Lecture – 9 **Date: 05.09.2016**

- AC Circuits: Steady State Analysis (contd.)
- AC Power Analysis

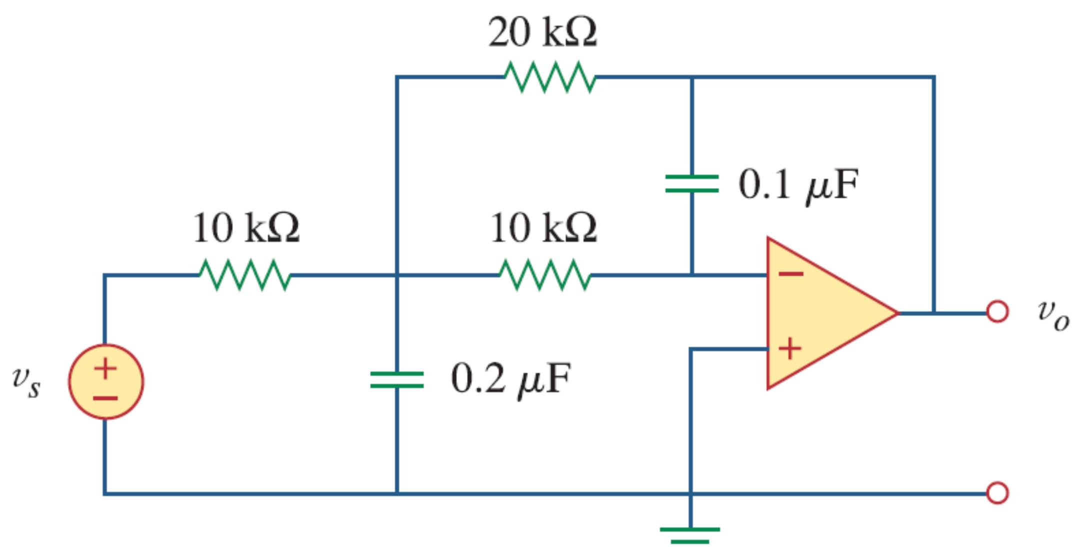
Op Amp AC Circuits Analysis Steps

1. Transfer the circuit to the phasor domain
2. Solve the circuit (using Mesh, Nodal techniques etc.)
3. Convert the results into time domain

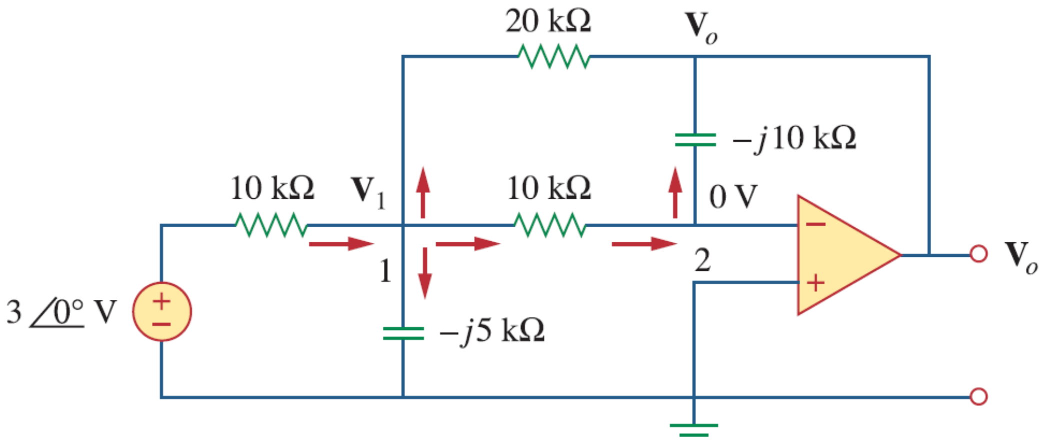
It is assumed that the op amps are ideal, i.e.,

- No current enters either of its input terminals.
- The voltage across its input terminals is zero.

- Determine $v_o(t)$ for this op amp circuit if $v_s = 3cos1000t\ V$.



Op Amp AC Circuits (contd.)



KCL at node-2

$$\frac{V_1 - 0}{10} = \frac{0 - V_o}{-j10}$$

$$V_1 = -jV_o$$

KCL at node-1

$$\frac{3\angle 0° - V_1}{10} = \frac{V_1}{-j5} + \frac{V_1 - 0}{10} + \frac{V_1 - V_o}{20}$$

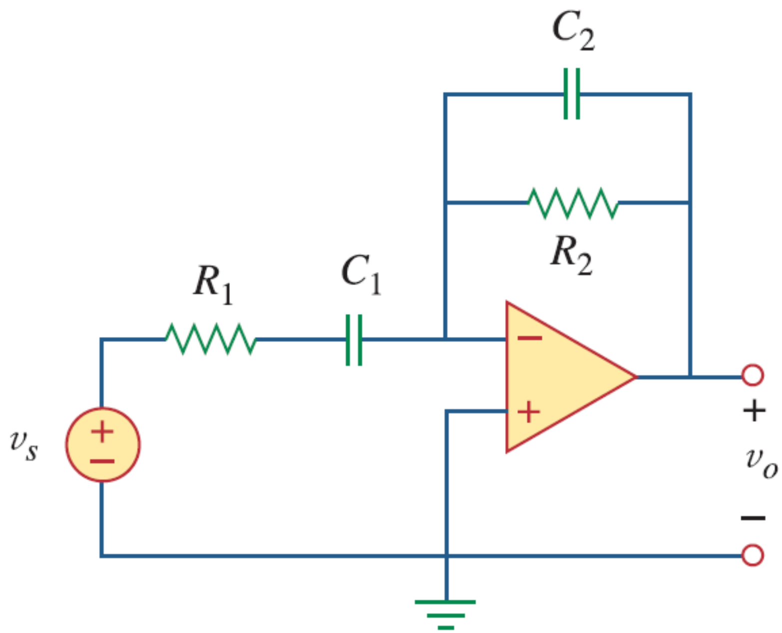
$$6 = (5 + j4)V_1 - V_o$$

$$V_o = \frac{6}{3 - j5} = 1.029\angle 59.04°$$

$$v_o(t) = 1.029 \cos(1000t + 59.04°) \text{ V}$$

Example – 1

Compute the closed-loop gain and phase shift assuming that $R_1 = R_2 = 10k\Omega$, $C_1 = 2\mu F$, $C_2 = 1\mu F$, and $\omega = 200\frac{rad}{s}$.



The feedback and input impedances are:

$$\mathbf{Z}_f = R_2 \left\| \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega R_2 C_2}\right.$$

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

The closed-loop gain is:

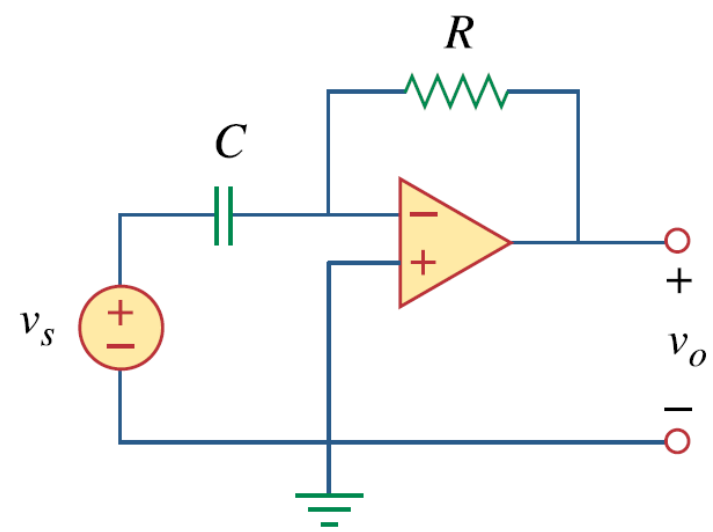
$$\mathbf{G} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{-j\omega C_1 R_2}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)}$$

$$\mathbf{G} = \frac{-j4}{(1 + j4)(1 + j2)} = 0.434\underline{/130.6^\circ}$$

Thus the closed-loop gain is 0.434 and phase shift is 130.6°

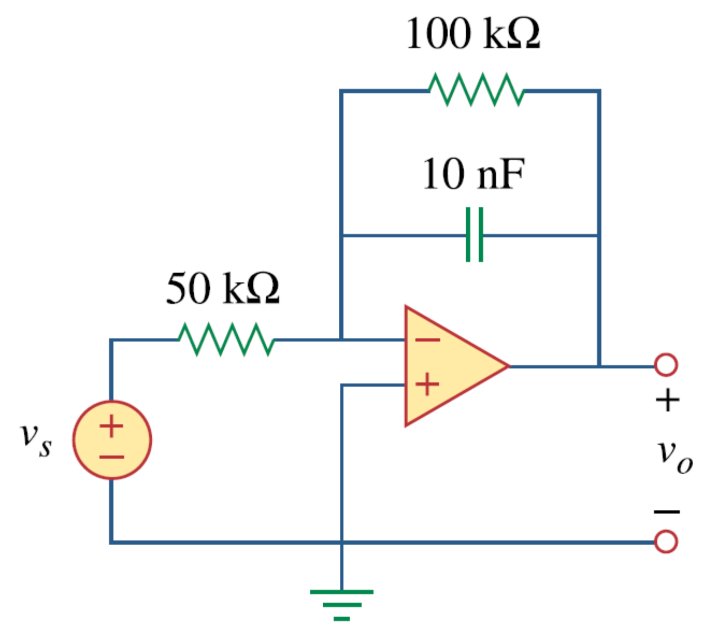
Example – 2

For the differentiator, obtain $\frac{V_0}{V_s}$. Find $v_o(t)$ when $v_{s(t)} = V_m sin\omega t$ and $\omega = \frac{1}{RC}$.



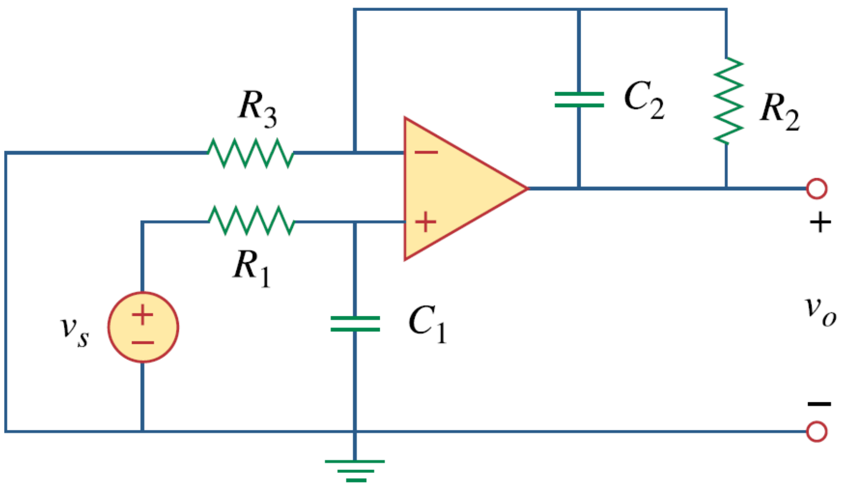
Example – 3

For this integrator with a feedback resistor, obtain $v_o(t)$ if $v_{s(t)} = 2cos4 \times 10^4 t$ V.



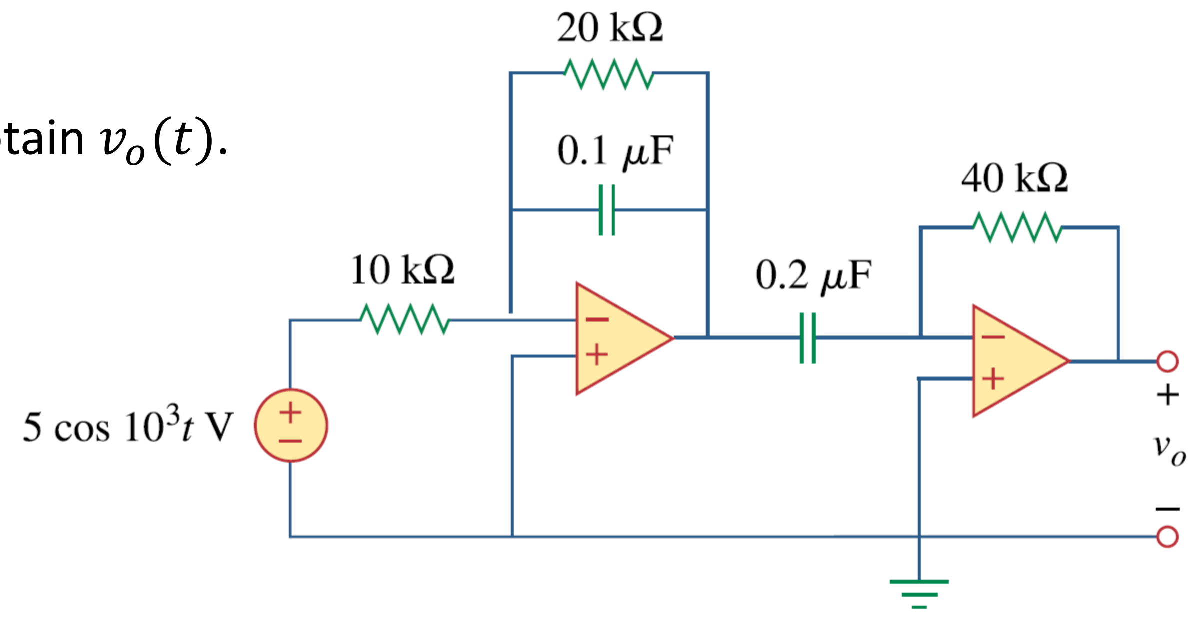
Example – 4

Derive the expression for $\frac{V_0}{V_s}$ for this op amp circuit.

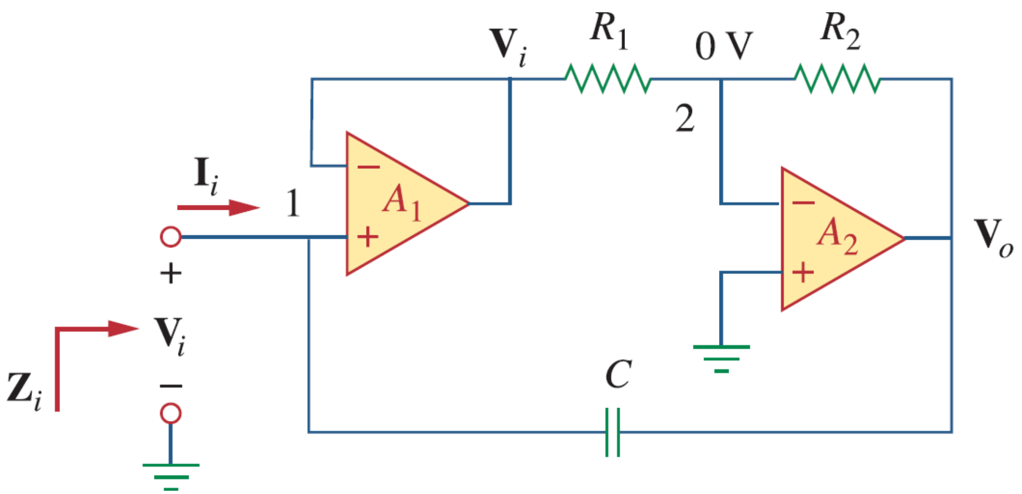


Example – 5

For this op amp circuit, obtain $v_o(t)$.



Capacitance Multiplier



- First op amp operates as a voltage follower.
- The second op amp is inverting amplifier
- No current enters the input terminal of op amp, the input current I_i flows through the feedback cap.

At the node 1:

$$\mathbf{I}_i = \frac{\mathbf{V}_i - \mathbf{V}_o}{1/j\omega C} = j\omega C(\mathbf{V}_i - \mathbf{V}_o)$$

KCL at the node 2 gives:

$$\frac{\mathbf{V}_i - 0}{R_1} = \frac{0 - \mathbf{V}_o}{R_2} \Rightarrow \mathbf{V}_o = -\frac{R_2}{R_1}\mathbf{V}_i$$

$$\mathbf{I}_i = j\omega C\left(1 + \frac{R_2}{R_1}\right)\mathbf{V}_i$$

Capacitance Multiplier (contd.)

$$\frac{\mathbf{I}_i}{\mathbf{V}_i} = j\omega\left(1 + \frac{R_2}{R_1}\right)C$$

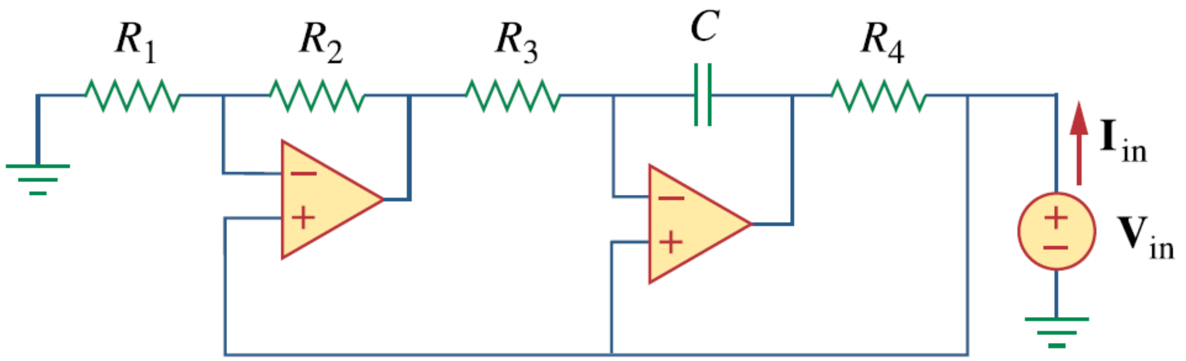
The input impedance: $\mathbf{Z}_i = \frac{\mathbf{V}_i}{\mathbf{I}_i} = \frac{1}{j\omega C_{eq}}$ ⇒ $C_{eq} = \left(1 + \frac{R_2}{R_1}\right)C$

- A proper selection of R_1 and R_2 can allow realization of desired capacitance between the input terminal and the ground.
- The size of effective capacitance is limited by the inverted output voltage limitation.
- The larger the capacitance multiplication, the smaller will be the allowable input voltage to prevent the op amp from reaching saturation.
- Similar op amp circuit can be designed to synthesize any desired inductance.
- On a similar line, resistance multiplier can be designed as well.

Example – 6

The op amp circuit given below is called an *inductance simulator*. Show that the input impedance is given by:

$$Z_{in} = \frac{V_{in}}{I_{in}} = j\omega L_{eq} \quad \text{Where:} \quad L_{eq} = \frac{R_1 R_3 R_4}{R_2} C$$



Self Study: Oscillators (Section: 10.9.2)

AC Power Analysis

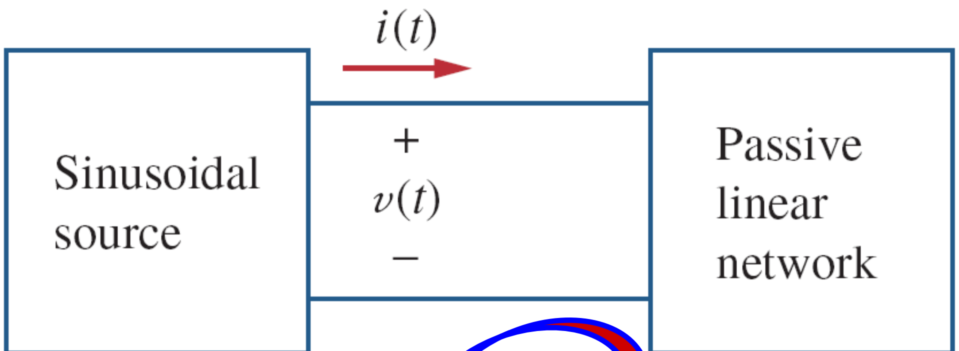
- So far in ac circuit analysis, focus has been mainly on calculating voltage and current.
- This chapter discusses power analysis. It is extremely important as power is the most important quantity in electric utilities, electronics, and communication systems as such systems involve transmission of power from one point to another.
- Also, every industrial and household electrical device—every fan, motor, lamp, pressing iron, TV, personal computer— has a power rating that indicates how much power the equipment requires; exceeding the power rating can do permanent damage to an appliance.
- The most common form of electric power is 50- or 60-Hz ac power. The choice of ac over dc allows high-voltage power transmission from the power generating plant to the consumer.

Instantaneous and Average Power

- The *instantaneous power* (in watts) is the power absorbed at any instant of time in a circuit.

$$p(t) = v(t)i(t)$$

It is the rate at which the circuit absorbs energy



Assume:

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

Instantaneous power absorbed:

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

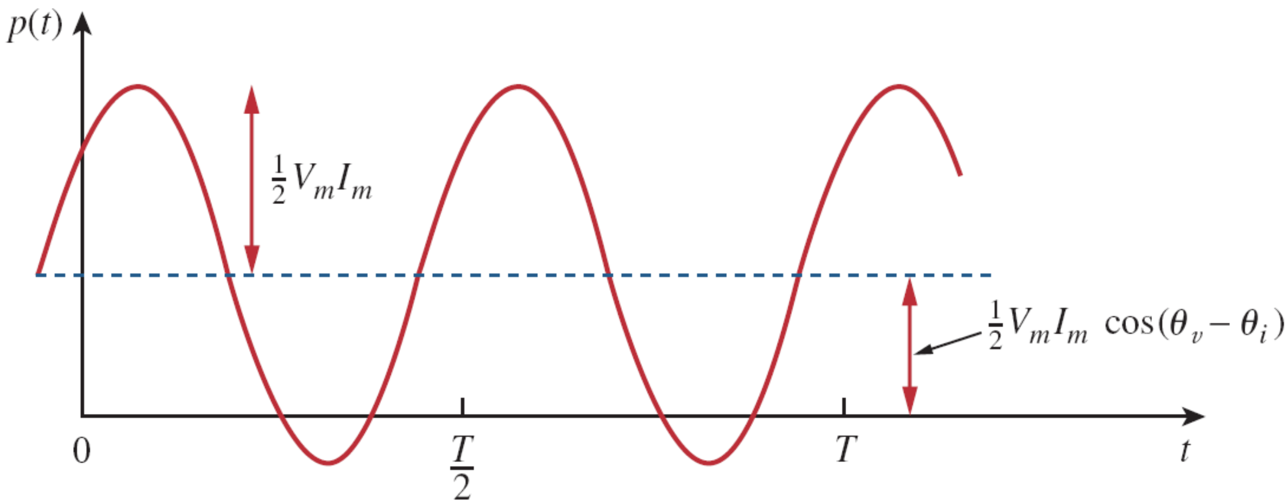
$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Instantaneous and Average Power (contd.)

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Constant and independent of time

Sinusoidal function of frequency 2ω



The instantaneous power is difficult to measure as it varies with time

- Instead its convenient to measure average power. Your watt meter measures the average power. Average power is the instantaneous power averaged over time period.

Instantaneous and Average Power (contd.)

Average Power → $$P = \frac{1}{T}\int_0^T p(t)\,dt$$

$$P = \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\,dt + \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt + \frac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt$$

Zero

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

In phasor form: $$\frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}V_m I_m \underline{/\theta_v - \theta_i}$$

$$= \frac{1}{2}V_m I_m[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)]$$

Instantaneous and Average Power (contd.)

Average Power → $$P = \frac{1}{2}\mathrm{Re}[\mathbf{VI}^*] = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}|\mathbf{I}|^2 R$$

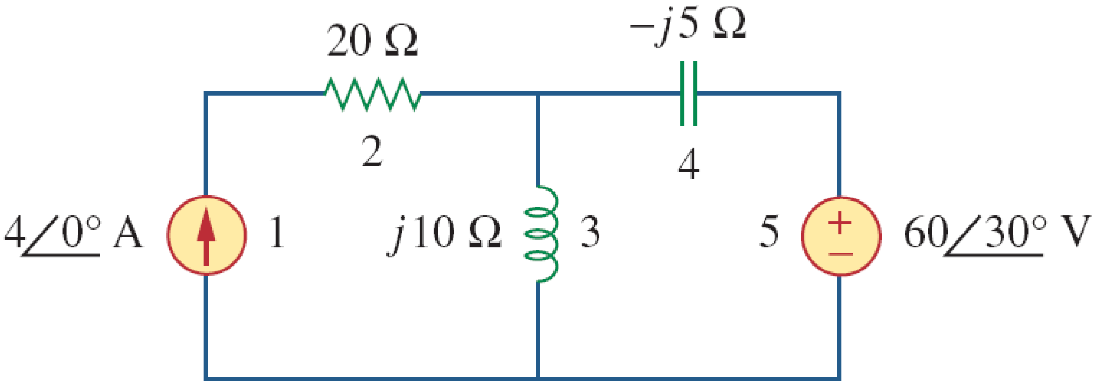
For current and voltage in phase, $\theta_v = \theta_i$: $P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}|\mathbf{I}|^2 R$

For current and voltage in quadrature, $\theta_v - \theta_i = \pm 90°$: $P = \frac{1}{2}V_m I_m \cos 90° = 0$

In summary, a resistive load (R) absorbs power at all times, while reactive load (L or C) absorbs zero average power.

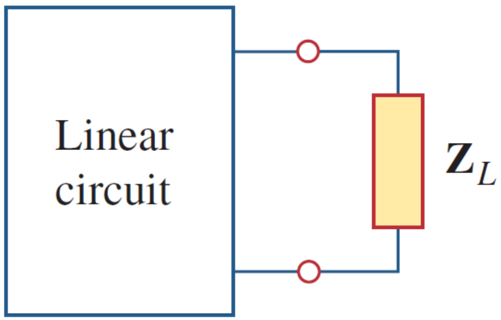
Example – 7

Determine the average power generated by each source and the average power absorbed by each passive element.

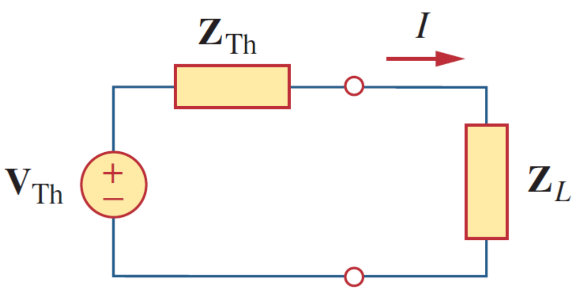


Maximum Average Power Transfer

- Let us consider an ac circuit terminated into a load Z_L.
- The Thevenin equivalent of the ac circuit is Z_{Th} and V_{Th}.



$$\mathbf{Z}_{\text{Th}} = R_{\text{Th}} + jX_{\text{Th}}$$
$$\mathbf{Z}_L = R_L + jX_L$$



The current through the load:

$$\mathbf{I} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{Z}_{\text{Th}} + \mathbf{Z}_L} = \frac{\mathbf{V}_{\text{Th}}}{(R_{\text{Th}} + jX_{\text{Th}}) + (R_L + jX_L)}$$

The average power delivered to the load:

$$P = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{|\mathbf{V}_{\text{Th}}|^2 R_L/2}{(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2}$$

Now the objective is to regulate R_L and X_L to maximize P.

Maximum Average Power Transfer (contd.)

- One can achieve that by setting $\partial P/\partial X_L$ and $\partial P/\partial R_L$ equal to zero.

$$\frac{\partial P}{\partial X_L} = -\frac{|\mathbf{V}_{\text{Th}}|^2 R_L (X_{\text{Th}} + X_L)}{[(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2]^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{|\mathbf{V}_{\text{Th}}|^2 [(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2 - 2R_L(R_{\text{Th}} + R_L)]}{2[(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2]^2}$$

- setting $\partial P/\partial X_L$ equal to zero gives: $X_L = -X_{Th}$
- setting $\partial P/\partial R_L$ equal to zero gives:

$$R_L = \sqrt{(R_{Th})^2 + (X_{Th} + X_L)^2}$$

$$\Rightarrow R_L = R_{Th}$$

$$\Rightarrow Z_L = R_L + jX_L = R_{Th} - jX_{Th} = (Z_{Th})^*$$

For maximum average power transfer, the load impedance $\mathbf{Z}_L$ must be equal to the complex conjugate of the Thevenin impedance $\mathbf{Z}_{\text{Th}}$.

Maximum Average Power Transfer (contd.)

$$Z_L = R_L + jX_L = R_{Th} - jX_{Th} = (Z_{Th})^*$$

This is the *maximum average power transfer theorem* for the sinusoidal steady state

$$P = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{|\mathbf{V}_{\text{Th}}|^2 R_L/2}{(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2}$$

$$P_{\max} = \frac{|\mathbf{V}_{\text{Th}}|^2}{8R_{\text{Th}}}$$

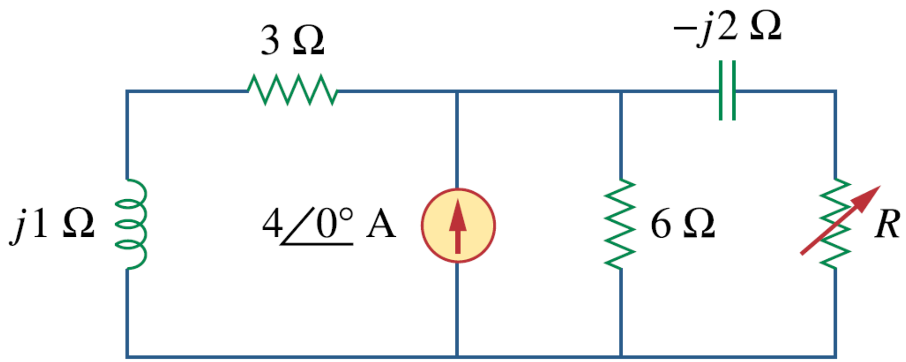
- For a purely real load, the condition for maximum power transfer is:

$$R_L = \sqrt{R_{\text{Th}}^2 + X_{\text{Th}}^2} = |\mathbf{Z}_{\text{Th}}|$$

for maximum average power transfer to a purely resistive load, the load impedance (or resistance) is equal to the magnitude of the Thevenin impedance.

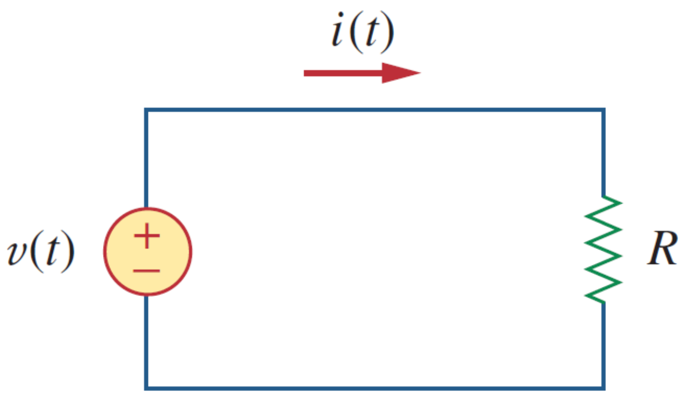
Example – 8

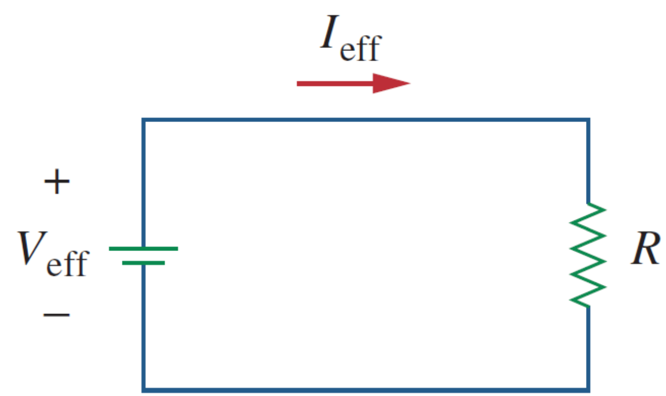
The variable resistor *R* in the following circuit is adjusted until it absorbs the maximum average power. Find *R* and the maximum average power absorbed.



Effective or RMS Value

The effective value of a ***periodic current*** is the ***dc current*** that delivers the same average power to a resistor as the periodic current.





$$P = \frac{1}{T}\int_0^T i^2 R\,dt = \frac{R}{T}\int_0^T i^2\,dt$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

$$P = I_{\text{eff}}^2 R$$

Similarly: $V_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$

Effective or RMS Value (contd.)

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} \qquad V_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T v^2\, dt}$$

The effective value is the (square) *root* of the *mean* (or average) of the *square* of the periodic signal. Therefore, the effective value is often known as the *root-mean-square* value (*rms* value).

$$I_{\text{eff}} = I_{\text{rms}}, \qquad V_{\text{eff}} = V_{\text{rms}}$$

- For a sinusoid, $i(t) = I_m cos\omega t$
  $$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2\omega t\, dt} = \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 + \cos 2\omega t)\, dt} = \frac{I_m}{\sqrt{2}}$$
- For a sinusoid, $v(t) = V_m cos\omega t$
  $$V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i)$$

Effective or RMS Value (contd.)

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i) \Rightarrow = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

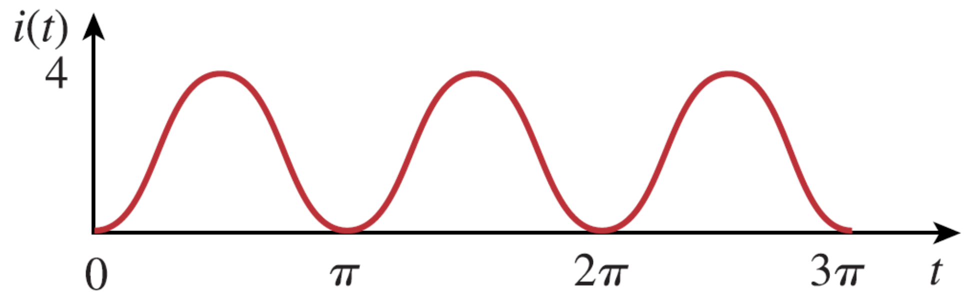
the average power absorbed by a resistor *R in the example circuit:*

$$P = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

- When a sinusoidal voltage or current is specified, it is often in terms of its maximum (or peak) value or its rms value, since its average value is zero.
- The power companies specify phasor magnitudes in terms of their rms values rather than the peak values.
- It is convenient in power analysis to express voltage and current in their rms values.
- Analog voltmeters and ammeters are designed to read directly the rms value of voltage and current.

Example – 9

Find the rms value of the current wave shown



Example – 10

Determine the rms value of this voltage.