Lecture – 14 **Date: 25.09.2017**

- Frequency Response

Introduction

- In sinusoidal circuit analysis, we learnt how to find voltages and currents in a circuit with a constant frequency source.
- However, if the amplitude of the sinusoidal source remain constant and the frequency is varied then one can obtain the circuit's *frequency response*.
- The frequency response may be regarded as a complete description of the sinusoidal steady-state behavior of a circuit as a function of frequency.
- The sinusoidal steady-state frequency responses of circuits are of significance in many applications, especially in communications and control systems.
- A specific application is in electric filters that block out or eliminate signals with unwanted frequencies and pass signals of the desired frequencies.
- Filters are used in radio, TV, and telephone systems to separate one broadcast frequency from another.

Transfer Function

- The transfer function (also called the *network function*) is a useful analytical tool for finding the frequency response of a circuit.
- It is represented by $H(\omega)$.
- Circuit's frequency response is essentially the plot of $H(\omega)$ when ω varies between 0 and ∞.

$$\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)}$$

It is the frequency-dependent ratio of a phasor output **Y**(ω) to a phasor input **X**(ω).

- Since the input and output can be either voltage or current at any place in the circuit, there are four possible transfer functions:

$$\mathbf{H}(\omega) = \text{Voltage gain} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Current gain} = \frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Transfer Impedance} = \frac{\mathbf{V}_o(\omega)}{\mathbf{I}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Transfer Admittance} = \frac{\mathbf{I}_o(\omega)}{\mathbf{V}_i(\omega)}$$

Being a complex quantity, $\boldsymbol{H}(\omega)$ has a magnitude $H(\omega)$ and a phase φ.

Transfer Function (contd.)

- The transfer function of a circuit can be obtained by first converting it to frequency-domain equivalent by replacing resistors, inductors, and capacitors with their impedances R, $j\omega L$ and $1/_{j\omega C}$.
- One can then use any circuit technique(s) to obtain the appropriate expressions.
- Can be simplified to:

$$\mathbf{H}(\omega) = \frac{\mathbf{N}(\omega)}{\mathbf{D}(\omega)}$$

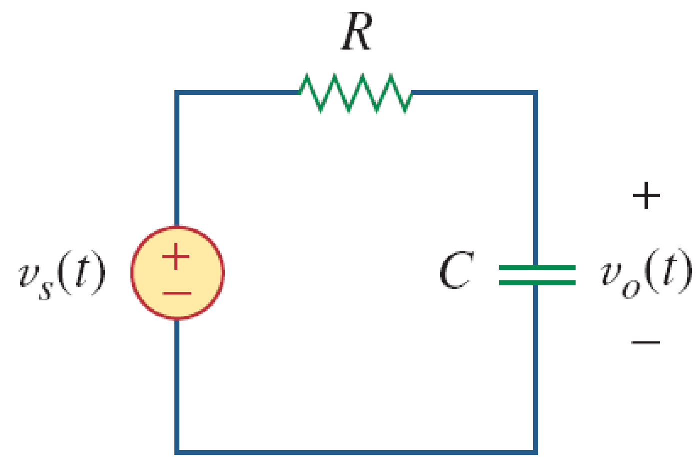
The roots of $\boldsymbol{N}(\omega)$ are called the *zeros* and are usually represented as $j\omega = z_1, z_2, \dots$ …Similarly, the roots of $\boldsymbol{D}(\omega)$ are the *poles* and are represented as $j\omega = p_1, p_2, \dots$ ….

A zero is a value that results in a zero value of the function. A pole is a value for which the function is infinite.

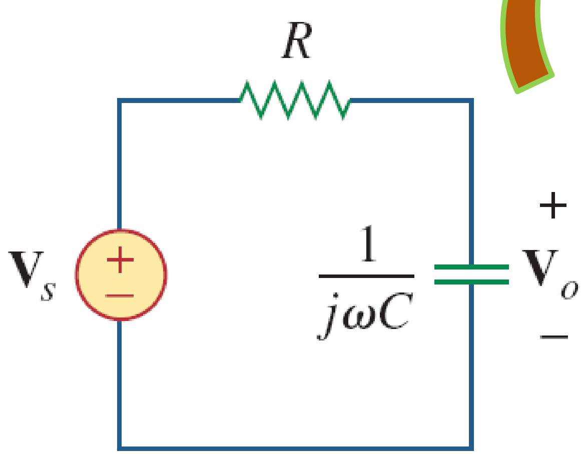
To avoid complex algebra, it is expedient to replace temporarily $j\omega$ with s when working with **H**(ω) and replace s with $j\omega$ at the end.

Example – 1

Find the transfer function $\frac{V_0}{V_s}$ and the corresponding frequency response of this *RC* circuit. Assume, $v_s = V_m cos\omega t$.



The frequency-domain equivalent of the circuit:



the transfer function

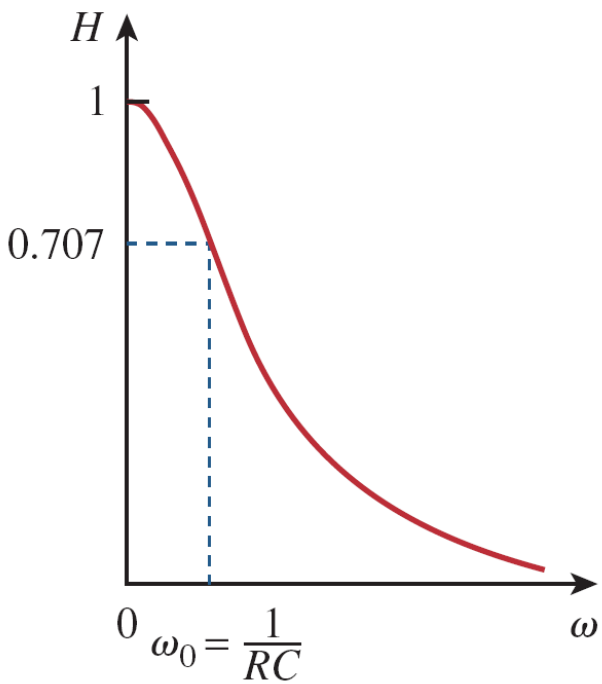
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

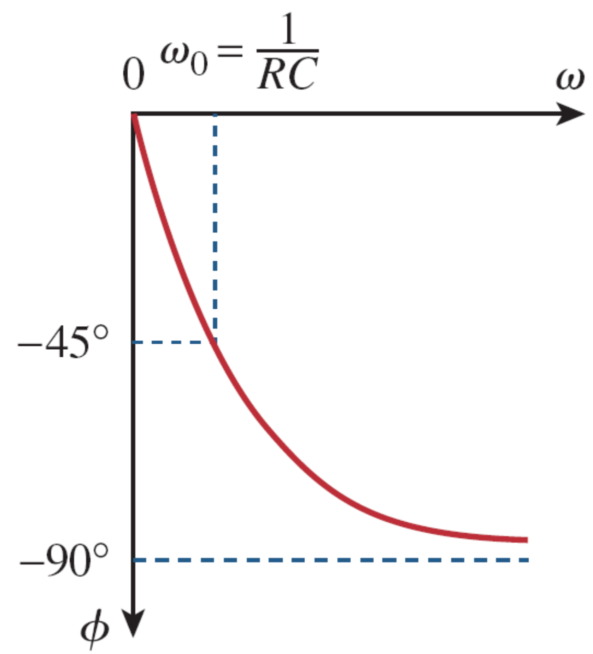
$$H = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}, \qquad \phi = -\tan^{-1}\frac{\omega}{\omega_0}$$

where $\omega_0 = \frac{1}{RC}$. For plotting *H* and φ for $0 < \omega < \infty$, we need values at some critical points.

Example – 1 (contd.

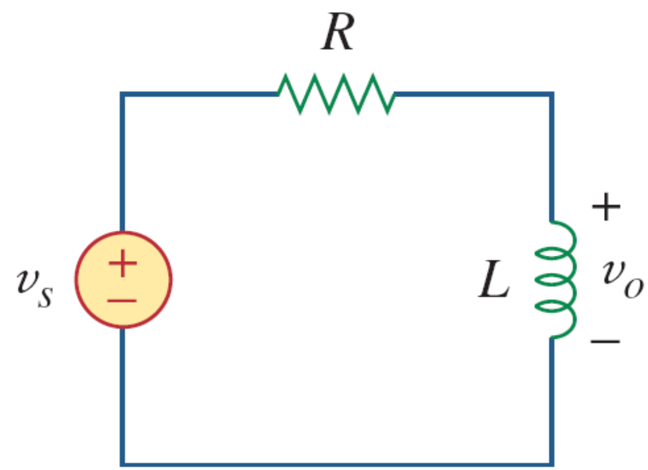
ω/ω_0	H	ϕ	ω/ω_0	H	ϕ
0	1	0	10	0.1	$-84°$
1	0.71	$-45°$	20	0.05	$-87°$
2	0.45	$-63°$	100	0.01	$-89°$
3	0.32	$-72°$	∞	0	$-90°$



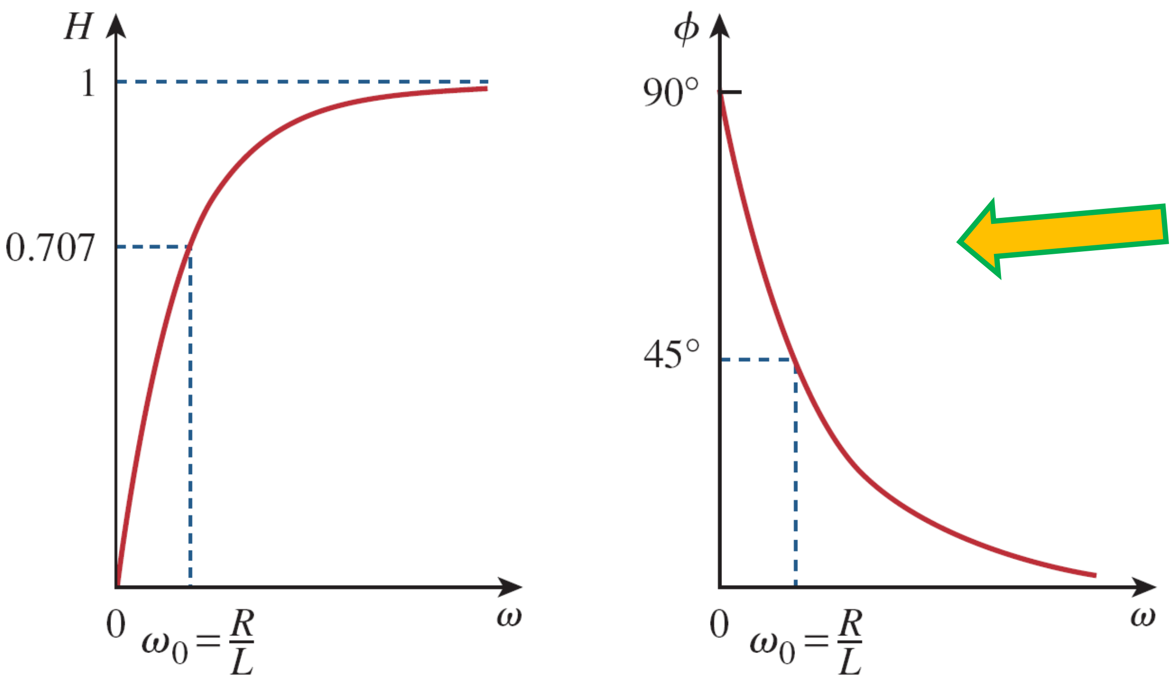


Example – 2

Find the transfer function $\frac{V_0}{V_s}$ and the corresponding frequency response of this *RL* circuit. Assume, $v_s = V_m cos\omega t$.

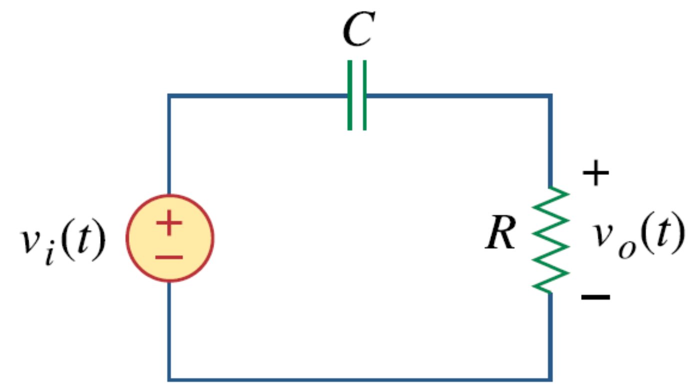


$$H(\omega) = \frac{V_0}{V_S} = \frac{j\omega L}{R + j\omega L}$$



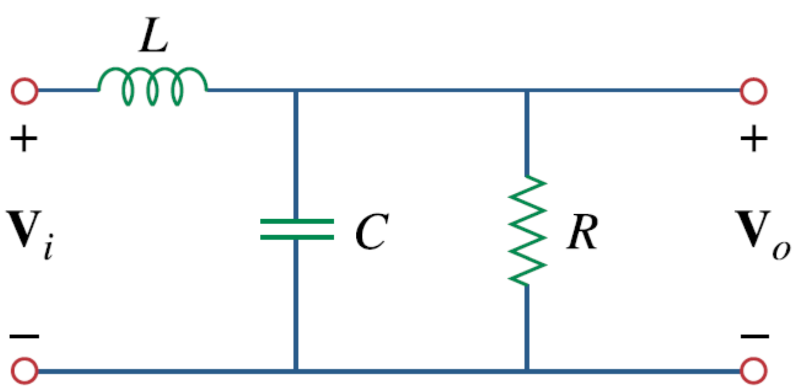
Example – 3

Find the transfer function $\frac{V_0}{V_s}$ and the corresponding frequency response of this *RC* circuit. Assume, $\omega_0 = \frac{1}{RC}$.

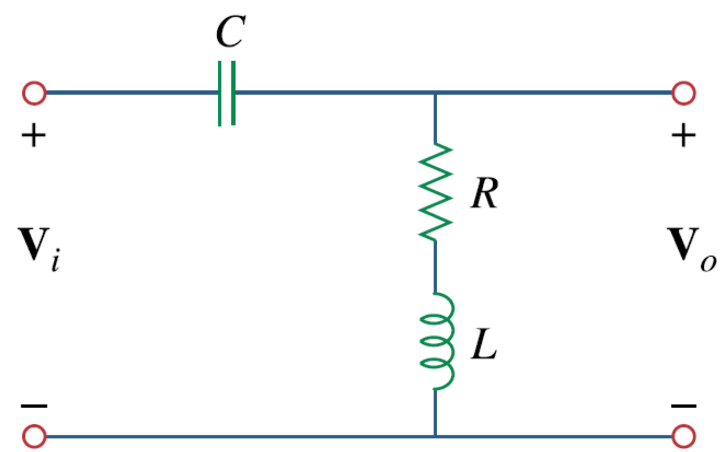


Example – 4

Find the transfer function $\frac{V_0}{V_i}$ of the following circuits.



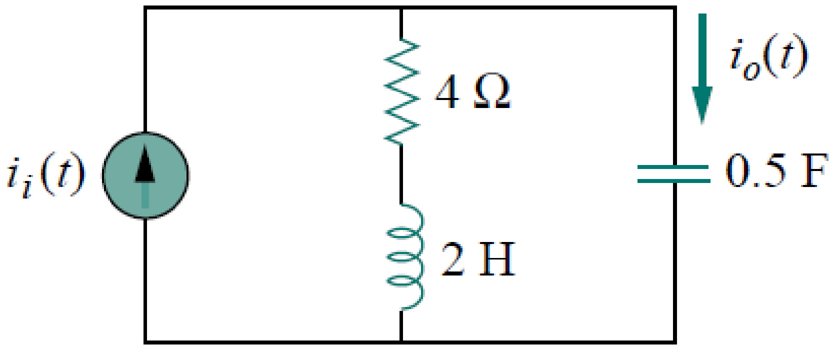
(a)



(b)

Example – 5

For this circuit, calculate the gain $\frac{I_0(\omega)}{I_i(\omega)}$ and its poles and zeros.



Example – 6

For this circuit, calculate the gain $\frac{V_0(\omega)}{I_i(\omega)}$ and its poles and zeros.