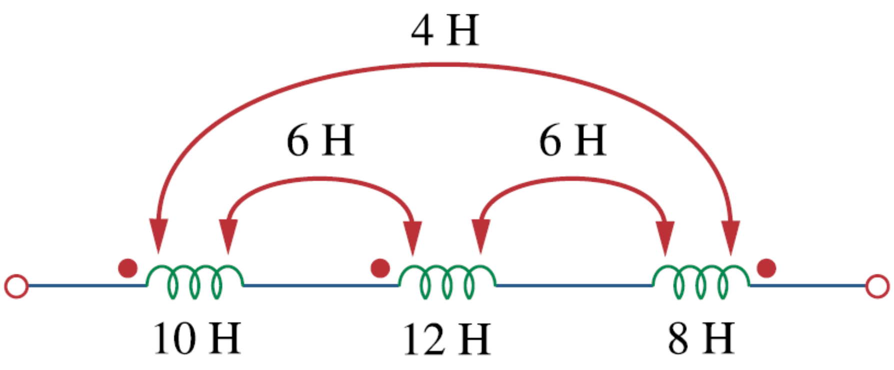
Lecture – 12 Date: 15.09.2016

- Magnetically Coupled Circuits: Energy in Coupled Circuits

Example – 1

Calculate the total inductance.



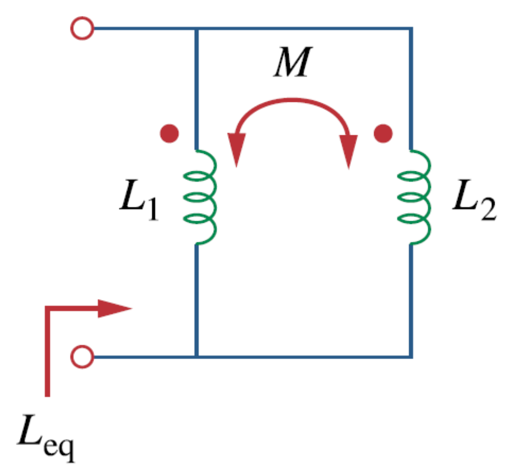
Example – 2

Two coils connected in series-aiding fashion have a total inductance of 250 mH. When connected in a series-opposing configuration, the coils have a total inductance of 150mH. If the inductance of one coil (L_1) is three times the other, find L_1, L_2, and M.

Example – 3

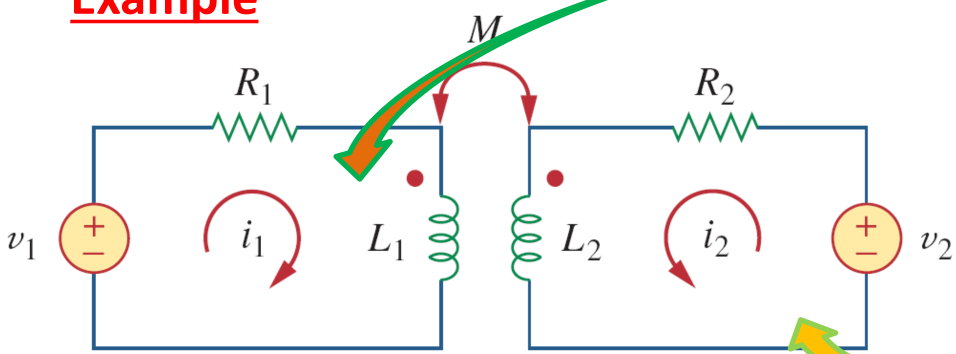
For the coupled coils, show that:

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$



Mutual Inductance (contd.)

Example



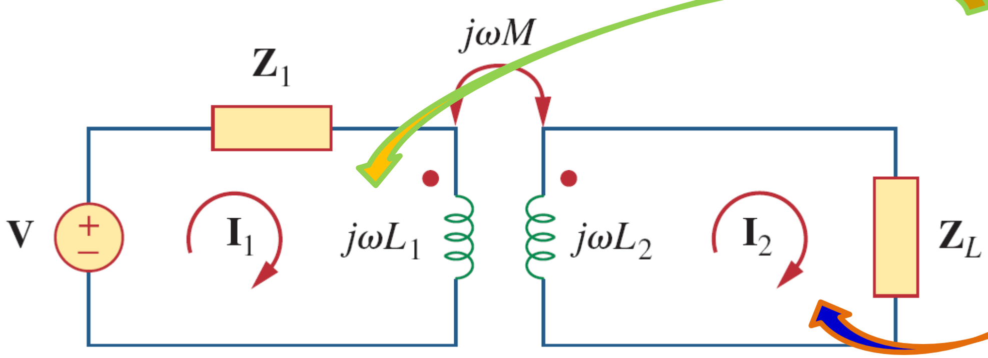
$$v_1 = i_1R_1 + L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2 = i_2R_2 + L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

- The KVL expressions in the frequency domain:

$$\mathbf{V}_1 = (R_1 + j\omega L_1)\mathbf{I}_1 + j\omega M\mathbf{I}_2 \qquad \mathbf{V}_2 = j\omega M\mathbf{I}_1 + (R_2 + j\omega L_2)\mathbf{I}_2$$

- **Another example:**

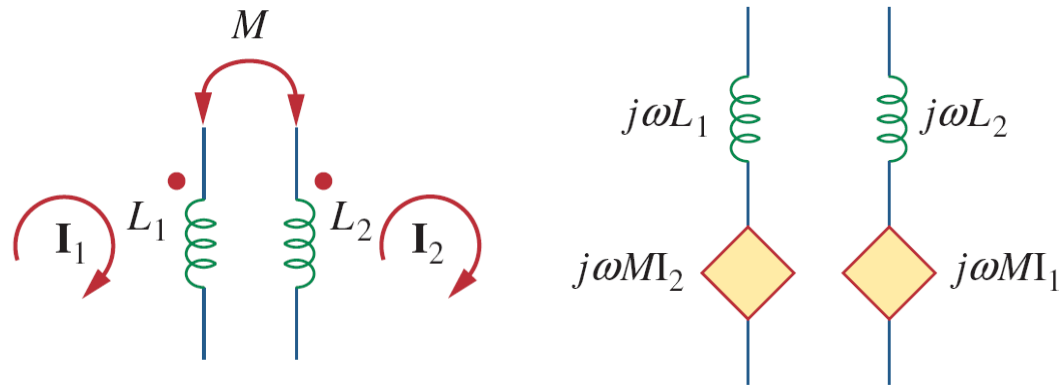


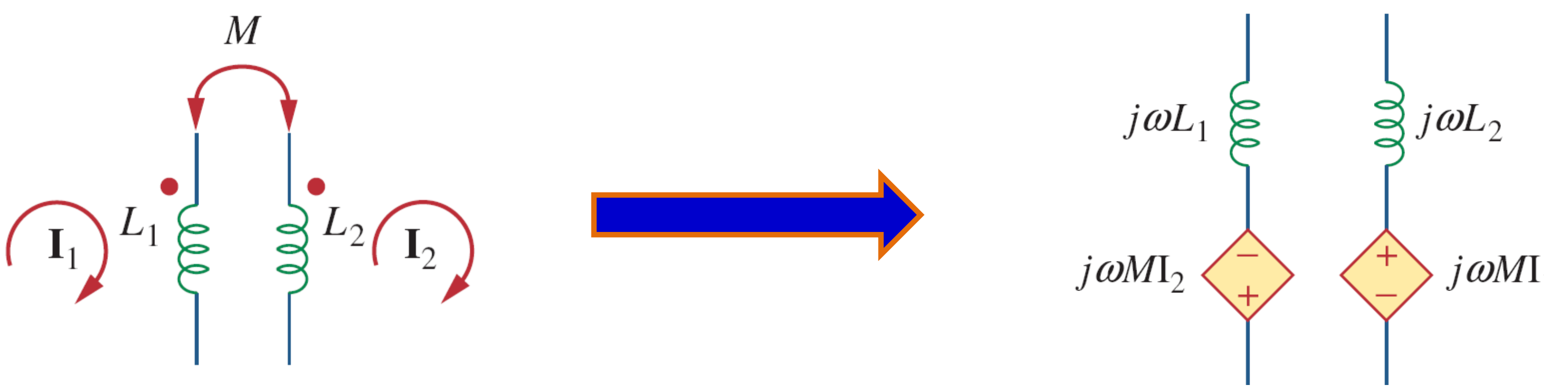
$$\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$$

$$0 = -j\omega M\mathbf{I}_1 + (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2$$

Mutual Inductance (contd.)

- Often, solving mutually coupled circuits requires tracking of two or more steps made at once regarding the sign and values of the mutually induced voltages.

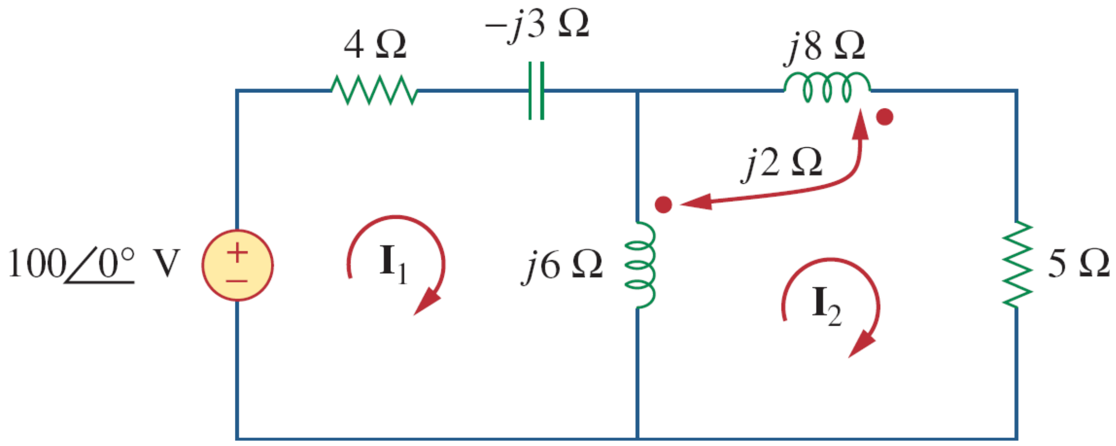




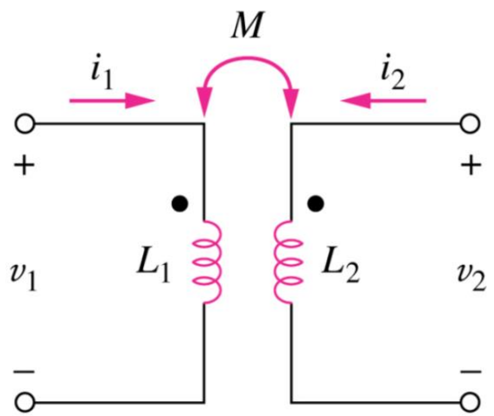
In this course, we are not concerned with the determination of the mutual inductances of the coils and their dot placements. We assume that the mutual inductance and the placement of the dots are the “givens’’ of the circuit problem, like the circuit components *R*, *L*, and *C*.

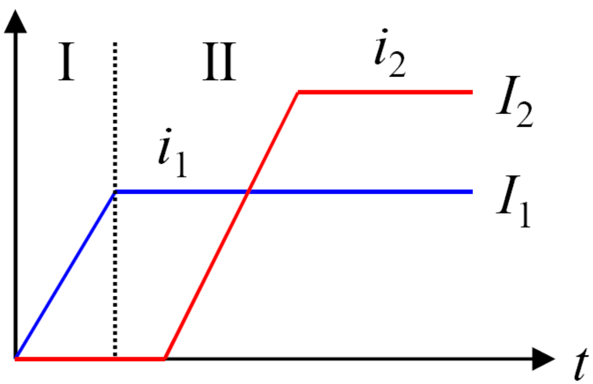
Example – 4

Calculate the mesh currents in this circuit.



Energy in Coupled Circuits





- To find the stored energy as $i_1 = I_1$ and $i_2 = I_2$:

<u>**Step-1:**</u> $i_2 = 0$ and i_1 increases from 0 to I_1 → the power in the circuit is

$$p_1(t) = v_1 i_1 = i_1 L_1 \frac{di_1}{dt}$$

<u>**the energy stored in the circuit**</u>

$$w_1 = \int p_1 dt = L_1 \int_0^{I_1} i_1 di_1 = \frac{1}{2} L_1 I_1^2$$

<u>**Step-2:**</u> $i_1 = I_1$ and i_2 increases from 0 to I_2 → the power in the circuit is

$$p_2(t) = i_1 v_1 + i_2 v_2 = I_1 M_{12} \frac{di_2}{dt} + i_2 L_2 \frac{di_2}{dt}$$

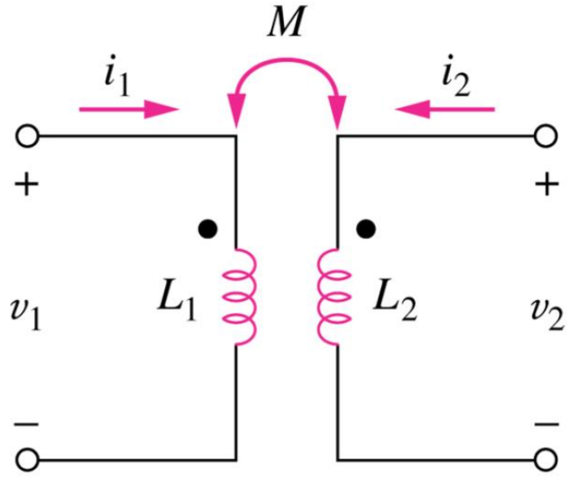
<u>**the energy stored in the circuit**</u> $= M_{12} I_1 I_2 + \frac{1}{2} L_2 I_2^2$

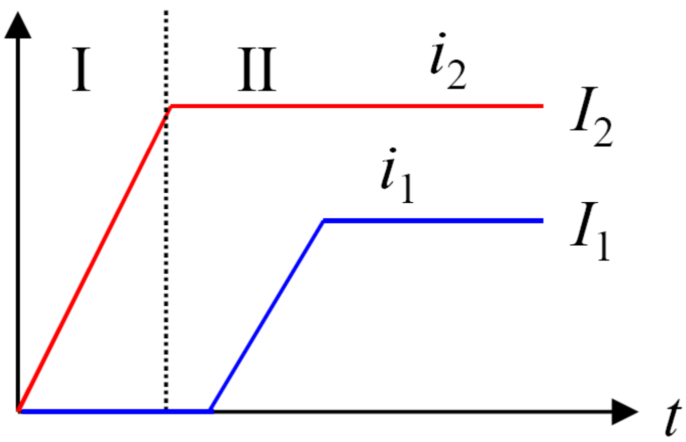
Energy in Coupled Circuits (contd.)

- The total energy stored in the coils when both and have reached constant i_1 and i_2 values is:

$$w = w_1 + w_2 = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M_{12} I_1 I_2$$

Alternatively:





$$w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M_{21} I_1 I_2$$

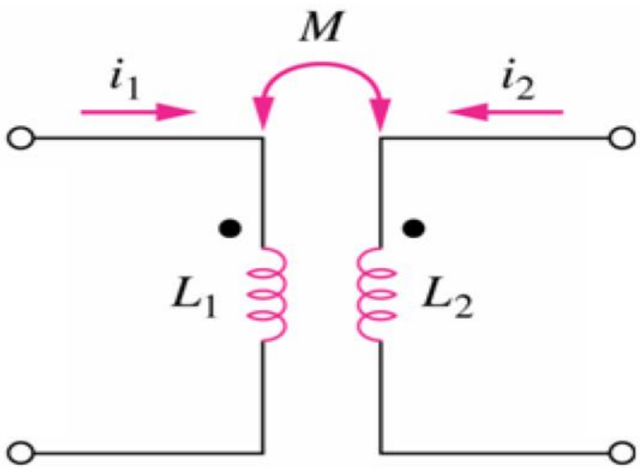
Energy in Coupled Circuits (contd.)

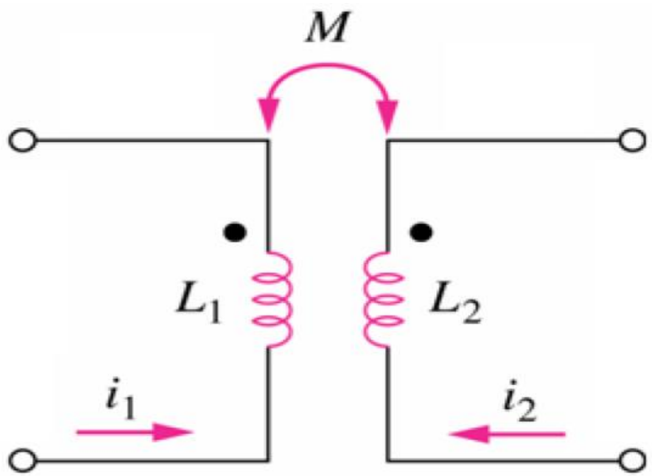
- But the total energy stored should be the same regardless of how we reach the final conditions.

$$M_{12} = M_{21} = M$$

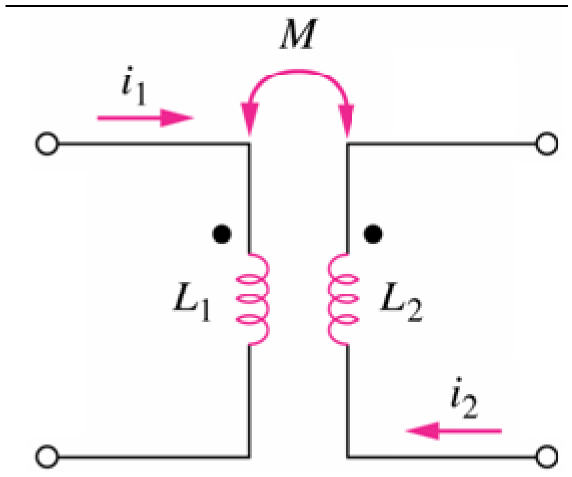
$$w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2$$

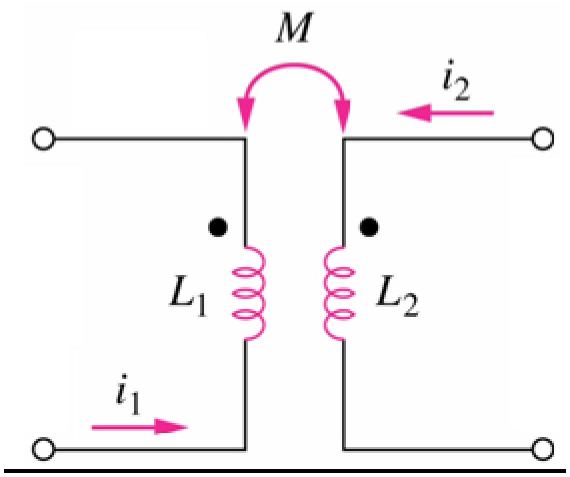
- This equation is applicable when the currents both enter the dotted terminals or undotted terminals.





Energy in Coupled Circuits (contd.)





$$w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 - M I_1 I_2$$

In general:

$$\boxed{w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2}$$

Energy in Coupled Circuits (contd.)

$$\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2 \geq 0$$

$$\frac{1}{2}(i_1\sqrt{L_1} - i_2\sqrt{L_2})^2 + i_1 i_2(\sqrt{L_1 L_2} - M) \geq 0$$

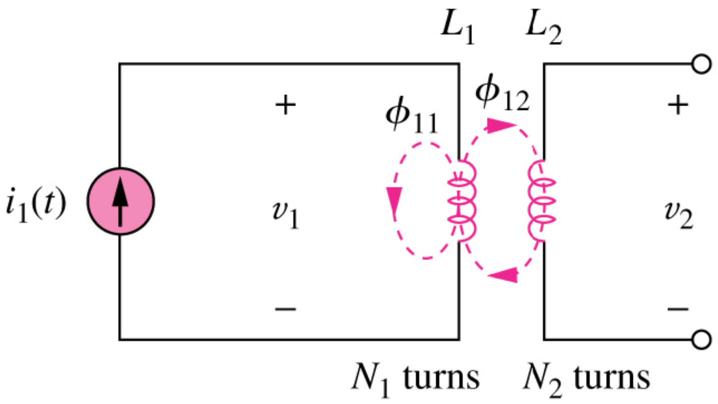
$$\sqrt{L_1 L_2} - M \geq 0 \quad \Rightarrow \quad M \leq \sqrt{L_1 L_2}$$

- The extent to which the mutual inductance *M* approaches the upper limit is specified by the *coefficient of coupling k*:

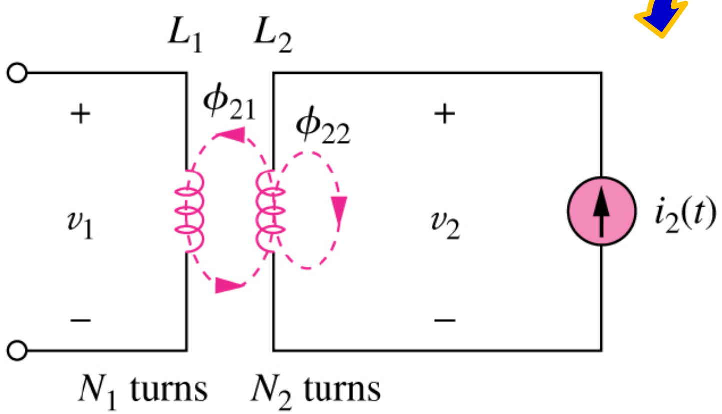
$$k = \frac{M}{\sqrt{L_1 L_2}}$$

The coupling coefficient is the fraction of the total flux emanating from one coil that links the other coil.

Energy in Coupled Circuits (contd.)



$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{12}}{\phi_{11} + \phi_{12}}$$



$$k = \frac{\phi_{21}}{\phi_2} = \frac{\phi_{21}}{\phi_{21} + \phi_{22}}$$

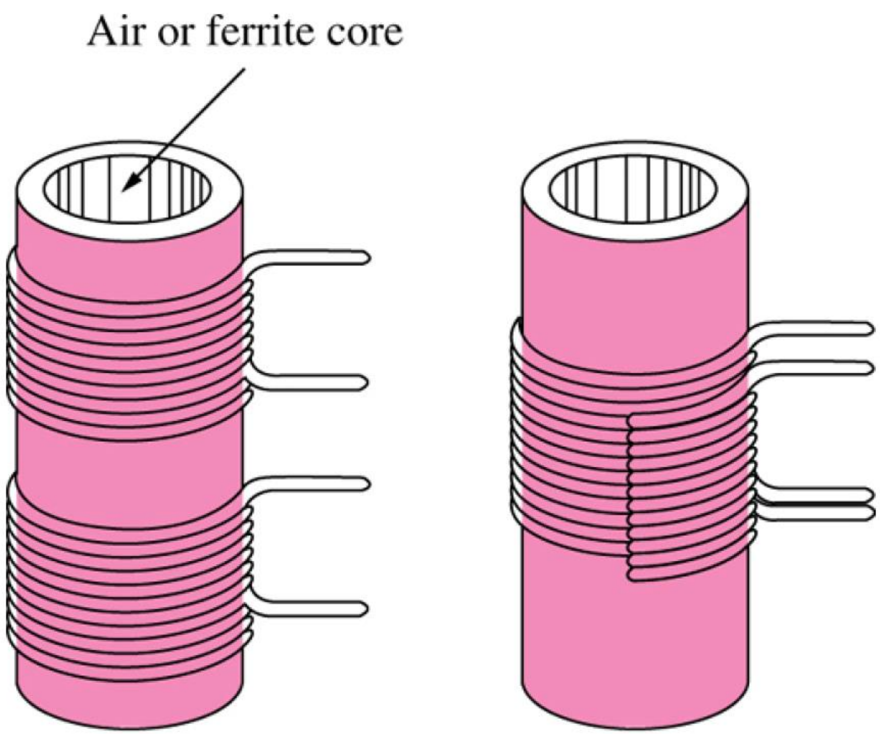
If $k = 1$, then we have 100 percent coupling, i.e., the entire flux produced by one coil links another coil:

$$\phi_{11} = \phi_{22} = 0$$

The coupling coefficient k is a measure of the magnetic coupling between two coils: $0 \leq k \leq 1$

Energy in Coupled Circuits (contd.)

- We expect k to depend on the closeness of the two coils, their core, their orientation, and their windings.

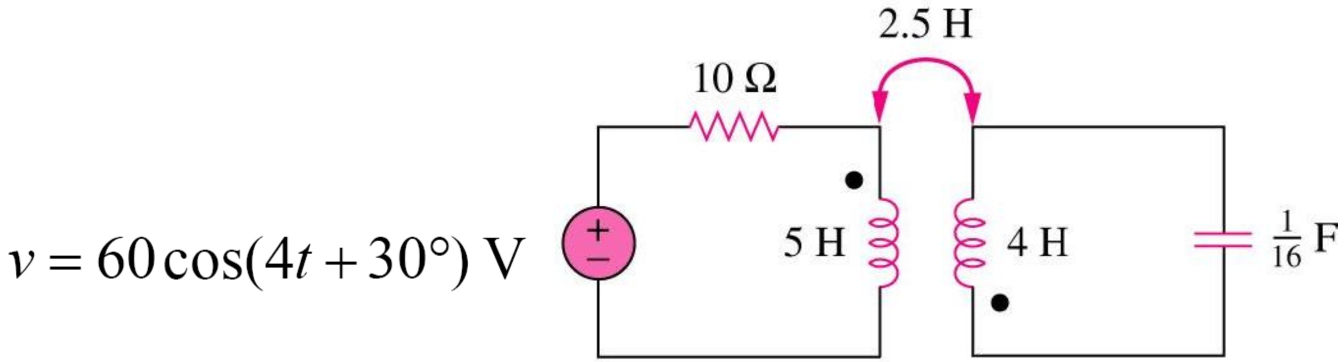


$k < 0.5$ $\quad$ $k > 0.5$

The air-core transformers used in radio frequency circuits are loosely coupled, whereas iron-core transformers used in power systems are tightly coupled.

Example – 5

Determine the coupling coefficient and calculate the energy stored in the coupled inductors at time $t = 1s$.



Example – 6

Given, L_1=40mH, L_2=5mH, and coupling coefficient k=0.6. Find $i_1(t)$ and $v_2(t)$, given that $v_1(t) = 10cos\omega t$ and $i_2(t) = 2sin\omega t$, $\omega = 2000\frac{rad}{s}$.