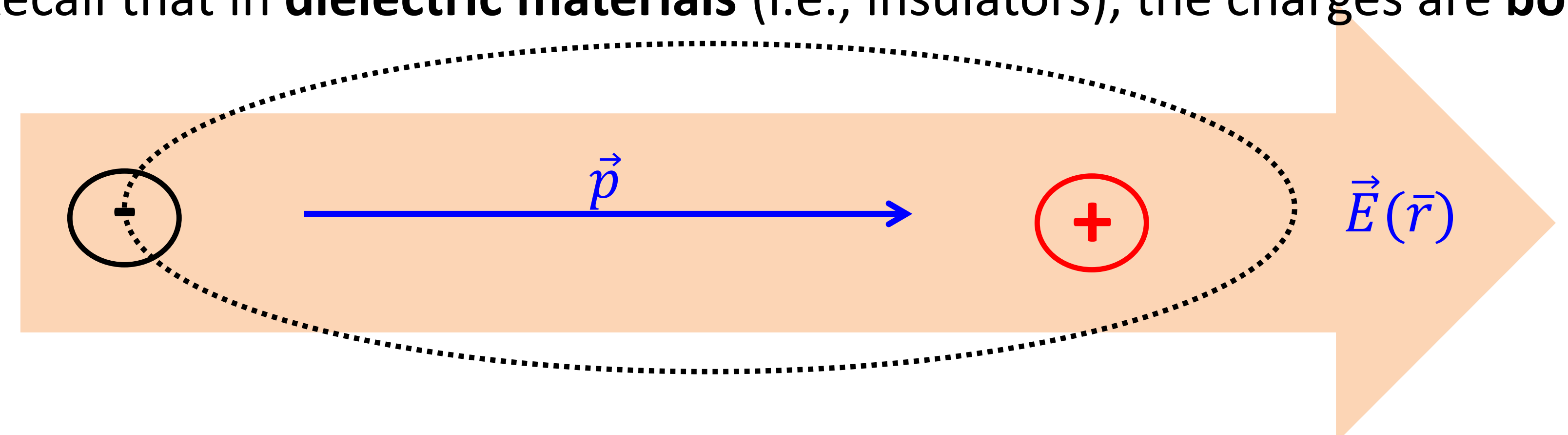
Lecture – 15 Date: 29.02.2016

- Polarization Vector
- Electrostatic Boundary Conditions

If a **dielectric** material is immersed in an **electric field**, each atom/molecule in the material will form an **electric dipole!**

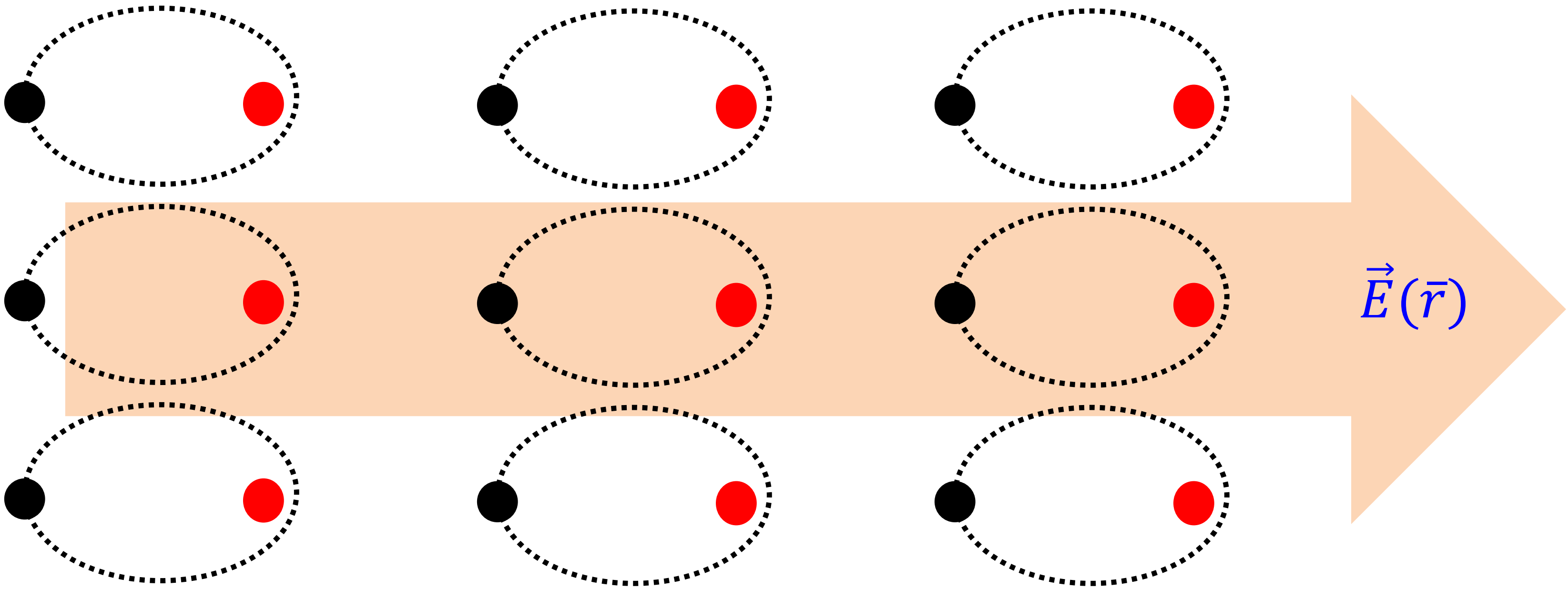
- Recall that in **dielectric materials** (i.e., insulators), the charges are **bound**.



As a result, atoms/molecules form **electric dipoles** when an electric field is present!

The Polarization Vector

- Note that even for some **small** volume Δv, there are **many** atoms/molecules present; therefore there will be **many** electric dipoles.



- We define an **average** dipole moment, per unit volume, called the **Polarization Vector** $\vec{P}(\bar{r})$.

$$\boxed{\vec{P}(\bar{r}) \doteq \frac{\sum \vec{p}_n}{\Delta v}\left[\frac{dipole\_moment}{unit\_volume} = \frac{C}{m^2}\right]}$$

The Polarization Vector (contd.)

Q: How are vector fields $\vec{P}(\bar{r})$ and $\vec{E}(\bar{r})$ related??

A: The direction of each dipole moment is the same as the polarizing electric field. Thus $\vec{P}(\bar{r})$ and $\vec{E}(\bar{r})$ have the same direction. Magnitudes are related by a unitless scalar value $\chi_e(\bar{r})$, called **electric susceptibility:**

$$\boxed{\vec{P}(\bar{r}) = \varepsilon_0 \chi_e(\bar{r}) \vec{E}(\bar{r})}$$

Electric susceptibility is a **material parameter** indicating the "stretchability" of the dipoles.

Q: Can we determine the **fields** created by a polarized material?

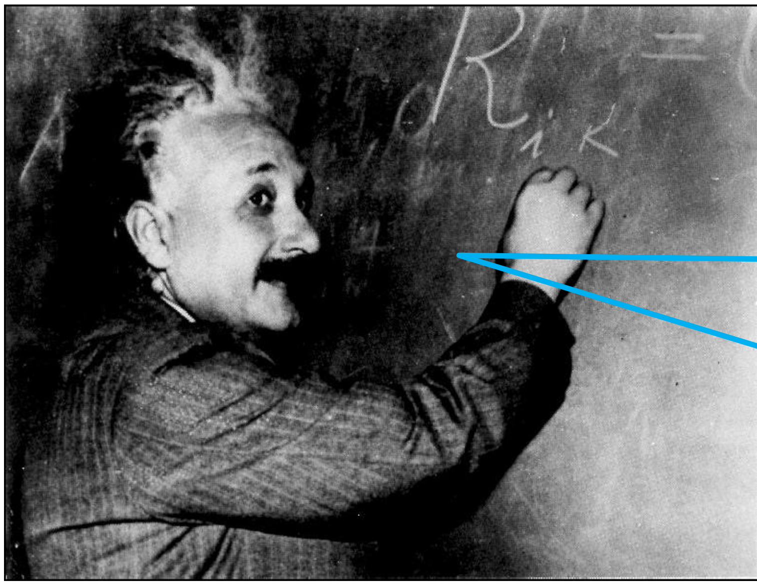
A: Recall the **electric potential field** created by **one** dipole is:

$$V(\bar{r}) = \frac{\vec{p}.\left(\bar{r} - \bar{r}'\right)}{4\pi\varepsilon_0 \left|\bar{r} - \bar{r}'\right|^3}$$

- Therefore, the electric potential field created by a **distribution of** dipoles (i.e., $\vec{P}(\bar{r})$) across some volume v is:

$$V(\bar{r}) = \iiint_v \frac{\vec{P}(\bar{r}').\left(\bar{r} - \bar{r}'\right)}{4\pi\varepsilon_0 \left|\bar{r} - \bar{r}'\right|^3} dv'$$

The Polarization Vector (contd.)



Q: But I thought **scalar** charge distributions $\rho_v(\bar{r})$ and $\rho_s(\bar{r})$created the electric potential field $V(\bar{r})$. Now you are saying that potential fields are created by the **vector** field $\vec{P}(\bar{r})$!?!

A: As we will soon see, the polarization **vector** $\vec{P}(\bar{r})$ creates equivalent charge **distributions**—we will get the correct answer for $V(\bar{r})$ from **either** source!

Polarization Charge Distributions

- Consider a chunk of **dielectric** material with volume v.

Say this dielectric material is immersed in an **electric field** $\vec{E}(\bar{r})$, therefore creating atomic **dipoles** with density $\vec{P}(\bar{r})$.

Q: What **electric potential field** $V(\bar{r})$ is created by these diploes?

A: We know that:

$$V(\bar{r}) = \iiint_v \frac{\vec{P}(\bar{r}').(\bar{r}-\bar{r}')}{4\pi\varepsilon_0 |\bar{r}-\bar{r}'|^3} dv'$$

Using Divergence Theorem

It can be shown that:

$$V(\bar{r}) = \iiint_v \frac{\vec{P}(\bar{r}').(\bar{r}-\bar{r}')}{4\pi\varepsilon_0 |\bar{r}-\bar{r}'|^3} dv' = \frac{1}{4\pi\varepsilon_0}\iiint_v \frac{-\nabla.\vec{P}(\bar{r}')}{|\bar{r}-\bar{r}'|} dv' + \frac{1}{4\pi\varepsilon_0}\oint\!\!\oint_S \frac{\vec{P}(\bar{r}').\hat{a}_n(\bar{r})}{|\bar{r}-\bar{r}'|} dS'$$

where S is the **closed** surface that surrounds volume v, and $\hat{a}_n(\bar{r})$ is the unit vector **normal** to surface S (pointing **outward**).

Polarization Charge Distributions (contd.)

$$V(\bar{r}) = \iiint_v \frac{\vec{P}(\bar{r}').(\bar{r}-\bar{r}')}{4\pi\varepsilon_0 |\bar{r}-\bar{r}'|^3} dv' = \frac{1}{4\pi\varepsilon_0} \iiint_v \frac{-\nabla.\vec{P}(\bar{r}')}{|\bar{r}-\bar{r}'|} dv' + \frac{1}{4\pi\varepsilon_0} \oiint_S \frac{\vec{P}(\bar{r}').\hat{a}_n(\bar{r})}{|\bar{r}-\bar{r}'|} dS'$$

This complicated result is only important when we compare it to the electric potential created by **volume** charge density $\rho_v(\bar{r})$ and **surface** charge density $\rho_s(\bar{r})$.

- If both volume and surface charge are present, the **total** electric potential field is:

$$V(\bar{r}) = \frac{1}{4\pi\varepsilon_0} \iiint_v \frac{\rho_v(\bar{r}')}{|\bar{r}-\bar{r}'|} dv' + \frac{1}{4\pi\varepsilon_0} \oiint_S \frac{\rho_S(\bar{r}')}{|\bar{r}-\bar{r}'|} dS'$$

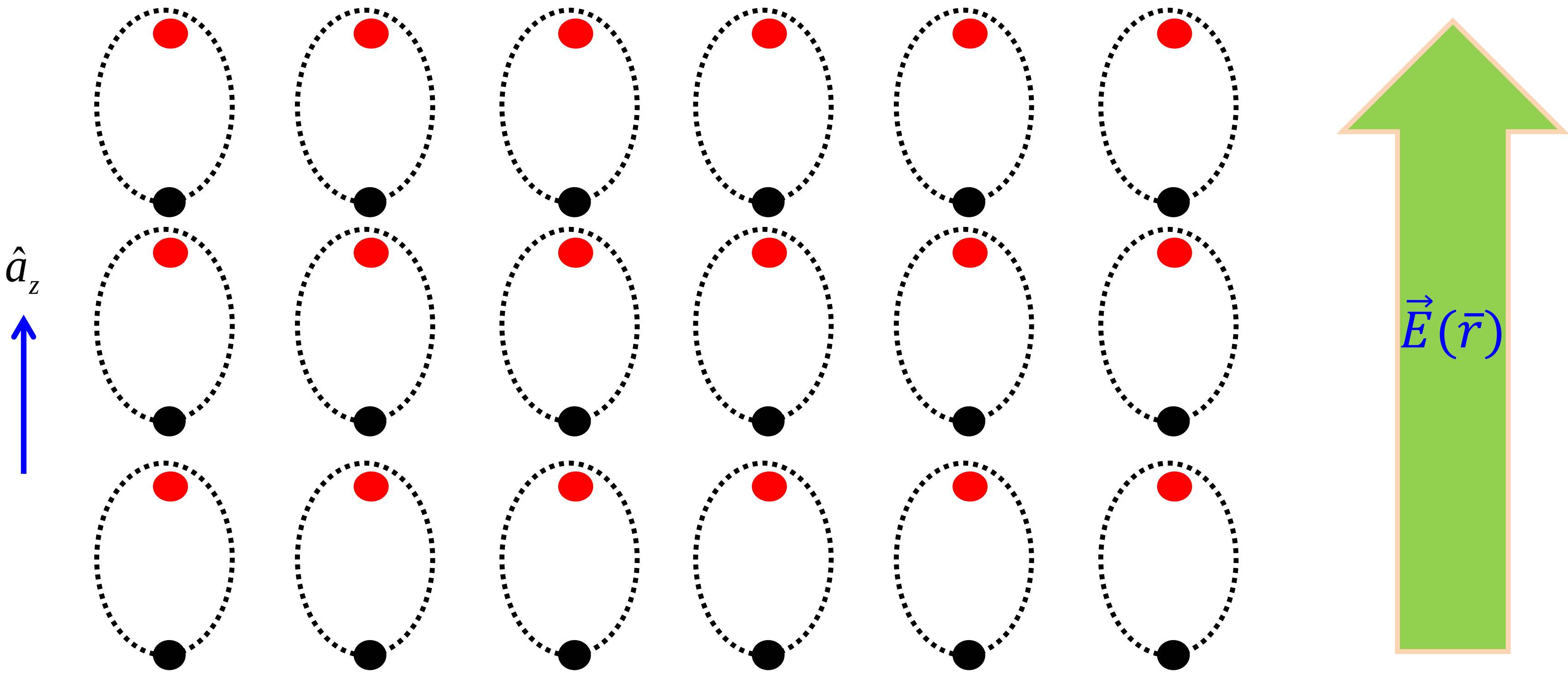
- The comparison gives:

$$\rho_{vp}(\bar{r}) = -\nabla.\vec{P}(\bar{r}) \qquad \rho_{sp}(\bar{r}) = \vec{P}(\bar{r}).\hat{a}_n$$

The subscript *p* (e.g., ρ_{vp}, ρ_{sp}) indicates that these functions represent **equivalent charge densities** due to the **dipoles** created in the dielectric.

Polarization Charge Distributions (contd.)

- Thus the electric potential field $V(\bar{r})$ (and thus electric field $\vec{E}(\bar{r})$) created by the dipoles in the dielectric (i.e., $\vec{P}(\bar{r})$) is **indistinguishable** from the electric potential field created by the equivalent charge densities $\rho_{vp}(\bar{r})$ and $\rho_{sp}(\bar{r})$!
- For example, consider a dielectric material immersed in an electric field, such that its polarization vector $\vec{P}(\bar{r})$ is: $\vec{P}(\bar{r}) = 3\hat{a}_z$ C/m²



Polarization Charge Distributions (contd.)

- Note since the polarization vector is a **constant**, the equivalent volume charge density is **zero**:

$$\rho_{vp}(\bar{r}) = -\nabla.\vec{P}(\bar{r}) = -\nabla.3\hat{a}_z = 0$$

- On the **top** surface of the dielectric ($\hat{a}_n = \hat{a}_z$), the equivalent **surface** charge is:

$$\rho_{sp}(\bar{r}) = \vec{P}(\bar{r}).\hat{a}_n = 3\hat{a}_z.\hat{a}_z = 3 \text{ C/m}^2$$

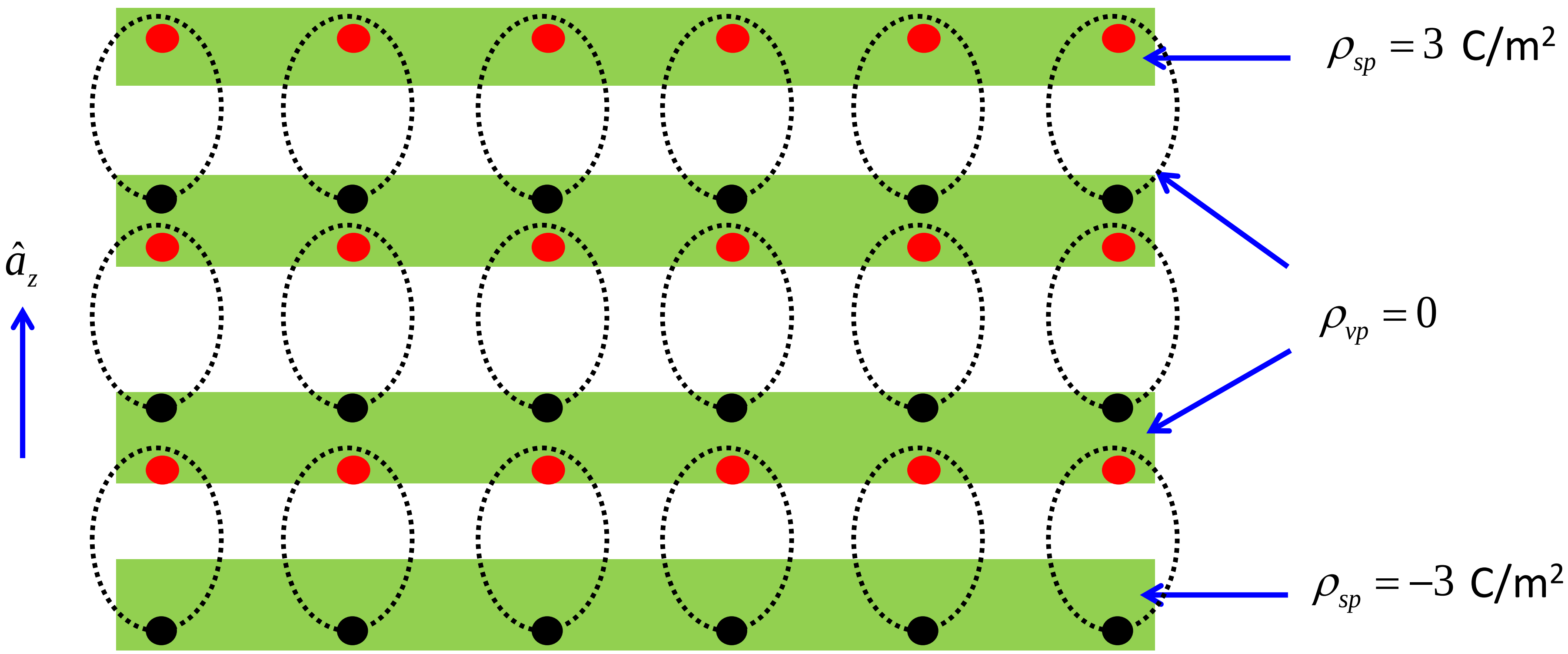
- On the **bottom** surface of the dielectric ($\hat{a}_n = -\hat{a}_z$), the equivalent **surface** charge is:

$$\rho_{sp}(\bar{r}) = \vec{P}(\bar{r}).\hat{a}_n = -3\hat{a}_z.\hat{a}_z = -3 \text{ C/m}^2$$

- On the **sides** of the dielectric material, the **surface** charge is **zero**, since ($\hat{a}_n.\hat{a}_z = 0$).

Polarization Charge Distributions (contd.)

- The result actually makes **physical** sense! Note at the **top** of dielectric, there is a layer of **positive** charge, and at the **bottom**, there is a layer of **negative** charge.



- In the **middle** of the dielectric, there are **positive** charge layers on top of **negative** charge layers. The two add together and **cancel** each other, so that equivalent **volume** charge density is **zero.**

Polarization Charge Distributions (contd.)

- Finally, recall that there is no perfect dielectric, all materials will have some non-zero conductivity $\sigma(\bar{r})$.
- As a result, we find that the total charge density within some material is the sum of the polarization charge density and the free charge (i.e., conducting charge) density:

$$\boxed{\rho_{vT}(\bar{r}) = \rho_v(\bar{r}) + \rho_{vp}(\bar{r})}$$

Where:

$\rho_{vT}(\bar{r}) \doteq$ total volume charge density

$\rho_v(\bar{r}) \doteq$ free charge density

$\rho_{vp}(\bar{r}) \doteq$ polarization charge density

- This is likewise (as well as **more frequently!**) true for **surface** charge density:

$$\boxed{\rho_{sT}(\bar{r}) = \rho_s(\bar{r}) + \rho_{sp}(\bar{r})}$$

Electric Flux Density

- Yikes! Things have gotten **complicated**!
- In free space, we found that charge $\rho_v(\bar{r})$ creates an electric field $\vec{E}(\bar{r})$.

Pretty **simple**! $\rho_v(\bar{r}) \Rightarrow \vec{E}(\bar{r})$

- But, if dielectric material is present, we find that charge $\rho_v(\bar{r})$ creates an **initial** electric field $\vec{E}_i(\bar{r})$. This electric field in turn **polarizes** the material, forming bound charge $\rho_{vp}(\bar{r})$. This bound charge, however, then creates its **own** electric field $\vec{E}_s(\bar{r})$ (sometimes called a **secondary** field), which modifies the initial electric field!

Not so simple! $\rho_v(\bar{r}) \Rightarrow \vec{E}_i(\bar{r}) \Rightarrow \rho_{vp}(\bar{r}) \Rightarrow \vec{E}_s(\bar{r})$

The **total** electric field created by free charge when dielectric material is present is thus $\vec{E}(\bar{r}) = \vec{E}_i(\bar{r}) + \vec{E}_s(\bar{r})$.

Electric Flux Density (contd.)

Q: Isn't there some **easier** way to account for the effect of dielectric material??

A: Yes there is! We use the concept of dielectric **permittivity**, and a new vector field called the **electric flux density** $\vec{D}(\bar{r})$**.**

- To see how this works, first consider the point form of **Gauss's Law**: $$\boxed{\nabla.\vec{E}(\bar{r}) = \frac{\rho_{vT}(\bar{r})}{\varepsilon_0}}$$

  $\rho_{vT}(\bar{r})$ is the **total** charge density, includes both the **free** $\rho_v(\bar{r})$ and **bound** $\rho_{vp}(\bar{r})$: $$\boxed{\rho_{vT}(\bar{r}) = \rho_v(\bar{r}) + \rho_{vp}(\bar{r})}$$

- Therefore, we can write Gauss's Law as: $$\boxed{\varepsilon_0 \nabla.\vec{E}(\bar{r}) = \rho_v(\bar{r}) + \rho_{vp}(\bar{r})}$$
- Recall the **bound** charge density is equal to: $\rho_{vp}(\bar{r}) = -\nabla.\vec{P}(\bar{r})$
- Therefore, expression for Gauss's Law becomes: $\varepsilon_0 \nabla.\vec{E}(\bar{r}) = \rho_v(\bar{r}) - \nabla.\vec{P}(\bar{r})$

$$\boxed{\nabla.\left(\varepsilon_0 \vec{E}(\bar{r}) + \vec{P}(\bar{r})\right) = \rho_v(\bar{r})}$$

Electric Flux Density (contd.)

- Let's define **electric flux density** $\vec{D}(\bar{r})$: $\boxed{\vec{D}(\bar{r}) \doteq \varepsilon_0 \vec{E}(\bar{r}) + \vec{P}(\bar{r})}$
- Therefore, we can write a **new** form of Gauss's Law: $\boxed{\nabla.\vec{D}(\bar{r}) = \rho_v(\bar{r})}$

This equation says that the electric flux density $\vec{D}(\bar{r})$ **diverges** from **free** charge $\rho_v(\bar{r})$. In other words, the source of electric flux density is free charge $\rho_v(\bar{r})$ --and free charge **only**!

- $\vec{E}(\bar{r})$ is created by **both free and bound charge** within the dielectric.
- $\vec{D}(\bar{r})$ is created by **free** charge **only**—the bound charge within the dielectric material makes no difference with regard to $\vec{D}(\bar{r})$!
- Recall that the polarization vector is related to electric field by susceptibility $\chi_e(\bar{r})$: $\boxed{\vec{P}(\bar{r}) = \varepsilon_0 \chi_e(\bar{r}) \vec{E}(\bar{r})}$
- Therefore the electric flux density is: $\boxed{\vec{D}(\bar{r}) = \varepsilon_0 \vec{E}(\bar{r}) + \varepsilon_0 \chi_e(\bar{r}) \vec{E}(\bar{r}) = \varepsilon_0 \left(1 + \chi_e(\bar{r})\right) \vec{E}(\bar{r})}$

Electric Flux Density (contd.)

- We can further simplify by defining the **permittivity** of the medium (the dielectric material):

$$\varepsilon(\bar{r}) = \varepsilon_0\left(1+\chi_e(\bar{r})\right)$$

- This enables us to define **relative** permittivity:

$$\varepsilon_r(\bar{r}) \doteq \frac{\varepsilon(\bar{r})}{\varepsilon_0} = 1+\chi_e(\bar{r})$$

- Results in **simple** relationship between electric flux density and electric field:

$$\vec{D}(\bar{r}) = \varepsilon(\bar{r})\vec{E}(\bar{r}) = \varepsilon_0\varepsilon_r(\bar{r})\vec{E}(\bar{r})$$

Like conductivity $\sigma(\bar{r})$, permittivity $\varepsilon(\bar{r})$ is a fundamental **material** parameter. Also like conductivity, it relates the electric field to another vector field.

Thus, we have an **alternative** way to view electrostatics:

1. Free charge $\rho_v(\bar{r})$ creates electric flux density $\vec{D}(\bar{r})$.
2. The electric field can be then determined by simply dividing $\vec{D}(\bar{r})$ by the material permittivity $\varepsilon(\bar{r})$ (i.e., $\vec{E}(\bar{r}) = \vec{D}(\bar{r})/\varepsilon(\bar{r})$.

$$\rho_v(\bar{r}) \Rightarrow \vec{D}(\bar{r}) \Rightarrow \vec{E}(\bar{r})$$

Electrostatic Field Equations in Dielectrics

- The electrostatic equations for fields in **dielectric materials** are:

$$\nabla \times \vec{E}(\bar{r}) = 0 \qquad \nabla . \vec{D}(\bar{r}) = \rho_v(\bar{r}) \qquad \vec{D}(\bar{r}) = \varepsilon(\bar{r})\vec{E}(\bar{r})$$

- In **integral** form, these equations are:

$$\oint_C \vec{E}(\bar{r}).\overline{dl} = 0 \qquad \oiint_S \vec{D}(\bar{r}).\overline{dS} = Q_{enc} \qquad \vec{D}(\bar{r}) = \varepsilon(\bar{r})\vec{E}(\bar{r})$$

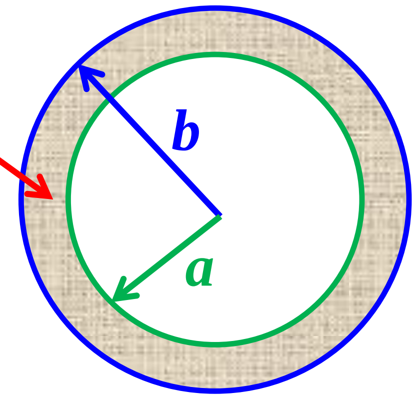
- Likewise, for free charge located in some **homogeneous** (i.e., constant) material with permittivity ε, we get the following relations:

$$\boxed{V(\bar{r}) = \frac{1}{4\pi\varepsilon}\iiint_v \frac{\rho_v(\bar{r}')}{|\bar{r}-\bar{r}'|}dv'} \qquad \boxed{\left|\vec{E}(\bar{r})\right| = \frac{1}{4\pi\varepsilon}\iiint_v \frac{\rho_v(\bar{r}')}{|\bar{r}-\bar{r}'|^2}dv'} \qquad \boxed{\nabla^2 V(\bar{r}) = \frac{-\rho_v(\bar{r})}{\varepsilon}}$$

In other words, for homogenous materials, **replace** ε_0 (the permittivity of free-space) with the more general permittivity value ε.

Example – 1

- At the center of a hollow dielectric sphere ($\varepsilon = \varepsilon_0 \varepsilon_r$) is placed a point charge Q. If the sphere has inner radius a and outer radius b, calculate $\vec{D}$, $\vec{E}$ and $\vec{P}$.



For $0 < r < a$

Gauss's law gives: $\vec{D} = \dfrac{Q}{4\pi r^2}\hat{a}_r$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r \Rightarrow \vec{P} = \vec{D} - \varepsilon_0 \vec{E} = 0$$

For $a < r < b$

Gauss's law gives: $\vec{D} = \dfrac{Q}{4\pi r^2}\hat{a}_r$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}\hat{a}_r$$

$$\vec{P} = \chi_e \varepsilon_0 \vec{E} = \frac{\varepsilon_r - 1}{\varepsilon_r}\frac{Q}{4\pi r^2}\hat{a}_r$$

Example – 1 (contd.)

For $r > b$

Gauss's law gives: $\vec{D} = \dfrac{Q}{4\pi r^2}\hat{a}_r \Rightarrow \vec{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r \Rightarrow \vec{P} = \vec{D} - \varepsilon_0\vec{E} = 0$

Therefore:

$$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r \qquad r > 0$$

$$\boxed{\vec{E} = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}\hat{a}_r & a < r < b \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r & \text{otherwise} \end{cases}}$$

$$\boxed{\vec{P} = \begin{cases} \dfrac{\varepsilon_r - 1}{\varepsilon_r}\dfrac{Q}{4\pi r^2}\hat{a}_r & a < r < b \\ 0 & \text{otherwise} \end{cases}}$$

Example – 2

Show that: $\vec{P} = (\varepsilon - \varepsilon_0)\vec{E}$ and $\vec{D} = \frac{\varepsilon_r}{\varepsilon_r - 1}\vec{P}$

Electrostatic Boundary Conditions

- A vector field is said to be spatially continuous if it doesn't exhibit abrupt changes in either magnitude or direction as a function of position.
- Even though the electric field may be continuous in adjoining dissimilar media, it may well be discontinuous at the boundary between them.
- Boundary conditions specify how the components of fields tangential and normal to an interface between two media relate across the interface.
- To determine boundary conditions, we need to use Maxwell's equations:

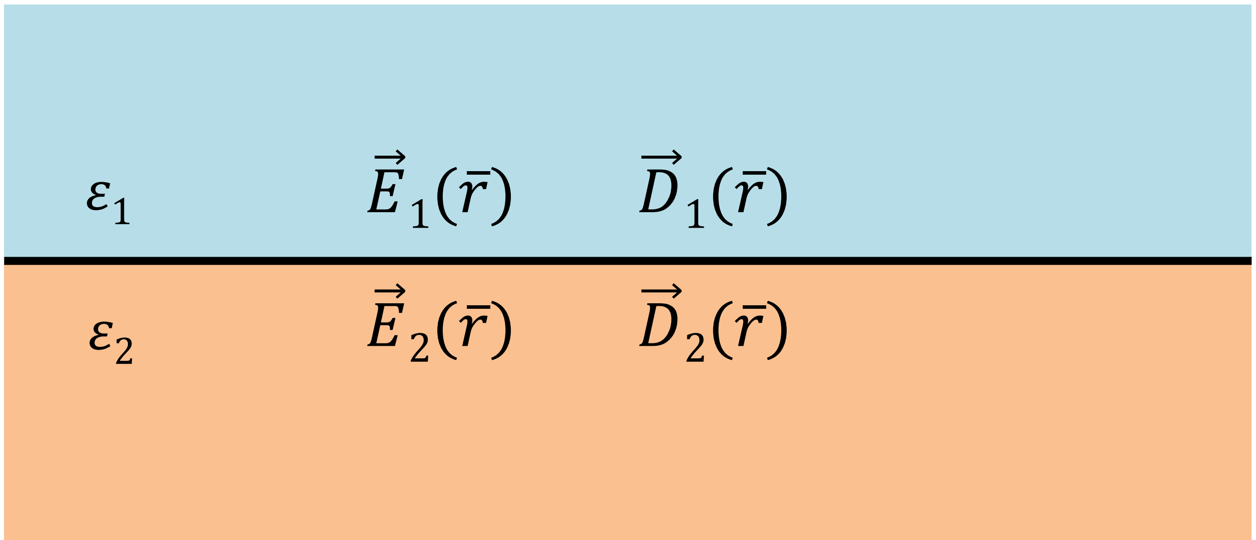
$$\oint \vec{E}.\overline{dl} = 0 \quad \Longleftrightarrow \quad \nabla \times \vec{E} = 0$$

$$\oint_S \vec{D}.\overline{dS} = Q_{enc} \quad \Longleftrightarrow \quad \nabla . \vec{D} = \rho_v$$

Needless to say, these boundary conditions are equally valid for Electrodynamics

Dielectric – Dielectric Boundary Conditions

- Consider the **interface** between two dissimilar **dielectric** regions:



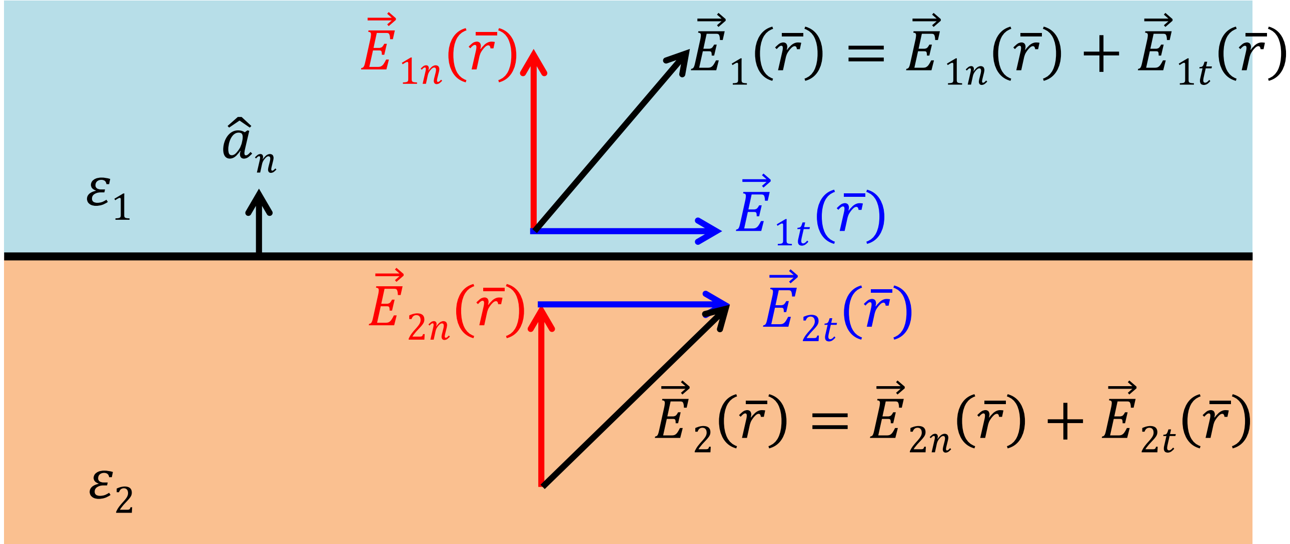
- Say that an **electric field** is present in both regions, thus producing also an electric flux density $\vec{D}(\bar{r}) = \varepsilon \vec{E}(\bar{r})$.

Q: How are the fields in dielectric **region 1** related to the fields in **region 2 ?**

A: They must satisfy the **dielectric boundary conditions** !

Dielectric – Dielectric Boundary Conditions (contd.)

- First, let's write the fields **at the dielectric interface** in terms of their **normal** $\vec{E}_n(\bar{r})$ and **tangential** $\vec{E}_t(\bar{r})$ vector components:



- Our first boundary condition states that the **tangential** component of the electric field is **continuous** across a boundary.

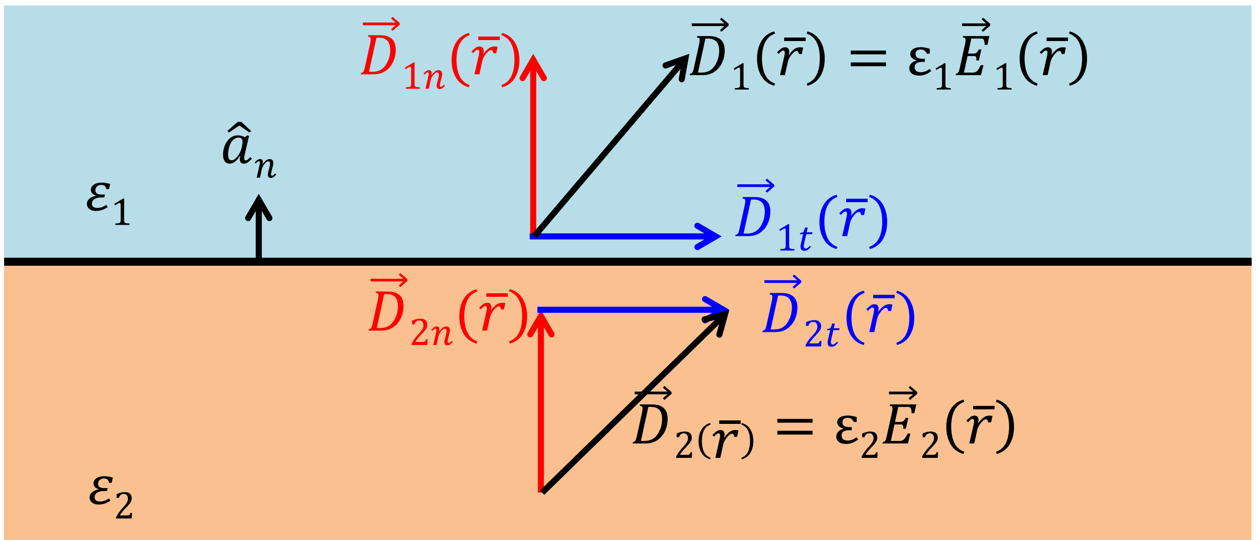
- In other words: $\boxed{\vec{E}_{1t}(\bar{r}_b) = \vec{E}_{2t}(\bar{r}_b)}$

where $\bar{r}_b$ denotes any point **on the boundary** (e.g., dielectric interface).

The **tangential** component of the electric field at **one** side of the dielectric boundary is **equal** to the tangential component at the **other** side !

Dielectric – Dielectric Boundary Conditions (contd.)

- We can likewise consider the **electric flux densities** on the dielectric interface in terms of their **normal** and **tangential** components:



- The second dielectric boundary condition states that the **normal** vector component of the **electric flux density** is **continuous** across the dielectric boundary.

- In other words: $\boxed{\vec{D}_{1n}(\bar{r}_b) = \vec{D}_{2n}(\bar{r}_b)}$

where $\bar{r}_b$ denotes any point **on the boundary** (e.g., dielectric interface).

Dielectric – Dielectric Boundary Conditions (contd.)

- Since $\vec{D}(\bar{r}) = \varepsilon\vec{E}(\bar{r})$, these boundary conditions can **likewise** be expressed as:

$$\boxed{\frac{\vec{D}_{1t}(\bar{r}_b)}{\varepsilon_1} = \frac{\vec{D}_{2t}(\bar{r}_b)}{\varepsilon_2}}$$

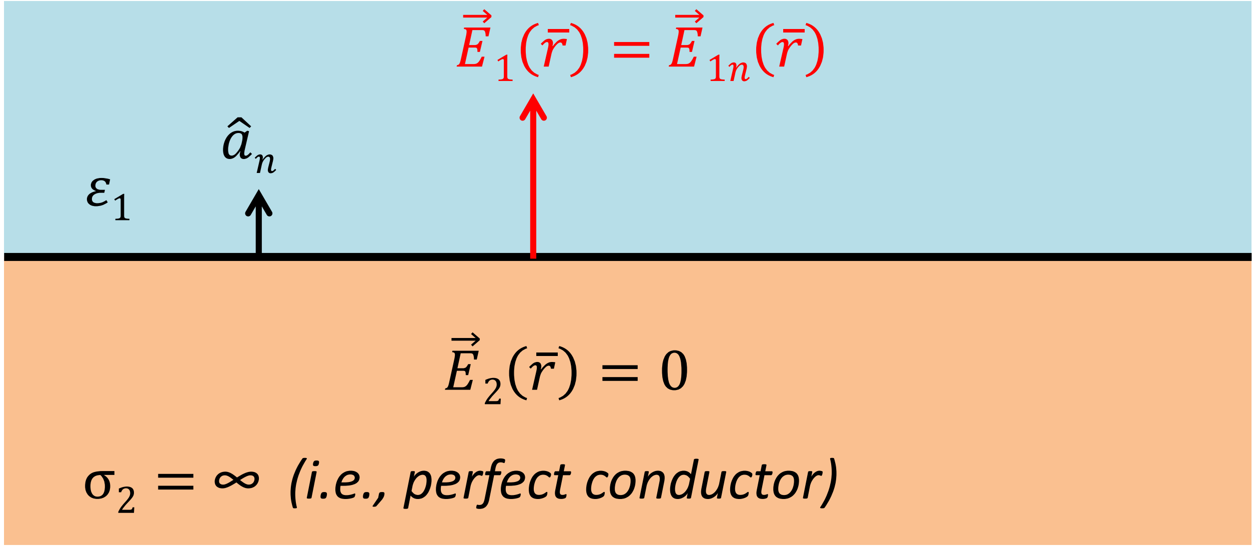
$$\boxed{\varepsilon_1 \vec{E}_{1n}(\bar{r}_b) = \varepsilon_2 \vec{E}_{2n}(\bar{r}_b)}$$

MAKE SURE <u>YOU</u> UNDERSTAND THIS:

These boundary conditions describe the relationships of the vector fields **at the dielectric interface** only (i.e., at points $r = \bar{r}_b$)!!!! They say **nothing** about the value of the fields at points above or below the interface.

Conductor – Dielectric Boundary Conditions

- Consider the case where region 2 is a **perfect conductor**:



- Recall $\vec{E}(\bar{r}) = 0$ in a perfect conductor. This of course means that **both** the tangential and normal component of $\vec{E}_2(\bar{r})$ are also equal to **zero**:

$$\boxed{\vec{E}_{2t}(\bar{r}) = 0 = \vec{E}_{2n}(\bar{r})}$$

- And, since the **tangential** component of the electric field is **continuous** across the boundary, we find that **at the interface**:

$$\boxed{\vec{E}_{1t}(\bar{r}_b) = \vec{E}_{2t}(\bar{r}_b) = 0}$$

Conductor – Dielectric Boundary Conditions (contd.)

- **Think about what this means!** The **tangential** vector component in the dielectric (at the dielectric/conductor boundary) is **zero**. Therefore, the electric field **at the boundary** only has a **normal** component:

$$\boxed{\vec{E}_1(\bar{r}_b) = \vec{E}_{1n}(\bar{r}_b)}$$

- Therefore, we can say:

The electric field on the surface of a conductor is orthogonal (i.e., normal) to the conductor.

Q1: What about the **electric flux density** $\vec{D}_1(\bar{r})$?

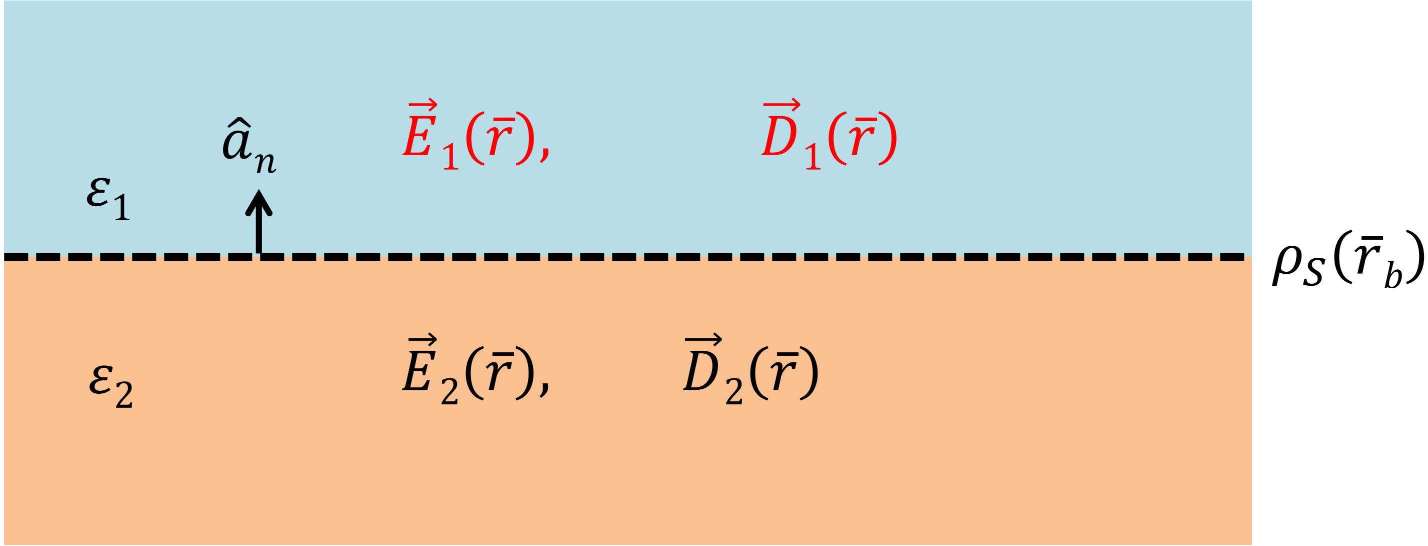
A1: The relation $\vec{D}_1(\bar{r}) = \varepsilon_1 \vec{E}_1(\bar{r})$ is still of course valid, so that the **electric flux density** at the surface of the conductor must **also be orthogonal** to the conductor. For boundary with surface charge density (ρ_s), $\vec{D}_{1n}(\bar{r}) = \varepsilon_1 \vec{E}_{1n}(\bar{r}) = \rho_s$.

Q2: But, we learnt that the **normal** component of the **electric flux density** is **continuous** across an interface. If $\vec{D}_{2n}(\bar{r}) = 0$, why isn't $\vec{D}_{1n}(\bar{r}) = 0$?

A2: Great question! The answer comes from a more **general** form of the **boundary condition**.

Conductor – Dielectric Boundary Conditions (contd.)

- Consider again the interface of two **dissimilar dielectrics**. This time, however, there is some **surface charge distribution** $\rho_S(\bar{r}_b)$ (i.e., **free** charge!) **at the dielectric interface**:



- The **boundary condition** for this situation turns out to be:

$$\boxed{\hat{a}_n.\left[\vec{D}_{1n}(\bar{r}_b)-\vec{D}_{2n}(\bar{r}_b)\right]=\rho_S(\bar{r}_b)} \Rightarrow \boxed{D_{1n}(\bar{r}_b)-D_{2n}(\bar{r}_b)=\rho_S(\bar{r}_b)}$$

- Note that **if** $\rho_S(\bar{r}_b)$ =0, this boundary condition returns (both physically and mathematically) to the case studied earlier—the **normal** component of the electric flux density is **continuous** across the interface.

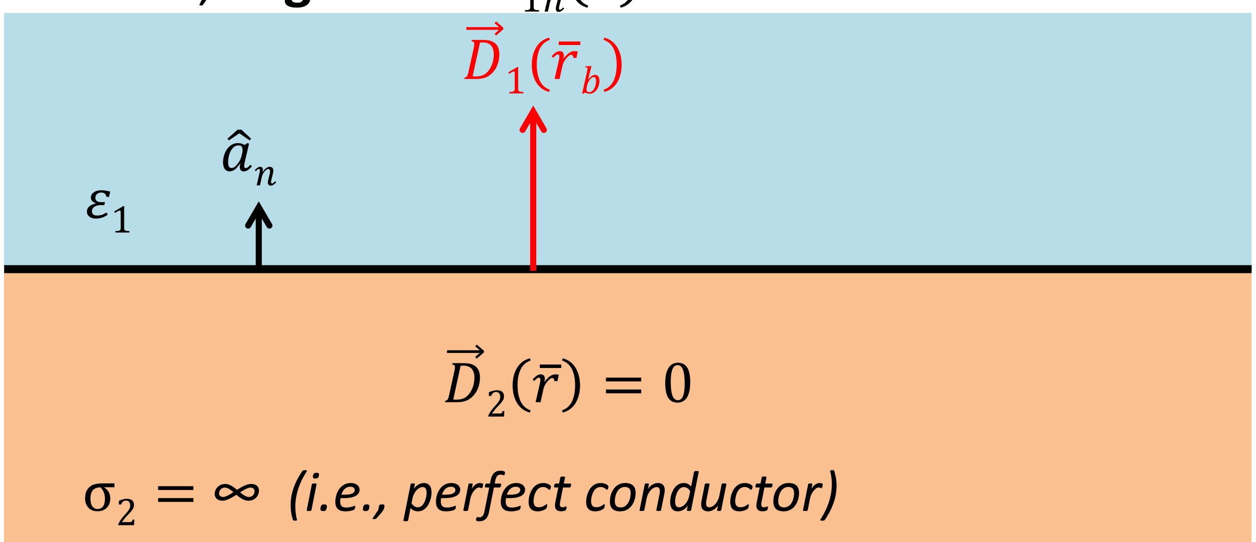
Conductor – Dielectric Boundary Conditions (contd.)

- This more **general** boundary condition is useful for the dielectric/**conductor** interface. Since $\vec{D}_2(\bar{r}) = 0$ in the conductor, we find:

$$\boxed{\hat{a}_n . \vec{D}_{1n}(\bar{r}_b) = \rho_S(\bar{r}_b)} \Rightarrow \boxed{D_{1n}(\bar{r}_b) = \rho_S(\bar{r}_b)}$$

the **normal** component of the **electric flux density** at the **conductor surface** is equal to the **charge density** on the conductor surface.

- In a perfect conductor, **plenty** of **free** charge available to form this charge density! Therefore, in **general** $\vec{D}_{1n}(\bar{r}) \neq 0$ at the surface of a conductor.



Conductor – Dielectric Boundary Conditions (contd.)

Summary: $\vec{E}_{1t}(\bar{r}_b)=0$ $\vec{D}_{1t}(\bar{r}_b)=0$ $\vec{D}_{1n}(\bar{r}_b)=\rho_S(\bar{r}_b)$ $\vec{E}_{1n}(\bar{r}_b)=\dfrac{\rho_S(\bar{r}_b)}{\varepsilon_1}$

These boundary conditions describe the fields **at the conductor/dielectric interface**. They say **nothing** about the value of the fields at locations above this interface.

- Thus under static conditions, the following conclusions can be made about a perfect conductor:

1. No electric field may exist within a conductor, i.e., $\rho_v = 0, \qquad \vec{E} = 0$
2. Since, $\vec{E} = -\nabla V = 0$, there can be no potential difference between any two points in the conductor; i.e,a conductor is an equipotential body.
3. An electric field must be external to the conductor and must be normal to its surface. i.e.,

$$\vec{D}_t = \varepsilon_0\varepsilon_r\vec{E}_t = 0, \qquad \vec{D}_n = \varepsilon_0\varepsilon_r\vec{E}_n = \rho_s$$

An important use of this concept is in the design of Electrostatic Shielding

Conductor – Free Space Boundary Conditions

- It is a special case of conductor-dielectric boundary conditions.
- Replace by $\varepsilon_r = 1$ in the expressions to get:

$$\vec{D}_t = \varepsilon_0 \vec{E}_t = 0, \qquad \vec{D}_n = \varepsilon_0 \vec{E}_n = \rho_s$$

It should be noted once again that the electric field must approach a conducting surface normally.