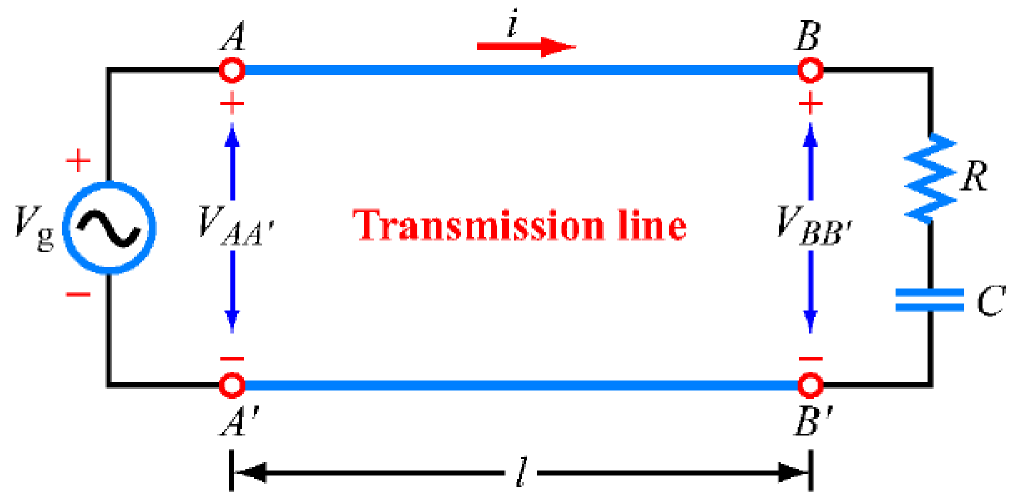
- Antenna in the case of Radar
- Input terminal of Amplifier
- Computer terminal in receiving mode

- In the case of dc: source is represented by Thevenin equivalent generator V_g and R_g.
- In the case of ac: the corresponding terms are $\widetilde{V}_g$ and Z_g.

Transmission Line (TL) (contd.)

The role of wavelength

- In low frequency circuits, circuit elements usually are interconnected using simple wires.



The pertinent questions:

- Is the pair of wires between terminals AA' and BB' a transmission line?
- If so, under what set of circumstances should we explicitly treat the pair of wires as a transmission line?

Answer:

- Yes
- The answer to second question depends on the length of the line l and the frequency f of the wave provided by the generator.

Transmission Line (TL) (contd.)

- Essentially, the determining factor is the ratio of length l and wavelength λ of the wave propagating on the transmission line.
- Let: $V'_{AA} = V_g(t) = V_0 cos\omega t$, and assume that the current flowing through the wires travel at the speed of light.
- Then $V'_{BB}(\mathrm{t})$ will appear after a delay of l/c.
- Therefore, if the wires are lossless: $V'_{BB}(\mathrm{t}) = V'_{BB}(\mathrm{t} - l/c)$

$$V'_{BB}(\mathrm{t}) = V_0 \cos(\omega t - l/c) \Rightarrow V'_{BB}(\mathrm{t}) = V_0 \cos(\omega t - \varphi_0)$$

Where:
$\boldsymbol{\varphi_0 = \omega l/c}$

- At t=0 and f=1kHz, $l = 5cm$

$$V'_{AA} = V_0 \qquad V'_{BB} = V_0 \cos\left(\frac{2\pi f l}{c}\right) = 0.999999998 V_0$$

- At t=0 and f=1kHz, $l = 20km$

$$V'_{AA} = V_0 \qquad V'_{BB} = V_0 \cos\left(\frac{2\pi f l}{c}\right) = 0.91 V_0$$

The value of V'_{BB} is controlled by $\boldsymbol{\varphi_0 = \omega l/c}$

Transmission Line (TL) (contd.)

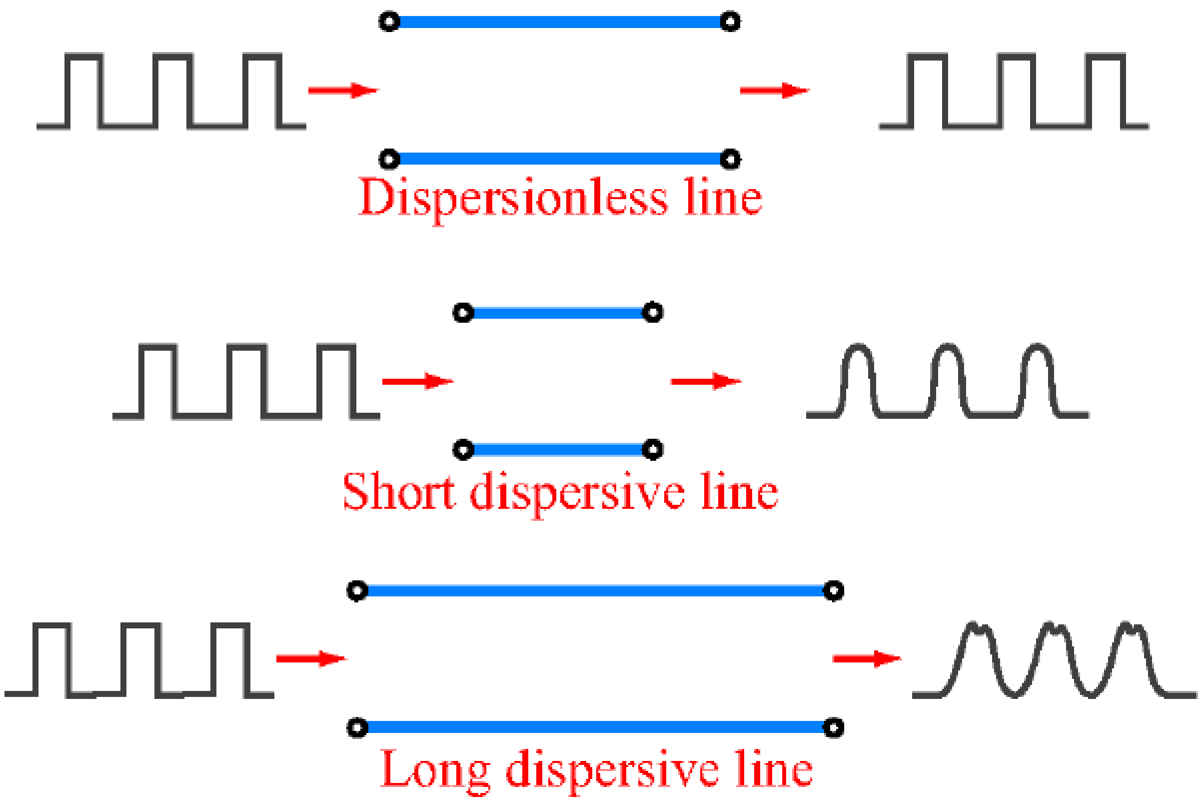
- The velocity of propagation is related to frequency as $u_p = f\lambda$ and in present case $u_p = c$.

$$\therefore \varphi_0 = \frac{\omega l}{c} = \frac{2\pi f l}{c} = 2\pi \frac{l}{\lambda} \quad \text{radians}$$

When l/λ is small, transmission-line effects may be ignored, but when $\frac{l}{\lambda} \geq 0.01$, it may be necessary to account not only for the phase shift due to the time delay, but also for the presence of reflected signals that may have been bounced back by the load toward the generator.

- **Power loss** on the line and **dispersive effects** may need to be considered as well.
- A dispersive line is one on which the wave velocity is not constant as a function of the frequency f.
- Therefore, the shape of a rectangular pulse, which can be decomposed into many sinusoidal waves of different frequencies, will be distorted on a dispersive TL.

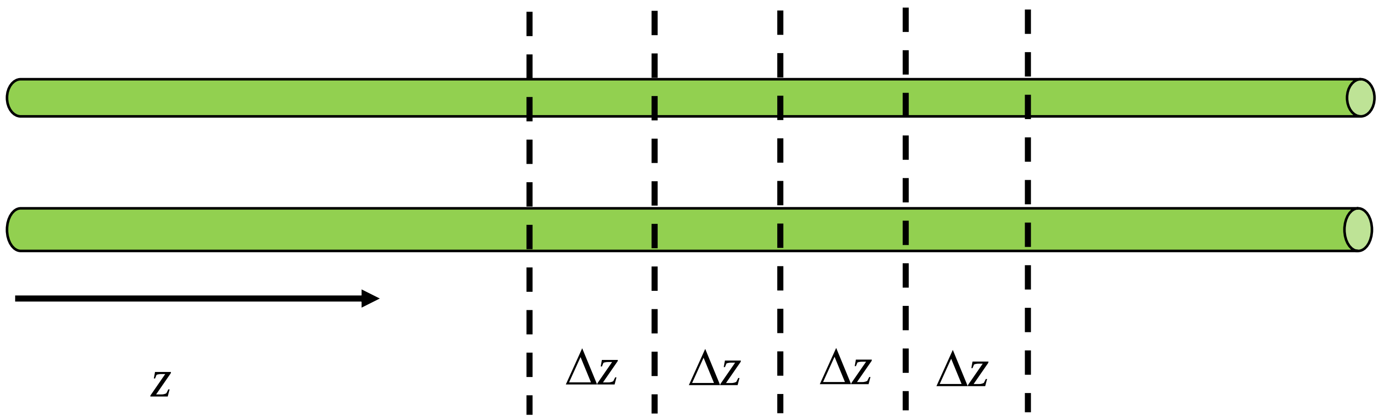
Transmission Line (TL) (contd.)



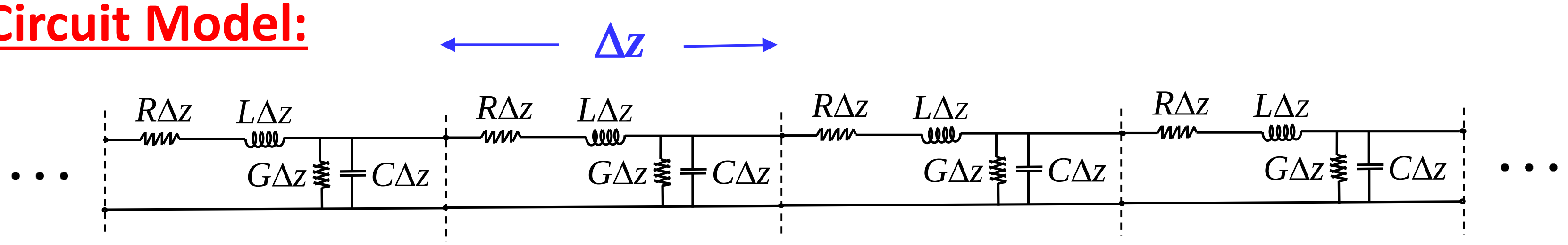
- Preservation of pulse shape is very important in high speed data transmission.
- For example, at 10GHz the wavelength is 3cm in air but only about 1cm in semiconductor.

Transmission Lines (TLs) (contd.)

- Variations in current and voltage across the circuit dimensions → KCL and KVL can't be directly applied → **This anomaly can be remedied** if the line is subdivided into elements of small (infinitesimal) length over which the current and voltage do not vary.

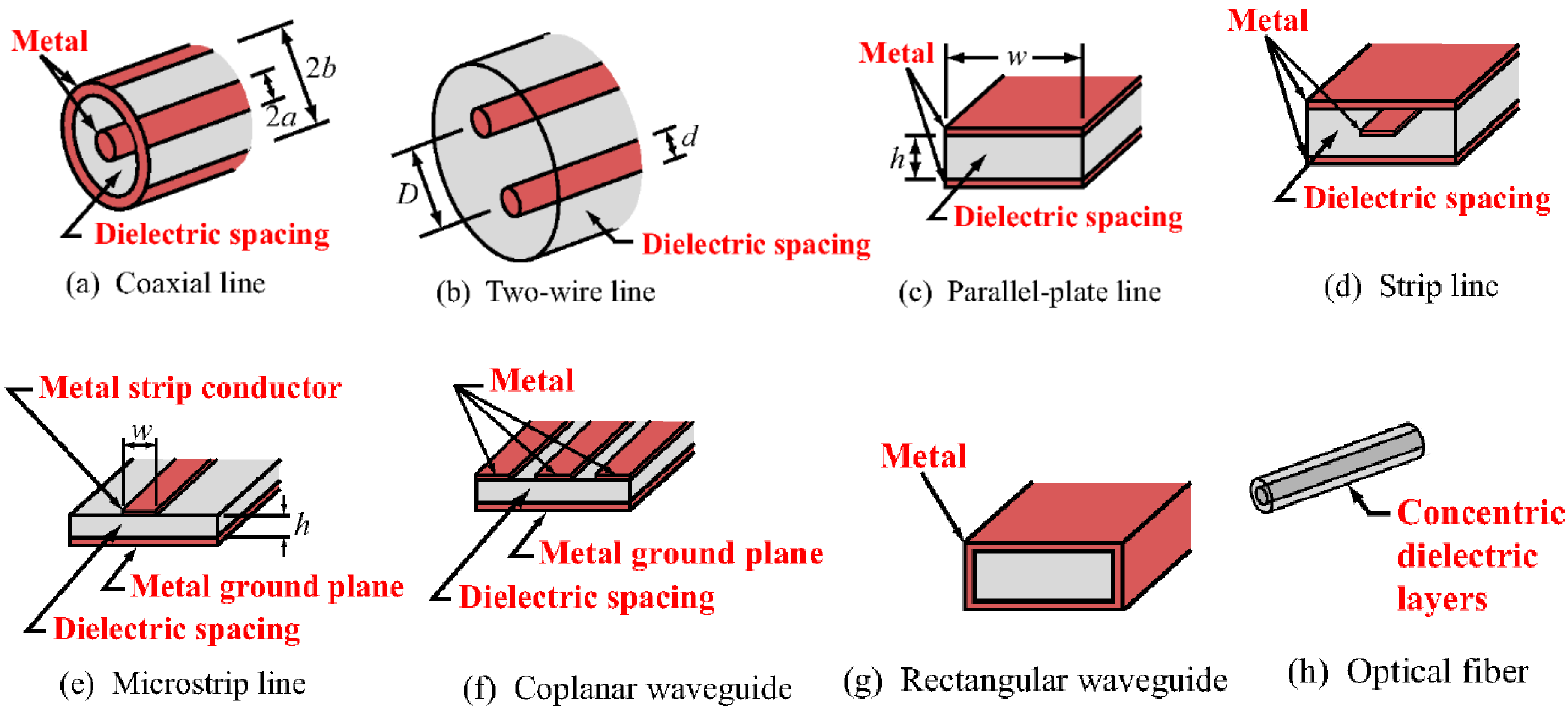


Circuit Model:



$$\lim_{\Delta z \to 0} \Rightarrow \text{Infinite number of infinitesimal sections}$$

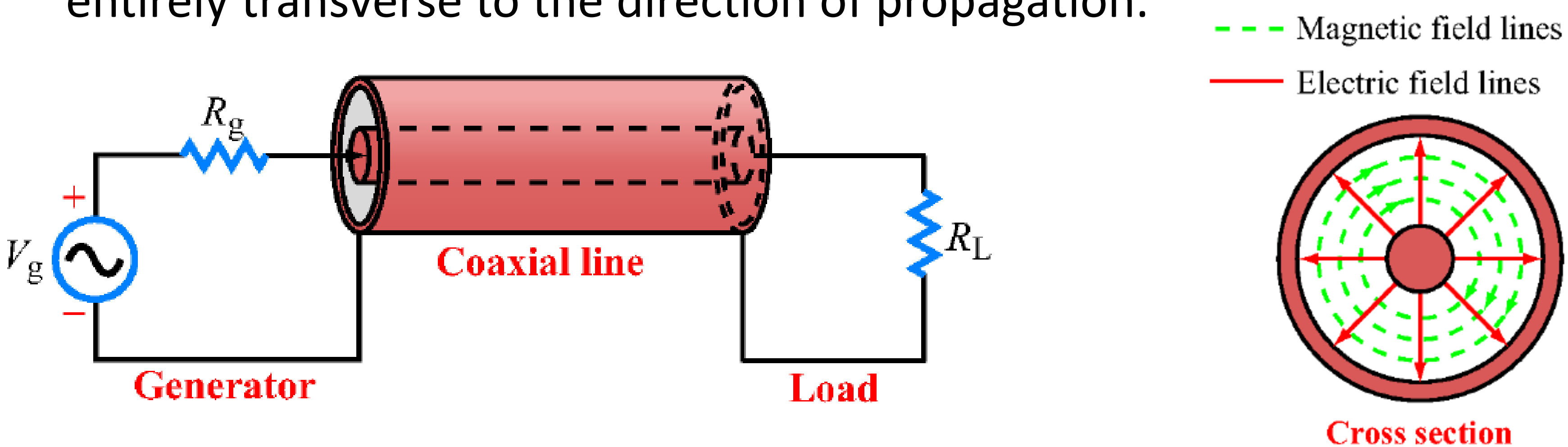
Example Transmission Line



(a) Coaxial line (b) Two-wire line (c) Parallel-plate line (d) Strip line (e) Microstrip line (f) Coplanar waveguide (g) Rectangular waveguide (h) Optical fiber

Example Transmission Line (contd.)

- **Transverse TEM Transmission Lines:** Electric and Magnetic fields are entirely transverse to the direction of propagation.

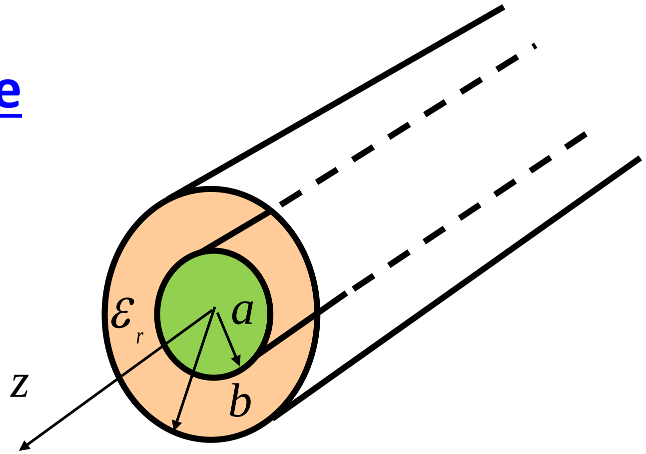


- The electric field is in the radial direction between the inner and outer conductors.
- The magnetic field circles the inner conductor.

- **Higher order Transmission Lines:** waves propagating along these lines have at least one significant field component in the direction of propagation. Hollow conducting waveguides, optical fiber etc belong to this class of lines.

Example of Transmission Lines

Coaxial Cable



σ_d = conductivity of dielectric [S/m].

σ_m = conductivity of metal [S/m].

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\mathrm{F/m}] \qquad G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad [\mathrm{S/m}]$$

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) \quad [\mathrm{H/m}] \qquad R = \frac{1}{\sigma_m\delta}\left(\frac{1}{2\pi a} + \frac{1}{2\pi b}\right) \quad [\Omega/\mathrm{m}]$$

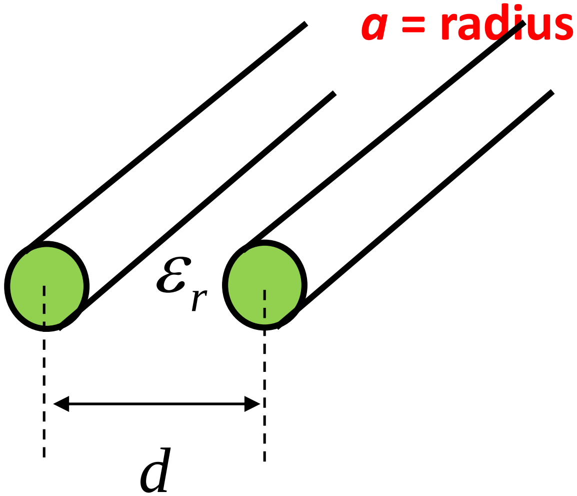
$$\delta = \sqrt{\frac{2}{\omega\mu\sigma_m}}$$

(skin depth of metal)

Example of Transmission Lines (contd.)

Twin Line

a = radius of wires



$$C = \frac{\pi\varepsilon_0\varepsilon_r}{\cosh^{-1}\left(\frac{d}{2a}\right)} \quad [\mathrm{F/m}]$$

$$L = \frac{\mu_0}{\pi}\cosh^{-1}\left(\frac{d}{2a}\right) \quad [\mathrm{H/m}]$$

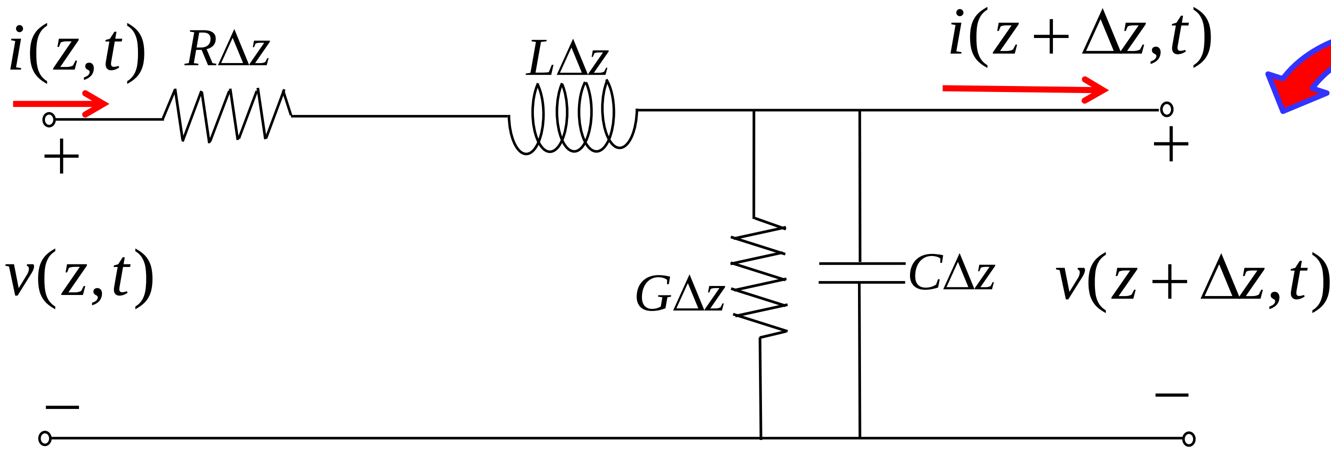
$$Z_0 = \frac{1}{\pi}\eta_0\frac{1}{\sqrt{\varepsilon_r}}\cosh^{-1}\left(\frac{d}{2a}\right) \quad [\Omega]$$

$$\cosh^{-1}x = \ln\left(x+\sqrt{x^2-1}\right) \xrightarrow{x\to\infty} \ln 2x$$

$$Z_0 \approx \frac{1}{\pi}\eta_0\frac{1}{\sqrt{\varepsilon_r}}\ln\left(\frac{d}{a}\right) \quad [\Omega]$$

$$a \ll d$$

Transmission Line Equations



Lossy
Transmission Line
Circuit Model

Apply KVL:

$$v(z,t)-v(z+\Delta z,t)=R\Delta z i(z,t)+L\Delta z\frac{\partial i(z,t)}{\partial t} \Rightarrow \frac{v(z,t)-v(z+\Delta z,t)}{\Delta z}=Ri(z,t)+L\frac{\partial i(z,t)}{\partial t}$$

Describes the voltage along the transmission lines

$$\therefore -\frac{\partial v(z,t)}{\partial z}=Ri(z,t)+L\frac{\partial i(z,t)}{\partial t}$$

For $\Delta z \rightarrow 0$

KCL on this line segment gives: $i(z,t)-i(z+\Delta z,t)=G\Delta z v(z+\Delta z,t)+C\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t}$

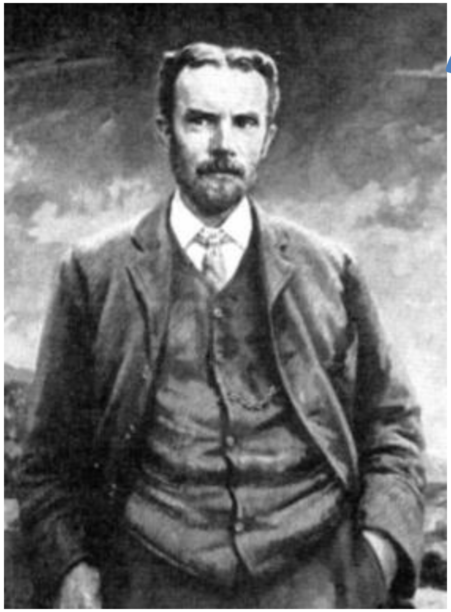
Transmission Line Equations (contd.)

Simplification results in:

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

For $\Delta z \to 0$

Describes the current along the transmission lines



These differential equations for current/voltages were derived by Oliver Heavyside. These equations are known as Telegrapher's Equations.

Transmission Line Equations (contd.)

- Let us define phasrors: $v(z,t) = Re\left[\tilde{V}(z)e^{j\omega t}\right]$ and $i(z,t) = Re\left[\tilde{I}(z)e^{j\omega t}\right]$

- With the substitution of phasors, the equations of voltage and current wave result in:

$$\mathrm{Re}\frac{\partial\left(\tilde{V}(z)e^{j\omega t}\right)}{\partial z} = -Re\left(R\tilde{I}(z)e^{j\omega t} + j\omega L\tilde{I}(z)e^{j\omega t}\right)$$

$$\mathrm{Re}\frac{\partial\left(\tilde{I}(z)e^{j\omega t}\right)}{\partial z} = -Re\left(G\tilde{V}(z)e^{j\omega t} + j\omega C\tilde{V}(z)e^{j\omega t}\right)$$

- The differential equations for current and voltage along the transmission line can be expressed in phasor form as:

$$Re\frac{d\left(\tilde{V}(z)e^{j\omega t}\right)}{dz} = -Re\left(R\tilde{I}(z)e^{j\omega t} + j\omega L\tilde{I}(z)e^{j\omega t}\right)$$

$$Re\frac{d\left(\tilde{I}(z)e^{j\omega t}\right)}{dz} = -Re\left(G\tilde{V}(z)e^{j\omega t} + j\omega C\tilde{V}(z)e^{j\omega t}\right)$$

As $I(z)$ and $V(z)$ are function of only position

Transmission Line Equations (contd.)

- The equations can be simplified as:

$$\Rightarrow Re\left[\left(\frac{d\left(\tilde{V}(z)\right)}{dz}+R\tilde{I}(z)+j\omega L\tilde{I}(z)\right)e^{j\omega t}\right]=0$$

$$\Rightarrow \mathrm{Re}\left[\left(\frac{d\left(\tilde{I}(z)\right)}{dz}+G\tilde{V}(z)+j\omega C\tilde{V}(z)\right)e^{j\omega t}\right]=0$$

For further simplification

At $\omega t=0$, $e^{j\omega t}=1$:

$$\Rightarrow Re\left[\left(\frac{d\left(\tilde{V}(z)\right)}{dz}+R\tilde{I}(z)+j\omega L\tilde{I}(z)\right)\right]=0$$

At $\omega t=\pi/2$, $e^{j\omega t}=j$:

$$\Rightarrow Re\left[\left(\frac{d\left(\tilde{V}(z)\right)}{dz}+R\tilde{I}(z)+j\omega L\tilde{I}(z)\right)j\right]=0$$

Transmission Line Equations (contd.)

- Finally we can write:

$$\frac{d\,\tilde{V}(z)}{dz} = -\left(R + j\omega L\right)\tilde{I}(z)$$

$$\frac{d\,\tilde{I}(z)}{dz} = -\left(G + j\omega C\right)\tilde{V}(z)$$

These differential equations can be solved for the phasors along the transmission line

Differentiating with respect to ***z*** gives

$$\frac{d^2\,\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0$$

$$\frac{d^2\,\tilde{I}(z)}{dz^2} - \gamma^2\tilde{I}(z) = 0$$

Transmission Line Wave Equations

Here

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Complex Propagation Constant

$$\gamma = \alpha + j\beta$$

Attenuation Constant (nepers/m)

Phase Constant (radians/m)

Transmission Line Equations (contd.)

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0 \quad \Rightarrow \quad \boxed{\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}}$$

V_0^+ and V_0^- are complex constants

$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\tilde{I}(z) = 0 \quad \Rightarrow \quad \boxed{\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}}$$

I_0^+ and I_0^- are complex constants

$e^{-\gamma z}$ represents a wave propagating in $+z$ *direction* whereas $e^{+\gamma z}$ a wave propagating in $-z$ *direction*.

- There are four unknowns $(V_0^+, V_0^-, I_0^+, I_0^-)$. These can be related as:

$$\tilde{I}(z) = \frac{\gamma}{R + j\omega L}\left[V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right]$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

Transmission Line Equations (contd.)

- Comparison of current phasor solutions lead us to:

$$\frac{V_0^+}{I_0^+} = Z_0 = \frac{-V_0^-}{I_0^-}$$

Characteristic Impedance

$$Z_0 = \frac{R + j\omega L}{\gamma}$$

- It is equal to the ratio of the voltage amplitude to the current amplitude for each of the traveling waves individually (with an additional minus sign in the case of $-z$ propagating wave.
- It is not equal to the ratio of the total voltage $\tilde{V}(z)$ to the total current $\tilde{I}(z)$ unless one of the two is absent.

Definitely not an impedance in traditional sense ……