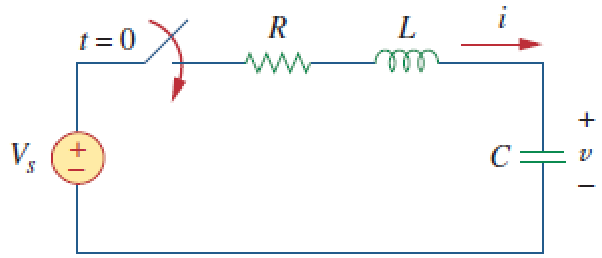
Lecture – 5 **Date: 18.08.2016**

- Second-Order Circuit (contd.)

Step Response of a Series RLC Circuit



- Apply KVL around the loop for t>0:

$$L\frac{di}{dt} + Ri + v = V_s \qquad i = C\frac{dv}{dt}$$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC}$$

solution has two components

$$v(t) = v_t(t) + v_{ss}(t)$$

- $v_t(t)$ dies out with time and is of the form:

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \text{(Overdamped)}$$

$$v_t(t) = (A_1 + A_2 t)e^{-\alpha t} \qquad \text{(Critically damped)}$$

$$v_t(t) = (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t} \qquad \text{(Underdamped)}$$

- $v_{ss}(t)$ is the final value of the capacitor voltage. $v_{ss}(t) = v(\infty) = V_s$

Step Response of a Series RLC Circuit (contd.)

- the complete solutions are:

$$v(t) = V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \text{(Overdamped)}$$

$$v(t) = V_s + (A_1 + A_2 t)e^{-\alpha t} \qquad \text{(Critically damped)}$$

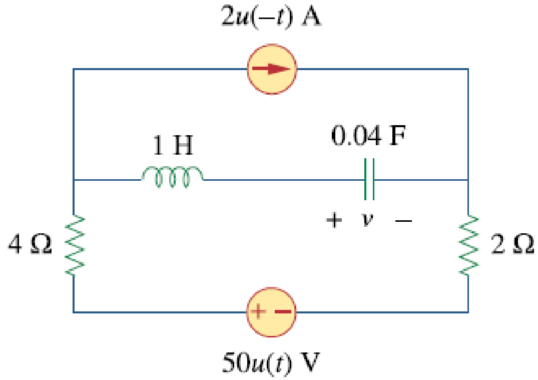
$$v(t) = V_s + (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t} \qquad \text{(Underdamped)}$$

- The values of A_1 and A_2 are obtained from the initial conditions: $v(0)$ and $dv(0)/dt$
- Just to reiterate: *v* and *i* are the voltage across the capacitor and the current through the inductor respectively.
- Above expressions help in finding *v*.
- once the capacitor voltage is $v_c = v$ is known, we can determine $i = C\,{}^{dv}/_{dt}$ which is the same current through the capacitor, inductor, and resistor.
- the voltage across the resistor is $v_R = iR$ while the inductor voltage is $L\,{}^{di}/_{dt}$.
- Alternatively, the complete response for any variable $x(t)$ can be found directly, because it has the general form:

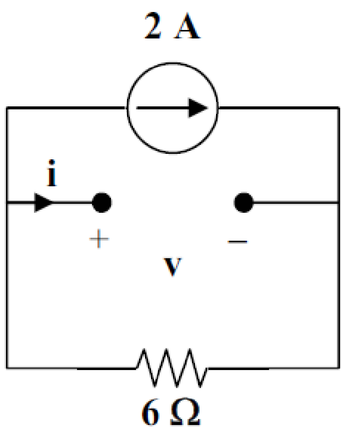
$$x(t) = x_{ss}(t) + x_t(t)$$

Example – 1

For this circuit, find v(t) for t >0.



- For t = 0-, the equivalent circuit is:



$$i(0-) = 0, \; v(0-) = -2x6 = -12V$$

Example – 1 (contd.)

- For t > 0, we have a series RLC circuit with a step input.

$$\alpha = R/(2L) = 6/2 = 3 \qquad \omega_o = 1/\sqrt{LC} = 1/\sqrt{0.04}$$

Underdamped

$$s = -3 \pm \sqrt{9-25} = -3 \pm j4$$

$$v(t) = V_f + [(A\cos 4t + B\sin 4t)e^{-3t}]$$

$$V_f = \text{final capacitor voltage} = 50\text{ V} \Rightarrow v(t) = 50 + [(A\cos 4t + B\sin 4t)e^{-3t}]$$

$$v(0) = -12 = 50 + A \Rightarrow A = -62$$

$$i(0) = 0 = Cdv(0)/dt$$

$$dv/dt = [-3(A\cos 4t + B\sin 4t)e^{-3t}] + [4(-A\sin 4t + B\cos 4t)e^{-3t}]$$

$$0 = dv(0)/dt = -3A + 4B \text{ or } B = (3/4)A = -46.5$$

$$v(t) = \underline{\{50 + [(-62\cos 4t - 46.5\sin 4t)e^{-3t}]\}\text{ V}}$$

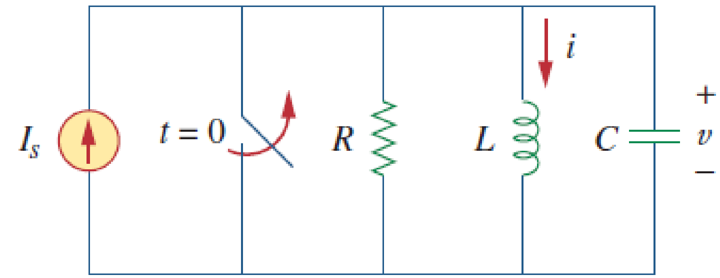
Practice Example

A series *RLC* circuit is described by: $$L\frac{d^2 i}{dt^2}+R\frac{di}{dt}+\frac{i}{C}=2$$

Find the response when $L=0.5H$, $R=4\Omega$, and $C=0.2F$. Let $i(0) = 1$, $di(0)/dt=0$.

Step Response of a Parallel RLC Circuit

- Apply KCL at the top node for t>0:



$$\frac{v}{R} + i + C\frac{dv}{dt} = I_s \qquad v = L\frac{di}{dt}$$

$$\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$$

$$i(t) = i_t(t) + i_{ss}(t)$$

$$i(t) = I_s + A_1e^{s_1t} + A_2e^{s_2t} \qquad \text{(Overdamped)}$$

$$i(t) = I_s + (A_1 + A_2t)e^{-\alpha t} \qquad \text{(Critically damped)}$$

$$i(t) = I_s + (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t} \qquad \text{(Underdamped)}$$

A_1 and A_2 to be determined from $i(0)$ and ${}^{di(0)}/_{dt}$.

$$i = i_L \qquad v = L\, di/dt$$

$$i_R = v/R$$

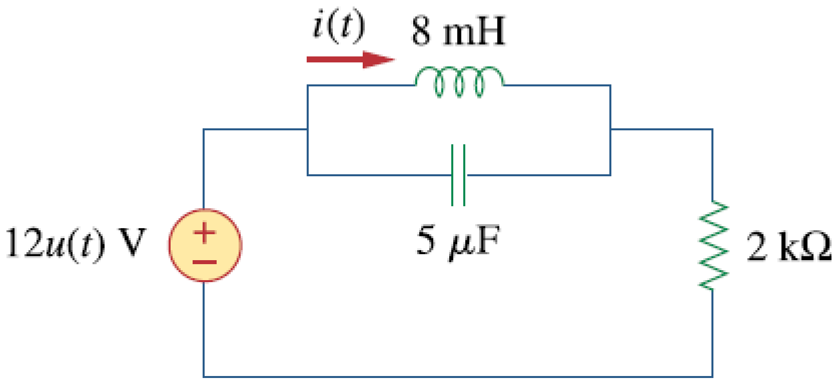
$$i_C = C\, dv/dt$$

- Alternatively, the complete response for any variable $x(t)$ can be found directly, because it has the general form:

$$x(t) = x_{ss}(t) + x_t(t)$$

Example – 2

find $i(t)$ *for* $t > 0$ in the following circuit:



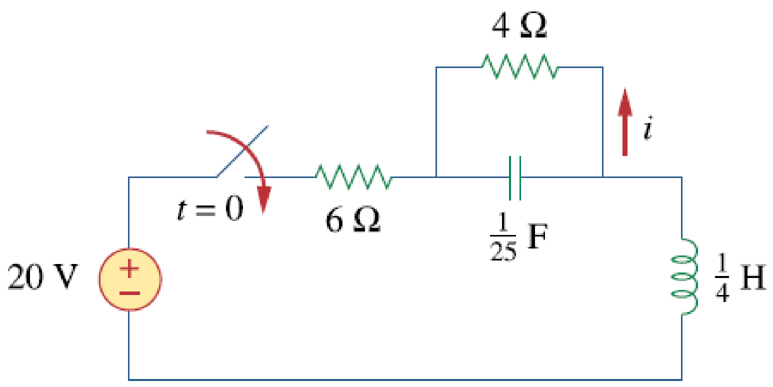
General Second-Order Circuits

- the series and parallel *RLC* circuits are the second-order circuits of greatest interest, other second-order circuits including op amps are also useful.
- for a second-order circuit, we determine its step response $x(t)$(which may be voltage or current) by taking the following four steps:

1. We first determine the initial conditions $x(0)$ and ${}^{dx(0)}/_{dt}$ and the final value $x(\infty)$.
2. We turn off the independent sources and find the form of the transient response $x_t(t)$ by applying KCL and KVL.
3. We obtain the steady-state response as: $x_{ss}(t) = x(\infty)$.
4. The total response is then found as the sum of the transient response and steady-state response.

Example – 3

In this circuit, find $i(t)$ *for* $t > 0$.

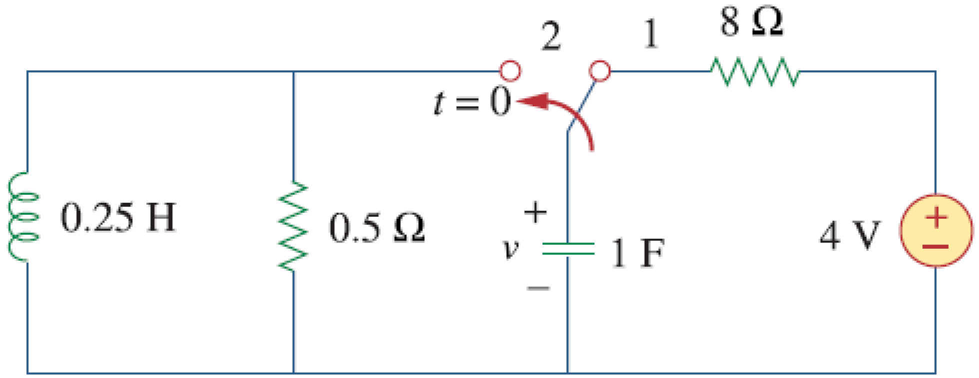


Example – 4

In the following circuit, the switch has been in position 1 for a long time but moved to position 2 at $t = 0$, Find:

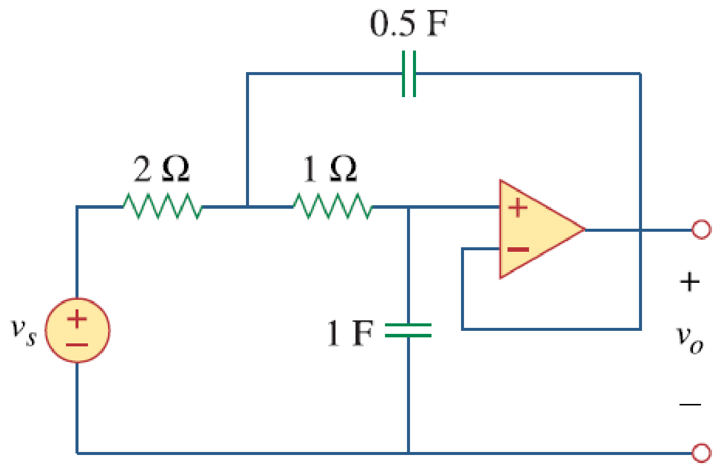
(a) $v(0^+)$, $dv(0^+)/dt$

(b) $v(t)$ for $t \geq 0$



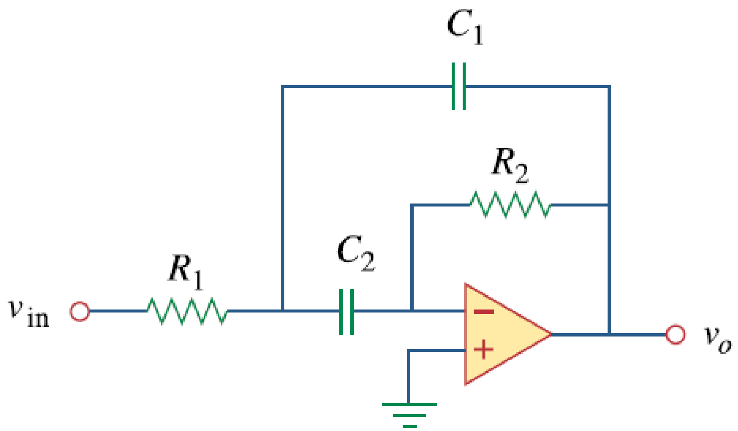
Example – 6

For the op amp circuit, derive the differential equation relating v_o *to* v_s.



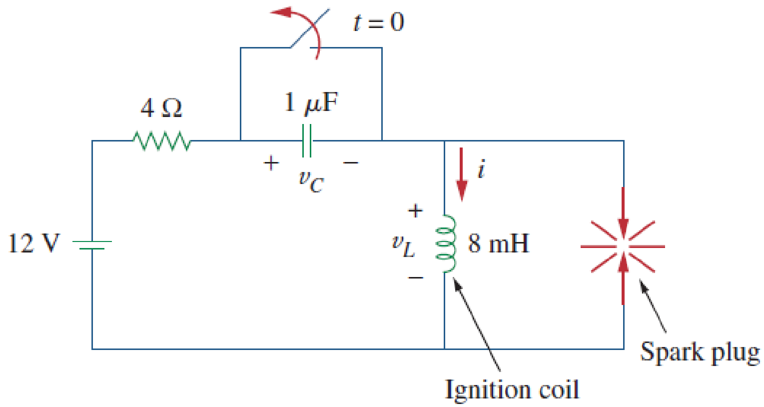
Example – 5

In the following op amp circuit, determine $v_0(t)$ for $t > 0$. Let $v_{in}(t) = u(t)V$, $R_1 = R_2 = 10\ k\Omega, C_1 = C_2 = 100\mu F$.



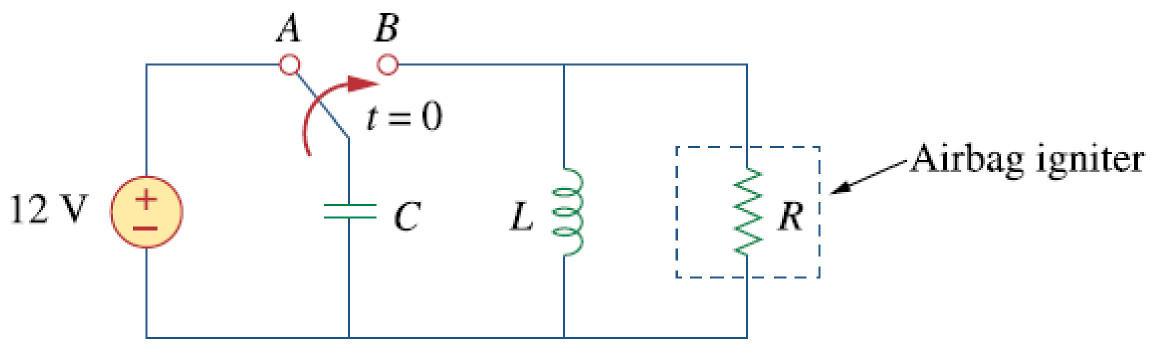
Automobile Ignition System

- Earlier, considered the automobile ignition system as a charging system. That was only a part of the system.
- another part—the voltage generating system.
- The 12-V source is due to the battery and alternator.
- The 4Ω resistor represents the resistance of the wiring.
- The ignition coil is modeled by the 8-mH inductor.
- The $1\mu F$ capacitor (known as the *condenser* to auto mechanics) is in parallel with the switch (known as the *breaking points* or *electronic ignition*).



Example – 7

An automobile airbag igniter is modeled by the following circuit. Determine the time it takes the voltage across the igniter to reach its first peak after switching from A to B. Let R= 3Ω, C=1/30pF, and L= 60mH.



Example – 8

A load is modeled as a 250-mH inductor in parallel with a 12Ω resistor. A capacitor is needed to be connected to the load so that the network is critically damped at 60 Hz. Calculate the size of the capacitor.