Lecture – 11

Date: 12.09.2016

- Magnetically Coupled Circuits: Mutual Inductance

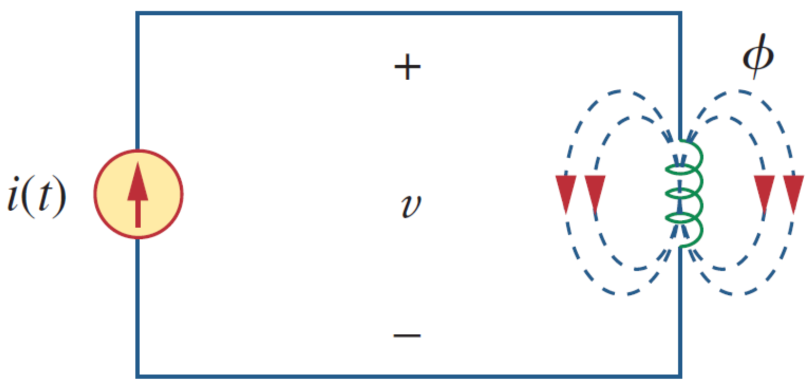
Magnetically Coupled Circuits

- So far, we have considered so far may be regarded as ***conductively coupled*** circuits → as one loop affects the neighboring loop through current conduction.
- When two loops with / without contacts between them affect each other through the magnetic field generated by one of them, they are said to be *magnetically coupled*.
- Transformer is an based on the concept of magnetic coupling → It uses magnetically coupled coils to transfer energy from one circuit to another
- These are used in power systems for stepping up or stepping down ac voltages or currents.
- They are used in electronic circuits such as radio and television receivers for varied purposes such as impedance matching, isolating one part of a circuit from another, and for stepping up or down ac voltages and currents.

Mutual Inductance

When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter. This phenomenon is known as *mutual inductance*.

Lets consider the following:



a single inductor (a coil with *N* turns)

- The flow of current $\boldsymbol{i}$ through the coil leads to the presence of a magnetic flux $\boldsymbol{\varphi}$ around it.
- According to Faraday's law, the voltage $\boldsymbol{v}$ induced in the coil is:

$$v = N\frac{d\varphi}{dt}$$

But the flux is produced by current and hence any change in flux is due to change in the current

Mutual Inductance (contd.)

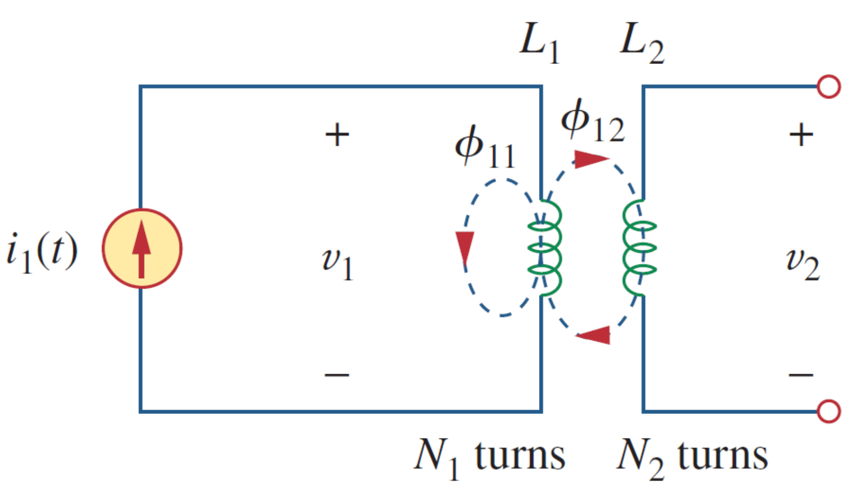
$$v = N\frac{d\varphi}{dt} \Rightarrow v = N\frac{d\varphi}{di}\frac{di}{dt} \Rightarrow v = L\frac{di}{dt}$$

the voltage-current relationship for the inductor

$$L = N\frac{d\varphi}{di}$$

L* *is called the self-inductance, because it relates the voltage induced in a coil by a time-varying current in the same coil

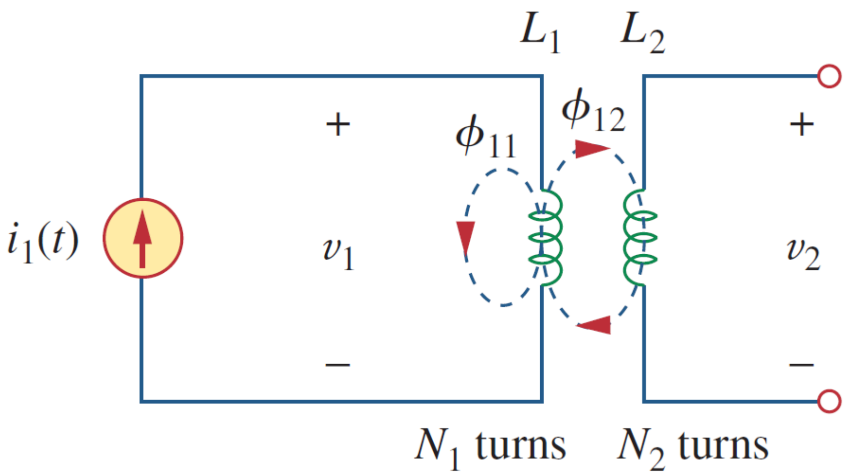
Now consider the following:



- For the sake of simplicity, assume that the second inductor carries no current.
- The magnetic flux φ_1 due to i_1 in coil 1 has two components: φ_{11} links only coil 1 whereas φ_{12} links both coils.

$$\varphi_1 = \varphi_{11} + \varphi_{12}$$

Mutual Inductance (contd.)



- the two coils are physically separated, they are said to be *magnetically coupled.*

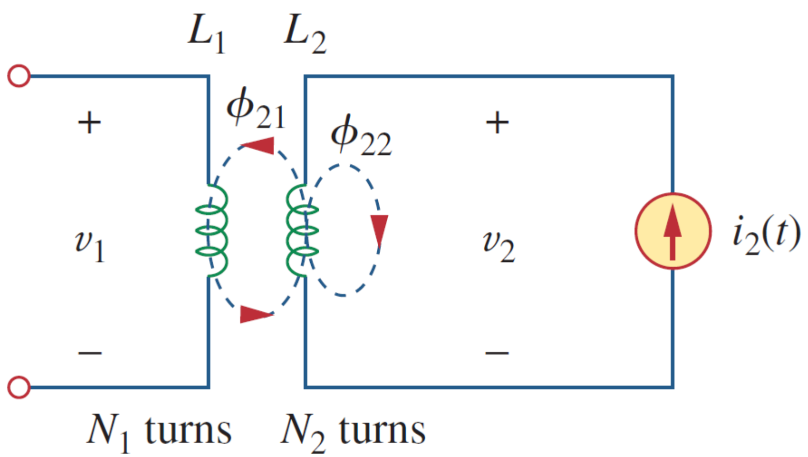
$$v_1 = N_1 \frac{d\varphi_1}{dt} \Rightarrow v_1 = N_1 \frac{d\varphi_1}{di_1}\frac{di_1}{dt}$$

$$v_2 = N_2 \frac{d\varphi_{12}}{dt} \Rightarrow v_2 = N_2 \frac{d\varphi_{12}}{dt}\frac{di_1}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt} \Leftarrow L_1 = N_1 \frac{d\varphi_1}{di_1} \Leftarrow$$ Self Inductance of coil 1

$$v_2 = M_{21} \frac{di_1}{dt} \Leftarrow M_{21} = N_2 \frac{d\varphi_{12}}{di_1} \Leftarrow$$ Mutual Inductance of coil 2

Mutual Inductance (contd.)



- now let current i_2 flow in coil 2, while coil 1 carries no current
- The magnetic flux φ_2 due to i_2 in coil 2 has two components: φ_{22} links only coil 2 whereas φ_{21} links both coils.

$$\varphi_2 = \varphi_{22} + \varphi_{21}$$

$$v_2 = N_2 \frac{d\varphi_2}{dt} = N_2 \frac{d\varphi_2}{di_2}\frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

L2 is Self Inductance of coil 2

$$v_1 = N_1 \frac{d\varphi_{21}}{dt} = N_1 \frac{d\varphi_{21}}{di_2}\frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

Mutual Inductance of coil 1

In general, $M_{21} = M_{12} = M$

Mutual Inductance between the coils

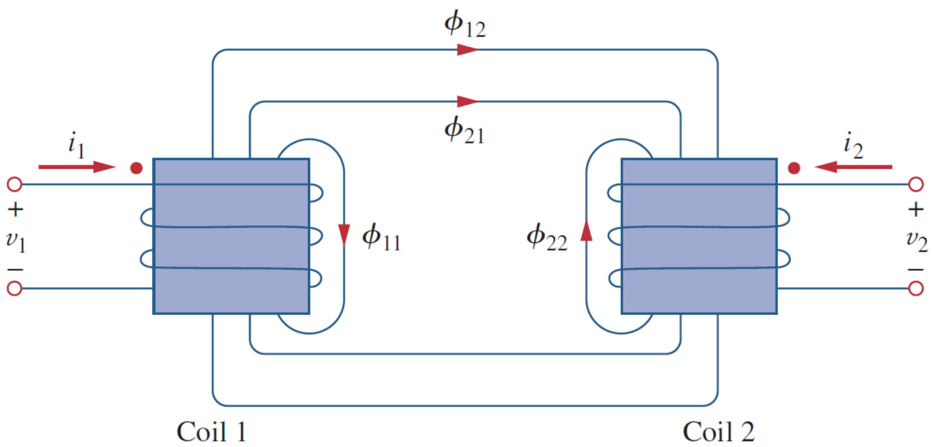
Mutual Inductance (contd.)

Summary: the mutual inductance results if a voltage is induced by a time-varying current in another circuit. It is the property of an inductor to produce a voltage in reaction to a time-varying current in another inductor near it.

Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H).

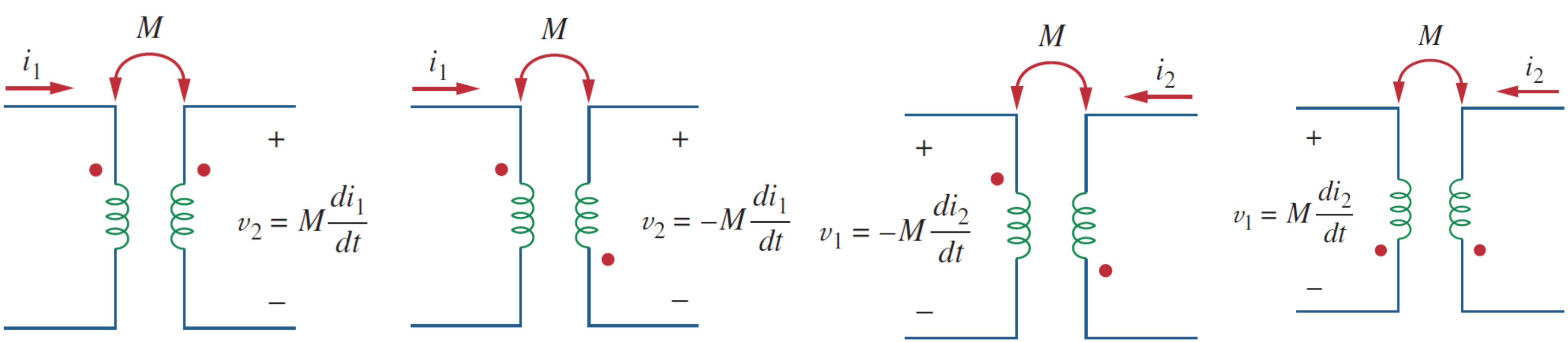
- Mutual inductance *M* is always a positive quantity, the mutual voltage $M\frac{di}{dt}$ may be negative or positive.
- The choice of the correct polarity for $M\frac{di}{dt}$ is made by examining the orientation or particular way in which both coils are physically wound and applying Lenz's law in conjunction with the right-hand rule.
- Since it is inconvenient to show the construction details of coils on a circuit schematic, the *dot convention* in circuit analysis is used. In this, a dot is placed in the circuit at one end of each of the two magnetically coupled coils to indicate the direction of the magnetic flux if current enters that dotted terminal of the coil.

Mutual Inductance (contd.)



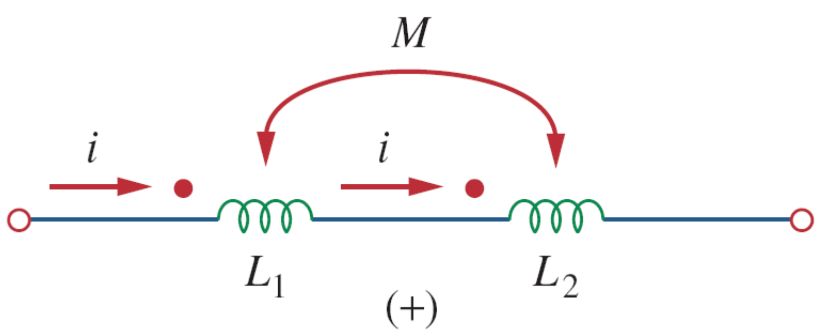
- If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

- If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.

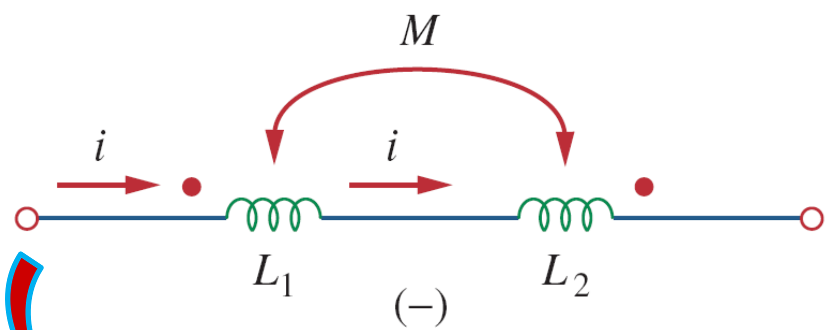


Mutual Inductance (contd.)

coupled coils in series:



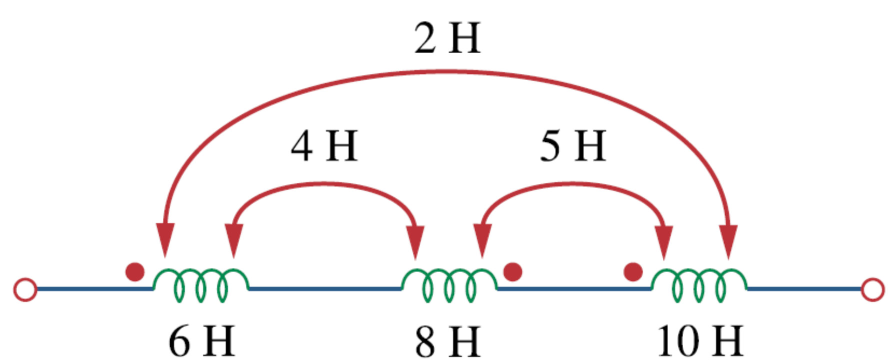
$$L = L_1 + L_2 + 2M$$



$$L = L_1 + L_2 - 2M$$

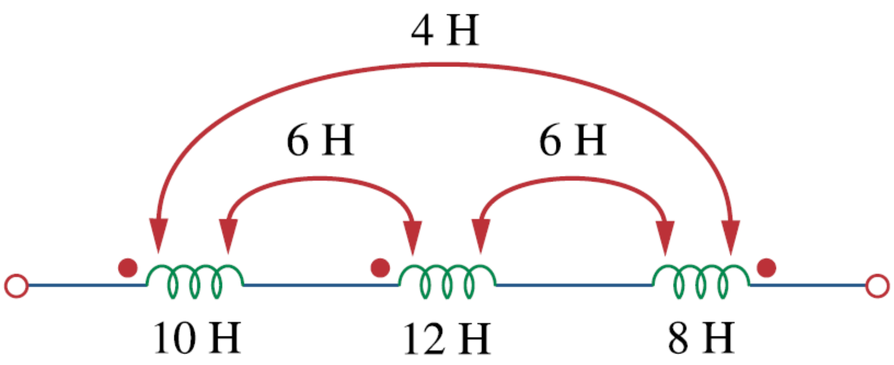
Example – 1

Calculate the total inductance.



Example – 2

Calculate the total inductance.



Example – 3

Two coils connected in series-aiding fashion have a total inductance of 250 mH. When connected in a series-opposing configuration, the coils have a total inductance of 150mH. If the inductance of one coil (L_1) is three times the other, find L_1, L_2, and M.