Lecture – 21 **Date: 07.11.2016**

- Fourier Transform Applications

Circuit Applications

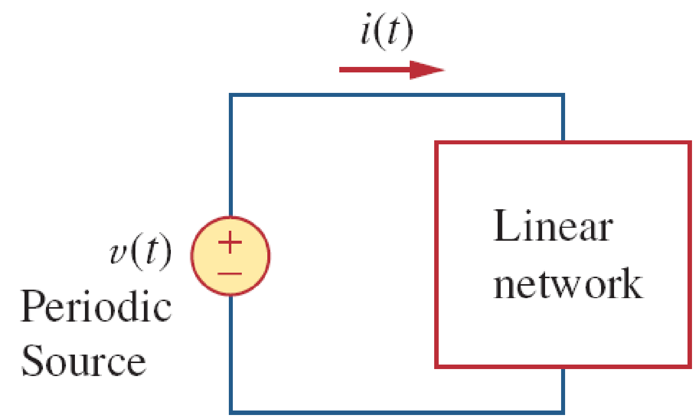
- Many circuits are driven by non-sinusoidal periodic functions. To find the steady-state response of a circuit to a non-sinusoidal periodic excitation requires the application of a Fourier series, ac phasor analysis, and the superposition principle. The procedure usually involves four steps.

Steps for Applying Fourier Series:

1. Express the excitation as a Fourier series.
2. Transform the circuit from the time domain to the frequency domain.
3. Find the response of the dc and ac components in the Fourier series.
4. Add the individual dc and ac responses using the superposition principle.

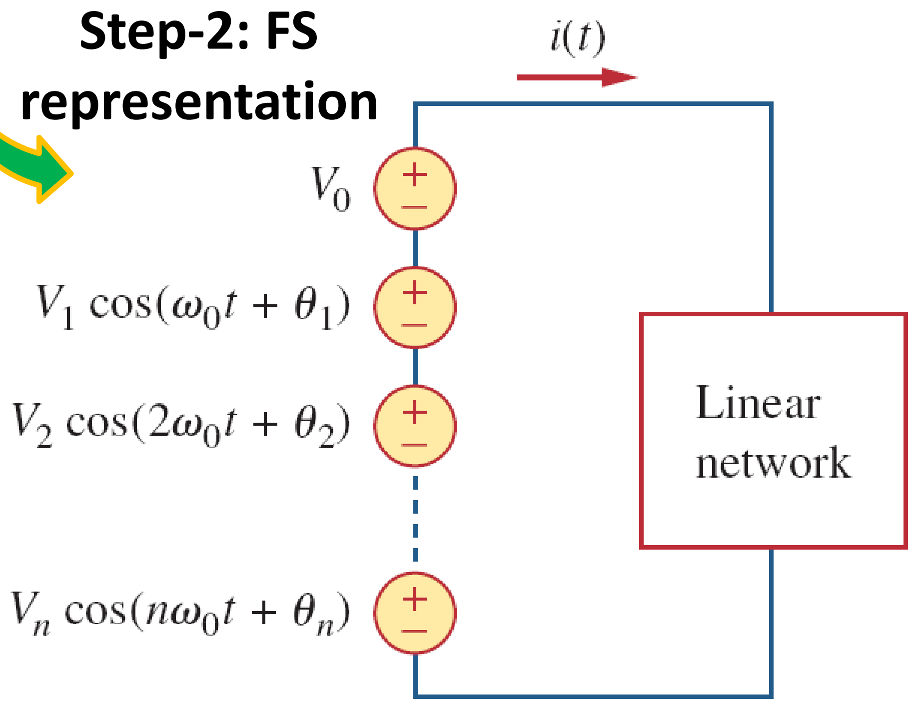
Circuit Applications (contd.)

- **First Step:** determine the Fourier series expansion of the excitation.



$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$

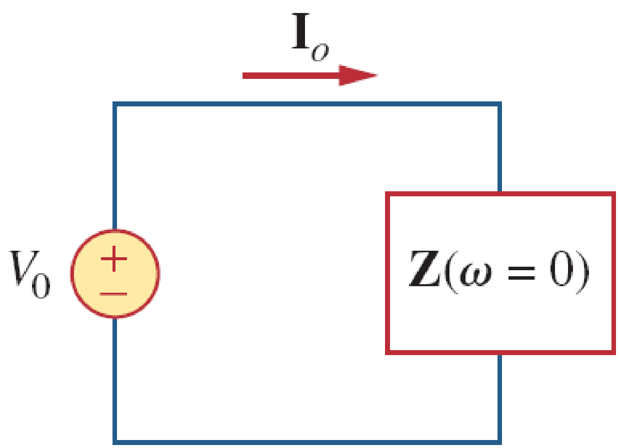
Step-2: FS representation

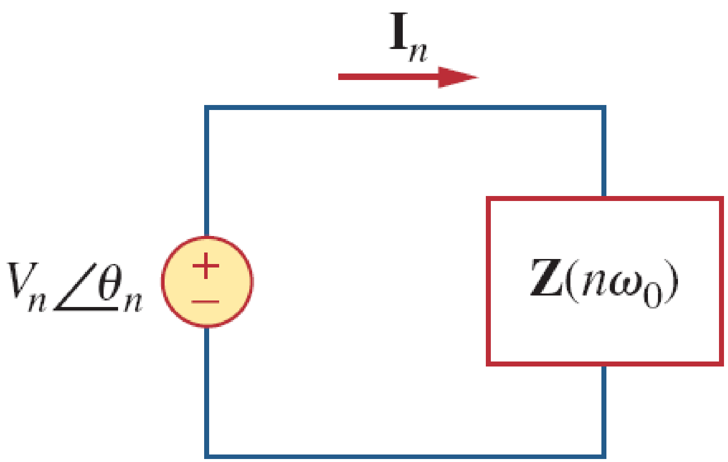


Third Step: find the response to each term in the Fourier series.

Circuit Applications (contd.)

- The response to the **dc component** can be determined in the frequency domain by setting $n = 0$ or $\omega = 0$, or in the time domain by replacing all inductors with short circuits and all capacitors with open circuits.
- The response to the ac component is obtained by applying the phasor techniques.





The network is represented by its impedance $\boldsymbol{Z}(n\omega_0)$ and it's the value when ω is everywhere replaced by $n\omega_0$

Circuit Applications (contd.)

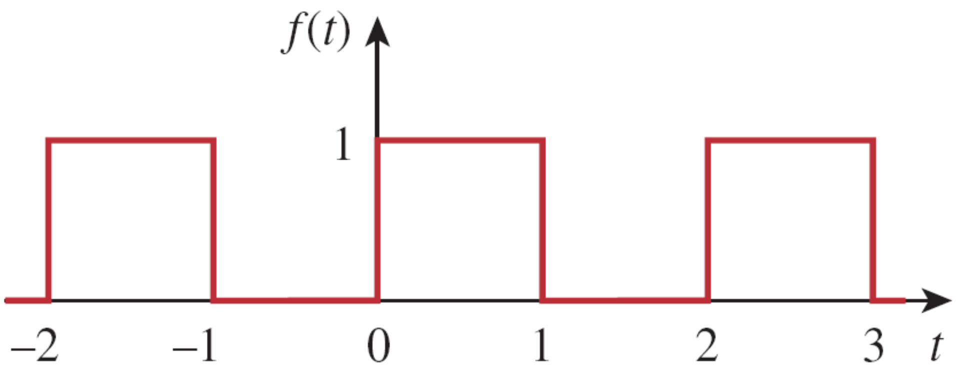
- **Step-4:** apply principle of superposition to obtain the total current.

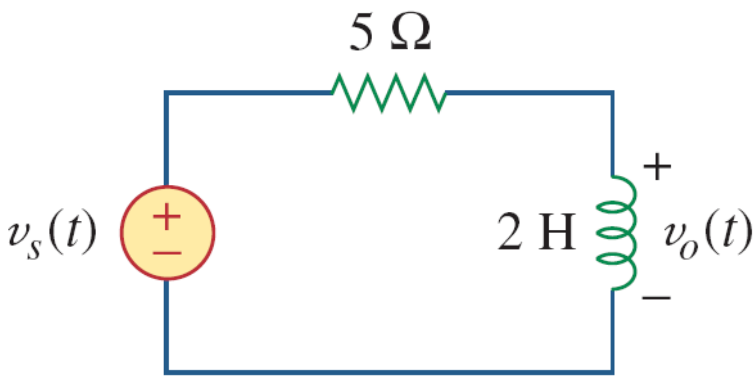
$$i(t) = i_0(t) + i_1(t) + i_2(t) + \cdots$$
$$= \mathbf{I}_0 + \sum_{n=1}^{\infty} |\mathbf{I}_n| \cos(n\omega_0 t + \psi_n)$$

where each component $\boldsymbol{I}_n$ with frequency $n\omega_0$ has been transformed to the time domain to get $i_n(t)$, and Ψ_n is the argument of $\boldsymbol{I}_n$.

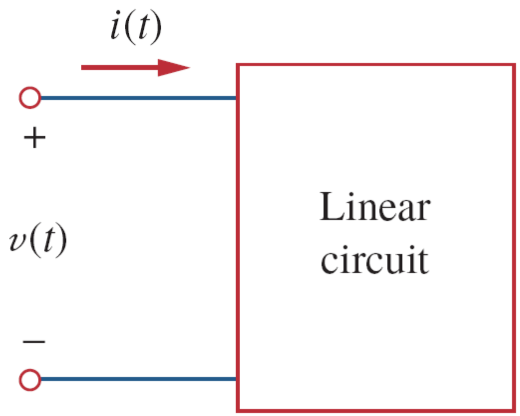
Example – 1

In this situation, $f(t)$ is the voltage source $v_s(t)$. Find the response of the circuit.





Average Power and RMS Power



- the periodic current and voltage in amplitude-phase form:

$$i(t) = I_{dc} + \sum_{m=1}^{\infty} I_m \cos(m\omega_0 t - \phi_m)$$

$$v(t) = V_{dc} + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t - \theta_n)$$

- the average power: $P = \frac{1}{T}\int_0^T vi\,dt$

$$P = \frac{1}{T}\int_0^T V_{dc} I_{dc}\,dt + \sum_{m=1}^{\infty} \frac{I_m V_{dc}}{T}\int_0^T \cos(m\omega_0 t - \phi_m)\,dt + \sum_{n=1}^{\infty} \frac{V_n I_{dc}}{T}\int_0^T \cos(n\omega_0 t - \theta_n)\,dt$$

$$+ \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{V_n I_m}{T}\int_0^T \cos(n\omega_0 t - \theta_n)\cos(m\omega_0 t - \phi_m)\,dt$$

Here, the second and third integrals vanish, due to the integration of the cosine over its period. All terms in the fourth integral are zero when $n \neq m$.

Average Power and RMS Power (contd.)

- **For $m = n$:** $P = V_{\mathrm{dc}} I_{\mathrm{dc}} + \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$

  the total average power is the sum of the average powers in each harmonically related voltage and current.

- For a periodic function $f(t)$, its rms value (or the effective value) is: $$F_{\mathrm{rms}} = \sqrt{\frac{1}{T} \int_0^T f^2(t)\, dt}$$

- In terms of Fourier coefficients, the rms value is: $$F_{\mathrm{rms}} = \sqrt{a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)}$$

- If $f(t)$ is the current through a resistor *R*, then: $$P = R F_{\mathrm{rms}}^2$$

Average Power and RMS Power (contd.)

- If $f(t)$ is the voltage across a resistor *R*, then:

$$P = \frac{F_{\text{rms}}^2}{R}$$

- The power dissipated by the 1Ω resistance is:

$$P_{1\Omega} = F_{\text{rms}}^2 = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

- This result is known as *Parseval's theorem*.
- Notice that $(a_0)^2$ is the power in the dc component, while $\frac{1}{2}\left[(a_n)^2 + (b_n)^2\right]$ is the ac power in the *n*th harmonic.
- Thus, Parseval's theorem states that the average power in a periodic signal is the sum of the average power in its dc component and the average powers in its harmonics.

Example – 2

- Determine the average power supplied to this circuit if $i(t) = 2 + 10\cos(t + 10°) + 6\cos(3t + 35°)A$