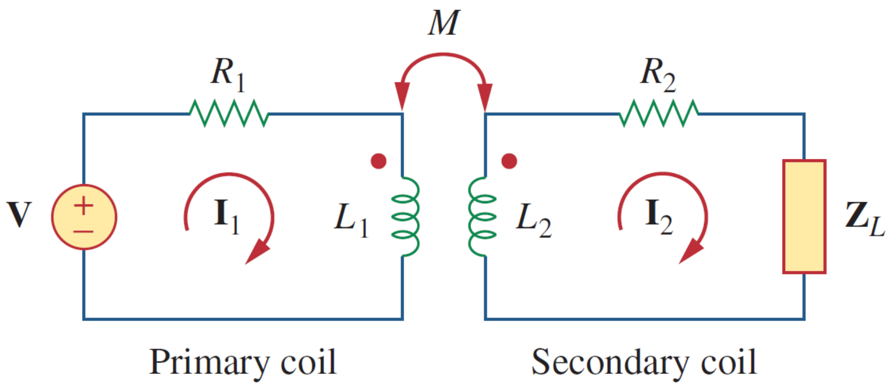
Lecture – 12 **Date: 12.09.2017**

- Transformer

Transformer

A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils.



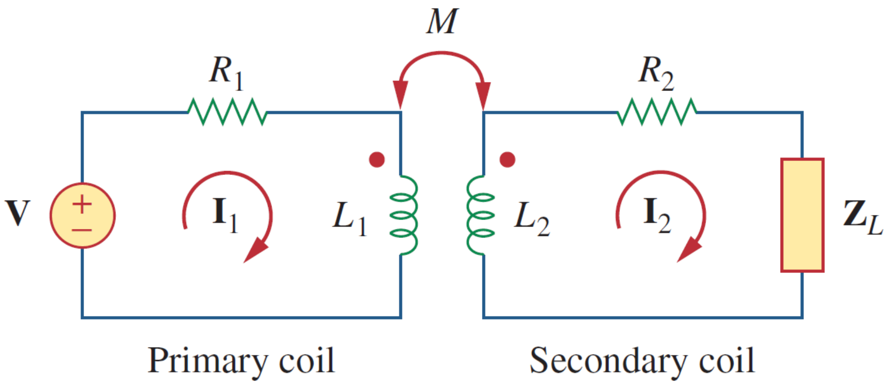
- The resistances R_1 and R_2 account for the losses (power dissipation) in the coils.

- The transformer is *linear* if the coils are wound on a magnetically linear material—a material for which the magnetic permeability is constant. Such materials include air, plastic, and wood.
- In fact, most materials are magnetically linear.

A linear transformer may also be regarded as one whose flux is proportional to the currents in its windings.

Linear transformers are sometimes called *air-core transformers*, although not all of them are necessarily air-core.

Transformer (contd.)



$$\mathbf{V} = (R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M \mathbf{I}_2$$

$$0 = -j\omega M \mathbf{I}_1 + (R_2 + j\omega L_2 + \mathbf{Z}_L)\mathbf{I}_2$$

- the input impedance $\boldsymbol{Z_{in}}$ as seen from the source:

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}}{\mathbf{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

primary impedance

due to the coupling between the primary and secondary windings

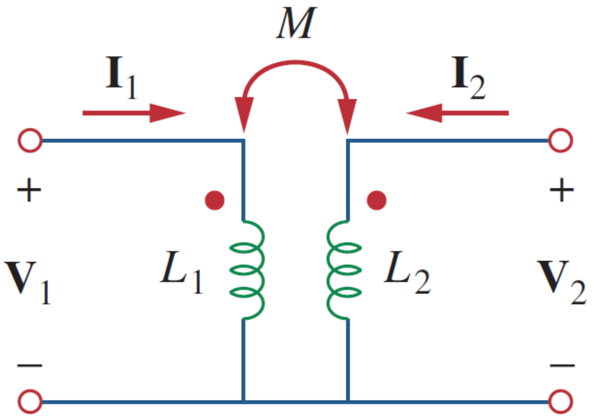
It is as though this impedance is reflected to the primary and is called *reflected impedance* $\boldsymbol{Z_R}$

$$\mathbf{Z}_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

Not affected by the location of the dots on the transformer

Transformer (contd.)

- it is sometimes convenient to replace a magnetically coupled circuit by an equivalent circuit with no magnetic coupling.

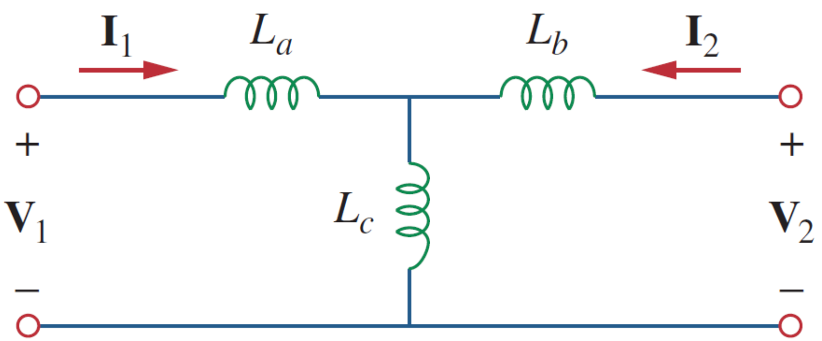


$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}$$

Matrix Inversion

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix} \dfrac{L_2}{j\omega(L_1L_2 - M^2)} & \dfrac{-M}{j\omega(L_1L_2 - M^2)} \\ \dfrac{-M}{j\omega(L_1L_2 - M^2)} & \dfrac{L_1}{j\omega(L_1L_2 - M^2)} \end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}$$

For a T-Network

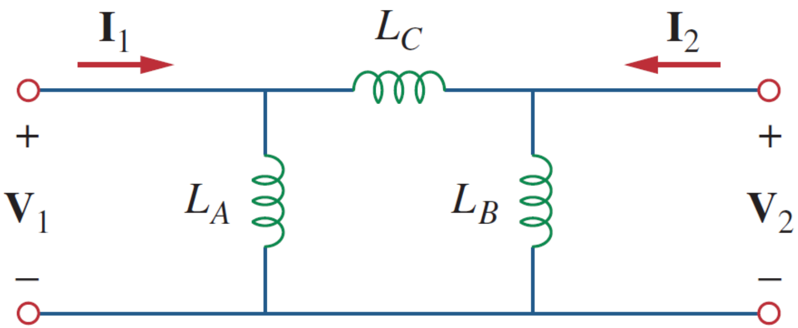


$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b + L_c) \end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}$$

- For the T-model to be equivalent to the linear transformer:

$$L_a = L_1 - M, \qquad L_b = L_2 - M, \qquad L_c = M$$

For a π-Network



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega L_A} + \frac{1}{j\omega L_C} & -\frac{1}{j\omega L_C} \\ -\frac{1}{j\omega L_C} & \frac{1}{j\omega L_B} + \frac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

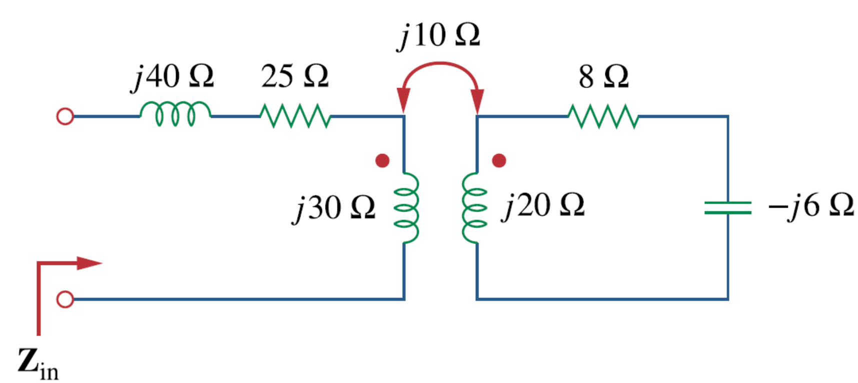
- For the π-model to be equivalent to the linear transformer:

$$L_A = \frac{L_1 L_2 - M^2}{L_2 - M}, \qquad L_B = \frac{L_1 L_2 - M^2}{L_1 - M} \qquad L_C = \frac{L_1 L_2 - M^2}{M}$$

In the T- and π- Models, the inductors are not magnetically coupled

Example – 1

(a) Find the input impedance of the circuit using the concept of reflected impedance.
(b) Obtain the input impedance by replacing the linear transformer by its T equivalent.

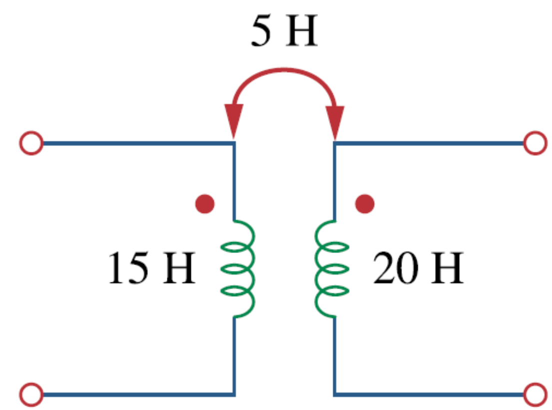


Example – 2

Find:

(a) the *T*-equivalent circuit,

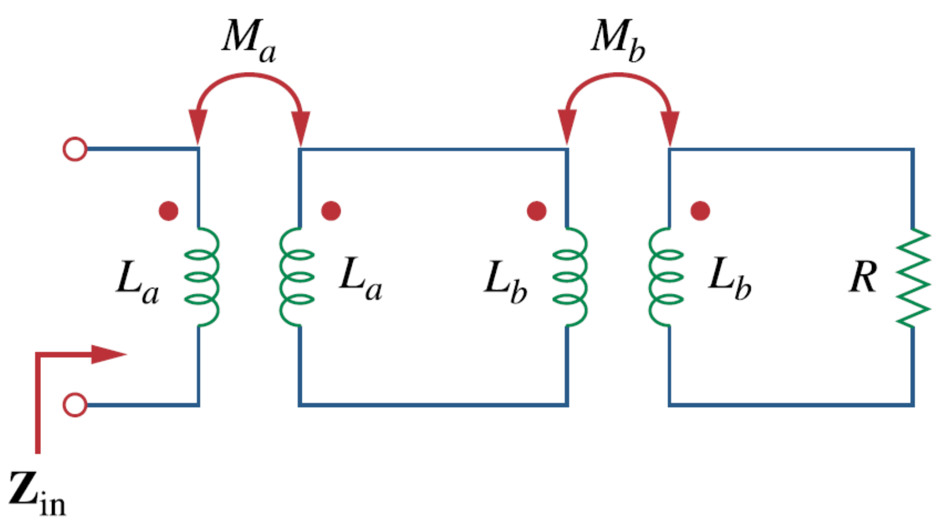
(b) the π-equivalent circuit.



Example – 3

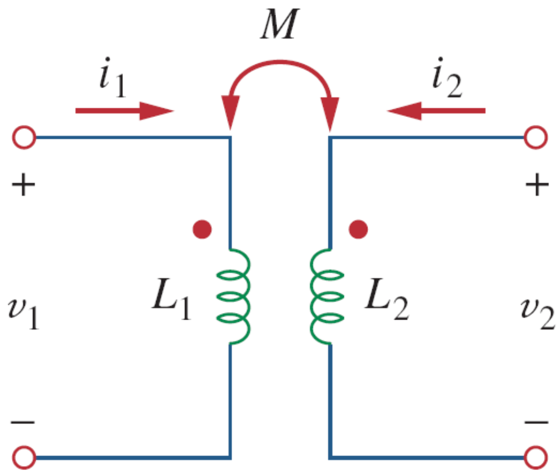
Two linear transformers are cascaded as shown. Show:

$$\mathbf{Z}_{\text{in}} = \frac{\omega^2 R(L_a^2 + L_a L_b - M_a^2 + j\omega^3 (L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2}{\omega^2 (L_a L_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}$$



Ideal Transformers

- An ideal transformer exhibits perfect coupling i.e., $k = 1$.
- It consists of two (or more) coils with a large number of turns wound on a common core of high permeability. Because of this high permeability of the core, the flux links all the turns of both coils, thereby resulting in a perfect coupling.



- In the frequency domain:

$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + \frac{M\mathbf{V}_1}{L_1} - \frac{j\omega M^2 \mathbf{I}_2}{L_1}$$

- We know for ideal transformer $k = 1$ and therefore $M = \sqrt{L_1 L_2}$

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + \frac{\sqrt{L_1 L_2}\mathbf{V}_1}{L_1} - \frac{j\omega L_1 L_2 \mathbf{I}_2}{L_1} = \sqrt{\frac{L_2}{L_1}}\mathbf{V}_1 = n\mathbf{V}_1$$

$$n = \sqrt{L_2/L_1}$$

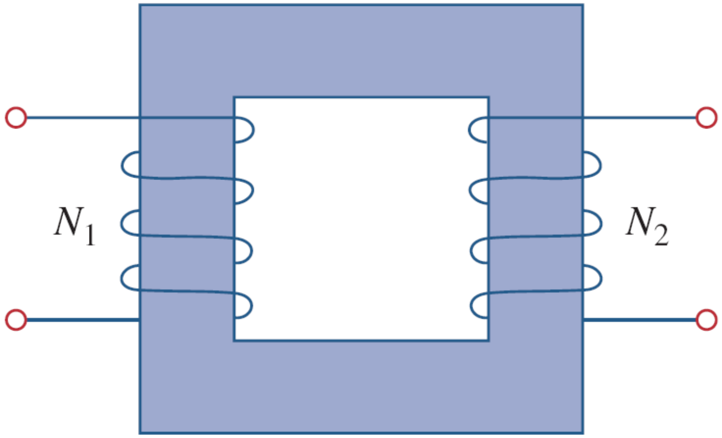
Ideal Transformers (contd.)

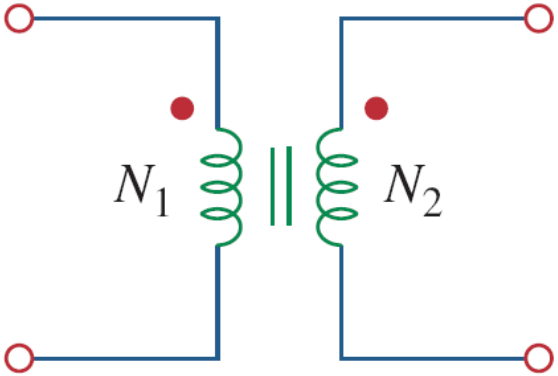
For $L_1, L_2, M \rightarrow \infty$, the turn ratio *n* remains the same and in such a scenario the coupled coils become an ideal transformer.

- **A transformer is said to be ideal if:**

1. Coils have very large reactances ($L_1, L_2, M \rightarrow \infty$)
2. Coupling coefficient is equal to unity (k=1)
3. Primary and secondary coils are lossless ($R_1 = 0 = R_2$).

Iron-core transformers are close approximations to ideal transformers.





$$v_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n \quad \Rightarrow \quad \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n$$

Ideal Transformers (contd.)

- For power conservation, the energy supplied to the primary must equal the energy absorbed by the secondary (there are no losses in an ideal transformer).

$$v_1 i_1 = v_2 i_2$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = n$$

the primary and secondary currents are related to the turns ratio in the inverse manner as the voltages.

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

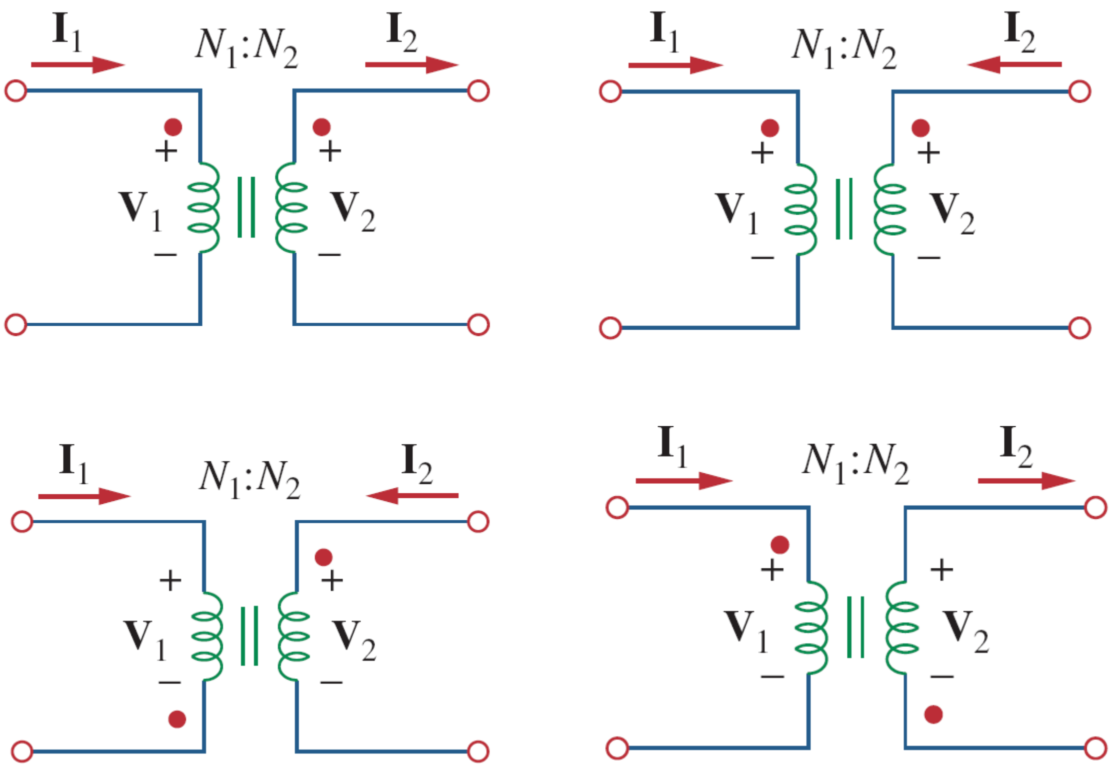
- For $n > 1$, we have a ***step-up transformer***, as the voltage is increased from primary to secondary ($V_2 > V_1$).
- For $n < 1$, the transformer is a ***step-down transformer***, as the voltage is decreased from primary to secondary ($V_2 < V_1$).

The ratings of transformers are usually specified as V_1 / V_2. A transformer with rating 2400/120 V should have 2400 V on the primary and 120 in the secondary (i.e., a step-down transformer). Keep in mind that the voltage ratings are in rms.

It is important to get the proper polarity of the voltages and the direction of the currents for the transformer.

Ideal Transformers (contd.)

- The two simple rules to follow are:
  - If $\mathbf{V_1}$ and $\mathbf{V_2}$ and are *both* positive or both negative at the dotted terminals, use +n in the equations. Otherwise, use –n.
  - If $\mathbf{I_1}$ and $\mathbf{I_2}$ and *both* enter into or both leave the dotted terminals, use +n. Otherwise use –n.



- we can always express:

$$\mathbf{V}_1 = \frac{\mathbf{V}_2}{n} \quad \text{or} \quad \mathbf{V}_2 = n\mathbf{V}_1$$

$$\mathbf{I}_1 = n\mathbf{I}_2 \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{I}_1}{n}$$

Ideal Transformers (contd.)

- The complex power in the primary winding is:

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \frac{\mathbf{V}_2}{n}(n\mathbf{I}_2)^* = \mathbf{V}_2\mathbf{I}_2^* = \mathbf{S}_2$$

the complex power supplied to the primary is delivered to the secondary without loss. The transformer absorbs no power. It is expected as the ideal transformer is lossless.

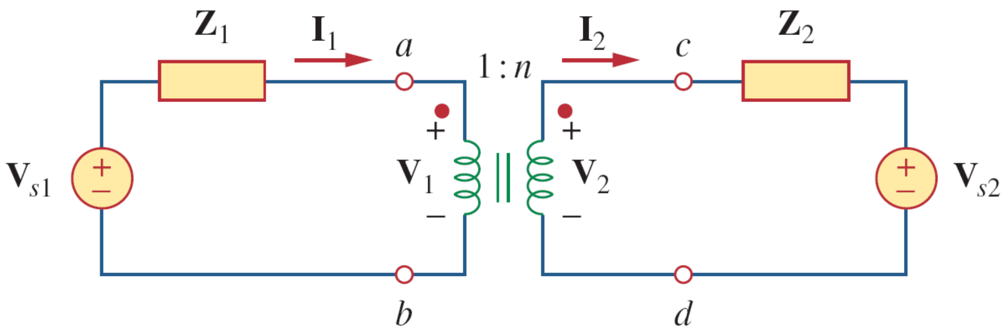
- The input impedance seen by the source:

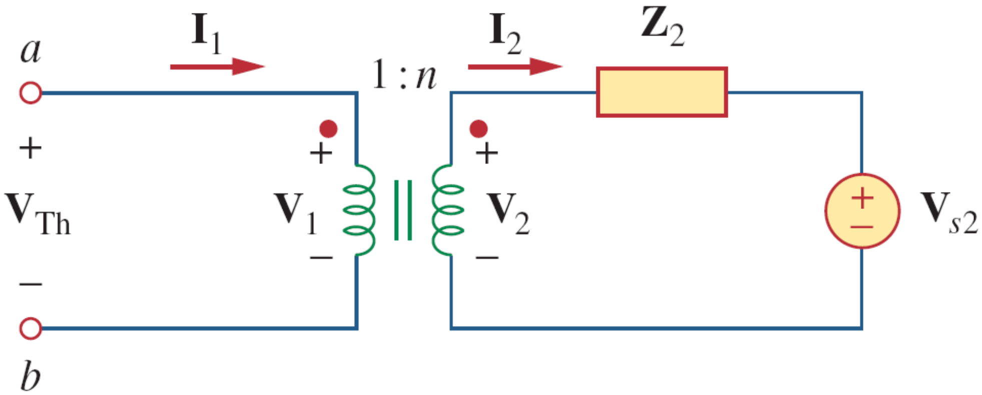
$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{1}{n^2}\frac{\mathbf{V}_2}{\mathbf{I}_2} \Rightarrow \mathbf{Z}_{\text{in}} = \frac{\mathbf{Z}_L}{n^2}$$

- The input impedance is also called the *reflected impedance*. The ability of the transformer to transform a given impedance into another impedance provides us a means of *impedance matching* to ensure maximum power transfer.
- In analyzing a circuit containing an ideal transformer, it is common practice to eliminate the transformer by reflecting impedances and sources from one side of the transformer to the other.

Ideal Transformers (contd.)

- suppose we want to reflect the secondary side of the circuit to the primary side i.e, *a-b*.

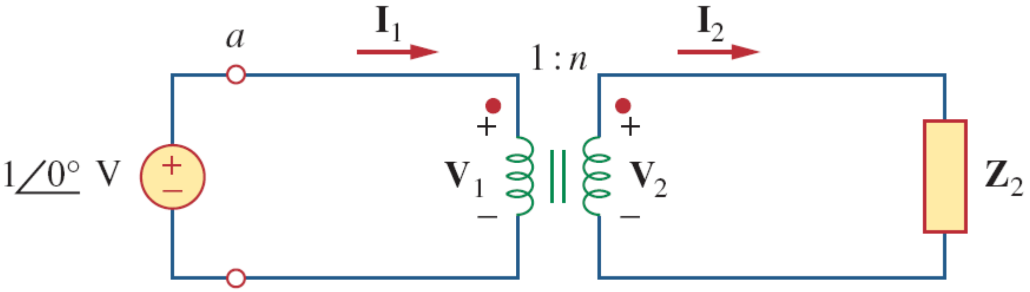




- First, obtain V_{TH} as the open-circuit voltage at terminals *a-b*.

- **Now, a-b is open: so $V_2 = V_{s2}$:** $\mathbf{V}_{\mathrm{Th}} = \mathbf{V}_1 = \dfrac{\mathbf{V}_2}{n} = \dfrac{\mathbf{V}_{s2}}{n}$

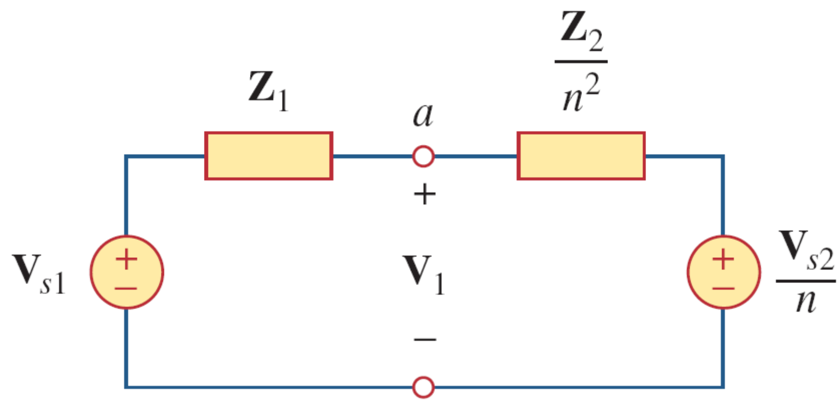
- For Z_{TH}, remove voltage source in secondary and excite the primary with a unit source.



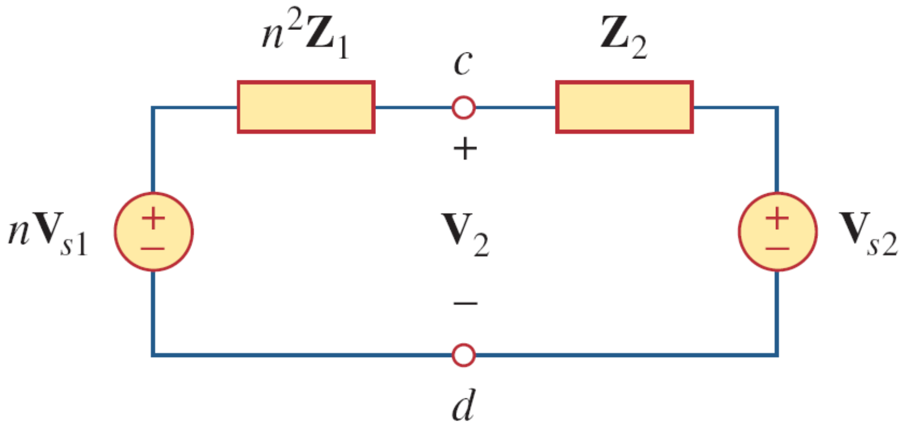
- $I_1 = nI_2$ and $V_1 = \frac{V_2}{n}$ and therefore: $\mathbf{Z}_{\mathrm{Th}} = \dfrac{\mathbf{V}_1}{\mathbf{I}_1} = \dfrac{\mathbf{V}_2/n}{n\mathbf{I}_2} = \dfrac{\mathbf{Z}_2}{n^2},$

Ideal Transformers (contd.)

- Once we have Z_{TH} and V_{TH}, we get the equivalent circuit as:



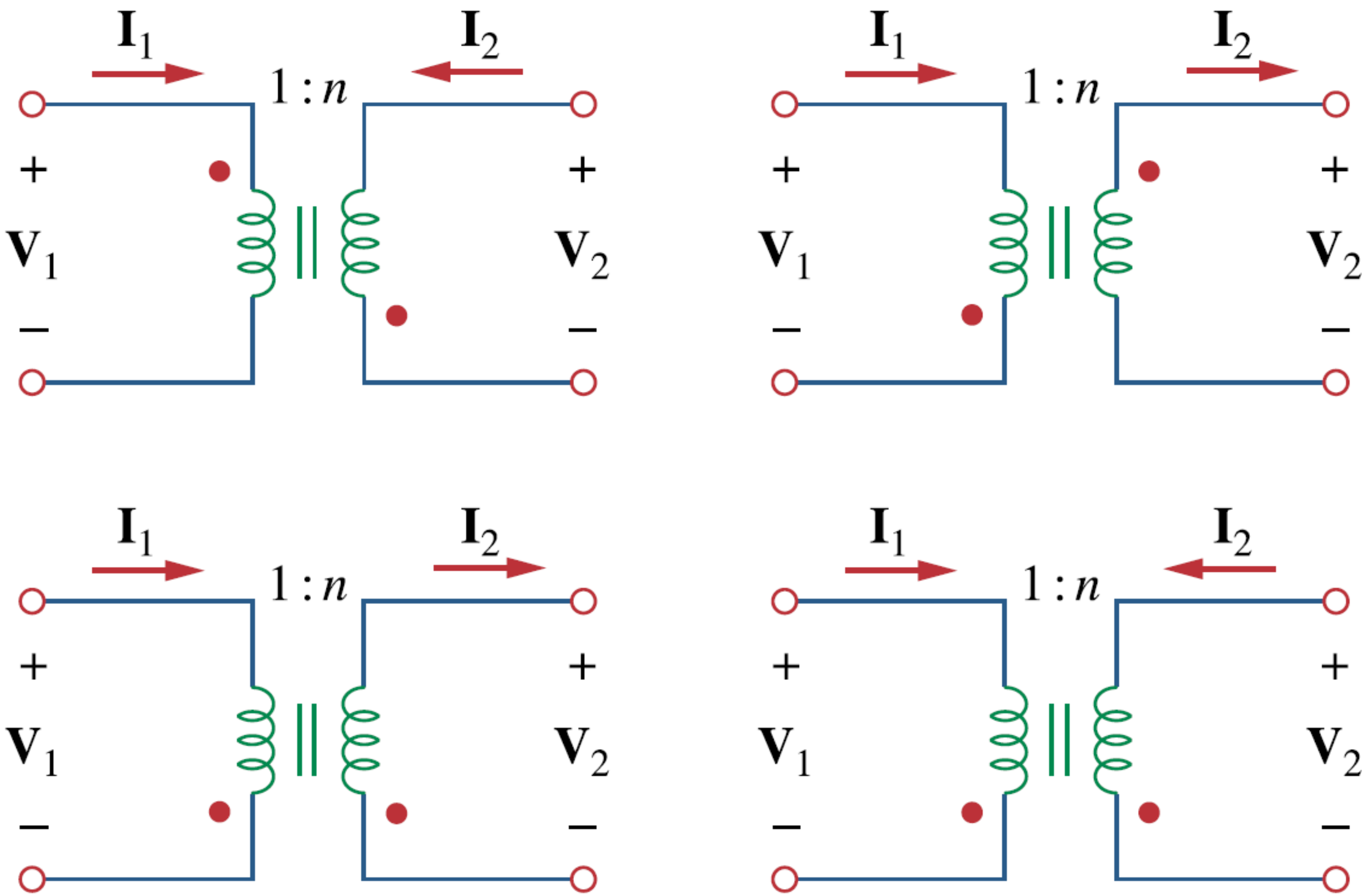
- We can also reflect the primary side of the circuit as:



- the power remains the same, whether calculated on the primary or the secondary side.
- However, this reflection approach only applies if there are no external connections between the primary and secondary windings.
- When we have external connections between the primary and secondary windings, we simply use regular mesh and nodal analysis.

Example – 4

- obtain the relationships between terminal voltages and currents for each of the ideal transformers given below

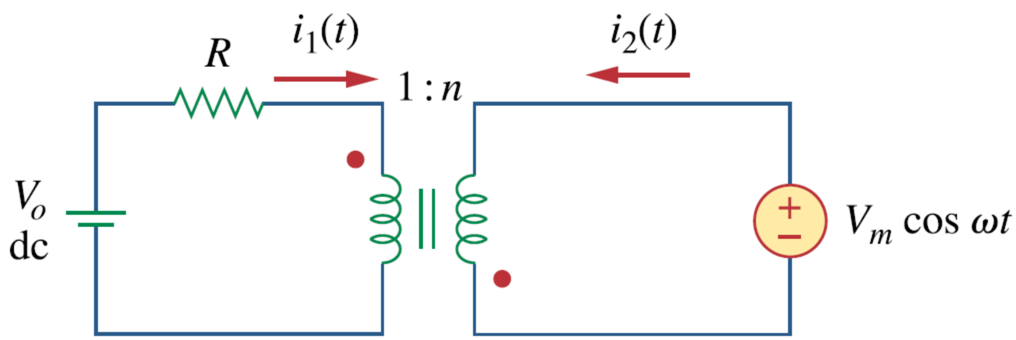


Example – 5

The primary of an ideal transformer with a turns ratio of 5 is connected to a voltage source with Thevenin parameters $v_{TH} = 10cos2000t$ V and $R_{TH} = 100\Omega$. Determine the average power delivered to a 200Ω load connected across the secondary winding.

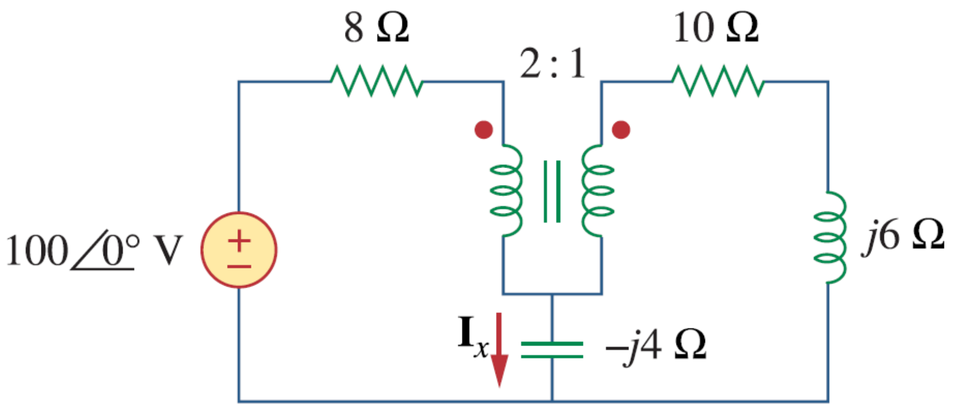
Example – 6

- In this ideal transformer circuit, find $i_1(t)$ and $i_2(t)$.



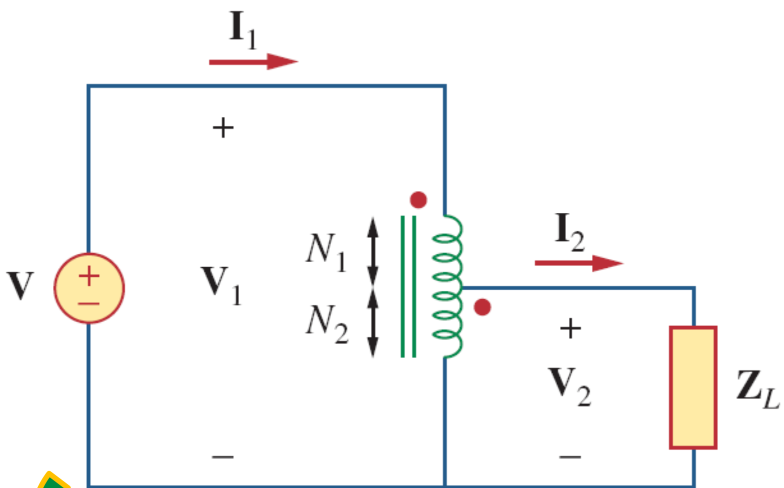
Example – 7

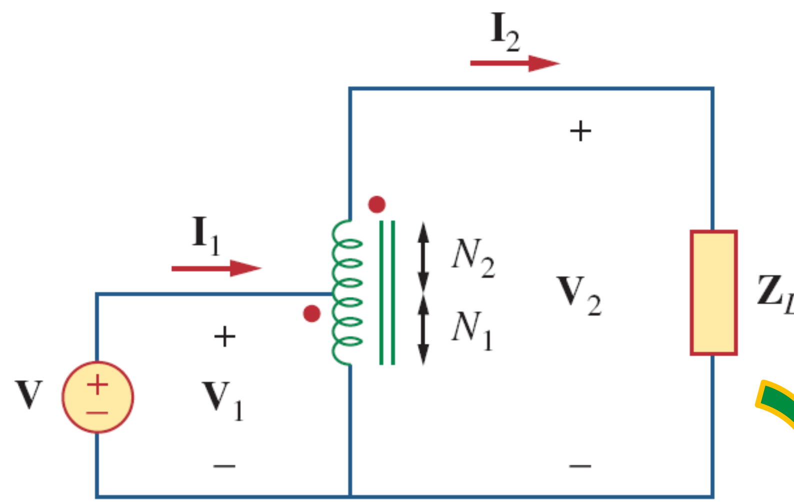
Find I_x in this ideal transformer circuit



Ideal Autotransformers

- An *autotransformer* has a single continuous winding with a connection point called a *tap* between the primary and secondary sides.
- The tap is often adjustable so as to provide the desired turns ratio for stepping up or stepping down the voltage. This way, a variable voltage can be provided to the load connected to the autotransformer.





Step-down $$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1 + N_2}{N_2} = 1 + \frac{N_1}{N_2}$$

Step-up $$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_1 + N_2}$$

Ideal Autotransformers

- an ideal autotransformer, there are no losses, so the complex power remains the same in the primary and secondary:

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^*$$

$$V_1 I_1 = V_2 I_2 \quad \Rightarrow \quad \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

Step-down

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_2}{N_1 + N_2}$$

Step-up

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_1 + N_2}{N_1} = 1 + \frac{N_2}{N_1}$$

- A major difference between conventional transformers and autotransformers is that the primary and secondary sides of the autotransformer are not only coupled magnetically but also coupled conductively.
- The autotransformer can be used in place of a conventional transformer when electrical isolation is not required.

Example – 8