Lecture – 18 **Date: 10.10.2017**

- Examples
- Active Filters
- Scaling

Passive Filter – Summary

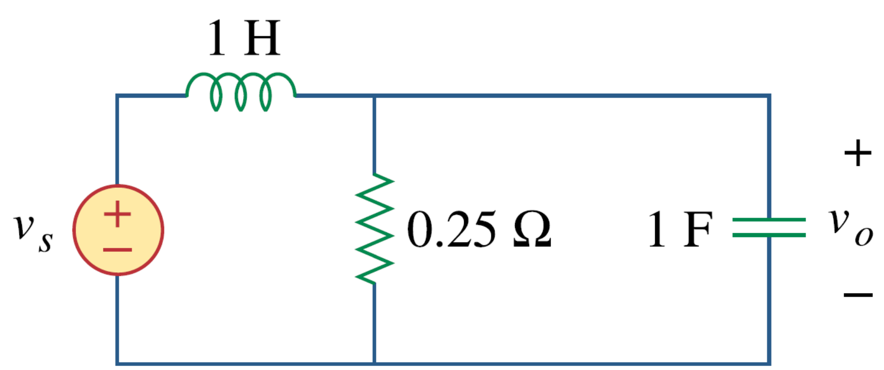
- the maximum gain of a passive filter is unity. To generate a gain greater than unity, one should use an active filter.
- There are other ways to get the types of filters.
- The filters discussed here are the simple types. Many other filters have sharper and complex frequency responses.

Example – 4

Show that a series *LR* circuit is a lowpass filter if the output is taken across the resistor. Calculate the corner frequency f_c if L = 2 mH and R = 10 k Ω.

Example – 5

Find the transfer function **V***o*/**V***s* of the circuit. Show that the circuit is a lowpass filter.



Example – 6

In a highpass *RL* filter with a cutoff frequency of 100 kHz, L = 40 mH. Find R.

Example – 7

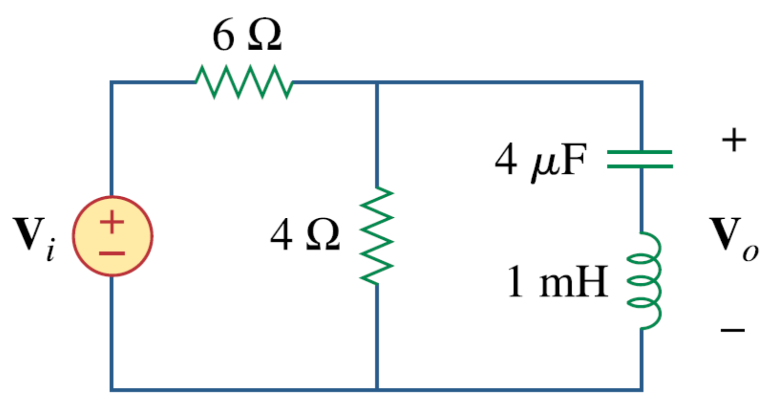
Design a series *RLC* type bandpass filter with cutoff frequencies of 10 kHz and 11 kHz. Assuming C = 80 pF, find R, L, and Q.

Example – 8

Determine the range of frequencies that will be passed by a series *RLC* bandpass filter with R = 10 Ω , L = 25mH, and C = 0.4 µF. Find the quality factor.

Example – 9

Find the bandwidth and center frequency of the bandstop filter



Active Filter

- There are three major limitations to the passive filters:
  - cannot generate gain greater than 1 → passive elements cannot add energy to the network.
  - may require bulky and expensive inductors.
  - perform poorly at freqs below the audio frequency range ($300Hz < f < 3000Hz$) → passive filters are useful at high freqs.

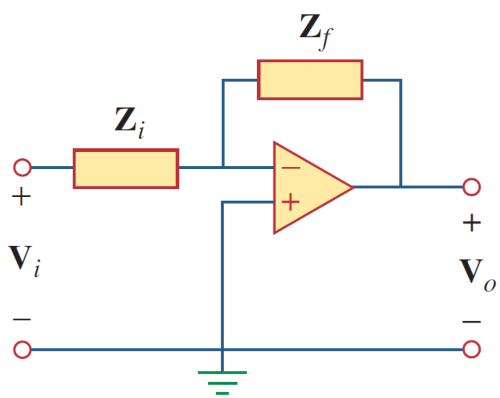
<u>Active filters:</u>

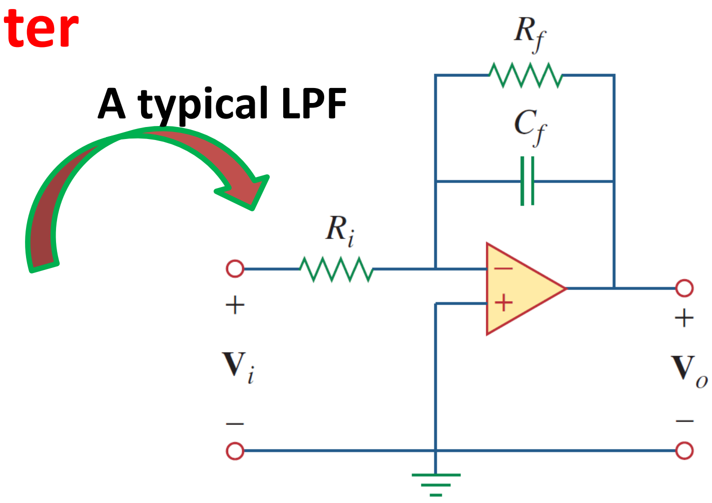
- consist of combinations of resistors, capacitors, and op amps.
- are often smaller and less expensive, because they do not require inductors → This enables realizations of filters ICs
- can provide gain in addition to providing the same frequency response as *RLC* filters.
- can be combined with buffer amplifiers (voltage followers) to isolate each stage of the filter from source and load impedance effects → allows designing the stages independently and then cascading them to realize the desired transfer function.

Active Filter

- Active filters are less reliable and less stable.
- The practical limit of most active filters is about 100 kHz → most active filters operate well below that frequency.
- These filters are often classified according to their order (or number of poles) or their specific design type.

First Order Low Pass Filter





$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i}$$

$$\mathbf{Z}_i = R_i$$

$$\mathbf{Z}_f = R_f \bigg\| \frac{1}{j\omega C_f} = \frac{R_f}{1 + j\omega C_f R_f}$$

- The components selected for Z_f and Z_i determine if its a lowpass or highpass
- but one of the components must be reactive.

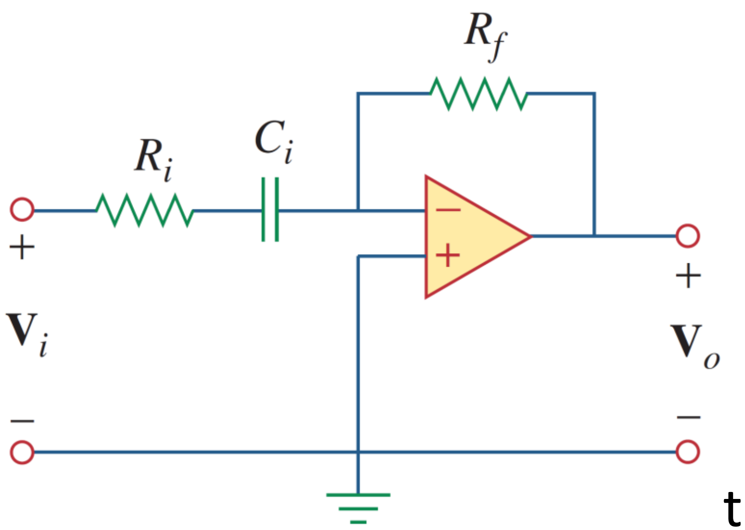
First Order Low Pass Filter

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_f R_f}$$

Similar to passive LPF expression → except that there is a low frequency ($\omega \rightarrow 0$) gain or dc gain of $-\frac{R_f}{R_i}$.

the corner frequency is: $\omega_c = \frac{1}{R_f C_f}$ ⇦ Independent of R_i

First Order High Pass Filter



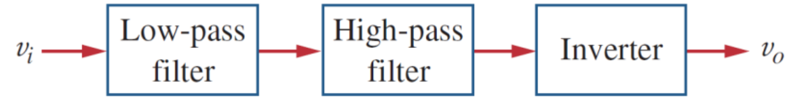
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i}$$

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i + 1/j\omega C_i} = -\frac{j\omega C_i R_f}{1 + j\omega C_i R_i}$$

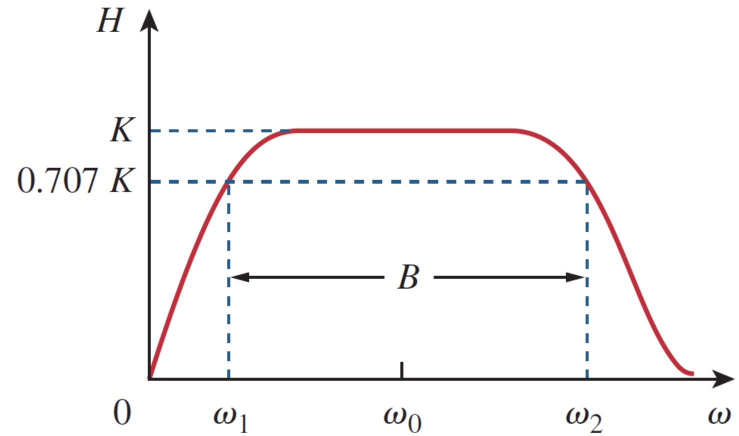
At high frequency ($\omega \rightarrow \infty$), the gain tends to $-\frac{R_f}{R_i}$.

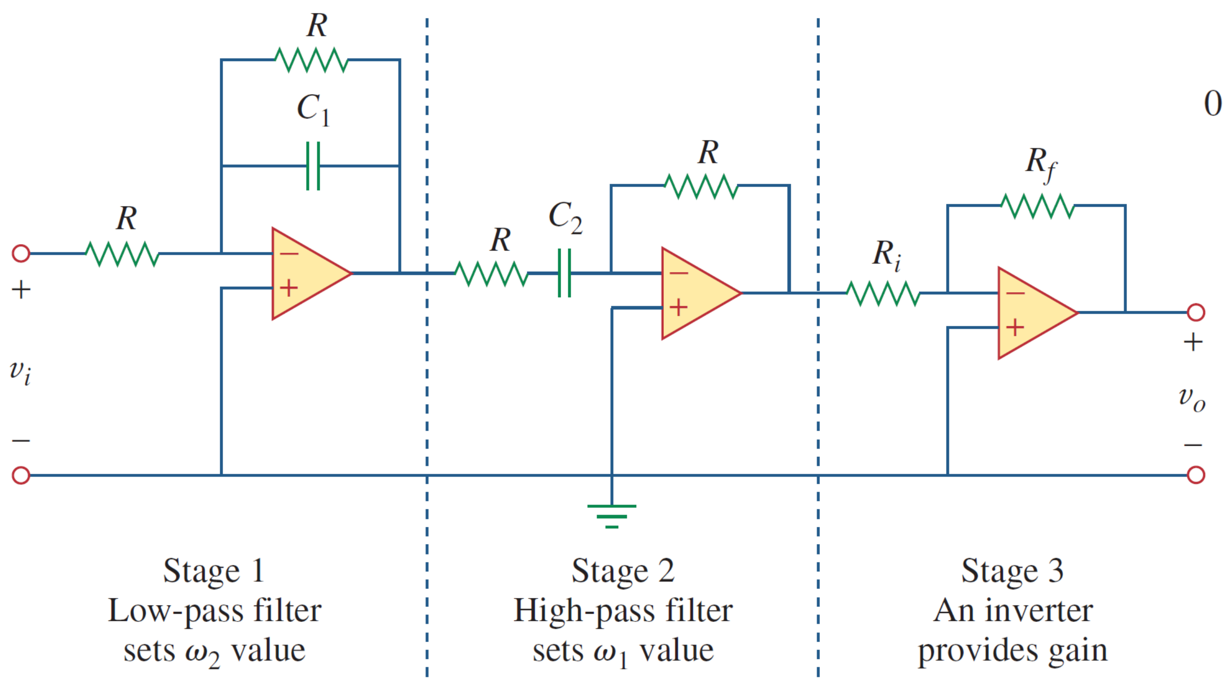
the corner frequency is: $\omega_c = \frac{1}{R_i C_i}$

Band Pass Filter



- LPF and HPF can be combined to form a BPF
- By cascading a unity-gain LPF, a unity-gain HPF, and an inverter with gain $-\frac{R_f}{R_i}$





$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$= \left(-\frac{1}{1 + j\omega C_1 R}\right)\left(-\frac{j\omega C_2 R}{1 + j\omega C_2 R}\right)\left(-\frac{R_f}{R_i}\right)$$

Band Pass Filter

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_1 R}\frac{j\omega C_2 R}{1 + j\omega C_2 R}$$

- The low pass section sets the upper corner frequency:

$$\omega_2 = \frac{1}{RC_1}$$

- the high pass section sets the lower corner frequency:

$$\omega_1 = \frac{1}{RC_2}$$

- Once the values of ω_1 and ω_2 are known, the center frequency, bandwidth, and quality factor are:

$$\omega_0 = \sqrt{\omega_1\omega_1} \qquad B = \omega_2 - \omega_1 \qquad Q = \frac{\omega_0}{B}$$

- Passband gain *K:* $\mathbf{H}(\omega) = \frac{-Kj\omega/\omega_1}{(1 + j\omega/\omega_1)(1 + j\omega/\omega_2)} = \frac{-Kj\omega\omega_2}{(\omega_1 + j\omega)(\omega_2 + j\omega)}$

- At ω_0: $H(\omega_0) = \left|\frac{-Kj\omega_0\omega_2}{(\omega_1 + j\omega_0)(\omega_2 + j\omega_0)}\right| = \frac{K\omega_2}{\omega_1 + \omega_2}$

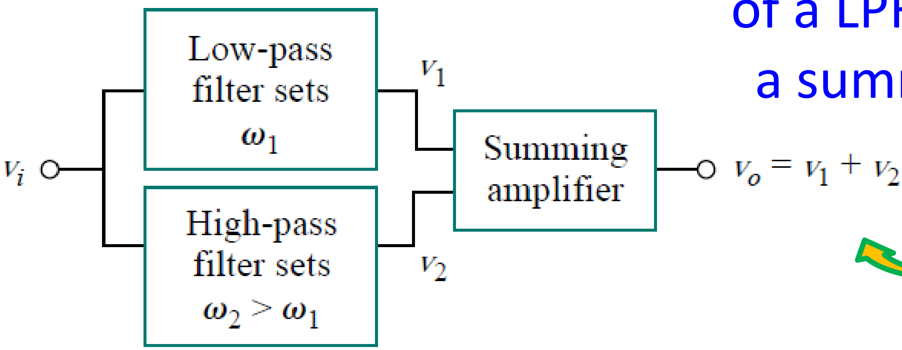
$$\frac{K\omega_2}{\omega_1 + \omega_2} = \frac{R_f}{R_i}$$

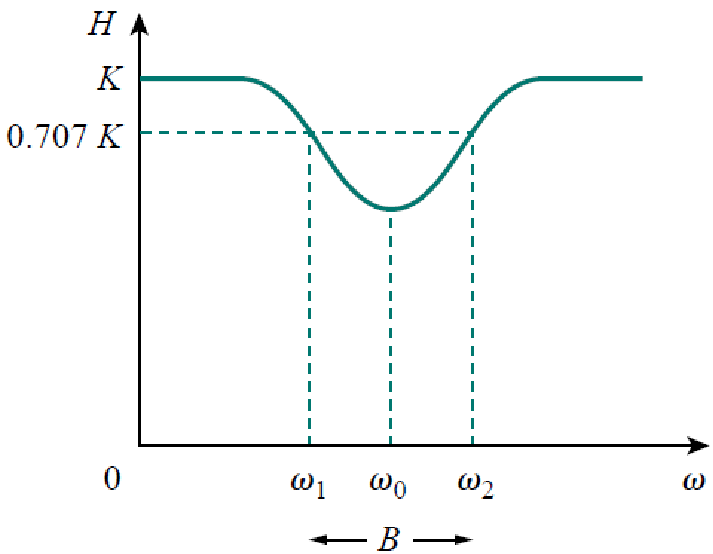
then gain *K* can be determined

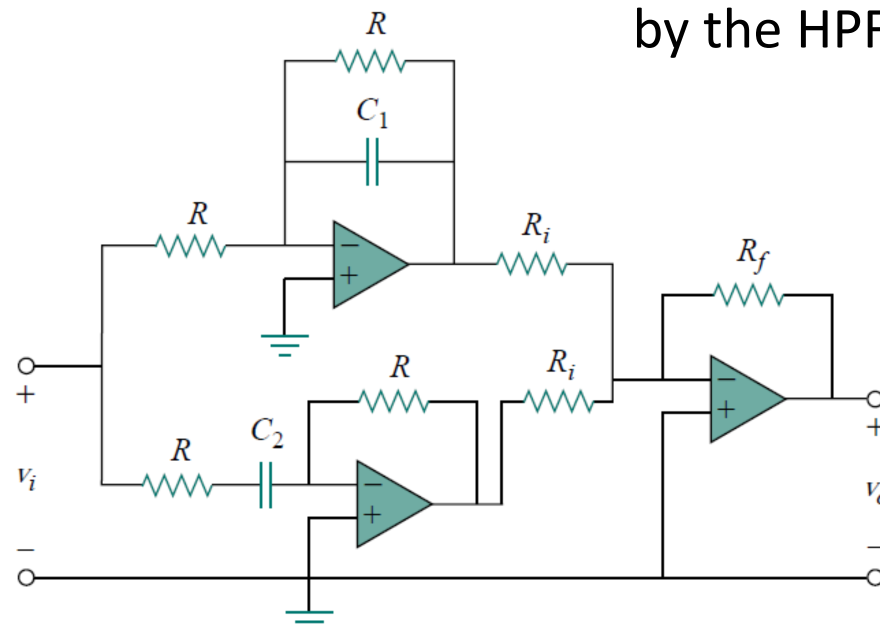
Band Reject Filter

parallel combination of a LPF and a BPF and a summing amplifier

You design it so that the lower cutoff frequency ω_1 is set by the LPF while the upper cutoff frequency ω_2 is set by the HPF.







$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{R_f}{R_i}\left(-\frac{1}{1 + j\omega C_1 R} - \frac{j\omega C_2 R}{1 + j\omega C_2 R}\right)$$

Band Reject Filter

The formulas for calculating the values of ω_1, ω_2, the center frequency, bandwidth, and quality factor are similar to the BPF

- To determine the pass band gain *K* of the filter:

$$\mathbf{H}(\omega) = \frac{R_f}{R_i}\left(\frac{1}{1 + j\omega/\omega_2} + \frac{j\omega/\omega_1}{1 + j\omega/\omega_1}\right) = \frac{R_f}{R_i}\frac{(1 + j2\omega/\omega_1 + (j\omega)^2/\omega_1\omega_1)}{(1 + j\omega/\omega_2)(1 + j\omega/\omega_1)}$$

- For the two pass bands ($\omega \to 0$ and $\omega \to \infty$) the gain is: $K = \frac{R_f}{R_i}$

- We can also find the gain at the ω_0 by finding the magnitude:

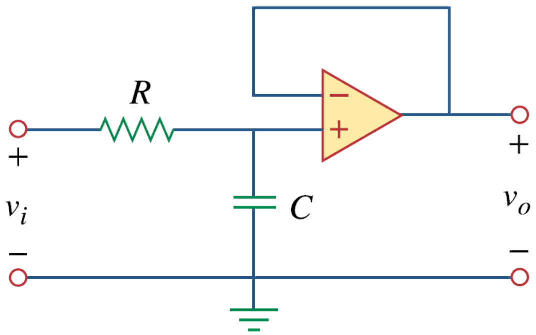
$$H(\omega_0) = \left|\frac{R_f}{R_i}\frac{(1 + j2\omega_0/\omega_1 + (j\omega_0)^2/\omega_1\omega_1)}{(1 + j\omega_0/\omega_2)(1 + j\omega_0/\omega_1)}\right|$$

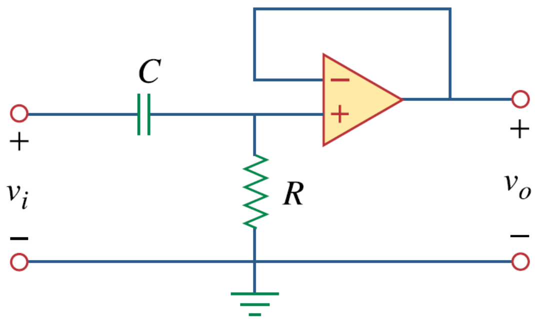
$$= \frac{R_f}{R_i}\frac{2\omega_1}{\omega_1 + \omega_2}$$

Again, the filters treated in this section are only typical. There are many other active filters that are more complex.

Example – 10

Find the transfer function for each of the following active filters





Example – 11

Design an active first-order high pass filter with:

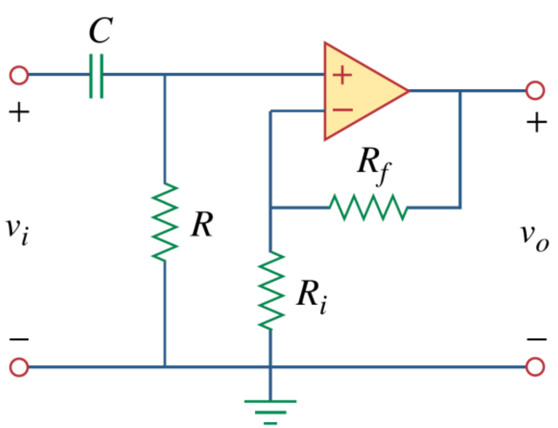
$$\mathbf{H}(s) = -\frac{100s}{s+10}, \qquad s = j\omega$$

Use a 1-μF capacitor.

Example – 12

A filter is given. Show that the transfer function is:

$$\mathbf{H}(\omega) = \left(1 + \frac{R_f}{R_i}\right)\frac{j\omega RC}{1 + j\omega RC}$$



What type of filter is this? What is the corner frequency

Example – 13

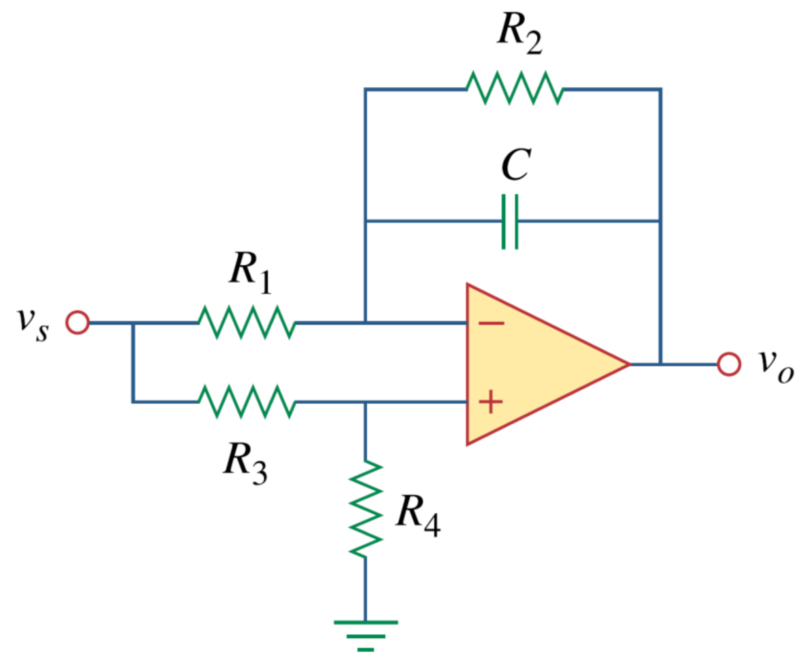
A "general" first-order filter is shown.

(a) Show that the transfer function is:

$$\mathbf{H}(s) = \frac{R_4}{R_3 + R_4} \times \frac{s + (1/R_1 C)[R_1/R_2 - R_3/R_4]}{s + 1/R_2 C}$$

(b) What condition must be satisfied for the circuit to operate as a highpass filter?

(c) What condition must be satisfied for the circuit to operate as a lowpass filter?

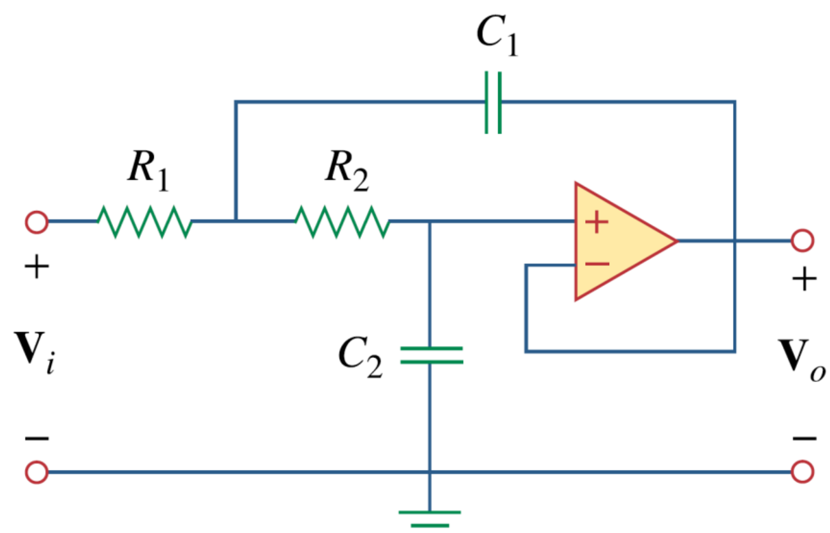


Example – 14

A second-order active filter is given.

(a) Find the transfer function.

(b) Show that it is a lowpass filter.



Scaling

- In the design and analysis of filters and resonant circuits, it is often convenient to work with element values of 1Ω, 1H, or 1F, and then transform the values to realistic values by ***scaling.***
- There are two ways of scaling a circuit: *magnitude* or *impedance scaling*, and *frequency scaling.*
- Magnitude scaling leaves the frequency response of a circuit unaltered, while frequency scaling shifts the frequency response up or down the frequency spectrum.

Magnitude Scaling

- Magnitude scaling is the process of increasing all impedances in a network by a factor, the frequency response remaining unchanged.
- The impedances of individual elements *R*, *L*, and *C* are:

$$\mathbf{Z}_R = R, \qquad \mathbf{Z}_L = j\omega L, \qquad \mathbf{Z}_C = \frac{1}{j\omega C}$$

- Then impedance of each circuit element is multiplied by a factor K_m and while the frequency remain constant.

$$\mathbf{Z}'_R = K_m \mathbf{Z}_R = K_m R,$$

$$\mathbf{Z}'_L = K_m \mathbf{Z}_L = j\omega K_m L$$

$$\mathbf{Z}'_C = K_m \mathbf{Z}_C = \frac{1}{j\omega C/K_m}$$

Magnitude Scaling

- In magnitude scaling, the new values of the elements and frequency are:

$$R' = K_m R, \qquad L' = K_m L$$

$$C' = \frac{C}{K_m}, \qquad \omega' = \omega$$

primed variables: new values

- For the series or parallel *RLC* circuit: $\omega_0' = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{K_m LC/K_m}} = \frac{1}{\sqrt{LC}} = \omega_0$

Similarly, it can be shown that the quality factor and the bandwidth are not affected by magnitude scaling.

Frequency Scaling

It is the process of shifting the frequency response of a network up or down the frequency axis while leaving the impedance the same.

It is achieved by multiplying the frequency by a factor K_f while keeping the impedance the same.

Frequency Scaling

Application of frequency scaling to $Z_L(\omega)$ and $Z_C(\omega)$ leads to:

$$\mathbf{Z}_L = j(\omega K_f)L' = j\omega L \quad \Rightarrow \quad L' = \frac{L}{K_f}$$

$$\mathbf{Z}_C = \frac{1}{j(\omega K_f)C'} = \frac{1}{j\omega C} \quad \Rightarrow \quad C' = \frac{C}{K_f}$$

- In frequency scaling, the new values of the elements and frequency are

$$R' = R, \qquad L' = \frac{L}{K_f}$$

$$C' = \frac{C}{K_f}, \qquad \omega' = K_f \omega$$

- For the series or parallel *RLC* circuit:

$$\omega_0' = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{(L/K_f)(C/K_f)}} = \frac{K_f}{\sqrt{LC}} = K_f \omega_0 \qquad B' = K_f B$$

the quality factor remains

Frequency and Magnitude Scaling

$$R' = K_m R, \qquad L' = \frac{K_m}{K_f} L$$

$$C' = \frac{1}{K_m K_f} C, \qquad \omega' = K_f \omega$$

We set $\boldsymbol{K_m = 1}$ when there is no magnitude scaling or $\boldsymbol{K_f = 1}$ when there is no frequency scaling.

Example – 15

What values of $\boldsymbol{K_m}$ and $\boldsymbol{K_f}$ will scale a 4-mH inductor and a 20-μF capacitor to 1H and 2F respectively?

Example – 16

(a) Find $\mathbf{Z}_{in}(s)$.

(b) Scale the elements by $K_m = 10$ and $K_f = 100$. Find $\mathbf{Z}_{in}(s)$ and ω_0.