Lecture – 20 **Date: 17.03.2016**

- Force Due to Magnetic Field
- Magnetic Torque and Moment
- Magnetic Dipole
- Magnetization in Materials
- Magnetic Field in Materials

Forces Due to Magnetic Fields

- Three possible ways for forces due to magnetic fields –
  - Due to moving charged particle
  - On a current element in another magnetic field $\vec{B}$
  - Between two current elements

Force on Charged Particle

- We know, the electric force $\vec{F}_e$ on a stationary or moving electric charge Q in an electric field is given by:

$$\vec{F}_e = Q\vec{E}$$

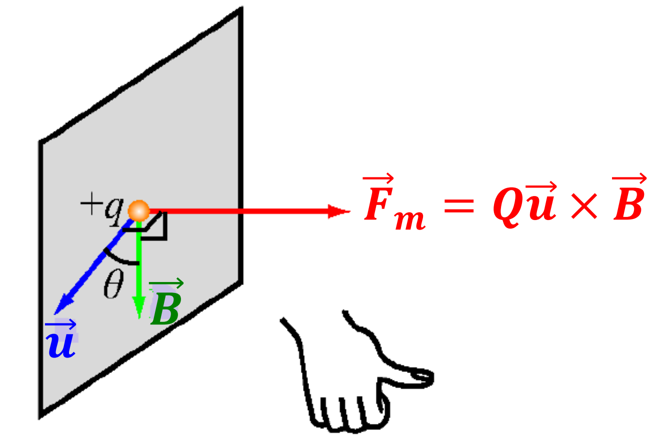
Therefore for positive *Q*: $\vec{F}_e$ and $\vec{E}$ have same directions

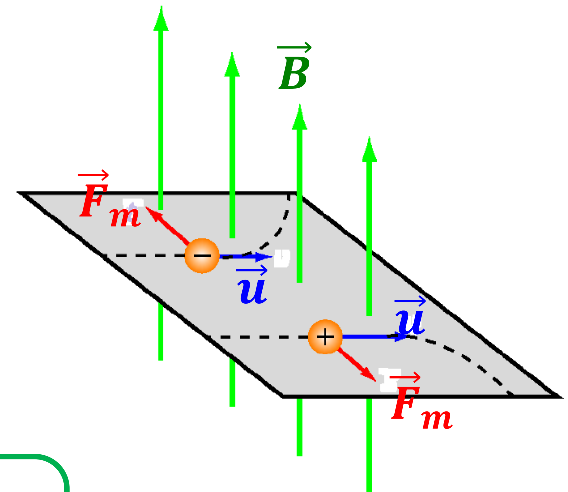
- A magnetic field can exert force only on moving charge. Experimentally it was observed that:

$$\vec{F}_m = Q\vec{u} \times \vec{B}$$

Obviously, $\vec{F}_m$ is perpendicular to both $\vec{u}$ and $\vec{B}$

Force on Charged Particle (contd.)





$\vec{F}_e = Q\vec{E}$ $\vec{F}_m = Q\vec{u} \times \vec{B}$

- $\vec{F}_e$ is independent of the velocity of charge and can perform work on the charge and change its kinetic energy.
- $\vec{F}_m$ depends on the velocity of charge and is normal to it → as a consequence $\vec{F}_m$ can't perform work because it is normal to the direction of motion of the charge ($\vec{F}_m . \vec{u} = 0$).
- The work performed when a particle is displaced by a differential distance $\vec{dl} = \vec{u}dt$ is: $dW = \vec{F}_m . \vec{dl} = (\vec{F}_m . \vec{u})dt = 0.$

Force on Charged Particle (contd.)

- Since no work is done, $\vec{F}_m$ doesn't cause any increase in the kinetic energy of the charge.
- The magnetic field can change the direction of motion of a charged particle, but not its speed.

The magnitude of $\vec{F}_m$ is generally small as compared to $\vec{F}_e$ except at high velocities.

- A charge Q moving in presence of both electric and magnetic fields experience a force:

$$\vec{F} = \vec{F}_e + \vec{F}_m = Q\left(\vec{E} + \vec{u} \times \vec{B}\right)$$

Lorentz Force Equation

- If the mass of moving charge Q is m then:

$$\vec{F} = m\frac{d\vec{u}}{dt} = Q\left(\vec{E} + \vec{u} \times \vec{B}\right)$$

- This equation gives the velocity of charge
- It is important to note that the energy transfer in the process is only due to electric field

Example – 1

- An electron moving in the positive x-direction perpendicular to the magnetic field is deflected in the negative z-direction. What is the direction of the magnetic field?

Example – 2

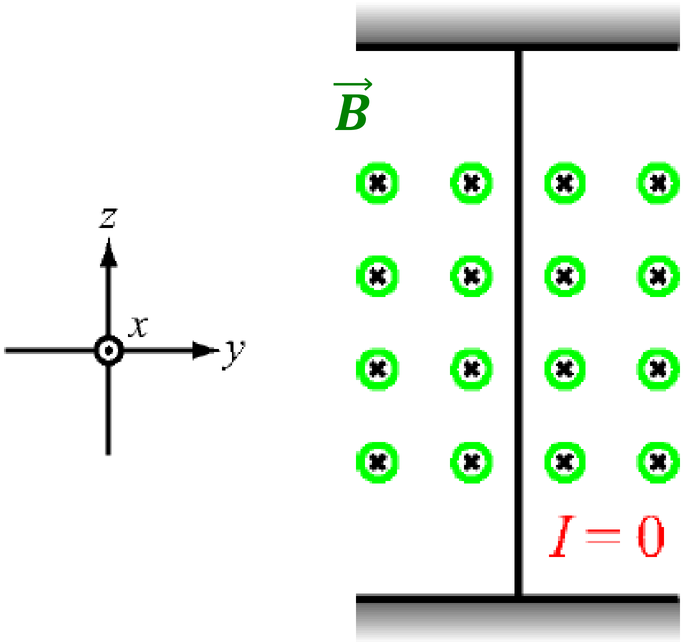
- A proton moving with a speed of $2 \times 10^6\ m/s$ through a magnetic field with magnetic flux density of 2.5T experiences a magnetic force of magnitude $4 \times 10^{-13} N$. What is the angle between the magnetic field and the proton's velocity?

Example – 3

A charged particle with velocity $\vec{u}$ is moving in a medium containing uniform fields $\vec{E} = E\hat{a}_x$ and $\vec{B} = B\hat{a}_y$. What should $\vec{u}$ be so that the particle experiences no net force on it?

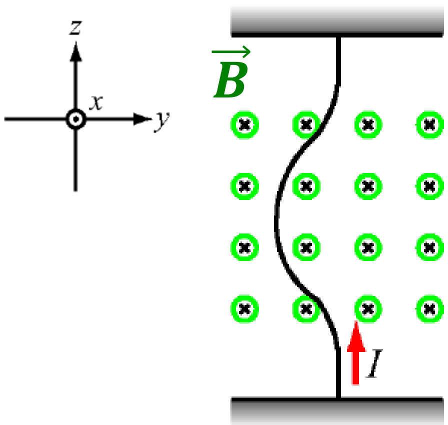
Force on a Current Element

- A current flowing through a conducting wire consists of charged particles drifting through the material of the wire.
- As a consequence, when a current carrying wire is a placed in a magnetic field, it will experience a force equal to the sum of the magnetic forces acting on the charged particles moving within it.

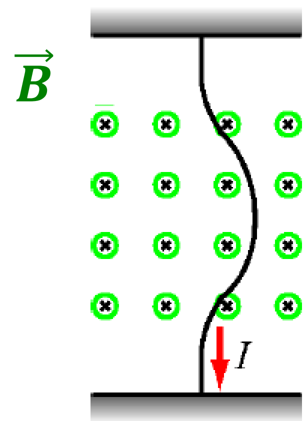


A vertical wire (carrying no current) oriented along the z-direction placed in a magnetic field $\vec{B}$ (in the $-\hat{a}_x$ direction) will experience no force.

Force on a Current Element (contd.)



The wire will deflect to $-\hat{a}_y$ direction if the direction of current flow is upward $(+\hat{a}_z)$ direction.



The wire will deflect to $+\hat{a}_y$ direction if the direction of current flow is downward $(-\hat{a}_z)$ direction.

- Mathematically, this phenomenon can be expressed as:

$$\overrightarrow{dF_m} = I\,\overline{dl} \times \vec{B}$$

force on a current element

- If the current is through a closed path C then:

$$\overrightarrow{F_m} = \oint_C I\,\overline{dl} \times \vec{B}$$

It should be noted that the magnetic field in this expression is due to another source i.e., it is external to the current element → Just for clarity, the magnetic field produced by the current element doesn’t exert a force on itself.

Force on a Current Element (contd.)

- **If the magnetic field is uniform then:**

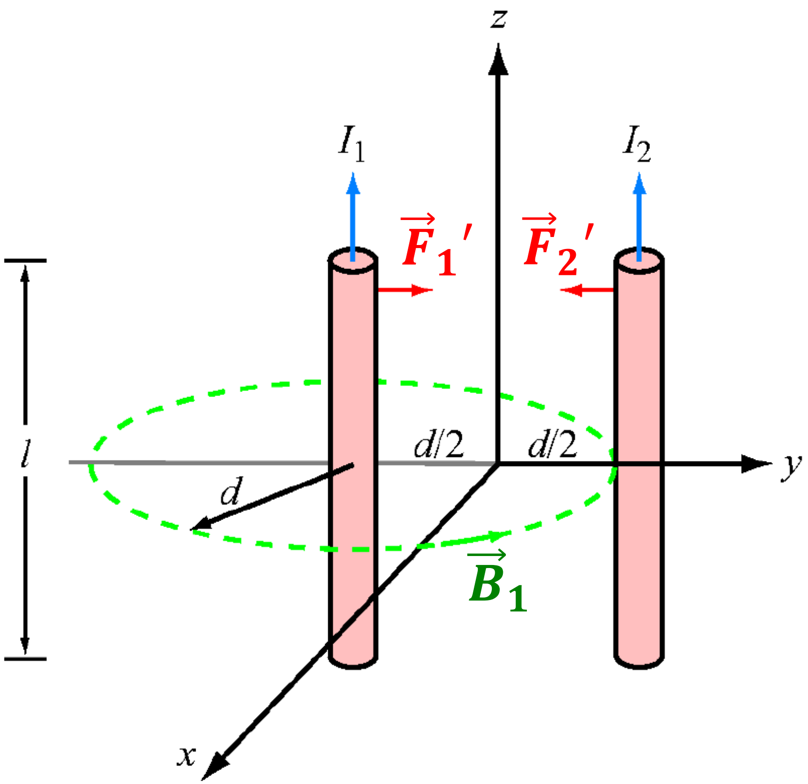
$$\overrightarrow{F_m} = I\left(\oint_C \overline{dl}\right) \times \vec{B} = 0$$

Conveys that the total magnetic force on any closed current loop in a uniform magnetic field is zero.

Example – 4

- A charged particle moves with a uniform velocity $4\hat{a}_x\ m/s$ in a region where $\vec{E} = 20\hat{a}_y\ V/m$ and $\vec{B} = B_0\hat{a}_z\ \frac{Wb}{m^2}$. Determine B_0 such that the velocity of the particle remains constant.

Force Between Two Current Elements



Let us consider two very long (or infinitely long) and straight parallel wires separated by a distance d and carrying currents I_1 and I_2 in the z-direction at $y = -\frac{d}{2}$ and $y = \frac{d}{2}$ respectively.

- Let us denote by $\vec{B}_1$ the magnetic field due to current I_1 at the location of the wire carrying current I_2, and conversely $\vec{B}_2$ the magnetic field due to I_2 at the location of the wire carrying current I_1.

Force Between Two Current Elements (contd.)

- We know:

$$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi d}\hat{a}_x$$

- Therefore the force $\vec{F}_2$ exerted on a length l of wire I_2 due to its presence in field $\vec{B}_1$ is:

$$\vec{F}_2 = I_2\left(l\hat{a}_z \times \vec{B}_1\right) = I_2\left(l\hat{a}_z \times \left(-\hat{a}_x\right)\frac{\mu_0 I_1}{2\pi d}\right) = -\hat{a}_y \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

- The corresponding force per unit length:

$$\vec{F}_2' = -\hat{a}_y \frac{\mu_0 I_1 I_2}{2\pi d}$$

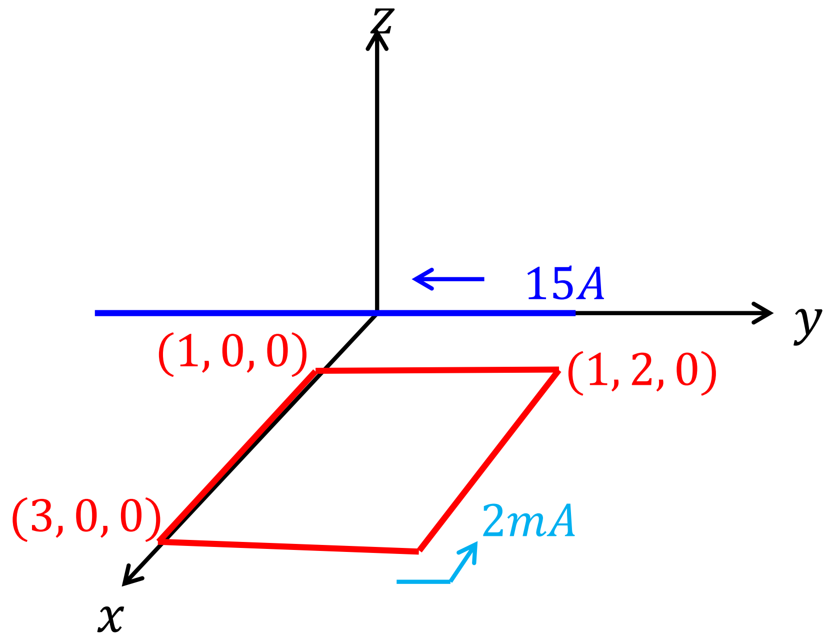
- Similarly, force per unit length exerted on wire carrying I_1 is:

$$\vec{F}_1' = \hat{a}_y \frac{\mu_0 I_1 I_2}{2\pi d}$$

Thus two parallel wires carrying currents in the same direction attract each other with equal force. If the currents are in opposite directions, the wires will repel one another with equal force.

Example – 5

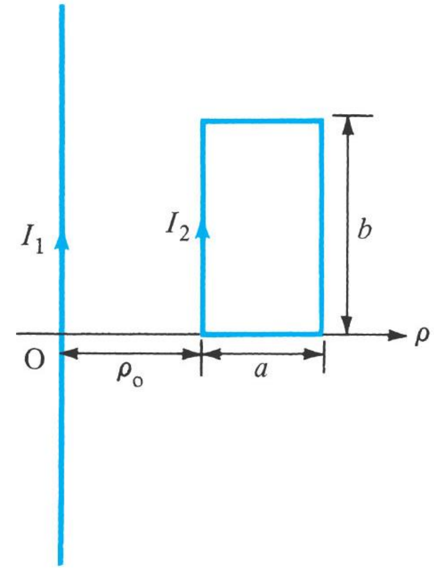
- There is a square loop of wire in the $z = 0$ plane carrying $2mA$ in the field of an infinite filament on the $y - axis$ as shown. Find the total force on the loop.



Example – 6

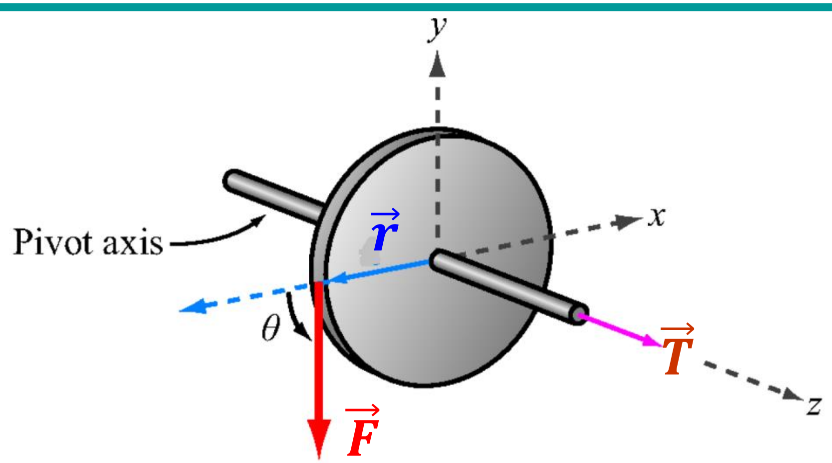
- A rectangular loop carrying current I_2 is placed parallel to infinitely long filamentary wire carrying current I_1 as shown in figure. Show that the force experienced by the loop is given by:

$$\vec{F}_m = -\frac{\mu_0 I_1 I_2 b}{2\pi}\left[\frac{1}{\rho_0} - \frac{1}{\rho_0 + a}\right]\hat{a}_\rho \ \textbf{N}$$



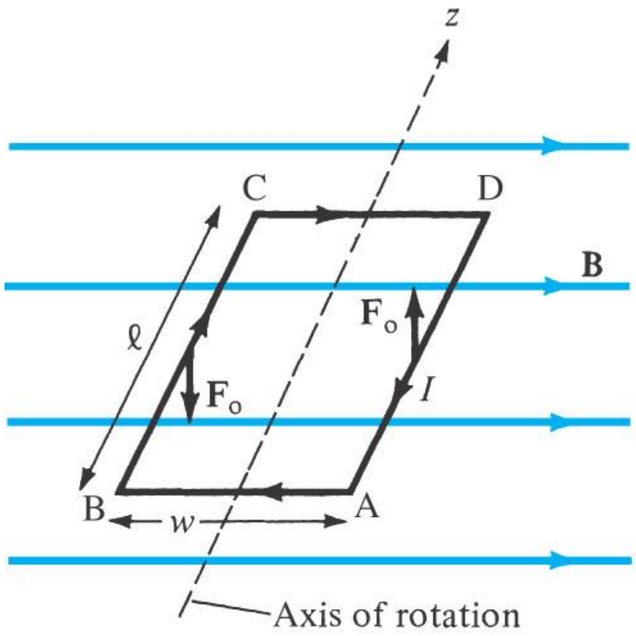
Magnetic Torque

- The torque $\vec{T}$ is the vector product of the force $\vec{F}$ and the moment arm $\bar{r}$.



$$\vec{T} = \vec{r} \times \vec{F}$$

- For the following configuration, the force on the loop is given by:



$$\vec{F} = I\int_B^C \overline{dl} \times \vec{B} + I\int_D^A \overline{dl} \times \vec{B}$$

$$\vec{F} = I\int_0^l dz\hat{a}_z \times \vec{B} + I\int_l^0 dz\hat{a}_z \times \vec{B}$$

$$\therefore \vec{F} = \vec{F}_0 + \left(-\vec{F}\right)_0 = 0$$

Magnetic Torque (contd.)

- Where: $\left|\vec{F}_0\right| = IBl$ ← $\vec{B}$ is considered uniform here

- Apparently no force is exerted on the loop → however, $\vec{F}_0$ and $-\vec{F}_0$ acts on two different points on the loop, thereby creating a couple.
- If normal to the loop plane makes an angle α with $\vec{B}$ then:

$$\left|\vec{T}\right| = BIlw\sin\alpha \quad \Rightarrow \quad \left|\vec{T}\right| = BIS\sin\alpha$$

Let us define a quantity: $\vec{m} = IS\hat{a}_n$ → Magnetic dipole moment

- Therefore: $\vec{T} = \vec{m}\times\vec{B}$

Although this expression is obtained for rectangular loop but is applicable for planar loop of any arbitrary shape.

Example – 7

- A rectangular coil of area $10\ cm^2$ carrying current $50A$ lies on plane $2x + 6y - 3z = 7$ such that the magnetic moment of the coil is directed away from the origin. Calculate its magnetic moment.

Example – 8

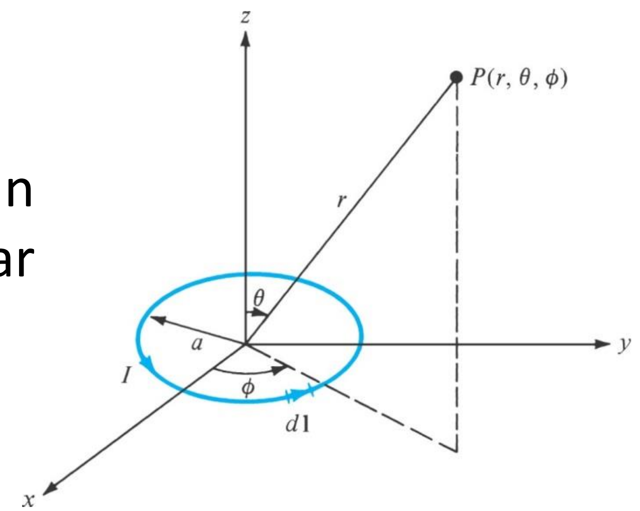
The coil of last example is surrounded by a uniform field $0.6\hat{a}_x + 0.4\hat{a}_y + 0.5\hat{a}_z\ Wb/m^2$.

(a) Find the torque on the coil.

(b) Show that the torque on the coil is maximum if placed on plane $2x - 8y + 4z = \sqrt{84}$. Calculate the magnitude of the maximum torque.

Magnetic Dipole

- A bar magnet or small filamentary current loop is usually referred to as a magnetic dipole.
- The reason will be soon apparent.
- Let us consider the magnetic field $\vec{B}$ at an observation point $P(r,\theta,\phi)$ due to a circular loop carrying current I.



- The magnetic vector potential at P is:

$$\vec{A}=\frac{\mu_0 I}{4\pi}\oint\frac{\overline{dl}}{r}$$

$$\vec{A}=\frac{\mu_0\vec{m}\times\hat{a}_r}{4\pi r^2}$$

$$\vec{B}=\nabla\times\vec{A}=\frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\hat{a}_r+\sin\theta\hat{a}_\theta\right)$$

These are similar to the expressions for V and $\vec{E}$ due to an electric dipole

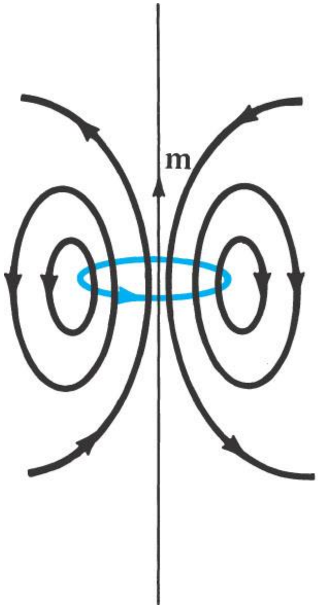
$$V=\frac{\vec{p}.\hat{a}_r}{4\pi\varepsilon_0 r^2}$$

$$\vec{E}=-\nabla V=\frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\hat{a}_r+\sin\theta\hat{a}_\theta\right)$$

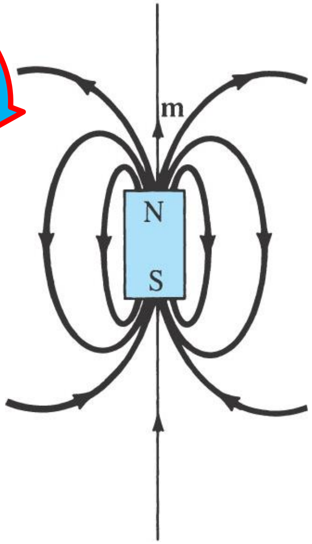
reasonable to regard a small current loop as a magnetic dipole

Magnetic Dipole (contd.)

- $\vec{B}$ lines around the magnetic dipole can be illustrated as:



- A short permanent magnet can also be considered as a magnetic dipole.
- The $\vec{B}$ lines due to bar are similar to those due to a small current loop.



Magnetic Materials

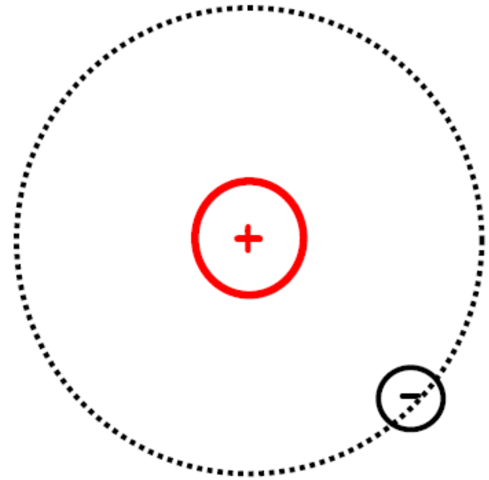
- Recall in dielectrics, electric dipoles were created when an $\vec{E} - field$ was applied.
- Therefore, we defined permittivity ε, electric flux density $\vec{D}$, and a new set of electrostatic equations.
- Recall that **atoms and molecules**, having both positive (i.e., protons) and negative (i.e., electron) charged particles can form **electric dipoles**.
- It will be apparent that that atoms and molecules can also form **magnetic dipoles**!

Q: How??

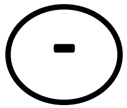
A: Recall a magnetic dipole is formed when current flows in a **small loop**. Current, of course, is **moving charge**, therefore charge moving around a small loop forms a magnetic dipole.

Molecules and atoms **often** exhibit electrons moving around in small loops!

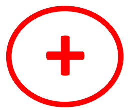
Magnetic Materials (contd.)



- Again, let us use our **ridiculously** simple model of an atom:

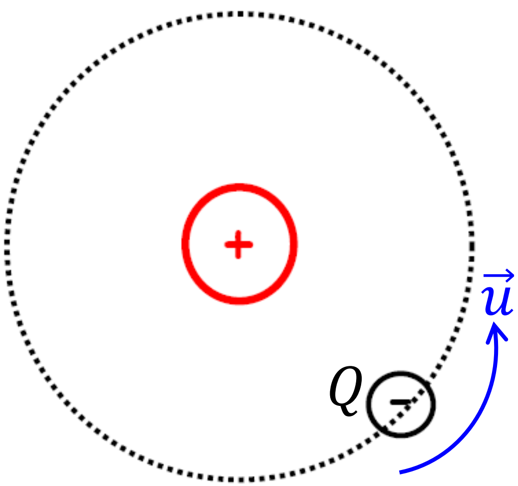


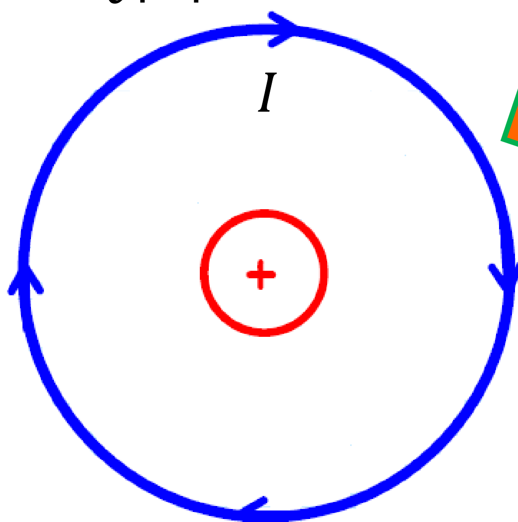
→ electron
(negative charge)



→ nucleus
(positive charge)

- An electron with charge Q orbiting around a nucleus at velocity $\vec{u}$ forms a **small current loop**, where $I = Q|\vec{u}|$.





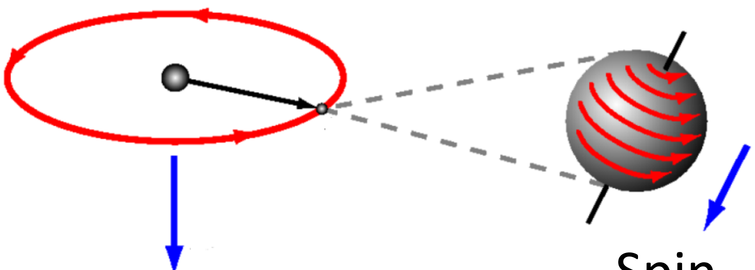
This forms a magnetic dipole

This is a very simple atomic explanation of how magnetic dipoles are formed in material.

Magnetic Materials (contd.)

- In reality, the physical mechanisms that lead to magnetic dipoles can be **far** more complex.

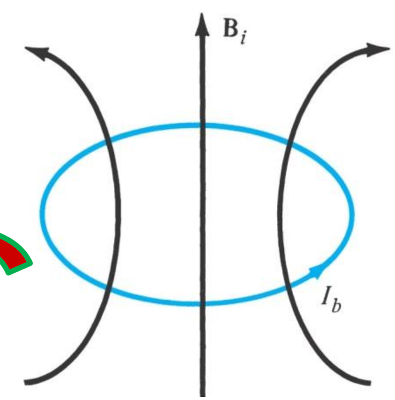
- For example, **electron spin** can also create a magnetic dipole moment.



Orbital magnetic moment

Spin magnetic moment

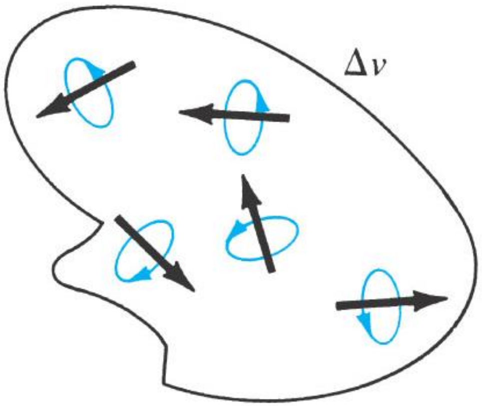
- Both these electronic motions produce internal magnetic fields $\vec{B}_i$ that are similar to the magnetic field produced by a current loop as shown.



This equivalent current loop has a magnetic moment of $\vec{m} = I_b S \hat{a}_n$, where S is the area of the loop and I_b is the bound current (bound to the atom).

Magnetic Materials (contd.)

- Typically, the atoms/molecules of materials exhibit either **no** magnetic dipole moment (i.e., $\vec{m} = 0$), or the dipole moments of each atom/molecule are **randomly oriented**, such that the **net** dipole moment is **zero**.



$$\vec{B} = 0, \vec{M} = 0$$

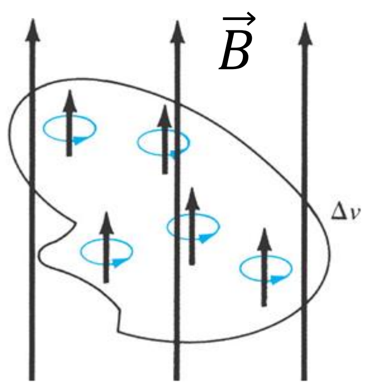
- Therefore, for N randomly oriented magnetic dipoles $\vec{m}_n$, we find:

$$\frac{1}{N}\sum_{n}\vec{m}_n = 0$$

- Similarly, the **total** magnetic flux density created by these magnetic dipoles is **also zero**:

$$\sum_{n}\vec{B}_n = 0$$

- However, sometimes the magnetic dipole moment of each atom/molecule is **not** randomly oriented, but in fact are **aligned**!



Magnetic Materials (contd.)

Q: Why would these magnetic dipoles be aligned?

A: Two possible reasons:

1) the material is a **permanent magnet**.

2) the material is immersed in some **magnetizing field** $\vec{B}$.

The Magnetization Vector

- Recall that we defined the **Polarization vector** of a dielectric material as the **electric dipole density**, i.e.:

$$\vec{P} \doteq \frac{\sum \vec{p}_n}{\Delta v}\left[\frac{dipole\_moment}{unit\_volume} = \frac{C}{m^2}\right]$$

- Similarly, we can define a **Magnetization vector** of a material to be the density of **magnetic** dipole moments:

$$\vec{M} \doteq \frac{\sum \vec{m}_n}{\Delta v \to 0}\left[\frac{magnetic\_dipole\_moment}{unit\_volume} = \frac{A}{m}\right]$$

A medium for which $\vec{M}$ is not zero everywhere is said to be magnetized

The Magnetization Vector (contd.)

- Note if the dipole moments of atoms/molecules within a material are **completely random**, the Magnetization vector will be **zero** (i.e., $\vec{M} = 0$).
- However, if the dipoles are **aligned**, the Magnetization vector will be **non-zero** (i.e., $\vec{M} \neq 0$).
- Furthermore, for a differential volume dv', the magnetic moment is $\overrightarrow{dm} = \vec{M} dv'$.
- Therefore the vector magnetic potential due to $\overrightarrow{dm}$ can be expressed as:

$$\overrightarrow{dA} = \frac{\mu_0 \vec{M} \times \hat{a}_R}{4\pi R^2} dv' \quad \Rightarrow \quad \overrightarrow{dA} = \frac{\mu_0 \vec{M} \times \vec{R}}{4\pi R^3} dv'$$

$$\therefore \vec{A} = \iiint_v \frac{\mu_0 \vec{M} \times \vec{R}}{4\pi R^3} dv'$$

The Magnetization Vector (contd.)

$$\therefore \vec{A} = \iiint_v \frac{\mu_0 \vec{M} \times \vec{R}}{4\pi R^3} dv'$$

Q: This is freaking me out!! I thought that **currents** $\vec{J}$ were responsible for creating magnetic vector potential. In fact, I could have sworn that:

$$\vec{A} = \iiint_v \frac{\mu_0 \vec{J}}{4\pi R} dv'$$

A: Relax, **both** expressions are correct!

The Magnetization Currents

$$\rho_{vp} = -\nabla.\vec{P}$$

$$\rho_{sp} = \vec{P}.\hat{a}_n$$

- Recall that we could attribute the electric field created by Polarization Vector $\vec{P}$ to **polarization** (i.e., bound) **charges** ρ_{vp} and ρ_{sp}.
- Similarly, we can **attribute** the magnetic vector potential (and therefore the magnetic flux density) created by Magnetization Vector $\vec{M}$ to **Magnetization Currents** $\vec{J}_b$ and $\vec{K}_b$, the bound volume current density (i.e., magnetization current density) and bound surface current density respectively.
- We have:

$$\vec{A} = \iiint_v \frac{\mu_0 \vec{M} \times \vec{R}}{4\pi R^3} dv'$$

- Earlier we came across the expression:

$$\frac{\vec{R}}{R^3} = \nabla'\left(\frac{1}{R}\right)$$

- Therefore:

$$\vec{A} = \frac{\mu_0}{4\pi} \iiint_v \vec{M} \times \nabla'\left(\frac{1}{R}\right) dv'$$

The Magnetization Currents (contd.)

- We can use the identity: $$\vec{M} \times \nabla'\left(\frac{1}{R}\right) = \frac{1}{R}\nabla' \times \vec{M} - \nabla' \times \frac{\vec{M}}{R}$$

- Therefore we can express: $$\vec{A} = \frac{\mu_0}{4\pi}\iiint_v \frac{\nabla' \times \vec{M}}{R} dv' + \frac{\mu_0}{4\pi}\oiint_S \frac{\vec{M} \times \hat{a}_n}{R} ds'$$

$$\vec{A} = \frac{\mu_0}{4\pi}\iiint_v \frac{\vec{J}_b}{R} dv' + \frac{\mu_0}{4\pi}\oiint_S \frac{\vec{K}_b ds'}{R}$$

where:

$$\vec{J}_b = \nabla \times \vec{M}$$

$$\vec{K}_b = \vec{M} \times \hat{a}_n$$

Therefore, we find that the magnetization of some material, as described by magnetization vector $\vec{M}$, creates **effective** currents $\vec{J}_b$ and $\vec{K}_b$. We call these effective currents **magnetization currents.**

$\vec{J}_b$ and $\vec{K}_b$ can be derived from $\vec{M}$ and hence are not commonly used

The Magnetic Field

- Now that we have defined **magnetization current**, we find that Ampere's Law for fields **within some material** becomes:

$$\nabla \times \vec{B} = \mu_0\left(\vec{J} + \vec{J}_b\right)$$

$$\nabla \times \vec{B} = \mu_0\left(\vec{J} + \nabla \times \vec{M}\right)$$

- This of course is **analogous** to the expression we derived for **Gauss's Law** in a dielectric media:

$$\nabla.\vec{E} = \frac{\rho_v + \rho_{vp}}{\varepsilon_0} = \frac{\rho_v - \nabla.\vec{P}}{\varepsilon_0}$$

Recall that we **removed** the polarization charge from this expression by defining a **new** vector field $\vec{D}$, leaving us with the more **general** expression of Gauss's Law:

$$\nabla.\vec{D} = \rho_v$$

The Magnetic Field (contd.)

Q: Can we similarly define a **new** vector field to "take care" of **magnetization** current ??

A: Yes! We call this vector field the **magnetic field** $\vec{H}$.

- Let's begin by **rewriting** Ampere's Law as:

$$\nabla \times \vec{B} - \mu_0 \vec{J}_b = \mu_0 \vec{J}$$

- Yuck! Now we see clearly the problem. In **free space**, if we know current distribution $\vec{J}$, we can find the resulting magnetic flux density $\vec{B}$ using the **Biot-Savart** Law:

$$\vec{B} = \frac{\mu_0}{4\pi} \iiint_v \frac{\vec{J} \times \vec{R}}{R^3} dv'$$

this is the solution for current in **free space**! It is **no longer valid** if some **material** is present!

The Magnetic Field (contd.)

Q: Why?

A: Because, the magnetic flux density produced by current $\vec{J}$ may **magnetize** the material (i.e., produce magnetic dipoles), thus producing **magnetization currents** $\vec{J}_b$.

These magnetization currents $\vec{J}_b$ will **also** produce a magnetic flux density—a **modification** of vector field $\vec{B}$ that is **not** accounted for in the Biot-Savart expression shown above!

- To determine the correct solution, we first recall that: $\vec{J}_b = \nabla \times \vec{M}$
- Therefore Ampere's Law is: $\nabla \times \vec{B} - \mu_0 \nabla \times \vec{M} = \mu_0 \vec{J}$

$$\nabla \times \left[\frac{\vec{B}}{\mu_0} - \vec{M} \right] = \vec{J}$$

- Now let's define a **new** vector field $\vec{H}$, called the **magnetic field**: $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$
- Therefore: $\nabla \times \vec{H} = \vec{J}$

The Magnetic Field (contd.)

- For most materials, it has been found that the magnetization vector $\vec{M}$ is directly **proportional** to the magnetic field $\vec{H}$:

$$\vec{M} = \chi_m \vec{H}$$

where the proportionality coefficient χ_m is the **magnetic susceptibility** of the material.

- Note that for a given magnetic field $\vec{H}$, as χ_m **increases**, the magnetization vector $\vec{M}$ **increases**.
- Magnetic susceptibility χ_m therefore indicates how **susceptible** the material is to **magnetization**.
- In other words, χ_m is a measure of how easily (or difficult) it is to create and align **magnetic dipoles** (from atoms/molecules) within the **material**.

Again, note the **analogy** to electrostatics. We defined earlier **electric** susceptibility χ_e, which indicates how susceptible a material is to **polarization** (i.e., the creation of **electric** dipoles).

The Magnetic Field (contd.)

- Therefore: $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ → $\vec{H} = \frac{\vec{B}}{\mu_0} - \chi_m \vec{H}$ → $\mu_0 (1 + \chi_m) \vec{H} = \vec{B}$

$$\therefore \mu = \mu_0 (1 + \chi_m)$$

Hey! We know the magnetic field $\vec{H}$ and magnetic flux density are related by a **simple constant**!

$$\vec{B} = \mu \vec{H}$$

$$\mu \doteq material\_permeability \left[\frac{Henrys}{meter} \right]$$

- The expression can be **further** simplified by defining a **relative** permeability:

$$\mu_r = 1 + \chi_m$$

The Magnetic Field (contd.)

- Therefore: $\vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H}$ ← Only valid for linear isotropic materials
- In other words, if the **relative** permeability of some material was, say, $\mu_r = 2$, then the **permeability** of the material is **twice** that of the permeability of **free space** (i.e., $\mu = 2\mu_0$). This perhaps is more readily evident when we write:

$$\mu_r = \frac{\mu}{\mu_0}$$

Note that μ and/or μ_r are **proportional** to magnetic susceptibility χ_m. As a result, permeability is likewise an indication of how **susceptible** a material is to **magnetization**.

- If $\mu_r = 1$, this susceptibility is that of **free space** (i.e., **none**!).
- Alternatively, a **large** μ_r indicates a material that is **easily magnetized**.
- For example, the relative permeability of **iron** is $\mu_r = 4000$!

The Magnetic Field (contd.)

- **Now**, we are **finally** able to determine the **magnetic flux density** in some **material,** produced by current density $\vec{J}$!
- Since $\vec{B} = \mu\vec{H}$ and:

$$\vec{H} = \frac{1}{4\pi}\iiint_v \frac{\vec{J}\times\vec{R}}{R^3}dv'$$

- we find the desired solution:

$$\vec{B} = \frac{\mu}{4\pi}\iiint_v \frac{\vec{J}\times\vec{R}}{R^3}dv'$$

Comparing this result with the Biot-Sarvart Law for **free space**, we see that the only difference is that μ_0 has been replaced with μ.

This last result is therefore a **more general** form of the Biot-Savart Law, giving the correct result for fields within some **material** with permeability μ. Of course, the "material" **could** be free space. However, the expression above will **still** provide the **correct** answer; because for free space $\mu = \mu_0$, thus returning the equation to its **original** (i.e., free space) form!

The Magnetic Field (contd.)

Summarizing, we can attribute the existence of a **magnetic field** $\vec{H}$ to **conduction** current $\vec{J}$, while we attribute the existence of **magnetic flux density** to the **total** current density, including the magnetization current.

Magnetic Materials

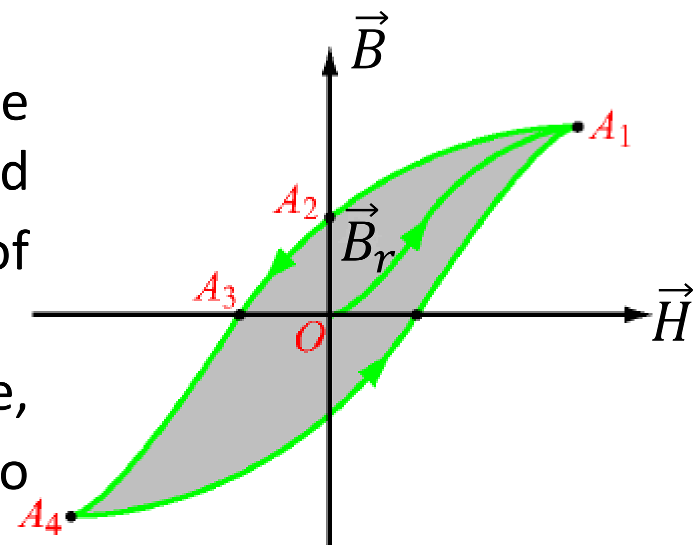
- A material usually is classified as diamagnetic, paramagnetic, or ferromagnetic on the basis of the value of its χ_m.
- **Diamagnetic** materials have negative susceptibilities **whereas** **paramagnetic** materials have positive susceptibilities.
- The absolute **magnitude of** $\boldsymbol{\chi_m}$ is of the **order of** $\mathbf{10^{-5}}$ for both classes of materials, which for most applications allows us to ignore χ_m.
- **Therefore,** $\boldsymbol{\mu_r \cong 1}$ **or** $\boldsymbol{\mu = \mu_0}$ **for diamagnetic and paramagnetic substances, which include dielectric materials and most metals.**
- In contrast, $|\boldsymbol{\mu_r}| \gg \mathbf{1}$ **for ferromagnetic materials.**

	Diamagnetism	Paramagnetism	Ferromagnetism
Common Substances	Bismuth, copper, diamond, gold, lead, mercury, silver, silicon	Aluminium, calcium, chromium, magnesium, niobium, platinum, tungsten	Iron, nickel, cobalt
Typical value of χ_m	$\cong -10^{-5}$	$\cong 10^{-5}$	$\gg 1$ *and hysteretic*
Typical value of μ_r	≈ 1	≈ 1	$\gg 1$ *and hysteretic*

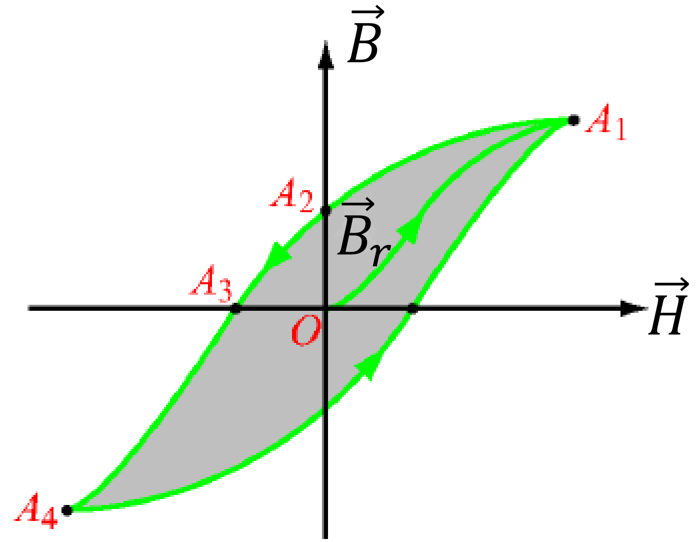
Magnetic Materials (contd.)

- Ferromagnetic materials exhibit unique magnetic properties due to the fact that their magnetic moments tend to readily align along the direction of external magnetic field.
- The magnetization behavior of a ferromagnetic material can be understood in terms of its $\vec{B} - \vec{H}$ magnetization curve.

- Suppose, we start with an unmagnetized sample of iron, denoted by point O.
- Increase in $\vec{H}$, for example by increasing the current passing through a wire wound around the sample, increases $\vec{B}$ up to point of saturation A_1.
- Removal of current through the wire (i.e, decrease in $\vec{H}$ to zero) doesn't bring back $\vec{B}$ to zero.
- Instead there remains a residual flux density $\vec{B}_r$



Magnetic Materials (contd.)

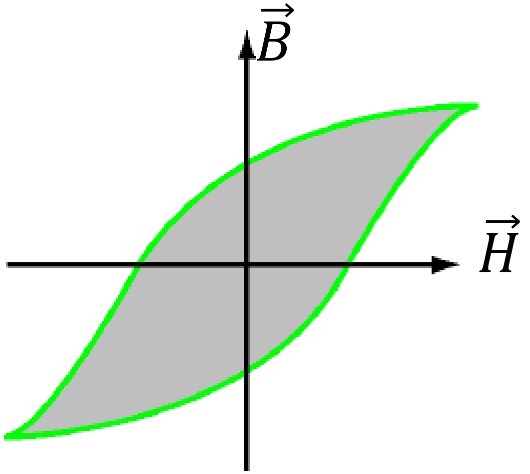


- The presence of $\vec{B}_r$ enables the iron sample to be used as permanent magnet.
- Reversing the direction of $\vec{H}$ and increasing its intensity causes decrease in $\vec{B}$ → further increase in $\vec{H}$ while maintaining the direction, the magnetization moves to saturation point A_4.
- Finally as $\vec{H}$ is made to return to zero and then increased again then the curve follows the path from A_4 to A_1.
- This overall process is called magnetic hysteresis.
- The existence of ***hysteresis loop*** implies that the magnetization process in ferromagnetic materials depends not only on the magnetic field $\vec{H}$, but also on the magnetic history of the material.
- The shape and extent of the hysteresis loop depend on the properties of the ferromagnetic material and the peak-to-peak range over which $\vec{H}$ is made to vary.

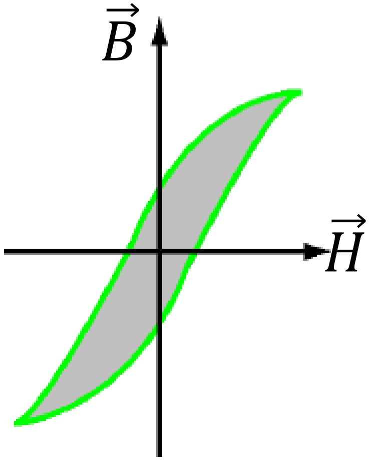
Magnetic Materials (contd.)

- Hard ferromagnetic materials are characterized by wide hysteresis loops.
- They can't be easily demagnetized by an external magnetic field because they have large residual magnetization $\vec{B}_r$.
- Hard ferromagnetic materials are used in the fabrication of permanent magnets for motors and generators.
- Soft ferromagnetic materials have narrow hysteresis loops and can be easily magnetized and demagnetized.

To demagnetize any ferromagnetic material, the material is subjected to several hysteresis cycles while gradually decreasing the peak-to-peak range of applied field



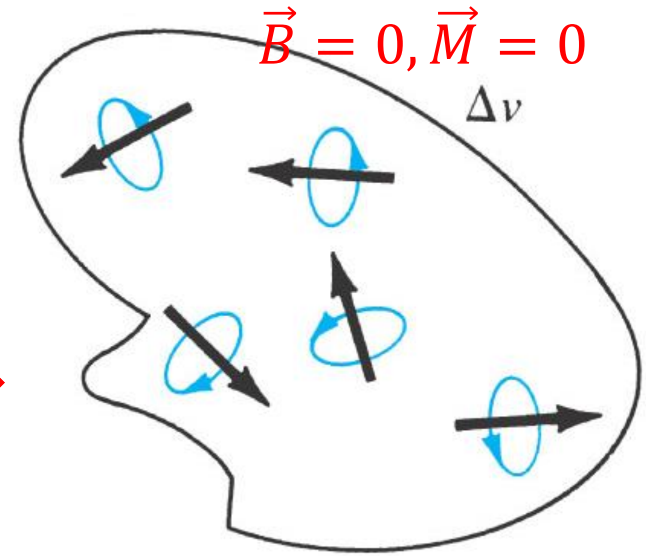
Hard material



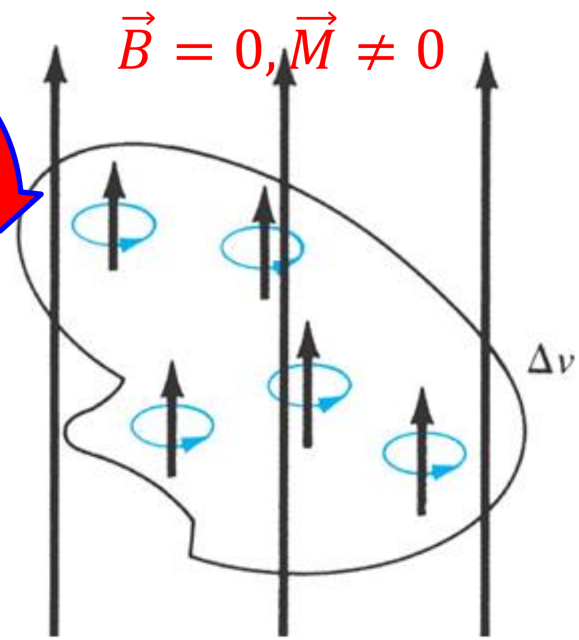
Soft material

Permanent Magnets

- For **most** magnetic material (i.e., where $\mu = \mu_0$), we find that the magnetization vector $\vec{M}$ will return to **zero** when a magnetization field $\vec{B}$ is removed. In other words, the **magnetic dipoles** will vanish, or at least return to their random state.



- However, some magnetic material, called **ferromagnetic** material, **retain** its dipole orientation, even when the magnetizing field is removed !



In this case, a **permanent magnet** is formed (just like the ones you stick on your fridge)!

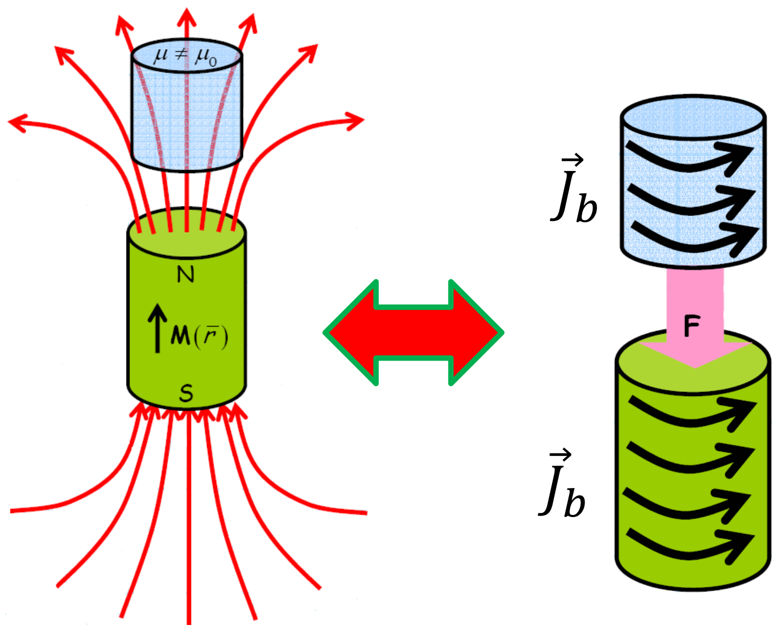
Permanent Magnets

- Ferromagnetic materials have **numerous applications**.
- For example, they will **attract** magnetic material.

Q: How?

A: A permanent magnet will produce **everywhere** a magnetic flux density $\vec{B}$, which we can **either** attribute to the magnetic **dipoles** within the material, **or** to the equivalent magnetic **current** $\vec{J}_b$.

The magnetic flux density produced by the magnet will act as a **magnetizing** field for some **other** magnetic material nearby, thus creating a **second** magnetization **current** $\vec{J}_b$ within the nearby material. The magnetization currents of the material and the magnet will **attract**!



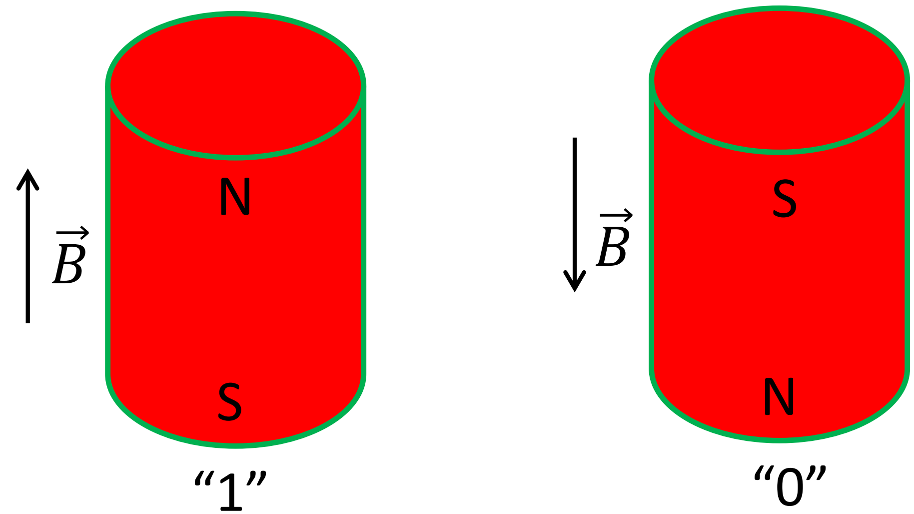
Permanent Magnets

- Another interesting application of ferromagnetic material is in non-volatile **data storage** (e.g., tape or disk). Ferromagnetics can be used as **binary memory** !

Q: How?

A: Recall that the magnetization vector in ferromagnetic material retains its direction after the magnetizing field $\vec{B}$ has been removed. In other words, it "**remembers**" the direction of the magnetizing field.

- We can assign each of **two** different magnetizing directions, therefore, a **binary** state:



Permanent Magnets

- If ferromagnetic material is **embedded** in a tape or disk, we can magnetize (e.g., **write**) small sections of the media, or detect the magnetization (e.g., **read**) small sections of the media.