Lecture – 9 **Date: 29.08.2017**

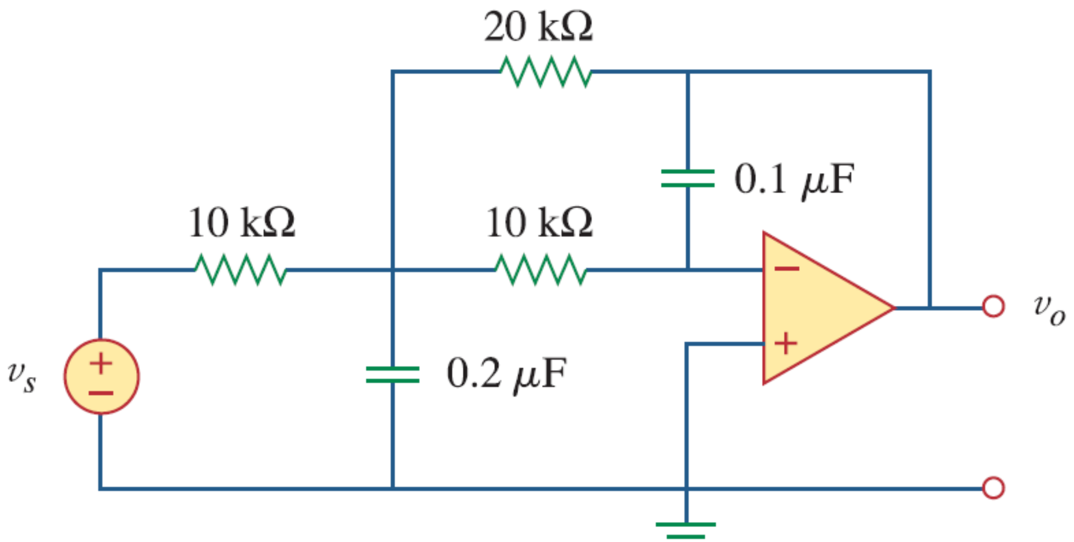
- AC Circuits: Steady State Analysis (contd.)
- AC Power Analysis

Op Amp AC Circuits Analysis Steps

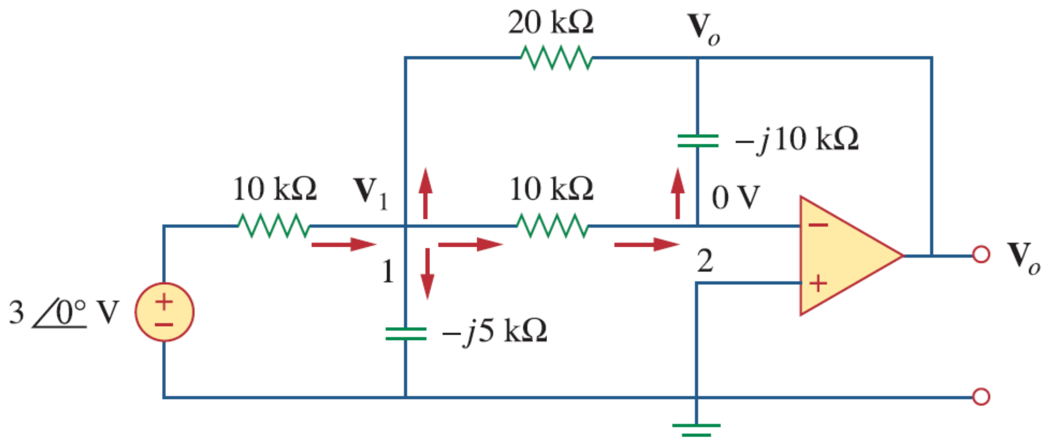
1. Transfer the circuit to the phasor domain
2. Solve the circuit (using Mesh, Nodal techniques etc.)
3. Convert the results into time domain

It is assumed that the op amps are ideal, i.e.,
- No current enters either of its input terminals.
- The voltage across its input terminals is zero.

- Determine $v_o(t)$ for this op amp circuit if $v_s = 3cos1000t\ V$.



Op Amp AC Circuits (contd.)



KCL at node-2

$$\frac{\mathbf{V}_1 - 0}{10} = \frac{0 - \mathbf{V}_o}{-j10}$$

$$\mathbf{V}_1 = -j\mathbf{V}_o$$

KCL at node-1

$$\frac{3\angle 0° - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j5} + \frac{\mathbf{V}_1 - 0}{10} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

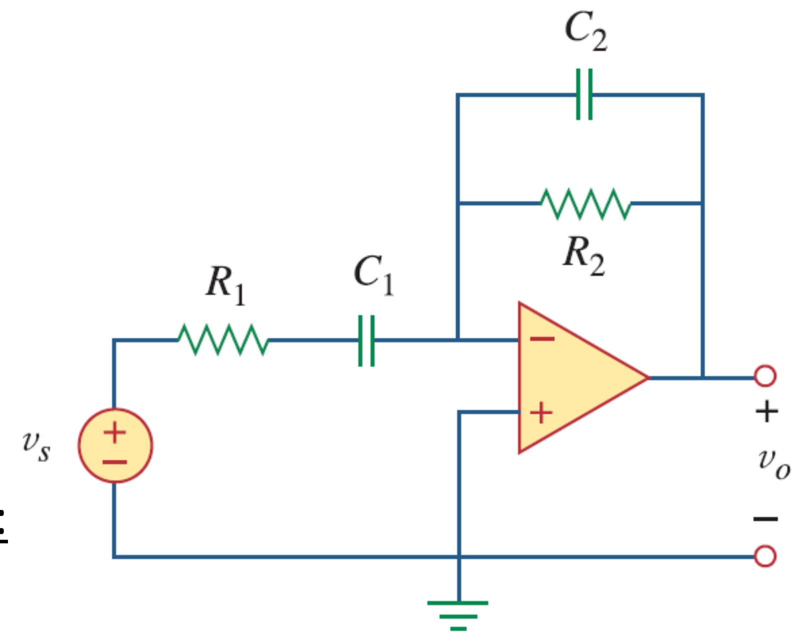
$$6 = (5 + j4)\mathbf{V}_1 - \mathbf{V}_o$$

$$\mathbf{V}_o = \frac{6}{3 - j5} = 1.029\angle 59.04°$$

$$v_o(t) = 1.029\cos(1000t + 59.04°)\ \text{V}$$

Example – 1

Compute the closed-loop gain and phase shift assuming that $R_1 = R_2 = 10k\Omega$, $C_1 = 2\mu F$, $C_2 = 1\mu F$, and $\omega = 200\frac{rad}{s}$.



The feedback and input impedances are:

$$\mathbf{Z}_f = R_2 \left\| \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega R_2 C_2}\right.$$

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

The closed-loop gain is:

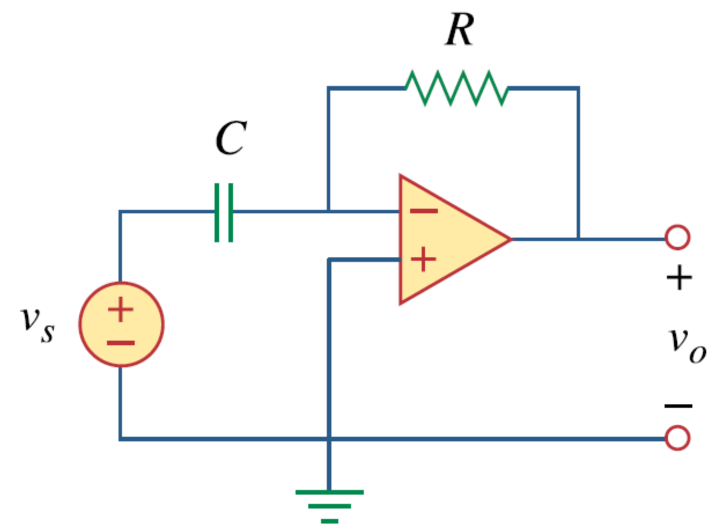
$$\mathbf{G} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{-j\omega C_1 R_2}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)}$$

$$\mathbf{G} = \frac{-j4}{(1 + j4)(1 + j2)} = 0.434\underline{/130.6^\circ}$$

Thus the closed-loop gain is 0.434 and phase shift is 130.6°

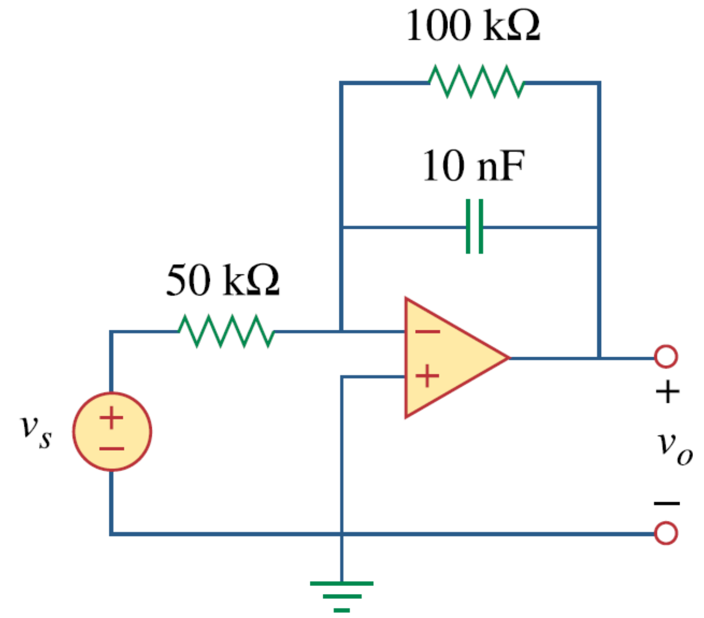
Example – 2

For the differentiator, obtain $\frac{V_0}{V_s}$. Find $v_o(t)$ when $v_{s(t)} = V_m sin\omega t$ and $\omega = \frac{1}{RC}$.

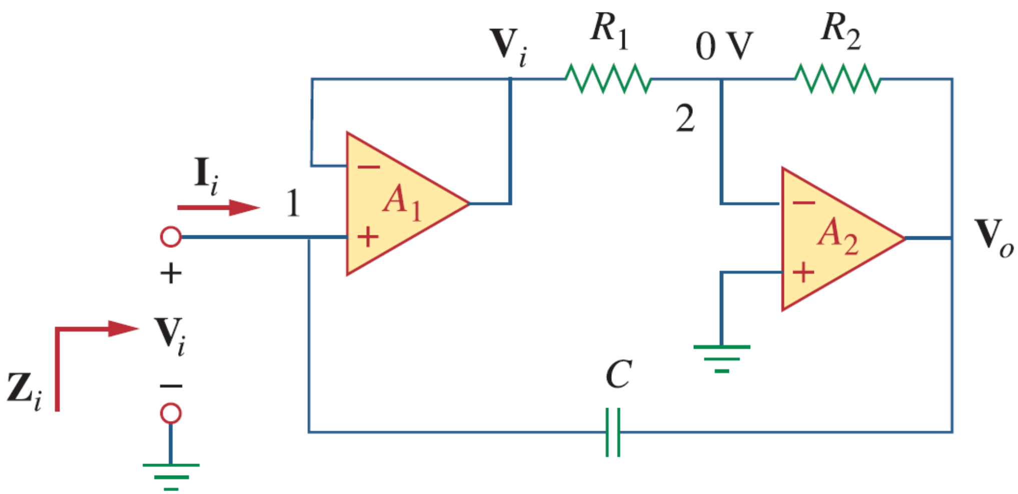


Example – 3

For this integrator with a feedback resistor, obtain $v_o(t)$ if $v_{s(t)} = 2cos4 \times 10^4 t$ V.



Capacitance Multiplier



- First op amp operates as a voltage follower.
- The second op amp is inverting amplifier
- No current enters the input terminal of op amp, the input current I_i flows through the feedback cap.

At the node 1:

$$\mathbf{I}_i = \frac{\mathbf{V}_i - \mathbf{V}_o}{1/j\omega C} = j\omega C(\mathbf{V}_i - \mathbf{V}_o)$$

KCL at the node 2 gives:

$$\frac{\mathbf{V}_i - 0}{R_1} = \frac{0 - \mathbf{V}_o}{R_2} \Rightarrow \mathbf{V}_o = -\frac{R_2}{R_1}\mathbf{V}_i$$

$$\mathbf{I}_i = j\omega C\left(1 + \frac{R_2}{R_1}\right)\mathbf{V}_i$$

The input impedance:

$$\mathbf{Z}_i = \frac{\mathbf{V}_i}{\mathbf{I}_i} = \frac{1}{j\omega C_{eq}} \Rightarrow C_{eq} = \left(1 + \frac{R_2}{R_1}\right)C$$

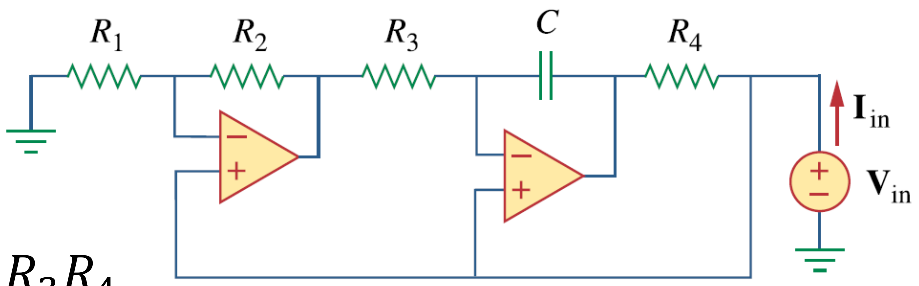
Capacitance Multiplier (contd.)

The input impedance: $\mathbf{Z}_i = \dfrac{\mathbf{V}_i}{\mathbf{I}_i} = \dfrac{1}{j\omega C_{eq}}$ ⇒ $C_{eq} = \left(1 + \dfrac{R_2}{R_1}\right) C$

- A proper selection of R_1 and R_2 can allow realization of desired capacitance between the input terminal and the ground.
- The size of effective capacitance is limited by the inverted output voltage.
- The larger the capacitance multiplication, the smaller will be the allowable input voltage to prevent the op amp from reaching saturation.
- Similar op amp circuit can be designed to synthesize any desired inductance.
- On a similar line, resistance multiplier can be designed as well.

Example – 4

The op amp circuit given below is called an *inductance simulator*. Show that the input impedance is given by:



$$Z_{in} = \frac{V_{in}}{I_{in}} = j\omega L_{eq} \quad \text{Where:} \quad L_{eq} = \frac{R_1 R_3 R_4}{R_2} C$$

Self Study: Oscillators (Section: 10.9.2)

AC Power Analysis

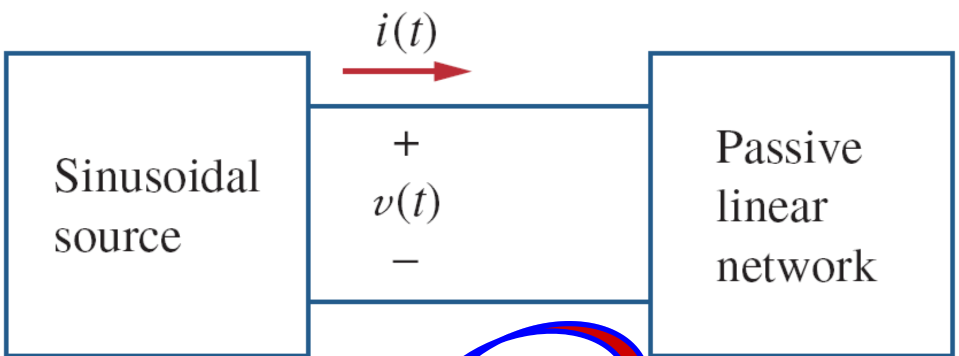
- So far in ac circuit analysis, focus has been on calculating voltage and current.
- Now lets discusses power analysis. Extremely important as power is the most important quantity in electric utilities, electronics, and communication systems as such systems involve transmission of power from one point to another.
- Every industrial and household electrical device—every fan, motor, lamp, pressing iron, TV, personal computer— has a power rating that indicates how much power the equipment requires; exceeding the power rating can do permanent damage to an appliance.
- Most common form of electric power is 50- or 60-Hz ac power. The choice of ac over dc allows high-voltage power transmission from the power generating plant to the consumer.

Instantaneous and Average Power

- The *instantaneous power* (in watts) is the power absorbed at any instant of time in a circuit.

$$p(t) = v(t)i(t)$$

It is the rate at which the circuit absorbs energy



Assume:

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

Instantaneous power absorbed:

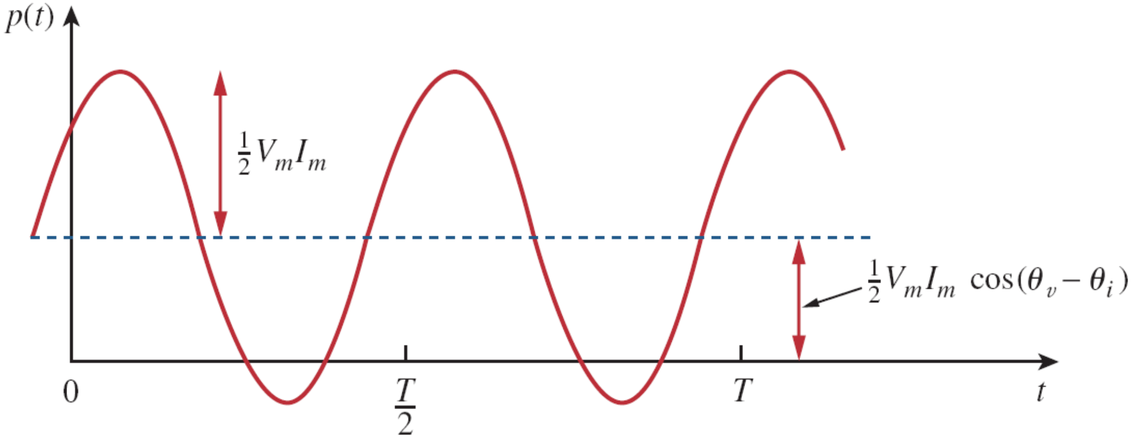
$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Constant and independent of time

Sinusoidal function of frequency 2ω

Instantaneous and Average Power (contd.)



The instantaneous power is difficult to measure as it varies with time

- Instead its convenient to measure average power. Your watt meter measures the average power, the instantaneous power averaged over time period.

Average Power

$$P = \frac{1}{T}\int_0^T p(t)\, dt$$

$$P = \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\, dt + \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\, dt$$

$$= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt + \frac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\, dt$$

Zero

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

Instantaneous and Average Power (contd.)

In phasor form: $\frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}V_m I_m \angle\theta_v - \theta_i$ $= \frac{1}{2}V_m I_m[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)]$

Average Power $P = \frac{1}{2}\text{Re}[\mathbf{V}\mathbf{I}^*] = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$

For current and voltage in phase, $\theta_v = \theta_i$: $P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}|\mathbf{I}|^2 R$

For current and voltage in quadrature, $\theta_v - \theta_i = \pm 90°$: $P = \frac{1}{2}V_m I_m \cos 90° = 0$

In summary, a resistive load (R) absorbs power at all times, while reactive load (L or C) absorbs zero average power.

Example – 5

Determine the average power generated by each source and the average power absorbed by each passive element.