ECE 215

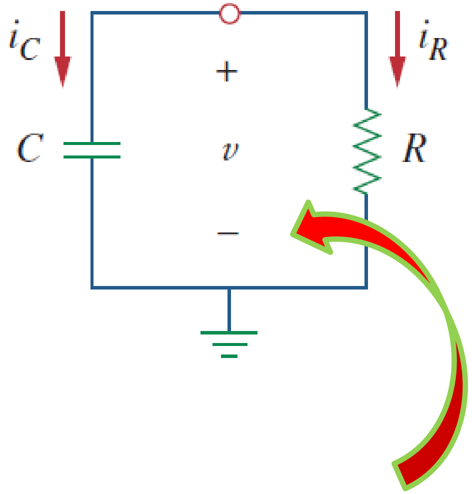
Lecture – 3 **Date: 07.08.2017**

- First-Order Circuit – Review

First Order Circuit

- A first order circuit is characterized by first-order differential equation in two types of configs:
  a) RL Circuit
  b) RC Circuit

Two possible scenarios: (a) without excitation, (b) with excitation



Need to determine $v(t)$, $i_c(t)$, and $i_R(t)$

This will result when the dc source is suddenly disconnected and in this scenario the energy stored in the capacitor gets released to the resistors.

Assume: $v(0) = V_0$ ⟸ Initial charging voltage of the capacitor

Therefore the energy stored in the capacitor: $w(0) = \frac{1}{2}CV_0^2$

KCL gives: $i_C + i_R = 0$ ⟹ $C\frac{dv}{dt} + \frac{v}{R} = 0$ ⟹ $\frac{dv}{v} = -\frac{1}{RC}dt$

⟹ $\ln v = -\frac{t}{RC} + \ln A$ ⟹ $v(t) = Ae^{-t/RC}$

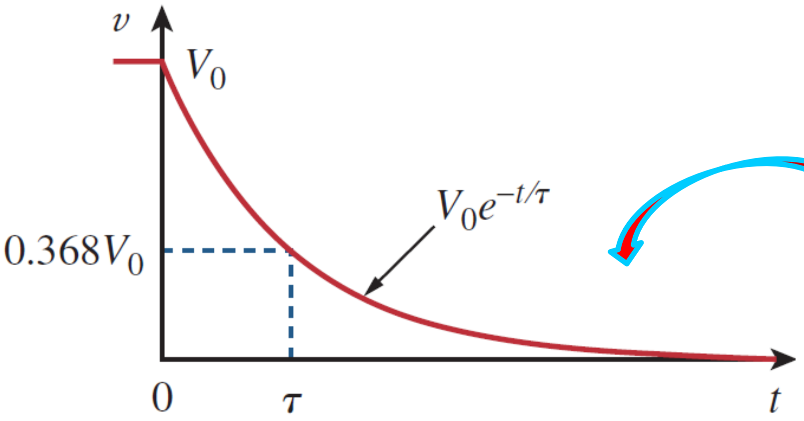
First Order Circuit (contd.)

From initial condition: $v(0) = A = V_0$

$$\therefore v(t) = V_0 e^{-t/RC}$$

Exponential Decay

Natural Response



The time constant of a circuit is the time required for the response to decay to a factor of $1/e$

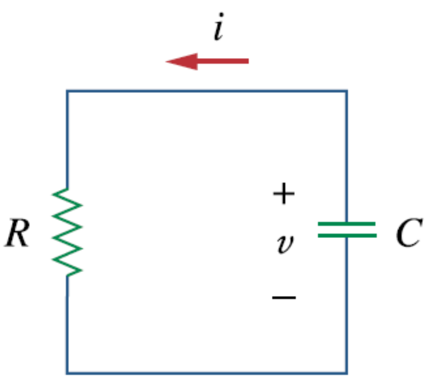
The power dissipated in the resistor

$$i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/RC} \quad \Rightarrow \quad p(t) = v i_R(t) = \frac{V_0^2}{R} e^{-2t/RC}$$

- The energy absorbed by the resistor up to time t is: $w_R(t) = \int_0^t p(j)dj = \frac{1}{2} C V_0^2 \left(1 - e^{-2t/\tau}\right)$

For $t \to \infty$ we get $w_R(\infty) \to \frac{1}{2} C V_0^2$

Example – 1



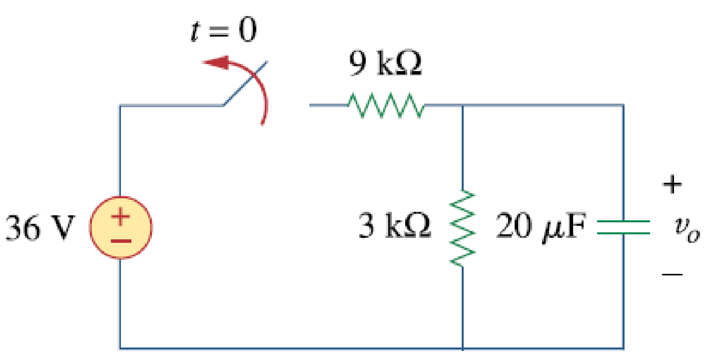
In the circuit shown:

$$v(t) = 56e^{-200t}\,\text{V}, \quad t > 0$$

$$i(t) = 8e^{-200t}\,\text{mA}, \quad t > 0$$

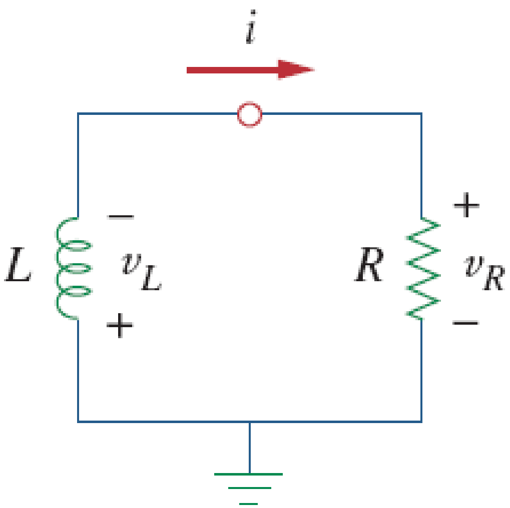
(a) Find the values of *R* and *C*.
(b) Calculate the time constant τ.
(c) Determine the time required for the voltage to decay half its initial value at *t* = 0.

Example – 2



Find $v_0(t)$ for $t > 0$. Determine the time necessary for the capacitor voltage to decay to one-third of its value at *t* = 0.

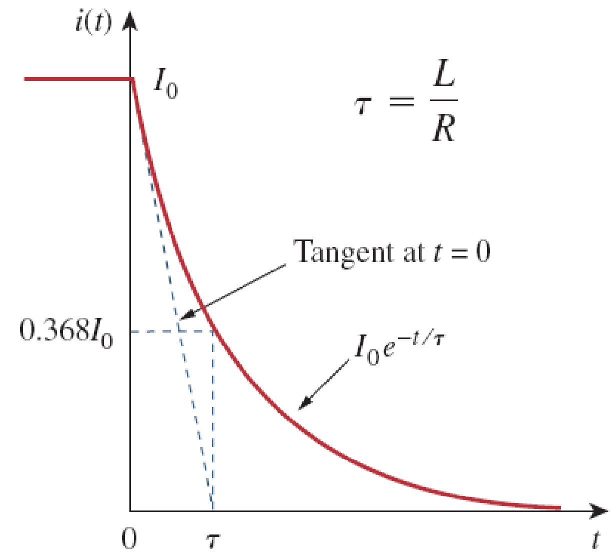
First Order Circuit (contd.)



Assume, at t = 0: $i(0) = I_0$

Stored energy in the inductor: $w(0) = \frac{1}{2}LI_0^2$

$$v_L + v_R = 0 \quad \Rightarrow \quad L\frac{di}{dt} + Ri = 0$$



$$\frac{di}{dt} + \frac{R}{L}i = 0 \quad \Rightarrow \quad \int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L}dt$$

$$\ln\frac{i(t)}{I_0} = -\frac{Rt}{L} \quad \Rightarrow \quad i(t) = I_0e^{-Rt/L}$$

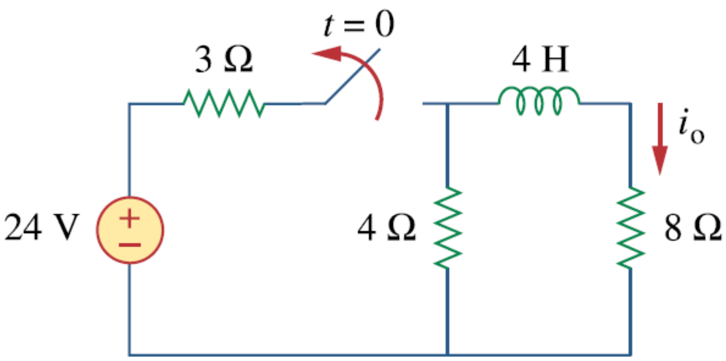
Power dissipated in R

$$v_R(t) = iR = I_0Re^{-t/\tau} \quad \Rightarrow \quad p = v_Ri = I_0^2Re^{-2t/\tau}$$

- The energy absorbed by R up to time t is: $w_R(t) = \int_0^t p(j)dj = \frac{1}{2}LI_0^2\left(1 - e^{-2t/\tau}\right)$

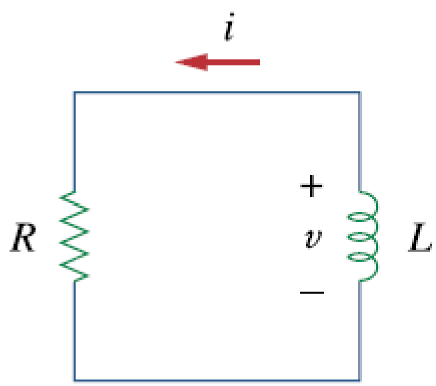
For $t \rightarrow \infty$ we get $w_R(\infty) \rightarrow \frac{1}{2}LI_0^2$

Example – 3



find i_0 for $t > 0$.

Example – 4



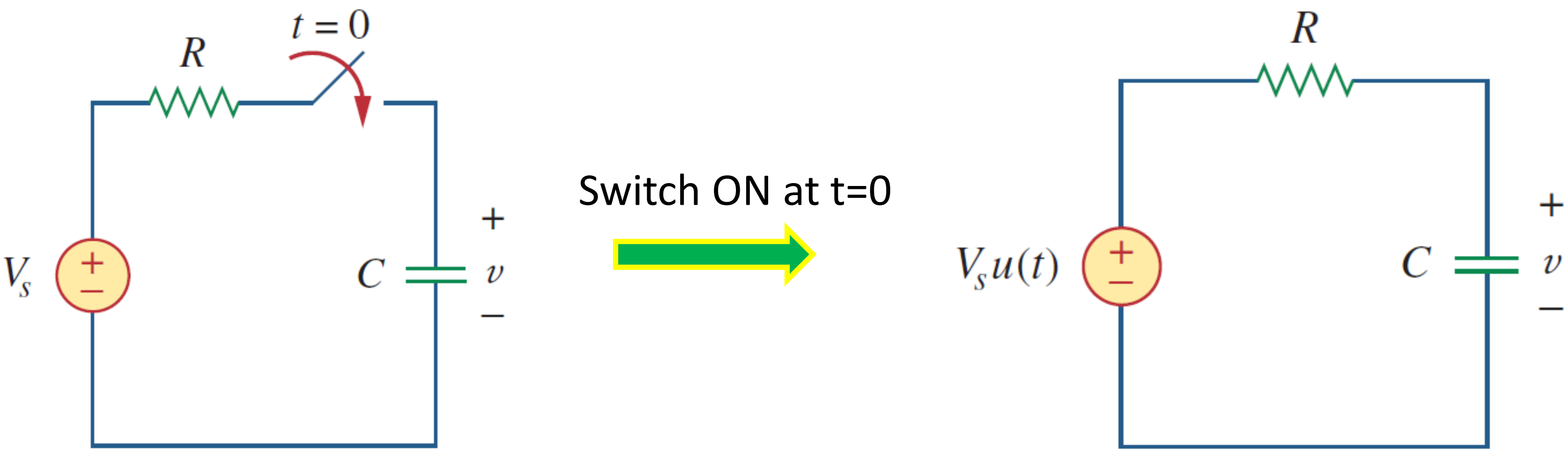
$$v(t) = 20e^{-10^3 t} \text{ V}, \qquad t > 0$$

$$i(t) = 4e^{-10^3 t} \text{ mA}, \qquad t > 0$$

(a) Find *R*, *L*, and τ .

(b) Calculate the energy dissipated in the resistance for $0 < t < 0.5$ ms.

Step Response of an RC Circuit



Assume an initial voltage V_0 on the capacitor

- Voltage on a capacitor cannot change instantaneously: $v(0^-) = v(0^+) = V_0$

$$C\frac{dv}{dt} + \frac{v - V_s u(t)}{R} = 0 \quad \Rightarrow \quad \frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC}u(t)$$

- For $t > 0$: $\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC} \quad \Rightarrow \quad \frac{dv}{v - V_s} = -\frac{dt}{RC}$

$$\ln(v - V_s)\Big|_{V_0}^{v(t)} = -\frac{t}{RC}\Big|_0^t \quad \Rightarrow \quad \ln(v(t) - V_s) - \ln(V_0 - V_s) = -\frac{t}{RC} + 0$$

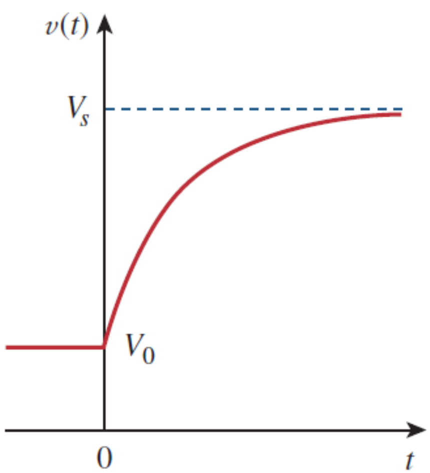
Step Response of an RC Circuit (contd.)

$$\ln \frac{v - V_s}{V_0 - V_s} = -\frac{t}{RC} \quad \Rightarrow \quad v - V_s = (V_0 - V_s)e^{-t/\tau}$$

$$v(t) = V_s + (V_0 - V_s)e^{-t/\tau}$$ For $t > 0$

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau}, & t > 0 \end{cases}$$

Complete Response



For $V_0 = 0$

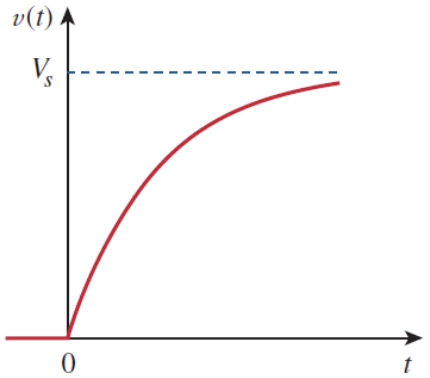
$$v(t) = \begin{cases} 0, & t < 0 \\ V_s(1 - e^{-t/\tau}), & t > 0 \end{cases} \quad \Rightarrow \quad v(t) = V_s(1 - e^{-t/\tau})u(t)$$

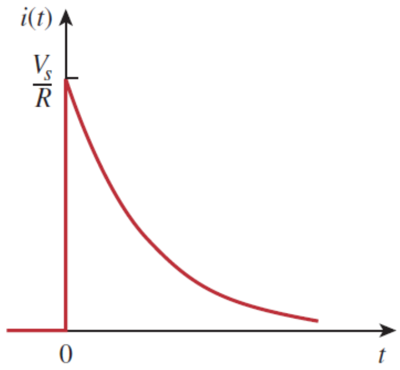
Step Response of an RC Circuit (contd.)

The current through the capacitor:

$$i(t) = C\frac{dv}{dt} = \frac{C}{\tau}V_s e^{-t/\tau}$$

$$i(t) = \frac{V_s}{R}e^{-t/\tau}u(t)$$





Complete response = natural response (stored energy) + forced response (independent source)

$$v = v_n + v_f$$

$$v_n = V_o e^{-t/\tau}$$

$$v_f = V_s(1 - e^{-t/\tau})$$

Complete response = transient response (temporary part) + steady-state response (permanent part)

$$v = v_t + v_{ss}$$

$$v_t = (V_o - V_s)e^{-t/\tau}$$

$$v_{ss} = V_s$$

Step Response of an RC Circuit (contd.)

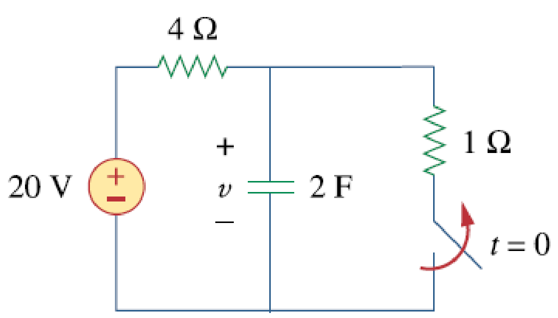
- Essentially we can express: $v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$
- For the determination of the step response of an *RC* circuit we require:

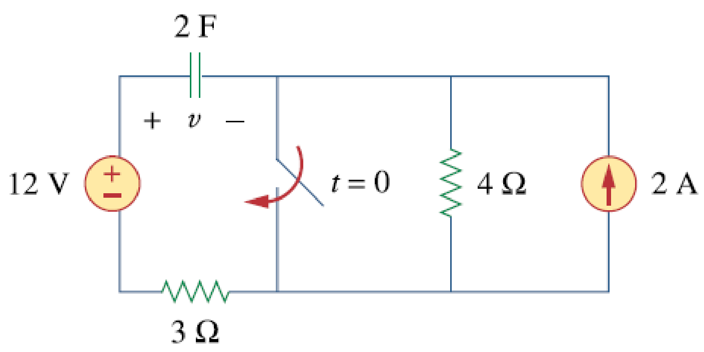
1. The initial capacitor voltage $v(0)$
2. The final capacitor voltage $v(\infty)$
3. The time constant τ.

We can obtain $v(0)$ from the given circuit for t<0 and $v(\infty)$and τ from the circuit for t>0.

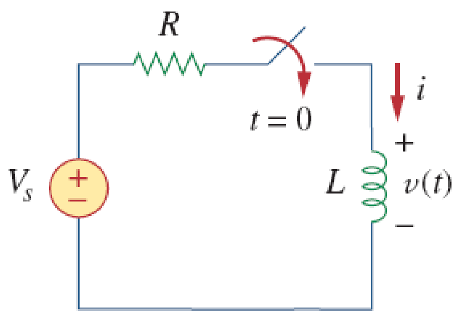
Example – 5

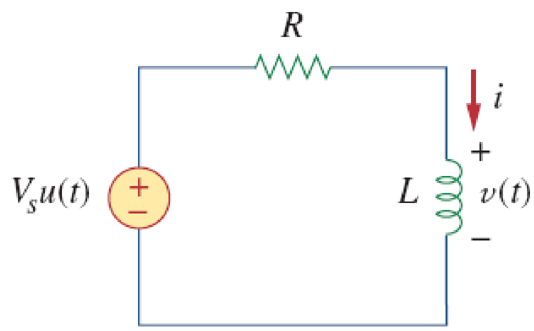
Calculate the capacitor voltage for *t* < 0 and *t* > 0 for both the circuits.





Step Response of an RL Circuit





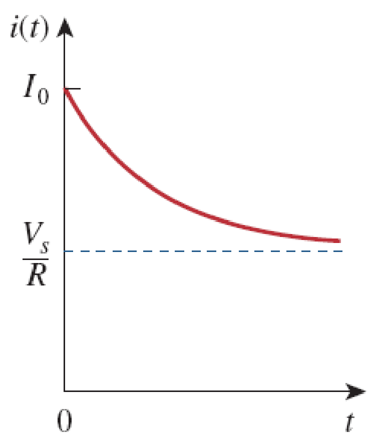
Complete Response: $i = i_t + i_{ss}$ $\quad i_t = Ae^{-t/\tau},$ $\quad \tau = \dfrac{L}{R}$ $\quad i_{ss} = \dfrac{V_s}{R}$

- Now the current through the inductor cannot change instantaneously:

$$i(0^+) = i(0^-) = I_0$$

At t=0: $I_0 = A + \dfrac{V_s}{R}$ ⇒ $A = I_0 - \dfrac{V_s}{R}$ ⇒ $i(t) = \dfrac{V_s}{R} + \left(I_0 - \dfrac{V_s}{R}\right)e^{-t/\tau}$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$



Step Response of an RL Circuit (contd.)

- For $I_0=0$

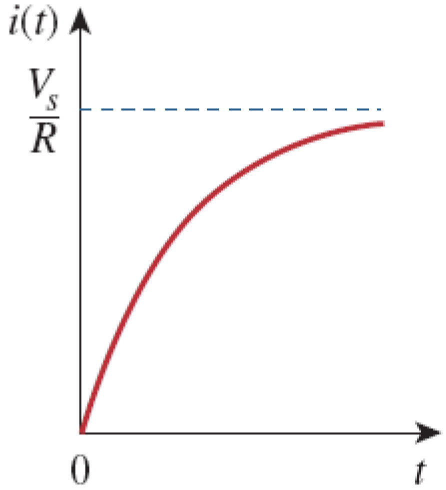
$$i(t) = \begin{cases} 0, & t < 0 \\ \frac{V_s}{R}(1 - e^{-t/\tau}), & t > 0 \end{cases}$$

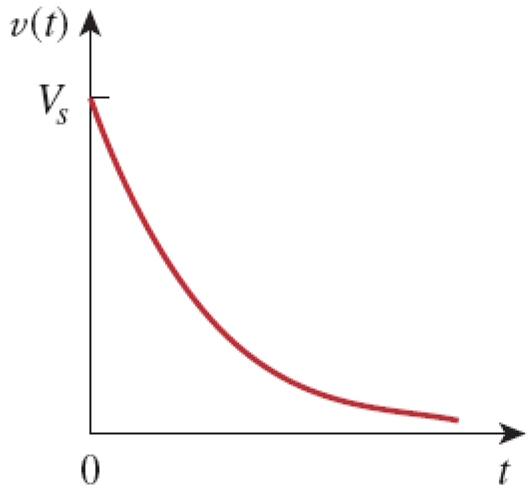
$$i(t) = \frac{V_s}{R}(1 - e^{-t/\tau})u(t)$$

- The voltage across the inductor:

$$v(t) = L\frac{di}{dt} = V_s\frac{L}{\tau R}e^{-t/\tau}, \qquad \tau = \frac{L}{R}, \qquad t > 0$$

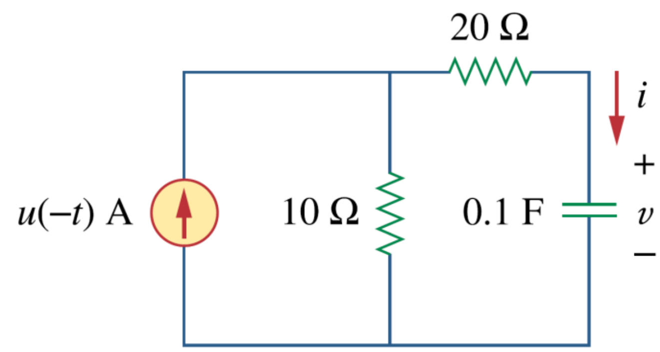
$$v(t) = V_s e^{-t/\tau}u(t)$$





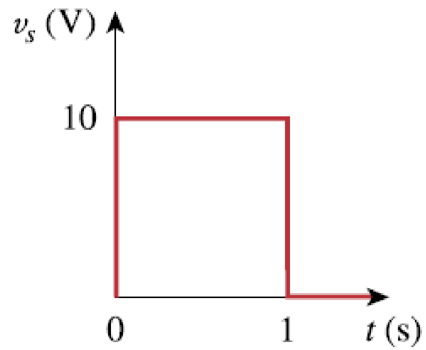
Example – 6

Find the $v(t)$ *and* $i(t)$ in this circuit.

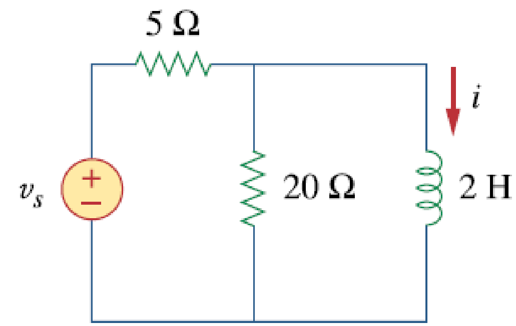


Example – 7

If the input pulse in Fig. (a) is applied to the circuit in Fig. (b), determine the response *i*(*t*).



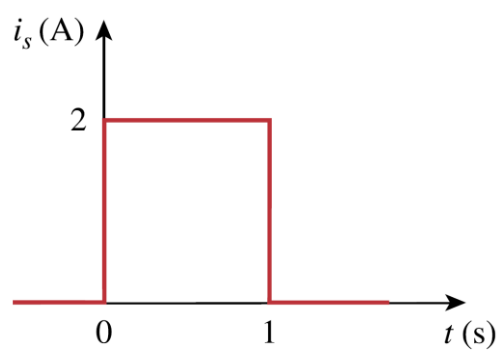
(a)



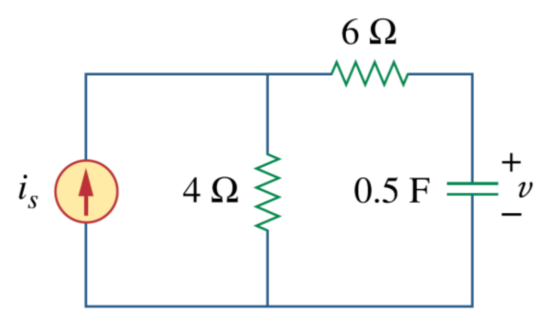
(b)

Example – 8

If the waveform in (a) is applied to the circuit of (b), find *v*(*t*). Assume *v*(0) = 0.

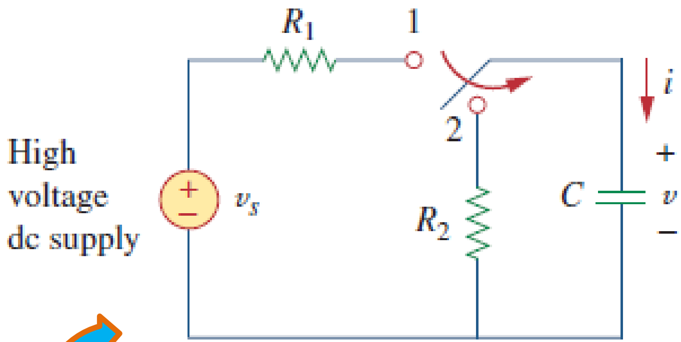


(a)



(b)

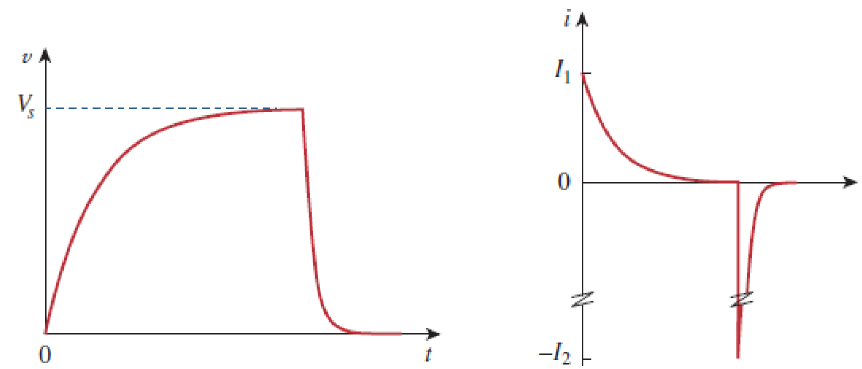
Photo Flash Unit



- An electronic flash unit is a common example of an *RC* circuit.
- This exploits the ability of the capacitor to oppose any abrupt change in voltage.

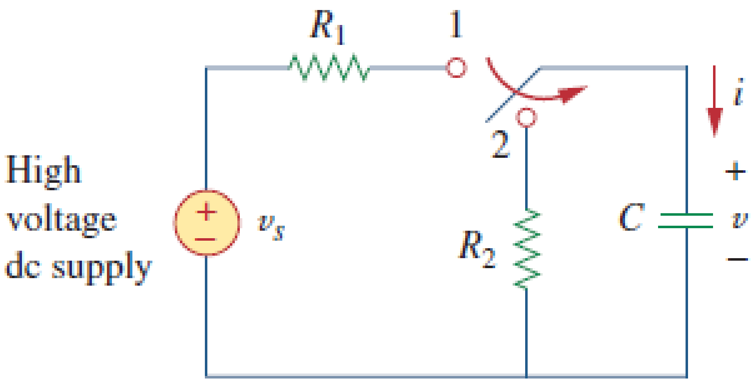
It consists of a high-voltage dc supply, a current-limiting large resistor R_1 and a capacitor *C* in parallel with the flash lamp of low Resistance R_2.

- When the switch is in position 1, the capacitor charges slowly due to the large time constant (R_1C).
- the capacitor voltage rises gradually from zero to V_s while its current decreases gradually from ($I_1 = V_s/R$) to zero.

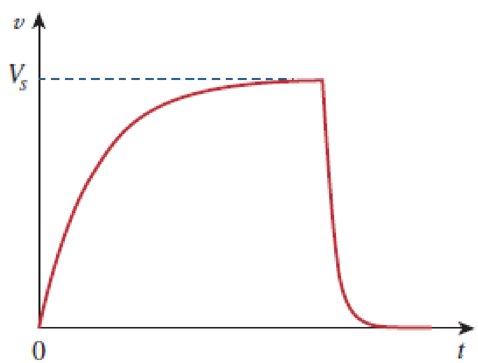


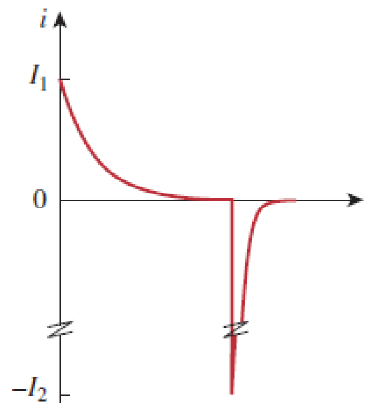
The charging time is approximately five times the time constant: $t_{charge} = 5R_1C$

Photo Flash Unit (contd.)



- With the switch in position 2, the capacitor voltage is discharged.
- The low resistance R_2 of the photo lamp permits a high discharge current with peak [$I_2 = V_s/R_2$] in a short duration



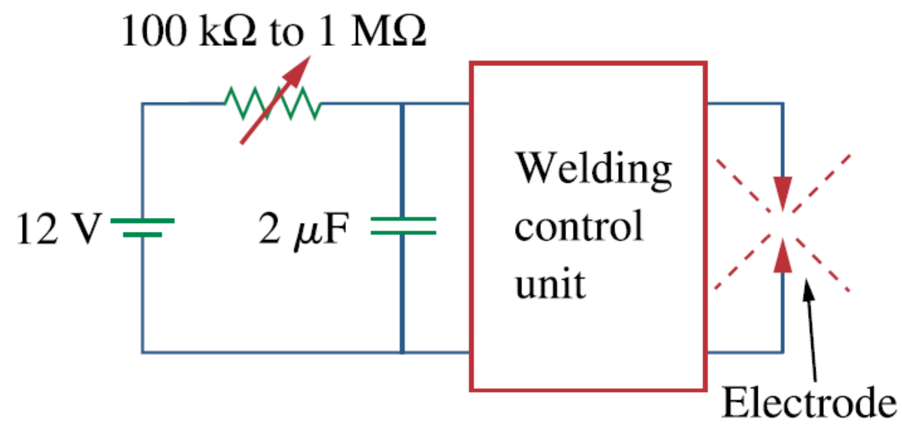


Discharging takes place in approximately five times the time constant: $t_{discharge} = 5R_2C$

- This simple *RC* circuit provides a short-duration and high current pulse.
- This circuit finds applications in electric spot welding and radar transmitter tube.

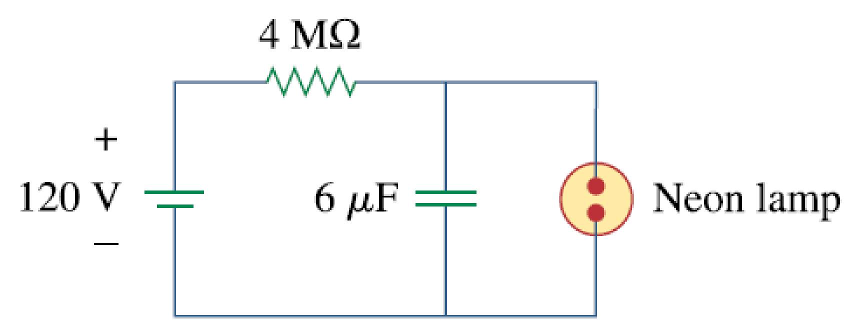
Example – 9

The Figure shows a circuit for setting the length of time voltage is applied to the electrodes of a welding machine. The time is taken as how long it takes the capacitor to charge from 0 to 8 V. What is the time range covered by the variable resistor?



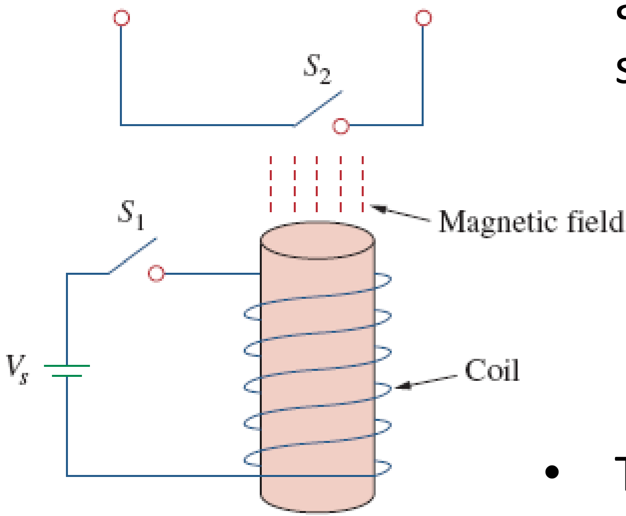
Example – 10

A simple relaxation oscillator circuit is shown. The neon lamp fires when its voltage reaches 75V and turns off when its voltage drops to 30V. Its resistance is 120Ω when on and infinitely high when off.

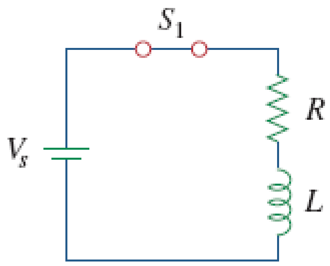


(a) For how long is the lamp on each time the capacitor discharges?
(b) What is the time interval between light flashes?

Relay Circuit



- Relay, a magnetically controlled switch, is essentially an electromagnetic device used to open or close a switch that controls another circuit.

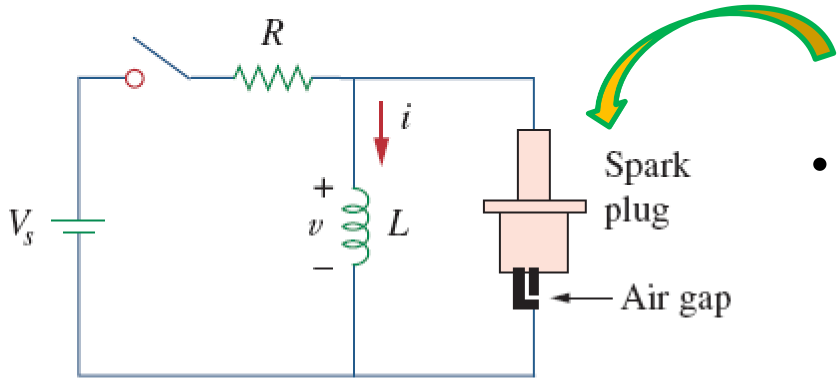


- The coil circuit is an *RL* circuit, where *R* and *L* are the resistance and inductance of the coil.

- the coil circuit is energized once switch S_1 is closed.
- The coil current gradually increases and produces a magnetic field. Eventually the magnetic field is sufficiently strong to pull the movable contact in the other circuit and close switch S_2.

Relays were used in the earliest digital circuits and are still used in high-power circuits.

Automobile Ignition Circuit



- It makes use of the ability of inductors to oppose rapid change in current as it makes them useful for spark generation.
- The ignition of fuel-air mixture in each cylinder at proper times is achieved by a spark plug, which consists of a pair of electrodes separated by an air gap.

- By creating a large voltage (thousands of volts) between the electrodes, a spark is formed across the air gap, thereby igniting the fuel. But how can such a large voltage be obtained from the car battery, which supplies only 12 V?
- This is achieved by means of an inductor (the spark coil) *L*. The voltage across the inductor $v = L\, {}^{di}/_{dt}$ can be made large by creating a large change in current in a very short time.
- When the switch is closed, the current through the inductor increases gradually and reaches the final value of ${}^{Vs}/_{R}$, where Vs is 12V.
- At steady state ${}^{di}/_{dt} = 0$ and hence the inductor voltage $v = 0$.
- When the switch suddenly opens, a large voltage is developed across the inductor (due to the rapid change in ${}^{di}/_{dt}$) causing a spark in the air gap.
- The spark continues until the energy stored in the inductor is dissipated in the spark discharge.