Lecture – 21 Date: 24.10.2017

- Fourier Series and Applications

Fourier Series and Circuit Applications

- So far, we have learnt the analysis of circuits with sinusoidal sources.
- However, many circuits are driven by non-sinusoidal periodic functions. The steady-state response of a circuit to a non-sinusoidal periodic excitation requires the application of a Fourier series, ac phasor analysis, and the superposition principle.
- The procedure usually involves four steps.

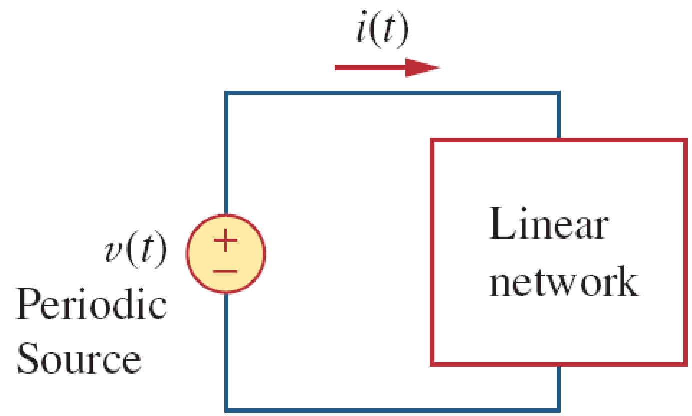
Steps for Applying Fourier Series:

1. Express the excitation as a Fourier series.
2. Transform the circuit from the time domain to the frequency domain.
3. Find the response of the dc and ac components in the Fourier series.
4. Add the individual dc and ac responses using the superposition principle.

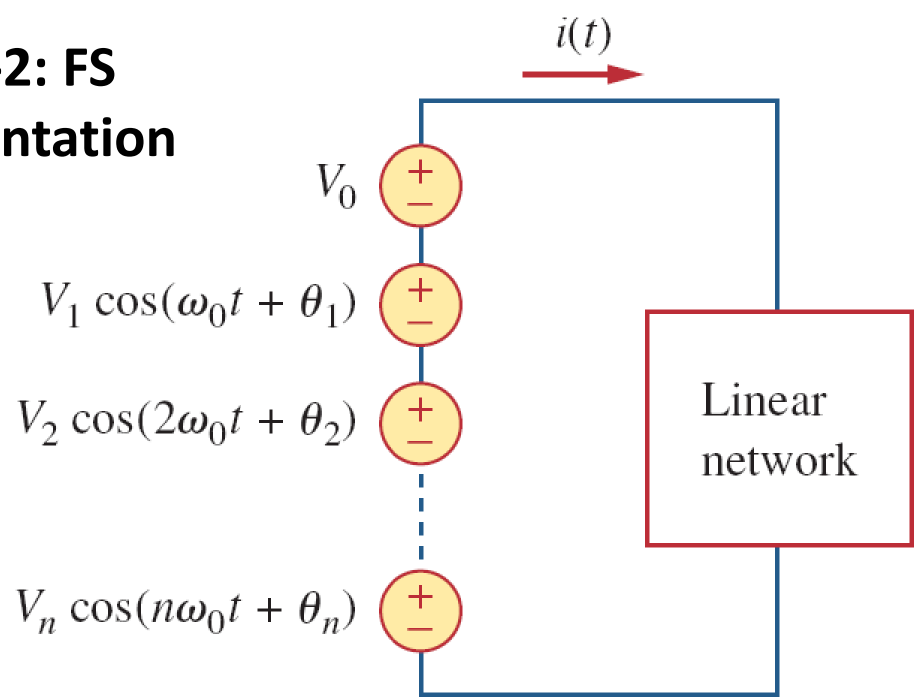
Circuit Applications (contd.)

- **First Step:** determine the Fourier series expansion of the excitation.

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$



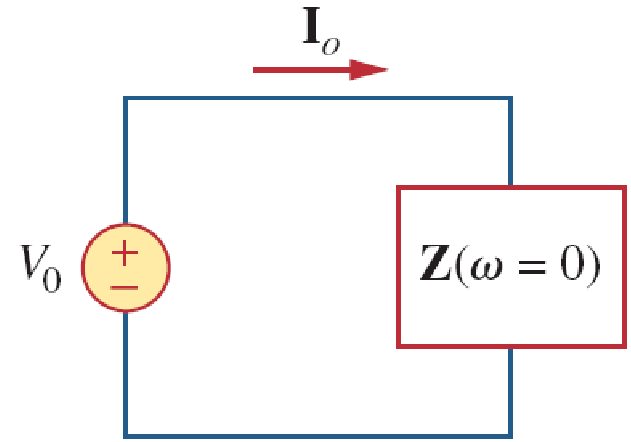
Step-2: FS representation



Third Step: find the response to each term in the Fourier series.

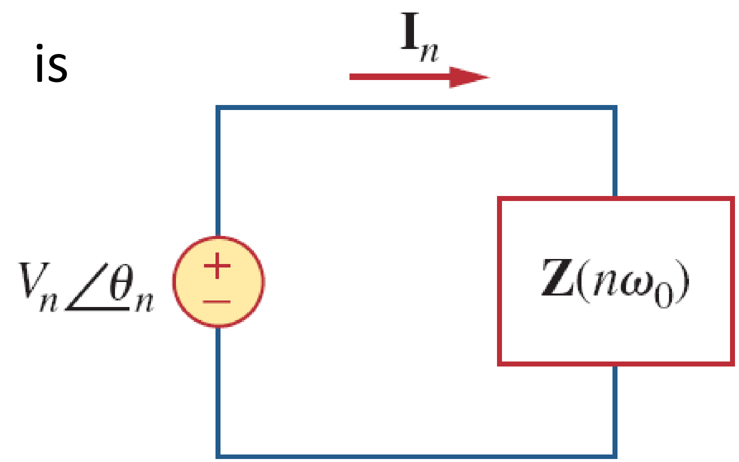
Circuit Applications (contd.)

- The response to the **dc component** is determined in the frequency domain by setting $n = 0$ or $\omega = 0$, or in the time domain by replacing all inductors with short circuits and all capacitors with open circuits.



- The response to the ac component is obtained by the phasor techniques.

The network is represented by its impedance $\boldsymbol{Z}(n\omega_0)$ and it's the value when ω is replaced by $n\omega_0$



- **Step-4:** apply principle of superposition to obtain the total current.

$$i(t) = i_0(t) + i_1(t) + i_2(t) + \cdots$$
$$= \mathbf{I}_0 + \sum_{n=1}^{\infty} |\mathbf{I}_n| \cos(n\omega_0 t + \psi_n)$$

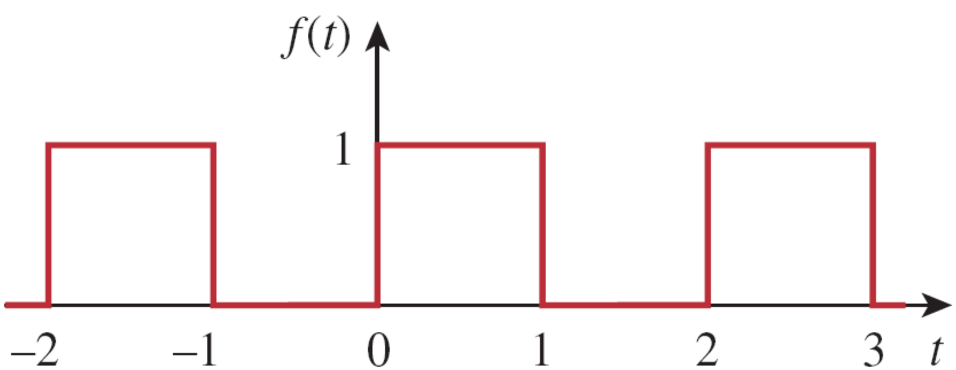
Circuit Applications (contd.)

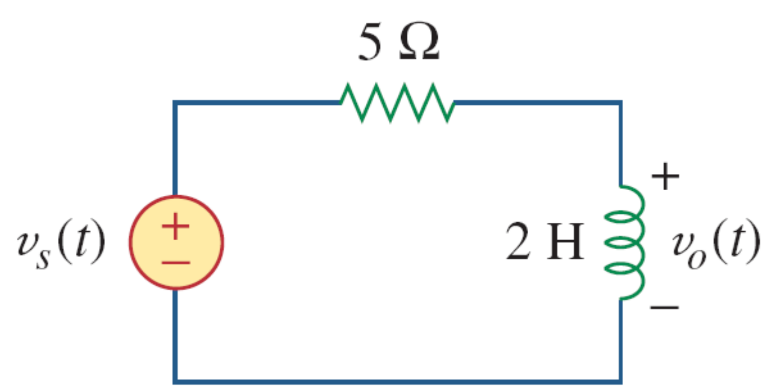
$$
\begin{aligned}
i(t) &= i_0(t) + i_1(t) + i_2(t) + \cdots \\
&= \mathbf{I}_0 + \sum_{n=1}^{\infty} |\mathbf{I}_n| \cos(n\omega_0 t + \psi_n)
\end{aligned}
$$

where each component $\boldsymbol{I}_n$ with frequency $n\omega_0$ has been transformed to the time domain to get $i_n(t)$, and Ψ_n is the argument of $\boldsymbol{I}_n$.

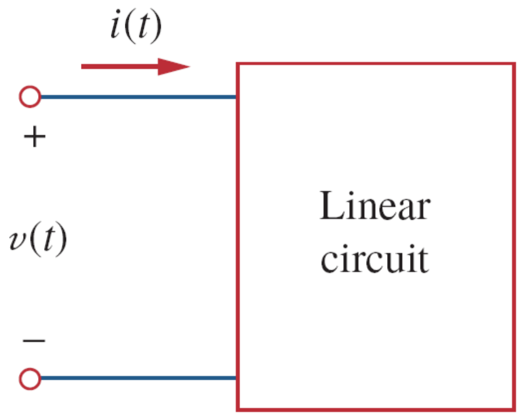
Example – 1

In this situation, $f(t)$ is the voltage source $v_s(t)$. Find the response of the circuit.





Average Power and RMS Power



- the periodic current and voltage in amplitude-phase form:

$$i(t) = I_{\text{dc}} + \sum_{m=1}^{\infty} I_m \cos(m\omega_0 t - \phi_m)$$

$$v(t) = V_{\text{dc}} + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t - \theta_n)$$

- the average power: $P = \frac{1}{T}\int_0^T vi\,dt$

$$P = \frac{1}{T}\int_0^T V_{\text{dc}} I_{\text{dc}}\,dt + \sum_{m=1}^{\infty} \frac{I_m V_{\text{dc}}}{T}\int_0^T \cos(m\omega_0 t - \phi_m)\,dt + \sum_{n=1}^{\infty} \frac{V_n I_{\text{dc}}}{T}\int_0^T \cos(n\omega_0 t - \theta_n)\,dt$$

$$+ \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{V_n I_m}{T}\int_0^T \cos(n\omega_0 t - \theta_n)\cos(m\omega_0 t - \phi_m)\,dt$$

Here, the second and third integrals vanish, due to the integration of the cosine over its period. All terms in the fourth integral are zero when $n \neq m$.

Average Power and RMS Power (contd.)

- **For $m = n$:** $P = V_{\text{dc}} I_{\text{dc}} + \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$

  the total average power is the sum of the average powers in each harmonically related voltage and current.

- For a periodic function $f(t)$, its rms value (or the effective value) is: $F_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T f^2(t)\, dt}$

- In terms of Fourier coefficients, the rms value is: $F_{\text{rms}} = \sqrt{a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)}$

- If $f(t)$ is the current through a resistor *R*, then: $P = R F_{\text{rms}}^2$

- If $f(t)$ is the voltage across a resistor *R*, then: $P = \frac{F_{\text{rms}}^2}{R}$

- The power dissipated by the 1Ω resistance is: $P_{1\Omega} = F_{\text{rms}}^2 = a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$

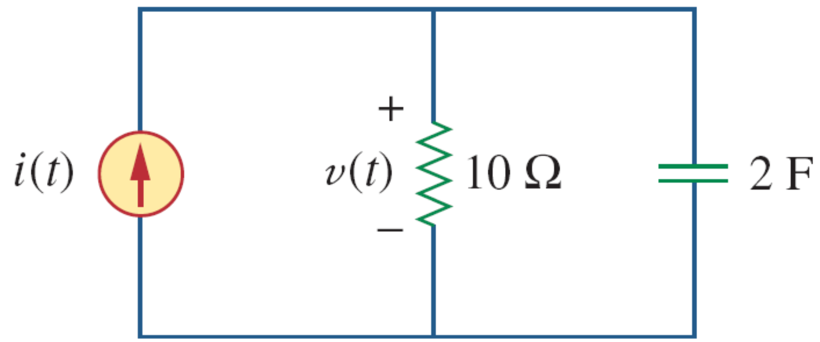
Average Power and RMS Power (contd.)

$$P_{1\Omega} = F_{\text{rms}}^2 = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

- This result is known as *Parseval′s theorem*.
- Notice that $(a_0)^2$ is the power in the dc component, while $\frac{1}{2}\left[(a_n)^2+(b_n)^2\right]$ is the ac power in the *n*th harmonic.
- Thus, Parseval's theorem states that the average power in a periodic signal is the sum of the average power in its dc component and the average powers in its harmonics.

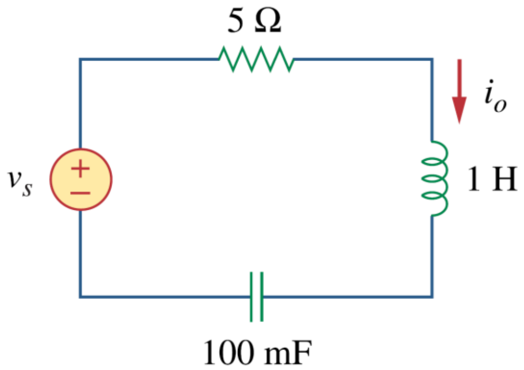
Example – 2

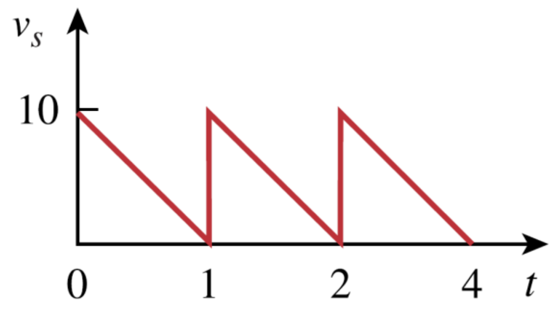
- Determine the average power supplied to this circuit if $i(t) = 2 + 10\cos(t + 10°) + 6\cos(3t + 35°)\text{A}$



Example – 3

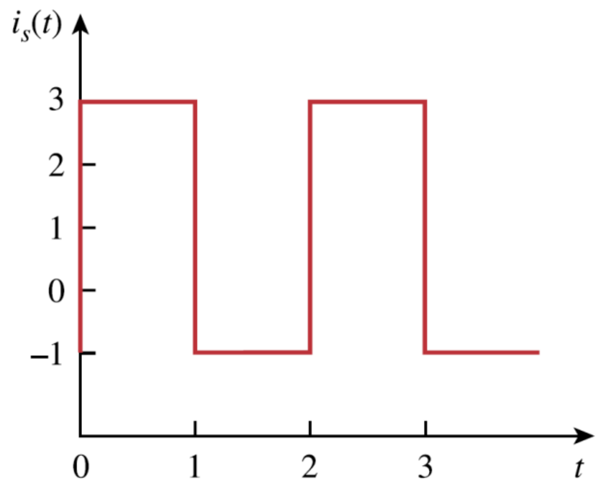
Find the response i_o for the circuit. The $v_s(t)$ is given in the waveform.

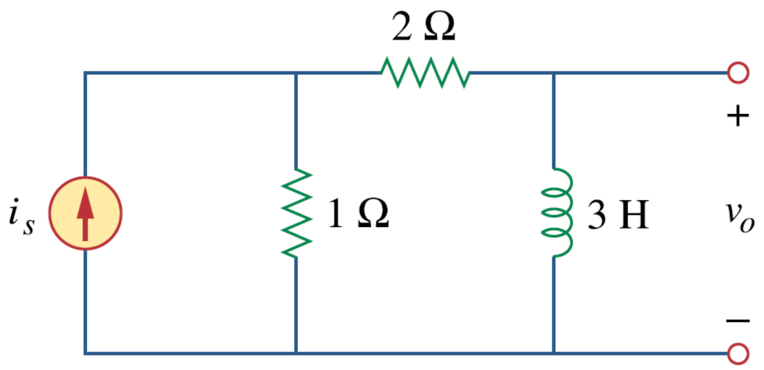




Example – 4

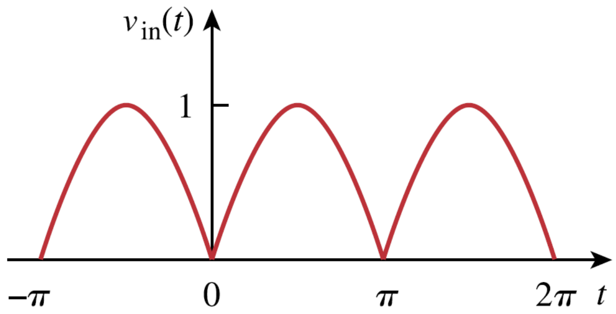
If the given periodic current waveform is applied to the circuit, find v_o.

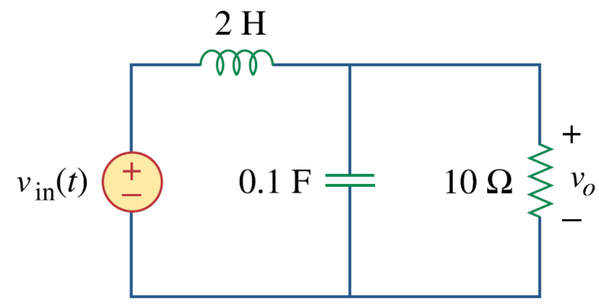




Example – 5

The full-wave rectified sinusoidal voltage is applied to the lowpass filter. Obtain the output voltage $v_o(t)$ of the filter.





Example – 6

A series *RLC* circuit has *R* = 10 Ω , *L* = 2 mH, and *C* = 40 μF. Determine the effective current and average power absorbed when the applied voltage is $v(t) = 100cos1000t + 50cos2000t + 25cos3000t$ V.

Spectrum Analyzers

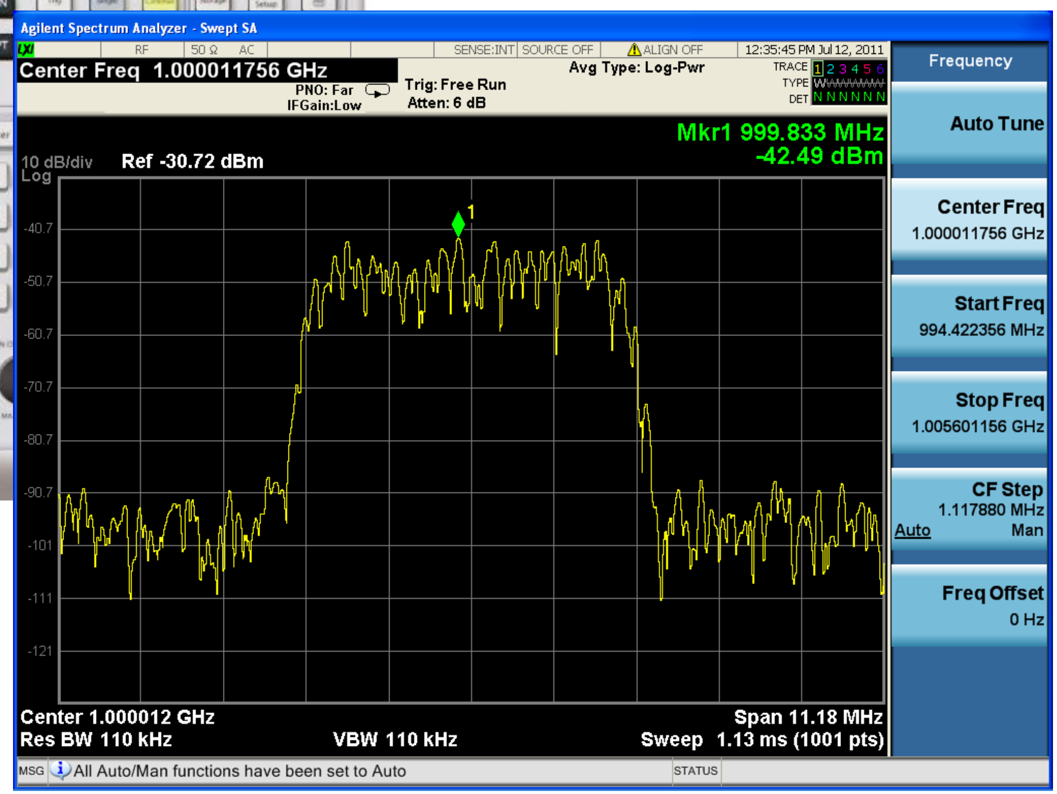
- The FS provides the spectrum of a signal → the spectrum consists of the amplitudes and phases of the harmonics versus frequency.
- By providing the spectrum of a signal $f(t)$, the FS helps in the identification of the pertinent features of the signal.
- It demonstrates which frequencies are playing an important role in the shape of the output and which ones are not.

- A *spectrum analyzer* is an instrument that displays the amplitude of the components of a signal versus frequency. It shows the various frequency components (spectral lines) that indicate the amount of energy at each frequency.

- It is unlike an oscilloscope, which displays the entire signal (all components) versus time.
- An oscilloscope shows the signal in the time domain, while the spectrum analyzer shows the signal in the frequency domain.

Spectrum Analyzers

- It can conduct noise and spurious signal analysis, phase checks, electromagnetic interference and filter examinations, vibration measurements, radar measurements, and more.

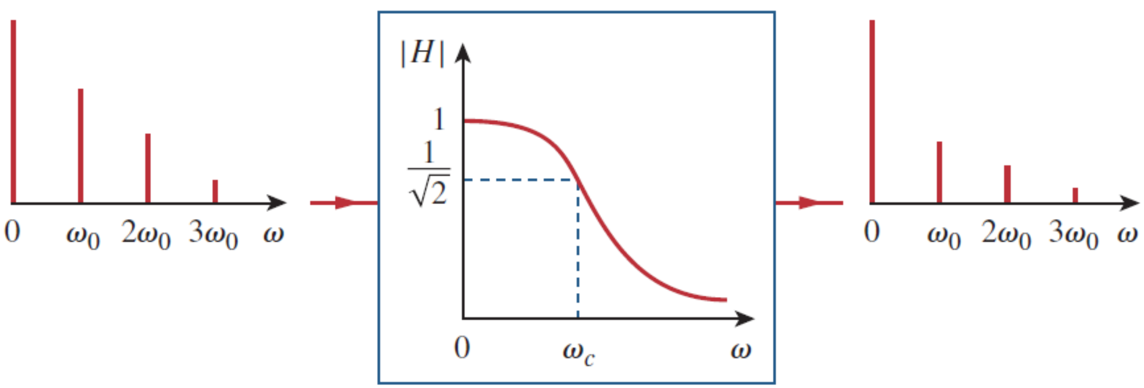


Filters

- Here, we investigate the techniques to design filters to select the fundamental component (or any desired harmonic) of the input signal and reject other harmonics.
- This filtering process cannot be accomplished without the Fourier series expansion of the input signal.

Low Pass Filter

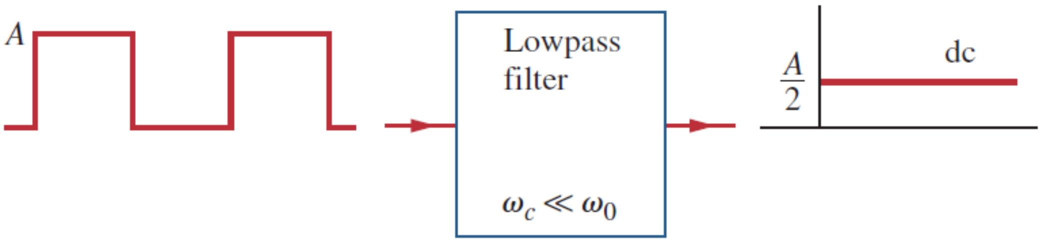
- The output of a low pass filter depends on the input signal, the transfer function $H(\omega)$ of the filter, and the corner or half-power frequency ω_c.
- We know $\omega_c = \frac{1}{RC}$ for an *RC* passive filter.



By making ω_c sufficiently large ($\omega_c \gg \omega$ e.g., by making *C* small), a large number of the harmonics can be passed.

Low Pass Filter

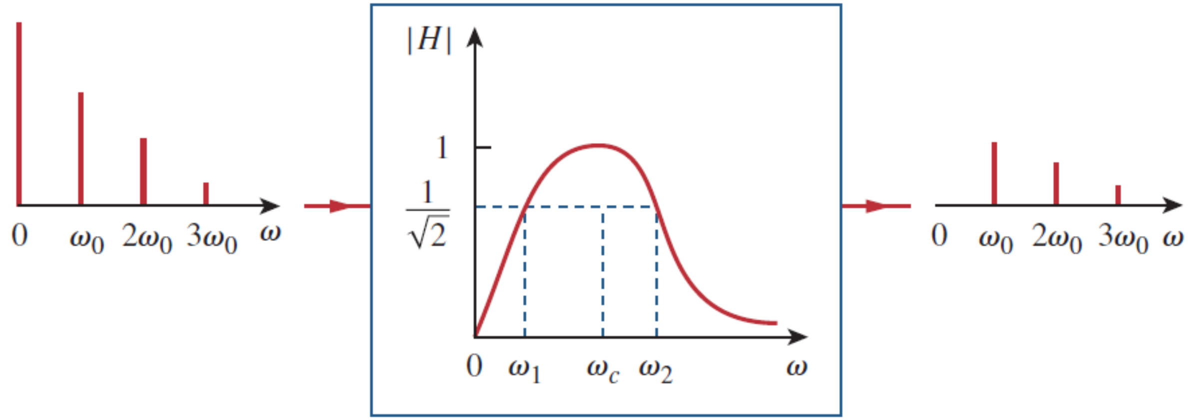
- the low-pass filter passes the dc and low-frequency components, while blocking the high-frequency components.



by making ω_c sufficiently small ($\omega_c \ll \omega$ e.g., by making C large), we can block out all the ac components and pass only dc

Band Pass Filter

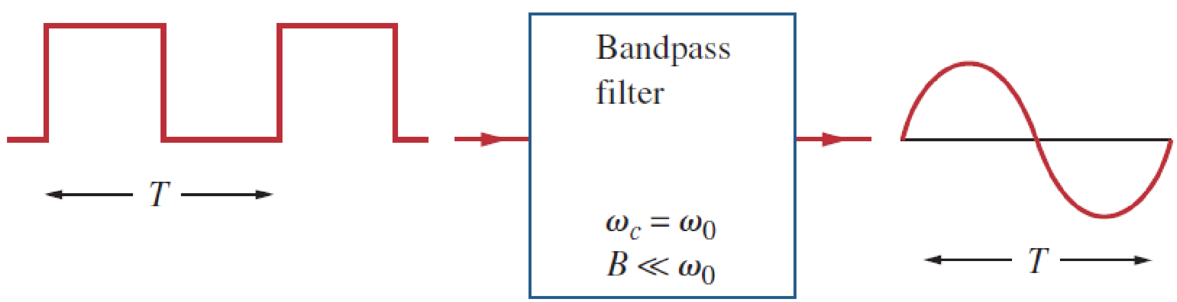
the output of a band pass filter depends on the input signal, the transfer function of the filter $H(\omega)$, its bandwidth B, and its center frequency ω_c.



It is assumed that ω_0, $2\omega_0$ and $3\omega_0$ are within that band

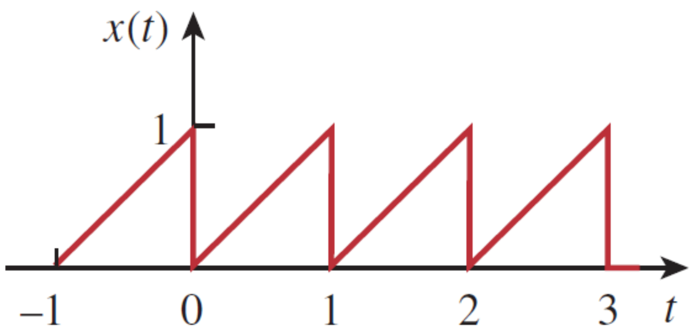
Band Pass Filter

- For highly selective filter ($B \ll \omega_0$) and $\omega_c = \omega_0$ where ω_0 is the fundamental frequency of the input signal → if the filter passes only the fundamental component (n=1) of the input and blocks out all higher harmonics → essentially, with a square wave as input, we obtain a sine wave of the same frequency as the output.

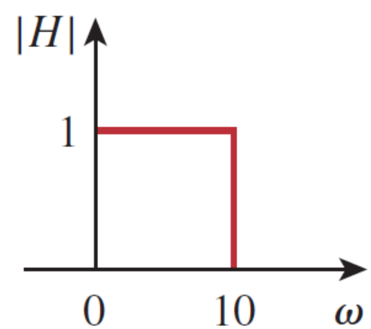


Example – 7

If the sawtooth waveform in Fig. (a) is applied to an ideal low pass filter with the transfer function shown in Fig. (b), determine the output.



(a)



(b)