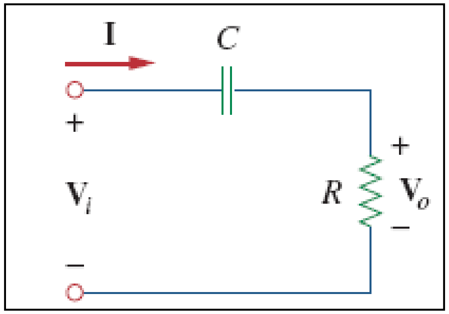
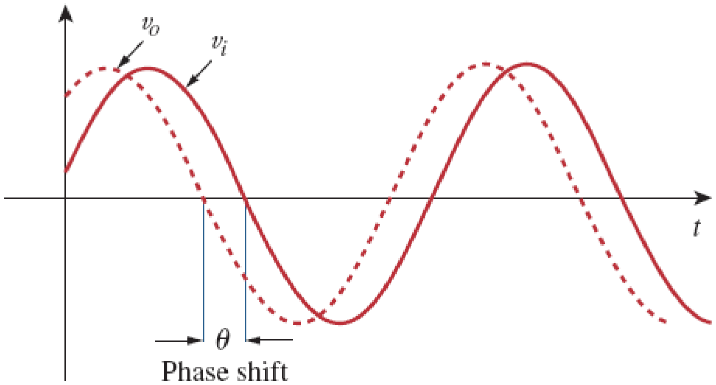
ECE 215

Lecture – 8 **Date: 28.08.2017**

- Phase Shifter, AC bridge
- AC Circuits: Steady State Analysis

Phase Shifter





the circuit current **I** leads the applied voltage by some phase angle θ, where $0 < \theta < 90^o$ depending on the values of *R* and *C*.

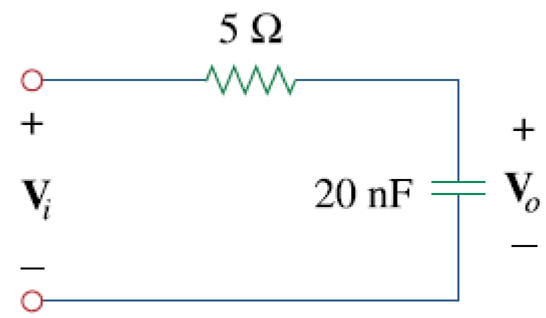
$$\mathbf{Z} = R + jX_C, \quad \Rightarrow \quad \theta = \tan^{-1}\frac{X_C}{R}$$

the amount of phase shift depends on the values of *R*, *C*, and the operating frequency.

- These simple single stage RC circuits are generally not used in practice.
- These RC circuits also work as voltage dividers. Therefore, as the phase shift approaches 90^o the output voltage approaches zero. Therefore, the simple *RC* circuits are used only when small amounts of phase shift are required.
- For large phase shifts, the *RC* networks are cascaded. This provides a total phase shift equal to the sum of the individual phase shifts.

Example – 1

For this RC circuit:
(a) Calculate the phase shift at 2 MHz.
(b) Find the frequency where the phase shift is 45^{o}.



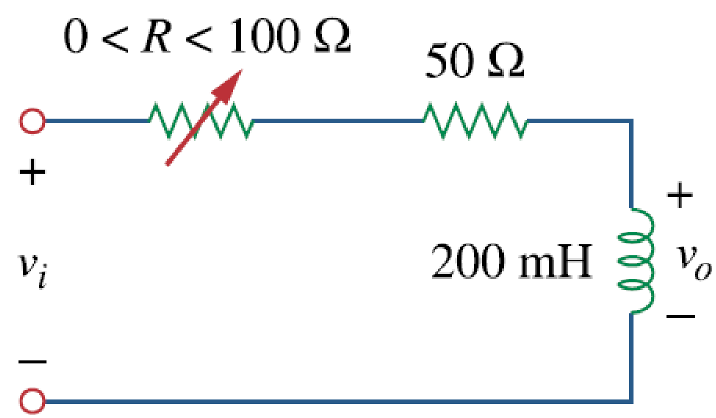
Example – 2

A coil with impedance 8 + *j*6 Ω is connected in series with a capacitive reactance *X*. The series combination is connected in parallel with a resistor *R*. Given that the equivalent impedance of the resulting circuit is $5\angle 0^o$ Ω, find the value of *R* and *X*.

Example – 3

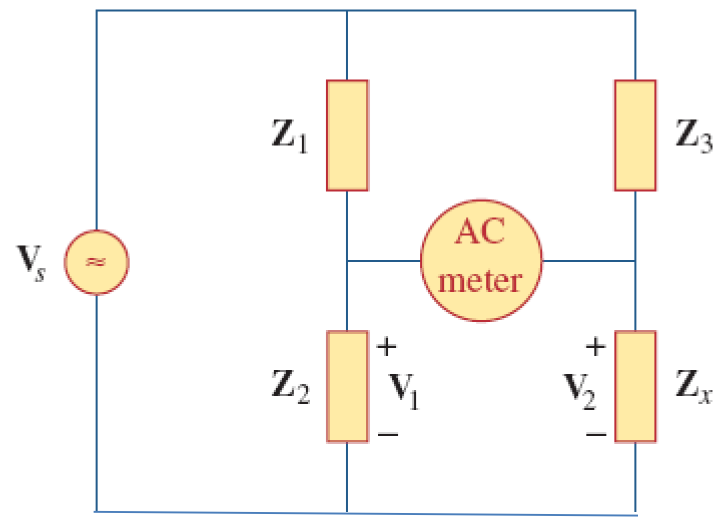
Consider this phase-shifting circuit for 60Hz and determine

(a) $\mathbf{V}_o$ when *R* is maximum
(b) $\mathbf{V}_o$ when *R* is minimum
(c) the value of *R* that will produce a phase shift of 45^o.



AC Bridges

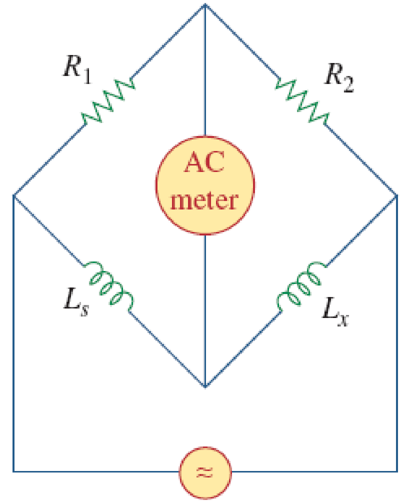
- An ac bridge circuit is used for measuring the inductance *L* of an inductor or the capacitance *C* of a capacitor.
- Similar to the Wheatstone bridge used for measuring an unknown resistance and follows the same principle.
- To measure *L* and *C*, however, an ac source is needed as well as an ac meter instead of the galvanometer.
- The ac meter may be a sensitive ac ammeter or voltmeter.



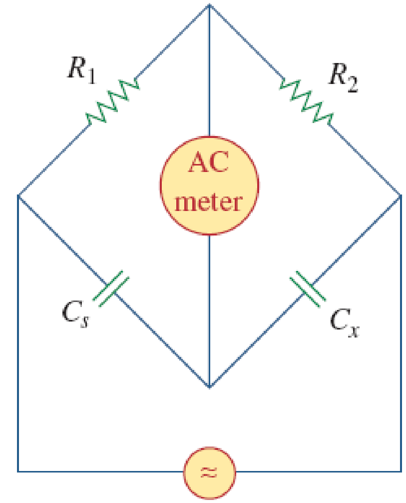
bridge is *balanced* when no current flows through the meter i.e., V1 = V2.

$$V_1 = \frac{Z_2}{Z_1 + Z_2}V_s = V_2 = \frac{Z_x}{Z_3 + Z_x}V_s \quad \Rightarrow \quad Z_x = \frac{Z_3}{Z_1}Z_2$$

AC Bridges (contd.)



$$L_x = \frac{R_2}{R_1} L_s$$

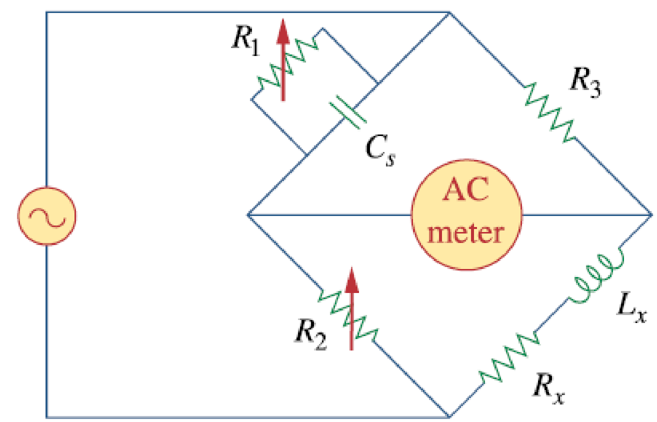


$$C_x = \frac{R_1}{R_2} C_s$$

Example – 4

This ac bridge is known as a *Maxwell bridge* and is used for accurate measurement of inductance and resistance of a coil in terms of a standard capacitance S_s. Show that when the bridge is balanced:

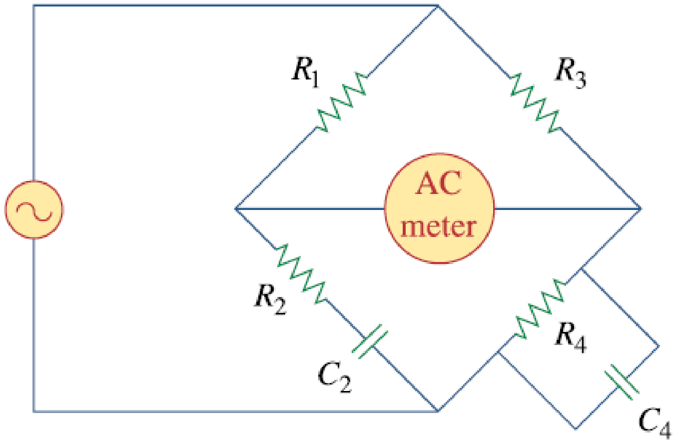
$$L_x = R_2 R_3 C_s \qquad R_x = \frac{R_2}{R_1} R_3$$



Example – 5

This ac bridge is called a *Wien bridge*. It is used for measuring the frequency of a source. Show that when the bridge is balanced:

$$f = \frac{1}{2\pi\sqrt{R_2 R_4 C_2 C_4}}$$



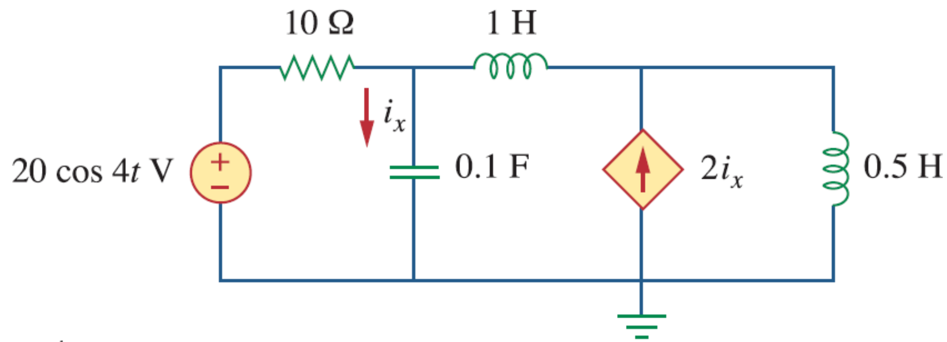
AC Circuit – Steady State Analysis

Analysis Steps

1. Transfer the circuit to the phasor domain
2. Solve the circuit (using Mesh, Nodal techniques etc.)
3. Convert the results into time domain

Nodal Analysis

Find i_x in this circuit.



Convert the quantities to frequency domain.

$$20 \cos 4t \Rightarrow 20\angle 0^\circ, \quad \omega = 4 \text{ rad/s}$$

$$1 \text{ H} \Rightarrow j\omega L = j4$$

$$0.5 \text{ H} \Rightarrow j\omega L = j2$$

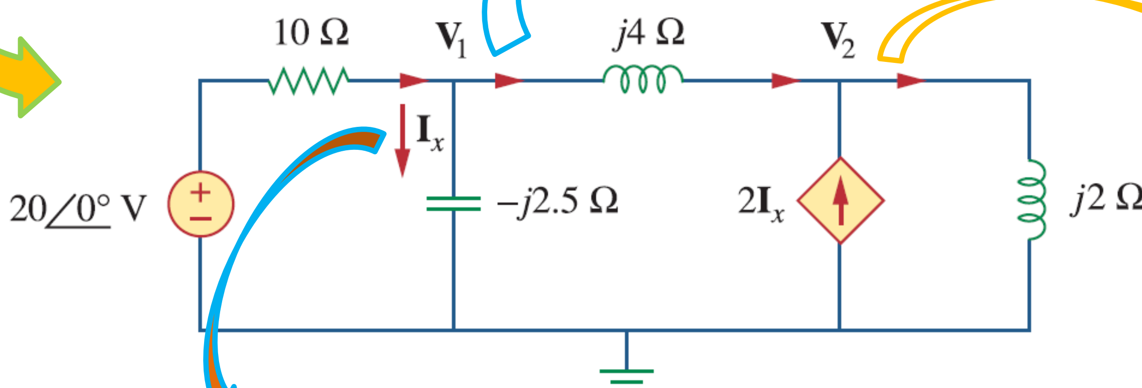
$$0.1 \text{ F} \Rightarrow \frac{1}{j\omega C} = -j2.5$$

KCL here

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

Freq Domain



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$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$\mathbf{I}_x = \mathbf{V}_1/-j2.5$$

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

Nodal Analysis (contd.)

- The two nodal equations can be expressed in matrix form:

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\underline{/18.43^\circ} \text{ V}$$

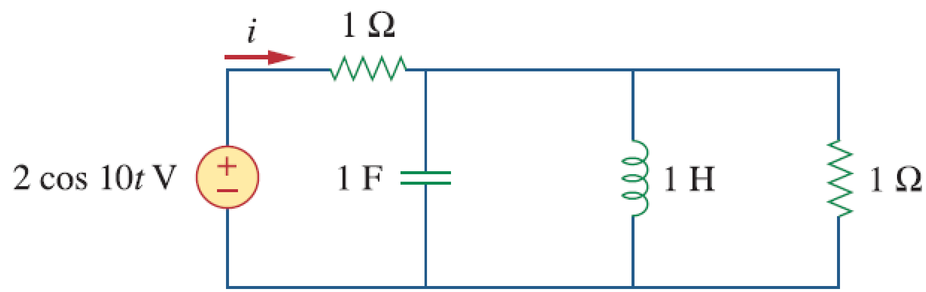
$$\Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\underline{/198.3^\circ} \text{ V}$$

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\underline{/18.43^\circ}}{2.5\underline{/-90^\circ}} = 7.59\underline{/108.4^\circ} \text{ A}$$

$$i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}$$

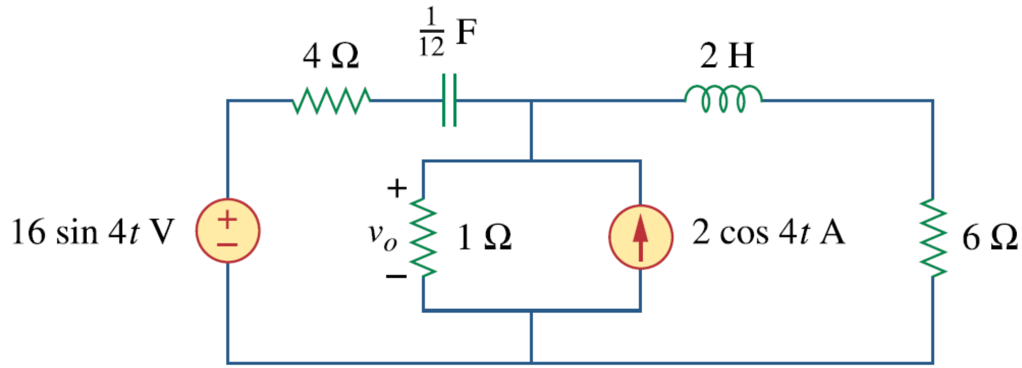
Practice: Find i in this circuit.



$$i(t) = \underline{1.9704 \cos(10t + 5.653^\circ) \text{ A}}$$

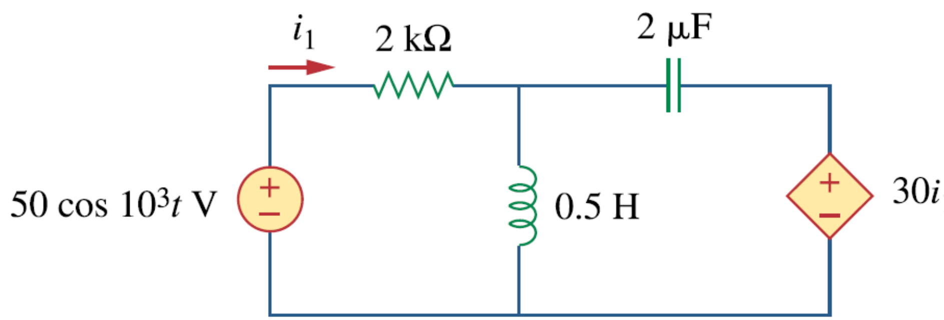
Example – 6

Determine v_0 in this circuit.



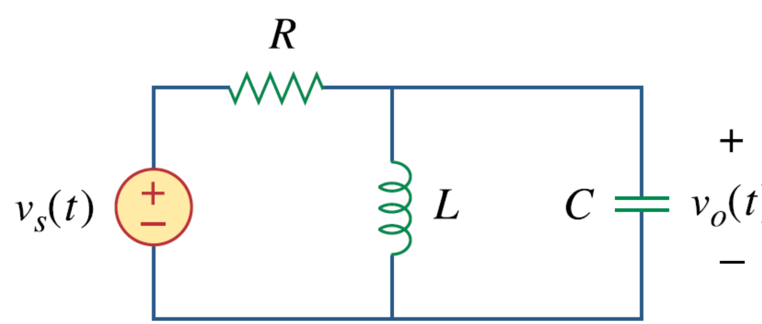
Example – 7

Determine i_1 in this circuit.



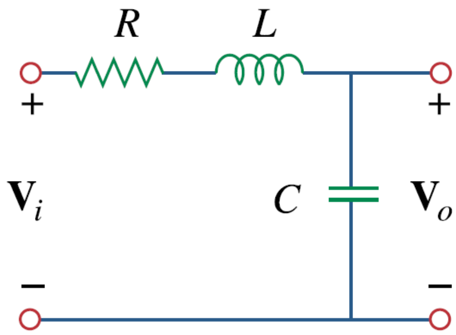
Example – 8

In this circuit if $v_s(t) = V_m Sin\omega t$ and $v_0(t) = A sin(\omega t + \varphi)$, derive the expressions for A and φ.



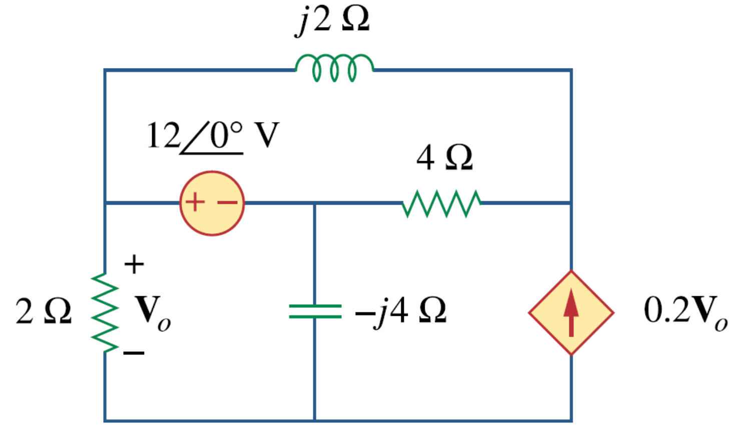
Example – 9

Find ${V_0}/{V_i}$ for $\omega = 0, \omega \to \infty$ and $\omega^2 = {1}/{LC}$.

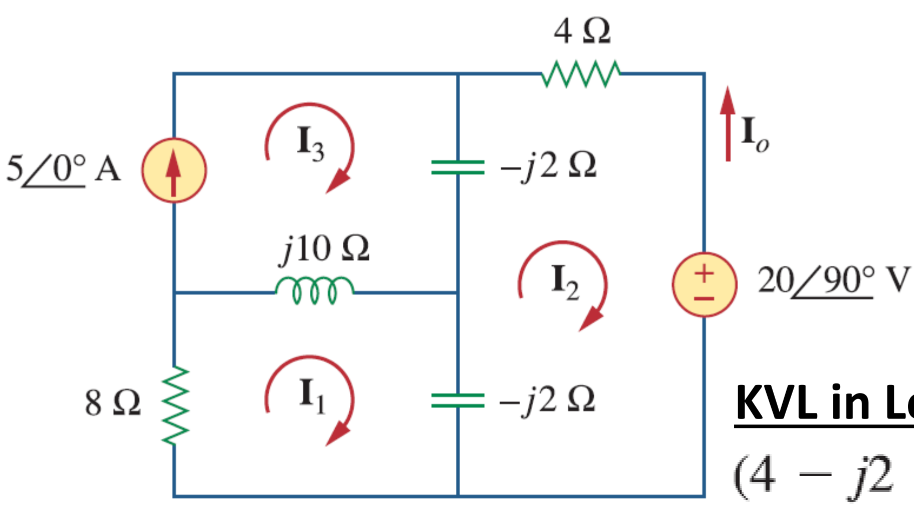


Example – 10

Use nodal analysis to find V_0.



Mesh Analysis → Determine I_0 using Mesh Analysis.



in Loop 3: $\mathbf{I}_3 = 5$

KVL in Loop 1:

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

KVL in Loop 2:

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\angle 90^\circ = 0$$

Mesh Analysis (contd.)

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50 \qquad j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10$$

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

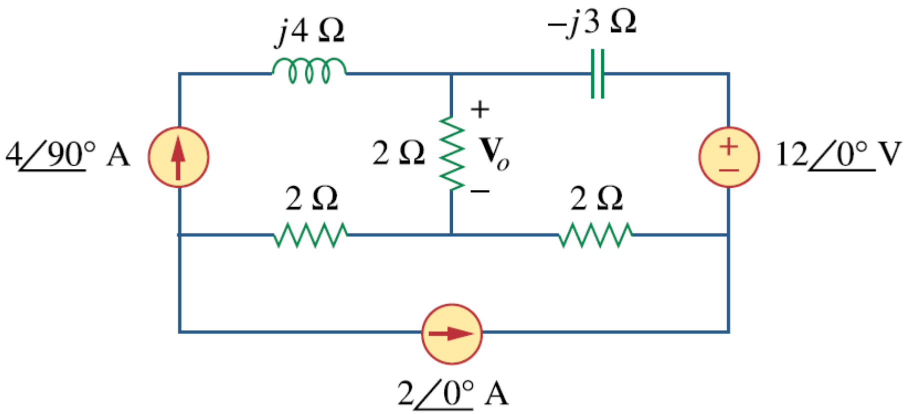
$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\angle -35.22^\circ$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\angle -35.22^\circ}{68} = 6.12\angle -35.22^\circ \text{ A}$$

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\angle 144.78^\circ \text{ A}$$

Example – 11

- Use mesh analysis to find v_0.

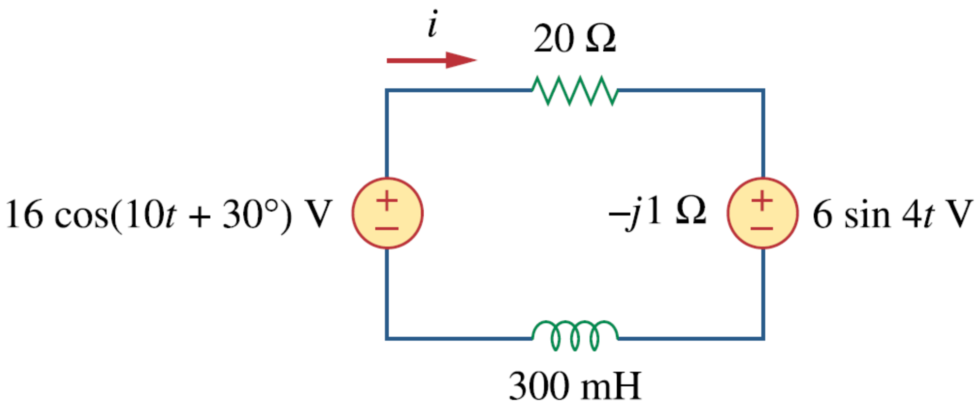


Superposition Theorem

- These circuits are linear and hence you can apply superposition theorem.
- If sources have different frequencies then individual response must be added in the time domain.
- You can't add them in phasors as they have different $e^{j\omega t}$.

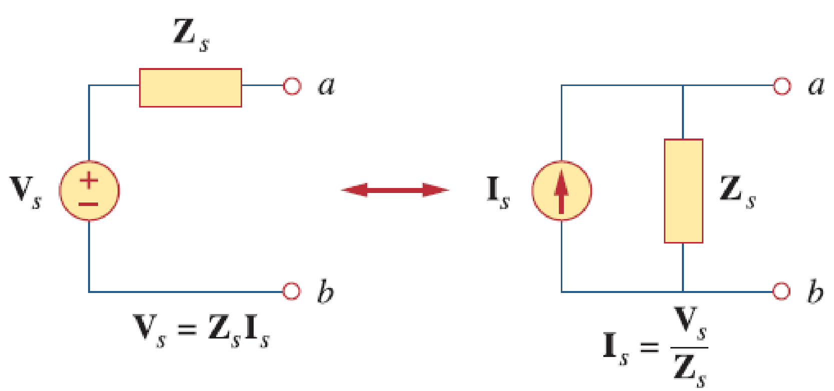
Example – 12

- Use superposition to find $i(t)$.



Source Transformation

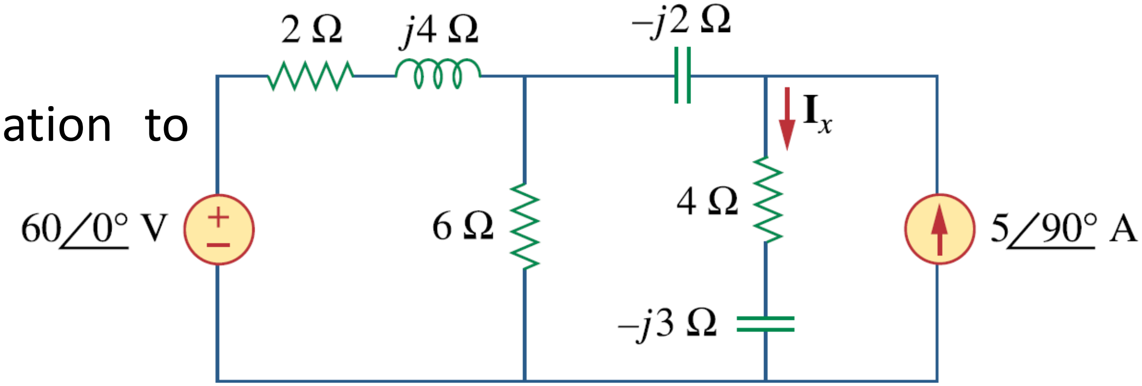
It involves transformation of **voltage source in series with an impedance** to a **current source in parallel with an impedance**, or vice versa.



$$\mathbf{V}_s = \mathbf{Z}_s\mathbf{I}_s \quad \Leftrightarrow \quad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}$$

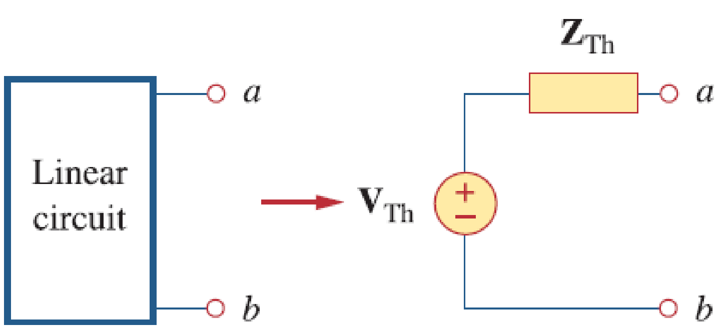
Example – 13

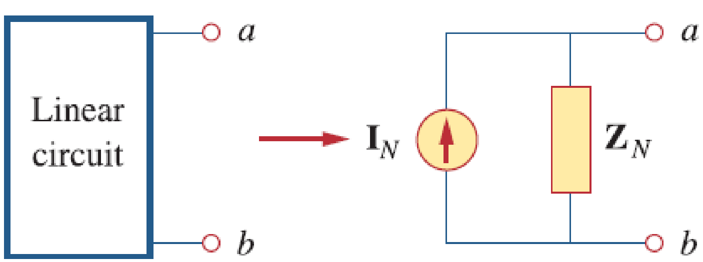
- Use source transformation to find I_x.



Thevenin and Norton Equivalent Circuits

- These theorems are applied to AC circuits similar to the way it is applied to DC circuits.
- You need to work with complex numbers in AC circuits.
- For sources with different frequencies, you will have different equivalent circuit for each frequency.

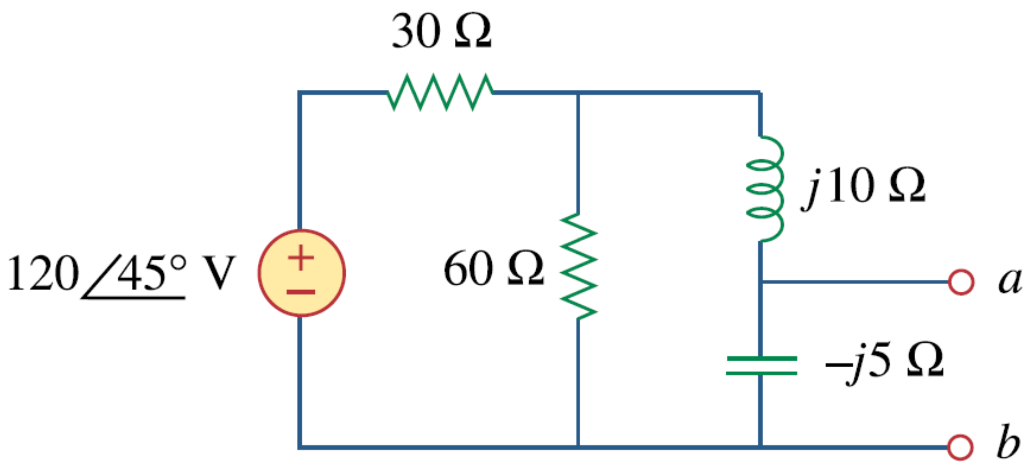




$$\mathbf{V}_{\mathrm{Th}} = \mathbf{Z}_N \mathbf{I}_N, \qquad \mathbf{Z}_{\mathrm{Th}} = \mathbf{Z}_N$$

Example – 14

- Obtain Thevenin and Norton equivalent circuits at terminal a-b.



Example – 15

- Using Thevenin Theorem, determine $v_o(t)$.