ECE315 / ECE515

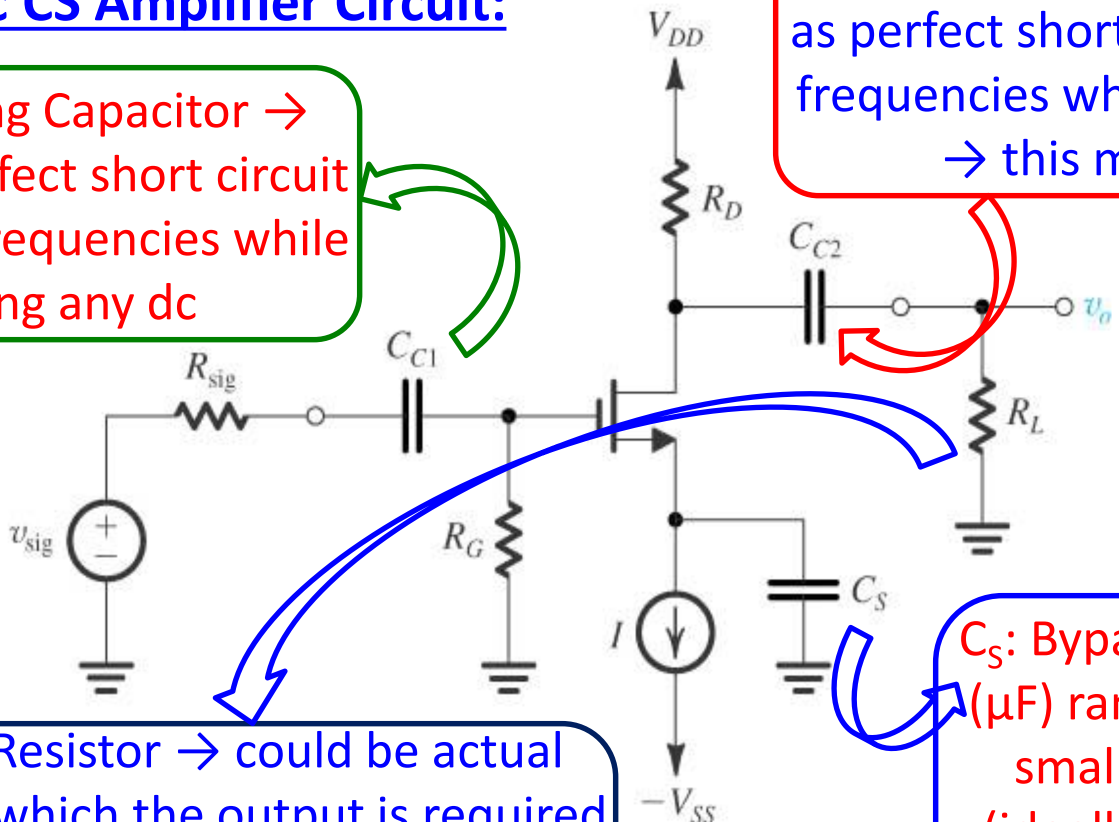
Lecture – 5

Date: 18.08.2016

- Common Source Amplifier
- MOSFET Amplifier Distortion

Example – 1

One Realistic CS Amplifier Circuit:



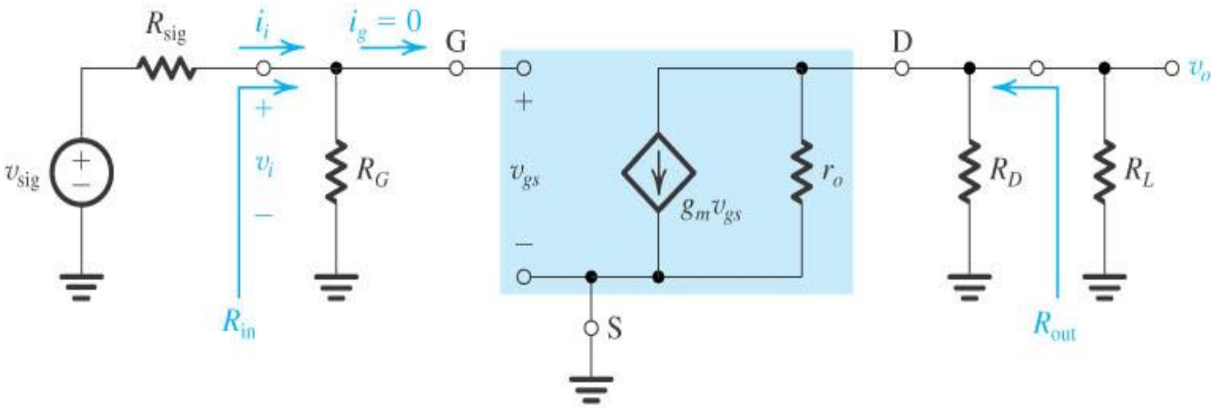
C_{c1}: Coupling Capacitor → serves as perfect short circuit at all signal frequencies while blocking any dc

C_{c2}: Coupling Capacitor → serves as perfect short circuit at all signal frequencies while blocking any dc → this makes $v_o = v_d$

R_L: Load Resistor → could be actual resistor to which the output is required or could be input resistance of another amplifier stage where more than one amplification stage is needed

C_S: Bypass Capacitor → (μF) range → provides small impedances (ideally perfect short circuit) at all signal frequencies

Example – 1 (contd.)



$$i_g = 0 \qquad R_{in} = R_G$$

$$v_i = v_{sig}\frac{R_{in}}{R_{in}+R_{sig}}$$

$$\Rightarrow v_i = v_{sig}\frac{R_G}{R_G+R_{sig}}$$

Usually, R_G is very high (of the order of MΩ) and therefore: $v_i \cong v_{sig}$

Now, $v_{gs} = v_i$

$$\Rightarrow v_o = -g_m v_{gs}\left(r_o \| R_D \| R_L\right)$$

$$\boxed{\therefore A_v = \frac{v_o}{v_{in}} = \frac{v_o}{v_{gs}} = -g_m\left(r_o \| R_D \| R_L\right)}$$

Open Loop Voltage Gain (ie, when there is no feedback loop from o/p to the i/p): $A_{vo} = -g_m\left(r_o \| R_D\right)$

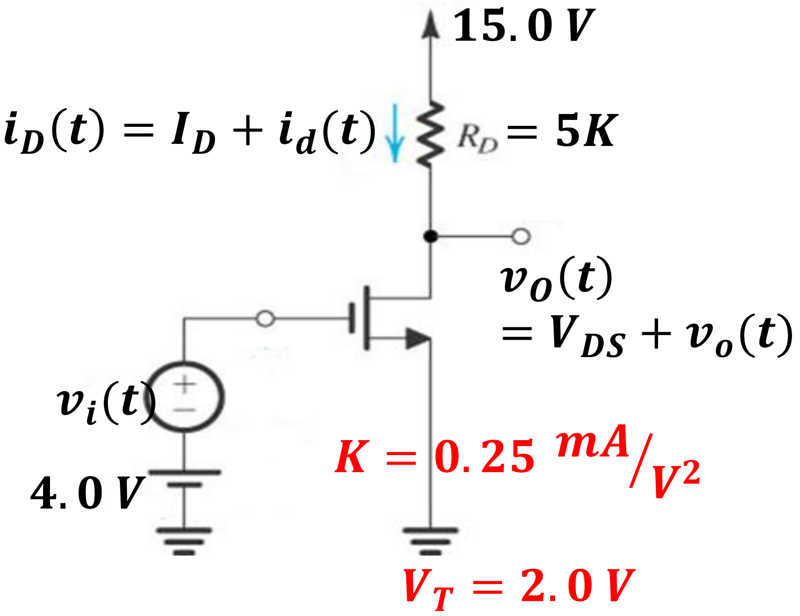
The overall voltage gain from the signal-source to the load is:

$$G_v = -\frac{R_G}{R_G+R_{sig}} g_m\left(r_o \| R_D \| R_L\right)$$

For the determination of R_{out}, the signal v_{sig} has to be set to zero (replace the signal generator with a short circuit) → simple inspection gives:

$$R_{out} = \left(r_o \| R_D\right)$$

MOSFET Amplifier Distortion



Lets look at the last example. You needed to perform a small-signal analysis to determine the small-signal open-circuit **voltage gain** $A_v = {}^{v_o(t)}/_{v_i(t)}$

- We found that the small-signal **voltage gain** is: $A_{vo} = \dfrac{v_o(t)}{v_i(t)} = -5.0$
- Say the **input** voltage to this amplifier is: $v_i(t) = V_i \cos\omega t$

Q: *What is the **largest** value that V_i can take without producing a **distorted** output?*

A: Well, we know that the **small-signal output** is:

$$v_o(t) = A_{vo}\, v_i(t)$$
$$= -5.0\, V_i \cos\omega t$$

BUT, this is **not** the output voltage!

→ The **total** output voltage is the **sum** of the **small-signal** output voltage and the **DC** output voltage!

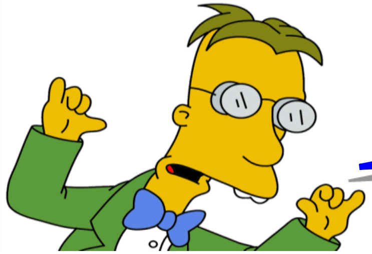
MOSFET Amplifier Distortion (contd.)

- Note for this example, the **DC output** voltage is the **DC drain** voltage, and that its value is:

$$V_O = V_D = 10 \text{ V}$$

  - Thus, the **total output voltage** is :

$$\begin{aligned} v_O(t) &= V_D + v_o(t) \\ &= 10.0 - 5.0 V_i \cos\omega t \end{aligned}$$



It is very important that you realize there is a ***limit*** *on both how high and how low the* ***total*** *output voltage $v_O(t)$ can go.*

That's right! If the ***total*** *output voltage $v_O(t)$ tries to exceed these limits—even for a moment—the MOSFET will leave* ***saturation*** *mode.*

And leaving saturation mode results in ***signal distortion****!*

MOSFET Amplifier Distortion (contd.)

- Let's break the problem down into **two** separate problems:

1) If **total** output voltage $v_O(t)$ becomes too small, the MOSFET will enter the **triode** mode

2) If **total** output voltage $v_O(t)$ becomes too large, the MOSFET will enter the **cutoff** mode

We'll first consider **problem 1.**

- For a MOSFET to remain in saturation, $v_{DS}(t)$ must remain greater than the excess gate voltage $V_{GS} - V_T$ all the time t.

$$v_{DS}(t) > V_{GS} - V_T$$

- Since the source terminal of the MOSFET in **this** circuit is connected to ground, we know that $V_S = 0$. Therefore:

$$v_{DS}(t) = v_D(t) = v_O(t)$$

$$V_{GS} = V_G$$

- And so the MOSFET will remain in saturation **only** if the total output voltage remains **larger** than $V_{GS} - V_T = V_G - V_T$.

$$v_O(t) > V_{GS} - V_T$$

- Thus, we conclude for this amplifier that the output "floor" is:

$$v_{DS}(t) > V_{GS} - V_T$$

MOSFET Amplifier Distortion (contd.)

- Here, $V_{GS} = 4.0\ V$ and $V_T = 2.0\ V$. Therefore: $L_- = V_G - V_t = 4 - 2 = 2.0\ V$
- Thus, to remain in saturation, the **total** output voltage must remain larger than the "floor" voltage at all time t. $v_O(t) > L_- = 2.0\ V$
- Since this **total** voltage is: $v_O(t) = 10.0 - 5.0\ V_i\ cos\omega t$
- we can determine the **maximum** value of small-signal **input** magnitude:

$$10.0 - 5.0 V_i\ cos\omega t > 2.0$$

$$\Rightarrow 8.0 > 5.0 V_i\ cos\omega t$$

$$\Rightarrow V_i\ cos\omega t < 1.6$$

- Since $cos\omega t$ can be as large as 1.0, we find that the **magnitude** of the **input** voltage can be **no larger** than 1.6 V, i.e.,

$$V_i < 1.6\ V$$

If the **input** magnitude exceeds this value, the MOSFET will (momentarily) leave the saturation region and enter the dreaded **triode** mode!

MOSFET Amplifier Distortion (contd.)

Now let's consider **problem 2**

- For the MOSFET to remain in saturation, the **drain** current must be **greater** than zero $(i_D > 0)$. Otherwise, the MOSFET will enter **cutoff** mode.
- Applying **Ohm's Law** to the drain resistor, we find the **drain current** is: $$i_D = \frac{V_{DD} - v_O}{R_C} = \frac{15 - v_O}{5}$$
- it is evident that drain current is **positive** only if: $v_O < 15 \text{ V}$
- In other words, the **upper** limit (i.e., the "ceiling") on the **total** output voltage is: $$L_+ = V_{DD} = 15.0 V$$
- Since this **total** voltage is: $v_O(t) = 10.0 - 5.0 V_i \cos\omega t$
- we can conclude that in order for the MOSFET to remain in **saturation** mode: $$10.0 - 5.0 V_i \cos\omega t > 15.0$$
- Therefore, we find: $$V_i \cos\omega t > \frac{-5.0}{5.0} = -1.0$$
- Since $cos\omega t$ can be as large as 1.0, we find that the **magnitude** of the **input** voltage can be **no larger** than: $$V_i < 1.0 \text{ V}$$

MOSFET Amplifier Distortion (contd.)

If the input magnitude **exceeds** 1.0 V, the MOSFET will (momentarily) leave the saturation and enter the **cutoff** region!

In summary:

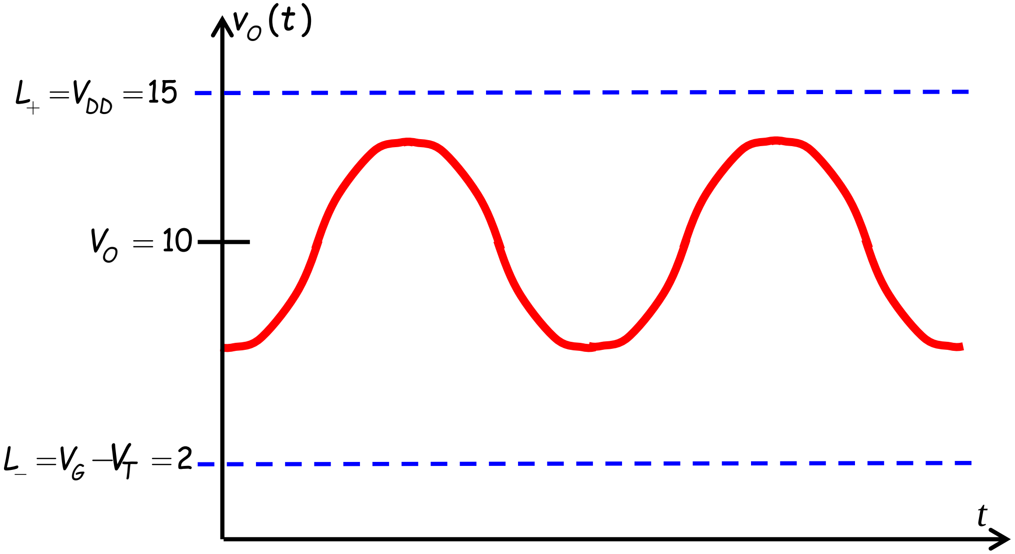
1) If, $V_i > 1.6\ V$, the MOSFET will at times enter **triode**, and **distortion** will occur!

2) If, $V_i > 1.0\ V$, the MOSFET will at times enter **cutoff**, and **even more** distortion will occur!

- To demonstrate this, let's consider **three** examples:

1. $\mathbf{V_i < 1.0\ V}$

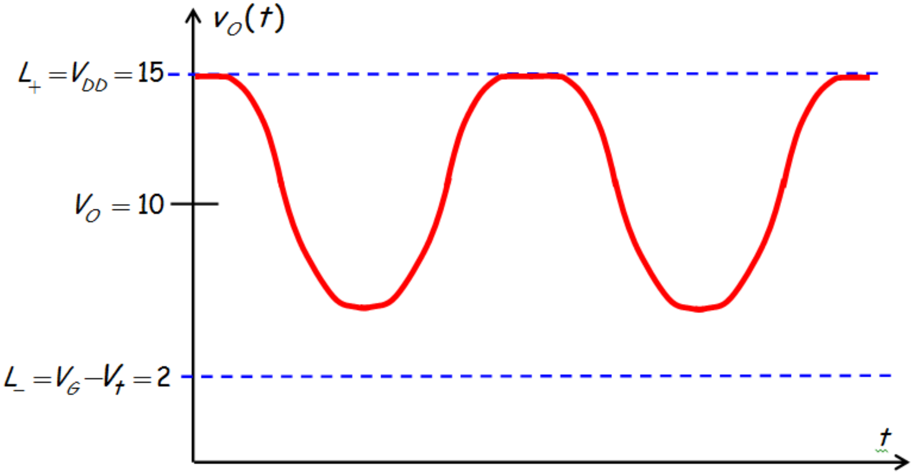
- The output signal in this case remains between $V_{DD} = 15.0V$ and $V_G - V_T = 2.0\ V$ for **all** time t. Therefore, the output signal is **not distorted**.



MOSFET Amplifier Distortion (contd.)

2. $1.6\ \mathrm{V} > \mathrm{V_i} > 1.0\ \mathrm{V}$

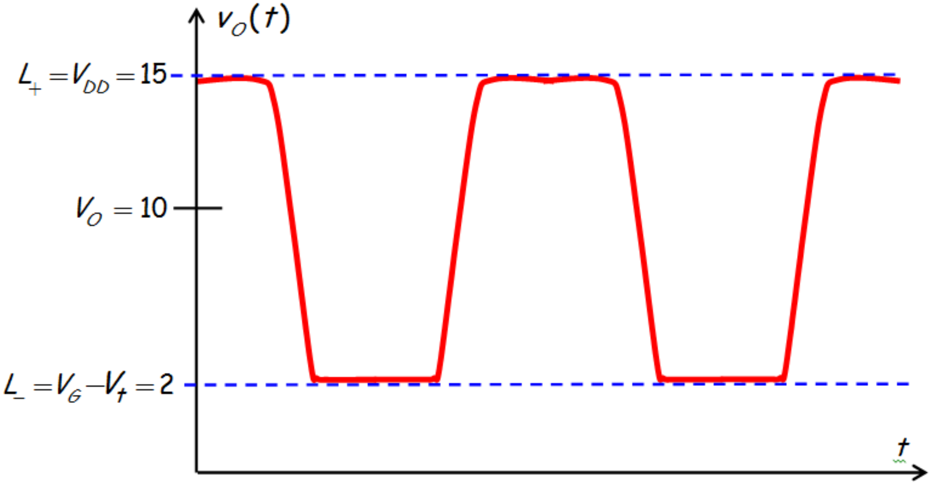
- The output signal in this case remains greater than $L_- = V_G - V_T = 2$ for all time t. However, the small-signal output is now large enough so that the total output voltage at times tries to **exceed** $L_+ = V_{DD} = 15$. For these times, the MOSFET will enter **cutoff**, and the output signal will be **distorted**.



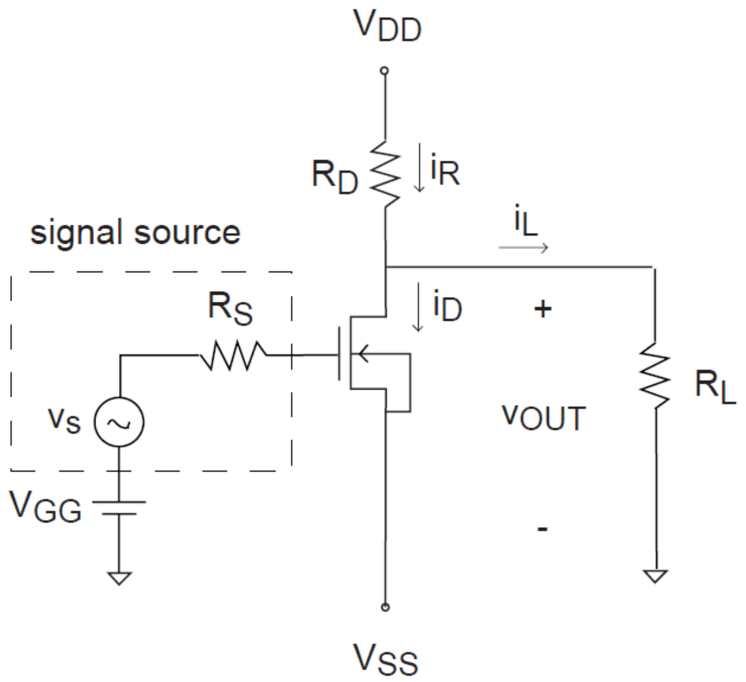
MOSFET Amplifier Distortion (contd.)

3. $\mathbf{V_i > 1.6\ V}$

- In this case, the small-signal input signal is sufficiently **large** so that the total output will attempt to exceed **both** limits (i.e., $V_{DD} = 15.0\ V$ and $V_G - V_T = 2.0\ V$). Therefore, there are periods of time when the MOSFET will be in **cutoff**, and periods when the MOSFET will be in **saturation**.



Effect of Input / Output Loading



Signal swing:

– Upswing limited by resistive divider:

$$v_{out,max} = V_{DD}\frac{R_L}{R_L + R_D}$$

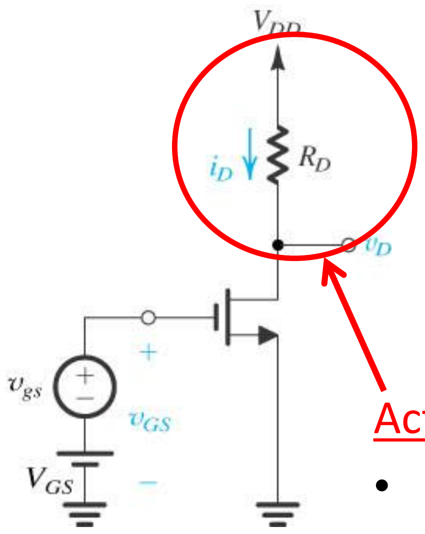
– Downswing not affected by loading

Voltage gain:

- input loading (R_S): no effect because gate does not draw current
- output loading (R_L): It detracts from voltage gain because it draws current.

$$|A_v| = g_m(r_o//R_D//R_L) < g_m(r_o//R_D)$$

Common Source (CS) Amplifier

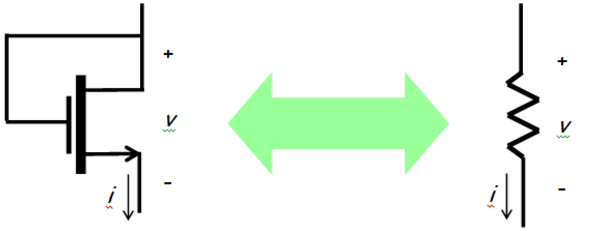


Major Limitations:

- Increase in A_v by increasing the R_D leads to smaller V_D i.e, the voltage V_{DS} → essentially limits the voltage swing
- R_D difficult to fabricate in smaller chip area → a major constraint for ICs
- Problems with the precision of R_D

Active loads overcome these problems

- Diode-connected load (FETs in which drain and gate are tied to work as resistors)
- Current source (such as a FET operating in saturation mode)
- FET operating in triode mode



Enhancement Load

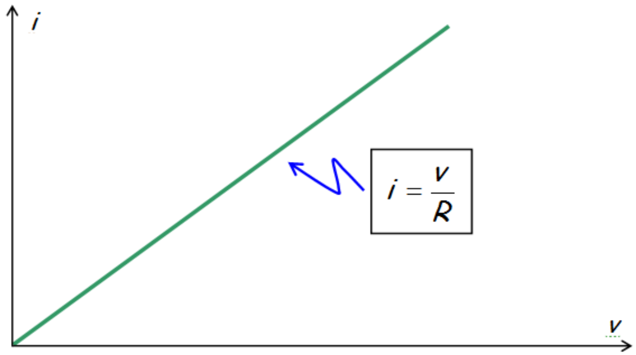
Resistor Load

- We can make a two terminal device from a MOSFET by **connecting** the gate and the drain!

Diode-Connected Load or Enhancement Load

Q: How does this “**enhancement load**” resemble a resistor?

A: For this we need to consider the i-v curves for both.



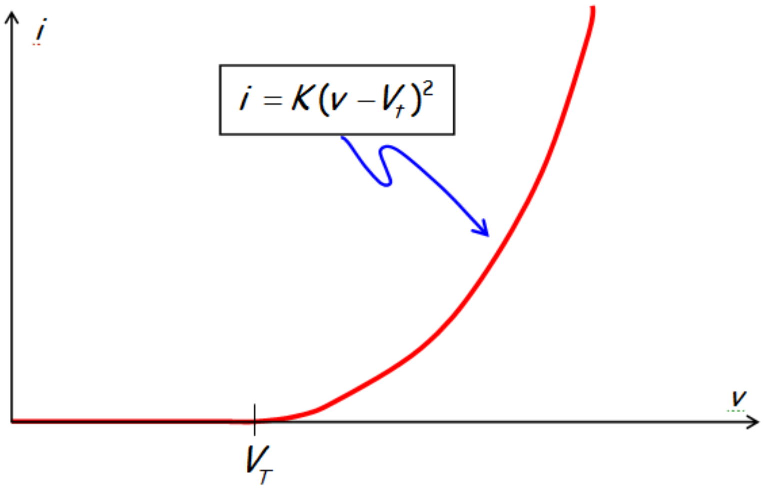
For a Resistor

- Now consider the same curve for an **enhancement load**.
  - Since the gate is tied to the drain, we find $v_G = v_D$, and thus $v_{GS} = v_{DS}$. As a result, we find that $v_{DS} > v_{GS} - V_T$ **always**.
  - Therefore, we find that if $v_{GS} > V_T$, the MOSFET will be in **saturation (**$i_D = K(v_{GS} - V_T)^2$**)**, whereas if $v_{GS} < V_T$, the MOSFET is in **cutoff (**$i_D = 0$**)**.

Diode-Connected Load (contd.)

- Since for enhancement load $i = i_D$ and $v = v_{GS}$, we can describe the enhancement load as:

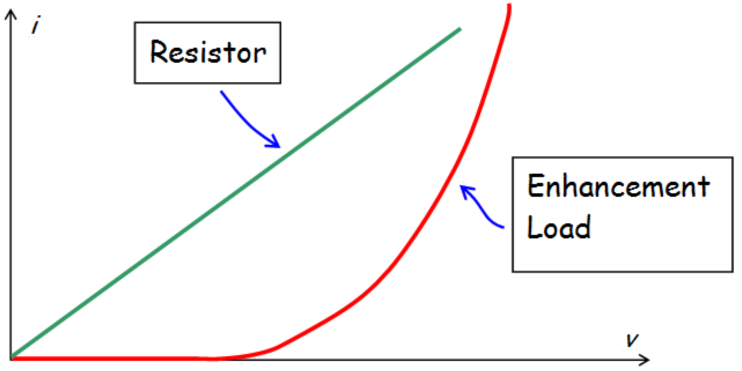
$$i = \begin{cases} 0 & \text{for } v < V_T \\ K\left(v - V_T\right)^2 & \text{for } v > V_T \end{cases}$$



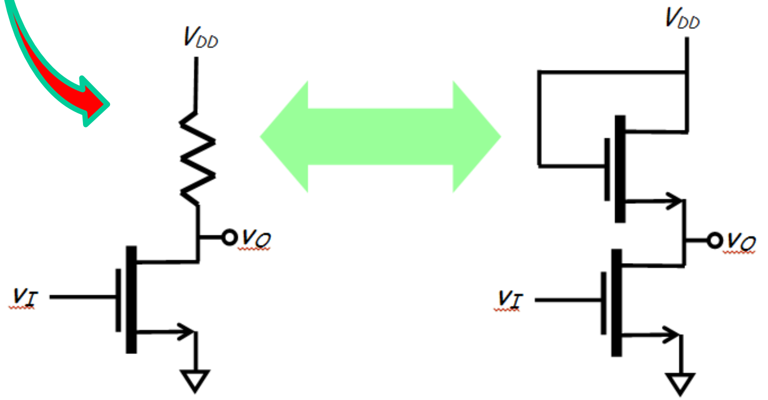
- So, resistors and enhancement loads are far from **exactly** the same, but:

1) They **both** have $i = 0$ when $v = 0$.
2) They **both** have increasing current *i* with increasing voltage *v*.

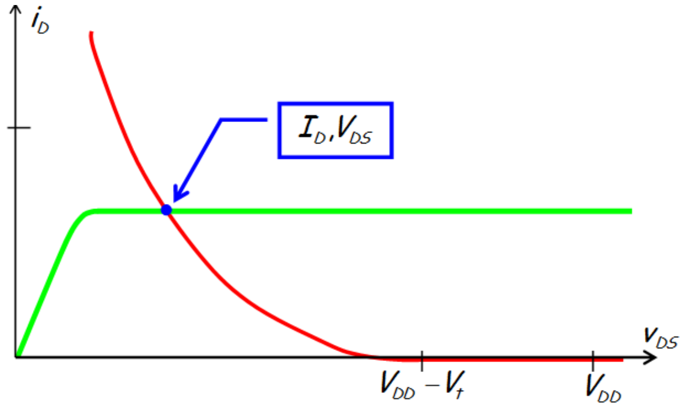
Diode-Connected Load (contd.)



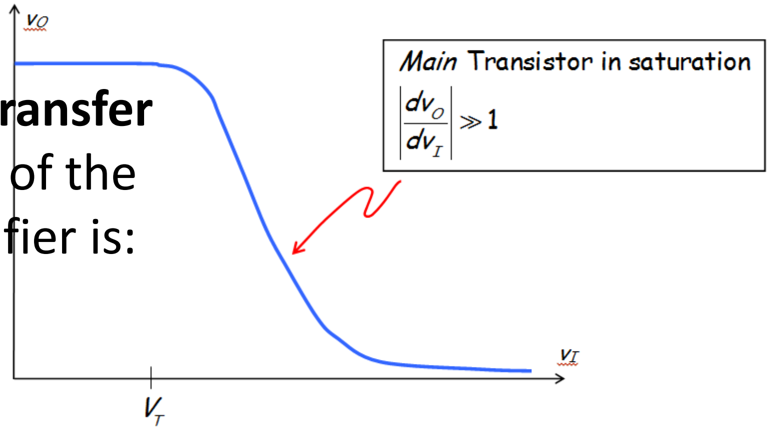
Therefore, we can build a **common source** amplifier with either a resistor, or in the case of an **integrated circuit**, an enhancement load.



For the diode-connected load amplifier, the **load line** is replaced with a **load curve** ($v = V_{DD} - v_{DS}$)!



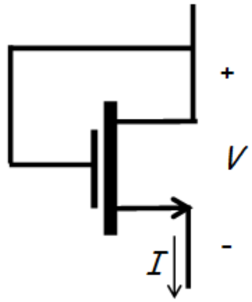
And the **transfer function** of the CS Amplifier is:



Diode-Connected Load - Analysis

Step 1 - DC Analysis

If, $V > V_T$ then $I = K(V - V_T)^2$ or: $V = \sqrt{\frac{I}{K}} + V_T$

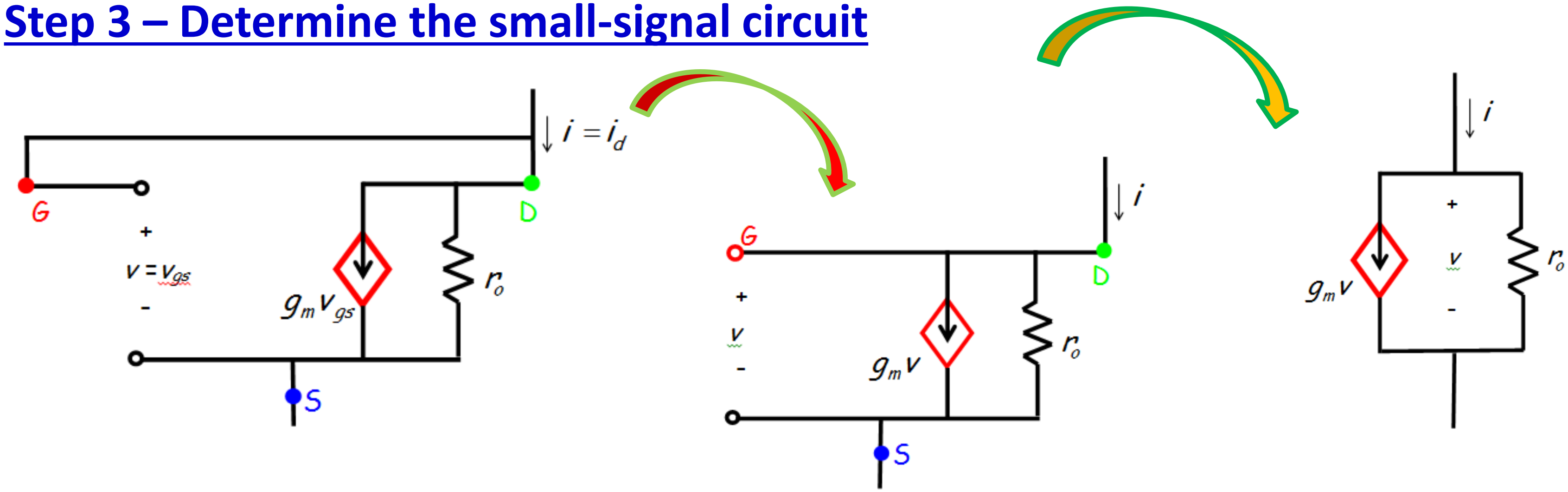


Step 2 – Determine g_m and r_o

$$g_m = 2K(V_{GS} - V_T) = 2K(V - V_T)$$

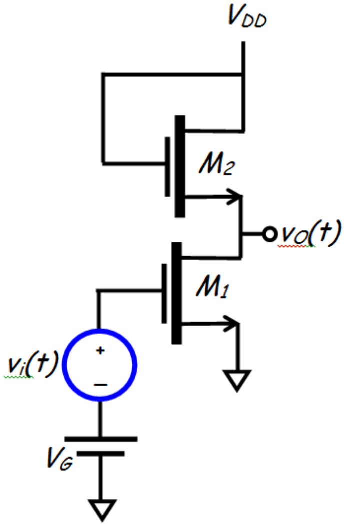
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{\lambda I} = \frac{1}{\lambda K(V - V_T)^2}$$

Step 3 – Determine the small-signal circuit



You need to replace all **enhancement loads** with this small-signal model whenever you are attempting to find the **small-signal circuit** of any MOSFET amplifier.

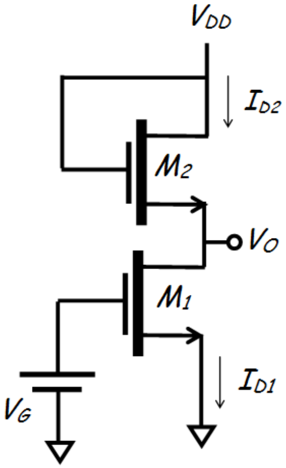
CS Amplifier with Diode-connected load



Q: What is the small-signal open-circuit voltage gain, input resistance, and output resistance of this amplifier?

A: The values that we will determine when we follow precisely the same steps as before!!

Step 1 – *DC Analysis*



Note that:

$$I_{D1} = I_{D2} \doteq I_D$$

and that:

$$V_{GS1} = V_G - 0 = V_G$$

and also that:

$$V_{DS2} = V_{GS2}$$

and finally that:

$$V_{DS1} = V_{DD} - V_{DS2}$$

- Let's **ASSUME** that both M_1 and M_2 are in saturation. Then we **ENFORCE**:

$$I_{D1} = K_1\left(V_{GS1} - V_{T1}\right)^2 = K_1\left(V_G - V_{T1}\right)^2$$

- Continuing with the **ANALYSIS**, we can find the drain current through the enhancement load (I_{D2}),

$$I_{D2} = K_2\left(V_{GS2} - V_{T2}\right)^2$$

CS Amplifier with Diode-connected Load (contd.)

$$I_{D1} = I_{D2}$$

$$K_1\left(V_G - V_{T1}\right)^2 = K_2\left(V_{GS2} - V_{T2}\right)^2 \Longrightarrow V_{GS2} = \sqrt{\frac{K_1}{K_2}}\left(V_G - V_{T1}\right) + V_{T2}$$

- Since $V_{DS2} = V_{GS2}$ and $V_{DS1} = V_{DD} - V_{DS2}$, we can likewise state that:

$$V_{DS2} = \sqrt{\frac{K_1}{K_2}}\left(V_G - V_{T1}\right) + V_{T2} \qquad V_{DS1} = V_{DD} - V_{T2} - \sqrt{\frac{K_1}{K_2}}\left(V_G - V_{T1}\right)$$

Now, we must **CHECK** to see if our assumption is correct.

- The saturation assumption will be correct if: $V_{DS2} = \sqrt{\frac{K_1}{K_2}}\left(V_G - V_{T1}\right) + V_{T2}$ $\quad V_{GS1} > V_{T1} \quad \therefore$ if $\quad V_G > V_{T1}$

$$g_{m1} = 2K_1\left(V_G - V_{T1}\right) \quad \text{and} \quad g_{m2} = 2K_2\left(V_{GS2} - V_{T2}\right)$$

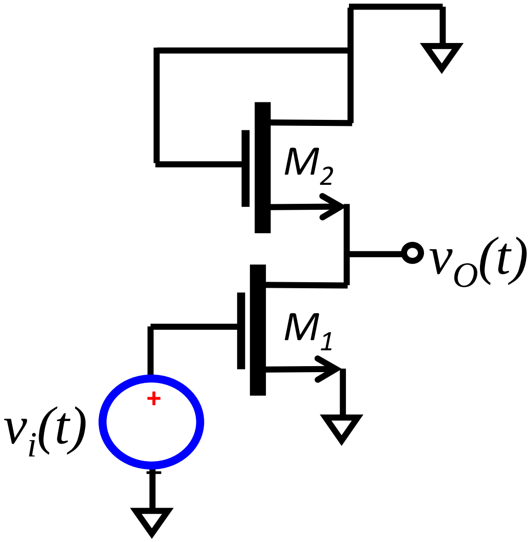
Step 2 – *Calculate small-signal parameters*

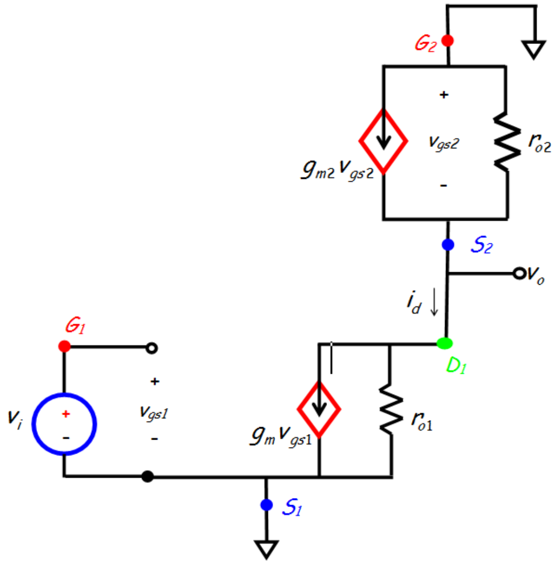
$$r_{o1} = \frac{1}{\lambda_1 I_D} \quad \text{and} \quad r_{o2} = \frac{1}{\lambda_2 I_D}$$

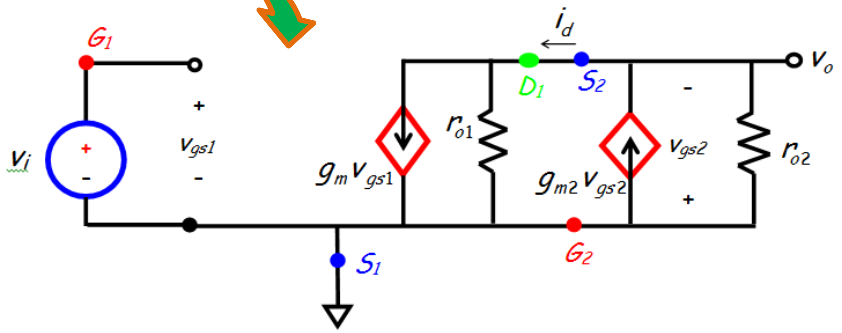
CS Amplifier with Diode-connected Load (contd.)

Step 3 – *Determine the small-signal circuit*

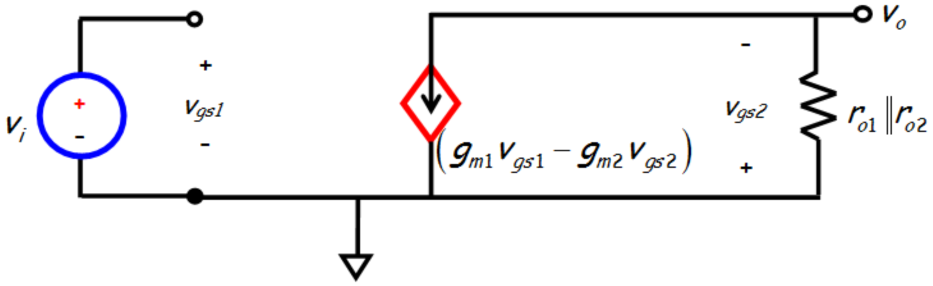
- First, let's turn off the DC sources:
- We now replace **MOSFET** M$_1$ with its equivalent small-signal model, **and** replace the **diode-connected load** with its equivalent small-signal model.







CS Amplifier with Diode-Connected Load (contd.)



- We find that: $v_{gs1} = v_i \qquad v_{gs2} = -v_o$

$$\begin{aligned} v_o &= -(g_{m1}v_{gs1} - g_{m2}v_{gs2})\,(r_{o1}\|r_{o2}) \\ &= -(g_{m1}v_i + g_{m2}v_o)\,(r_{o1}\|r_{o2}) \end{aligned}$$

- Rearranging, we find:

$$A_{vo} = \frac{v_o}{v_i} = \frac{-(r_{o1}\|r_{o2})g_{m1}}{1+(r_{o1}\|r_{o2})g_{m2}} \approx \frac{-g_{m1}}{g_{m2}}$$

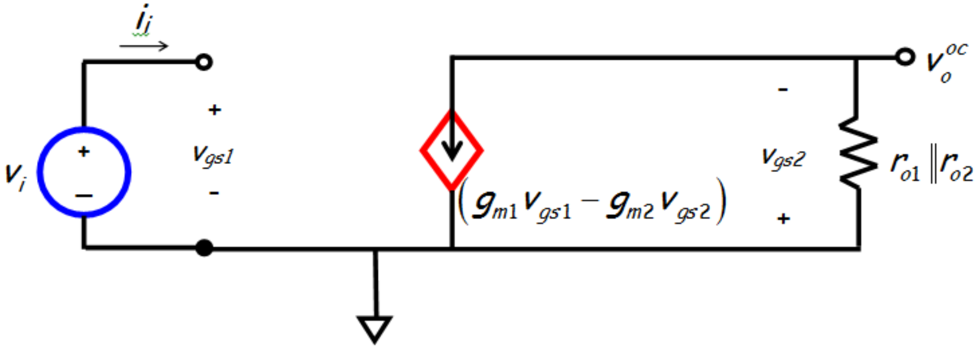
Strong main device and weak load device gives higher gain

- Therefore: $A_{vo} = \frac{-g_{m1}}{g_{m2}} = -\frac{2\sqrt{K_1}\sqrt{I_D}}{2\sqrt{K_2}\sqrt{I_D}} = -\sqrt{\frac{K_1}{K_2}} = -\frac{\sqrt{\left(W/L\right)_1}}{\sqrt{\left(W/L\right)_2}}$

In other words, we adjust the MOSFET channel geometry to set the small-signal gain of this amplifier!

CS Amplifier with Diode-Connected Load (contd.)

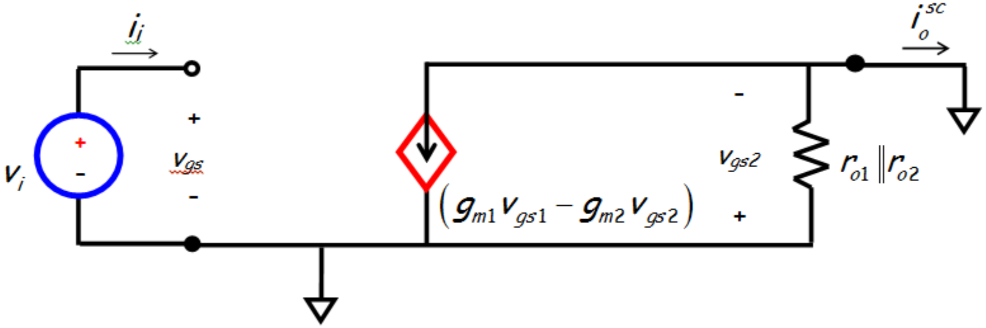
- Now let's determine the small-signal input and output resistances of this amplifier!



- It is evident that: $R_i = \frac{v_i}{i_i} = \infty$

- Now for the output resistance, we know that the open-circuit output voltage is:

$$v_o^{oc} = -(g_{m1}v_{gs1} - g_{m2}v_{gs2})(r_{o1}\|r_{o2})$$



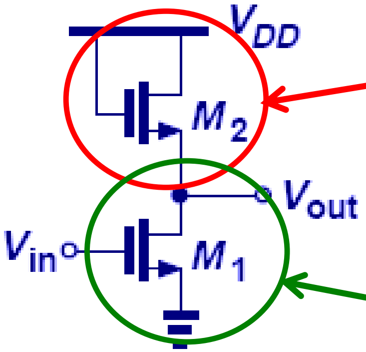
- Likewise, the short-circuit output current is:

$$i_{os} = -(g_{m1}v_{gs1} - g_{m2}v_{gs2})$$

- Thus, the small-signal output resistance of this amplifier is equal to:

$$R_o = \frac{v_o^{oc}}{i_o^{sc}} = \frac{-(g_{m1}v_{gs1} - g_{m2}v_{gs2})(r_{o1}\|r_{o2})}{-(g_{m1}v_{gs1} - g_{m2}v_{gs2})} = (r_{o1}\|r_{o2})$$

CS Amplifier with Diode-Connected Load (contd.)



Diode-connected load ⟹

$$R_D = \frac{1}{g_{m2} + g_{mb2}}$$

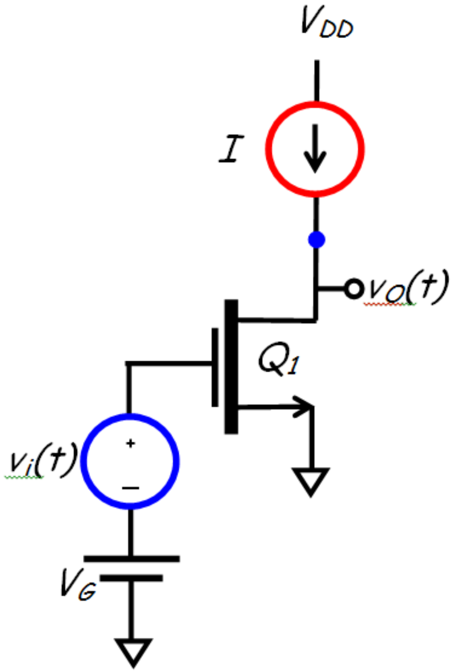
Main device

$$A_v = -g_m R_D = -g_{m1}\frac{1}{g_{m2} + g_{mb2}} \Rightarrow A_v = -\frac{g_{m1}}{g_{m2}}\frac{1}{1+\eta} \Longleftarrow \eta = \frac{g_{mb2}}{g_{m2}}$$

$$\boxed{\therefore A_v = -\frac{\sqrt{2\mu_n C_{ox}(W/L)_1 I_{D1}}}{\sqrt{2\mu_n C_{ox}(W/L)_2 I_{D2}}}\frac{1}{1+\eta}}$$

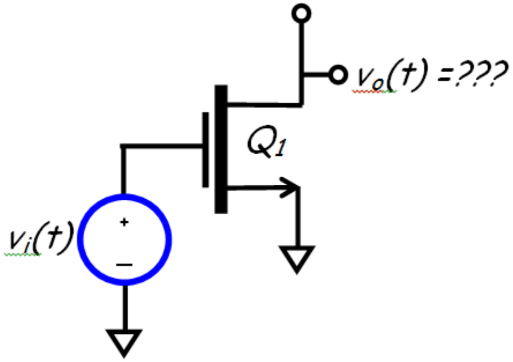
- If variation of η with the output voltage is neglected → the gain is independent of bias currents and voltages
- However, for this to happen the device M_1 has to remain in saturation → this ensures that current I_{D1} is constant → ensures constant g_{m1}
- In other words, the gain remains relatively constant for the variation in input and output signals → ensures that input-output relationship is linear

CS Amplifier with Constant Current Source

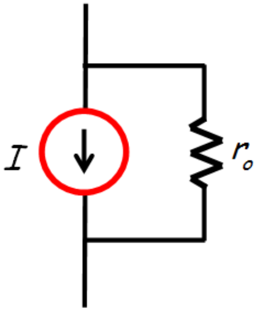


- Now consider this NMOS amplifier using a **current source**.
  - Note **no** resistors or capacitors are present!
  - This is a **common source** amplifier.
  - I_D stability **is not a problem!**

Q: I don't understand! Wouldn't the small-signal circuit be:



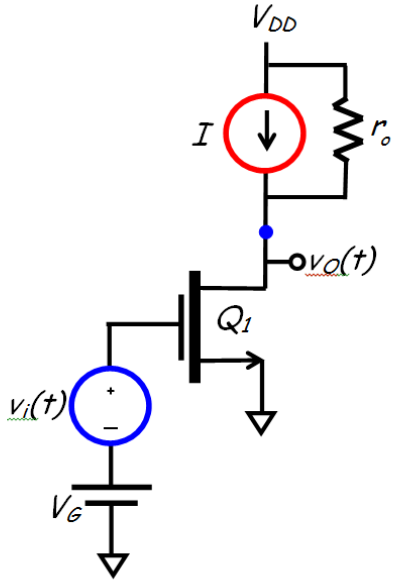
A: Remember, every **real** current source (as with every voltage source) has a **source resistance** r_o . A more **accurate** current source model is therefore:

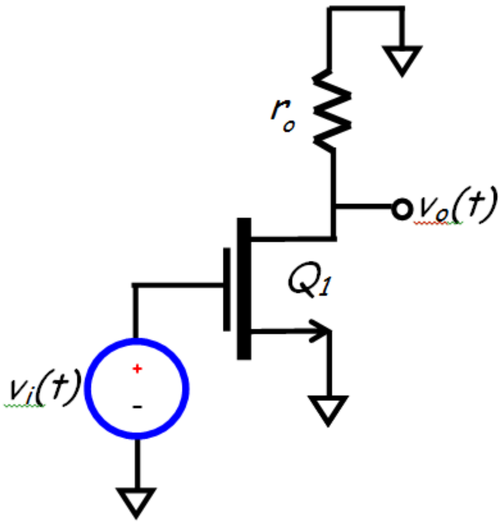


CS Amplifier with Constant Current Source (contd.)

Ideally, $r_o = \infty$. However, for good current sources, this output resistance is large (e.g., $r_o = 100\ k\Omega$). Thus, we mostly **ignore** this value (i.e., approximate it as $r_o = \infty$), but there are some circuits where this resistance makes quite a **difference**. → **This** is one of those circuits!

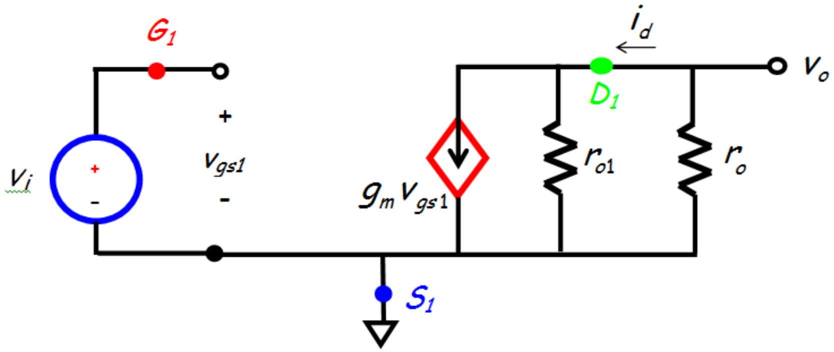
- Therefore, a more **accurate** amplifier circuit schematic is:
- And so the **small-signal circuit** becomes the familiar:





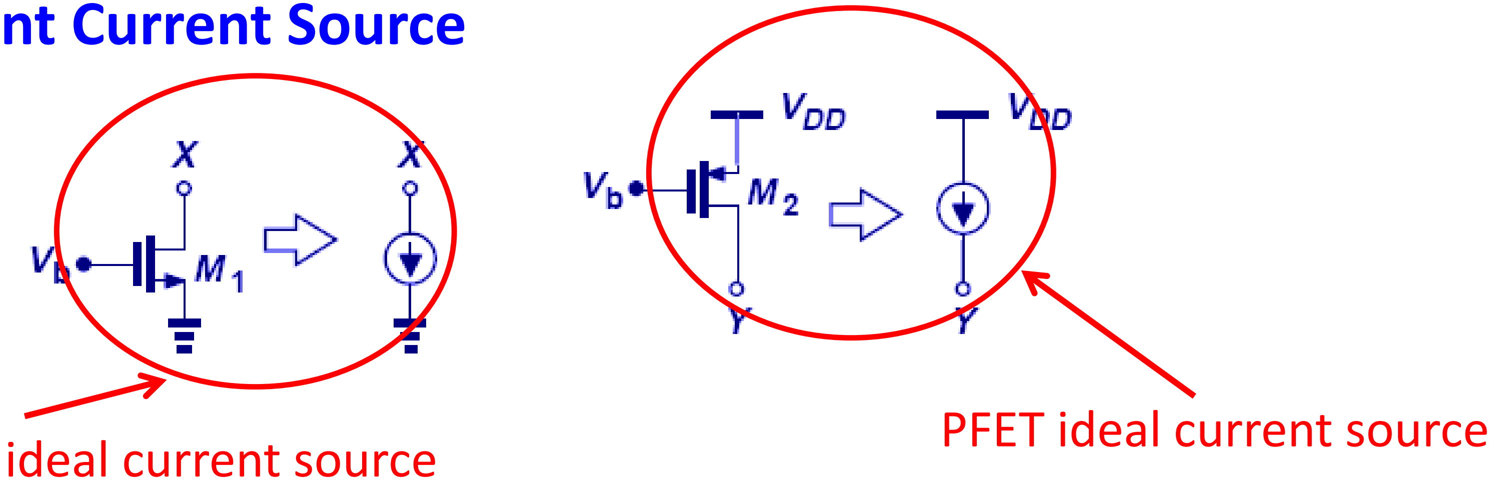
CS Amplifier with Constant Current Source (contd.)

- Therefore, the small signal model is:



<ins>Now go ahead and do the analysis</ins>

Constant Current Source



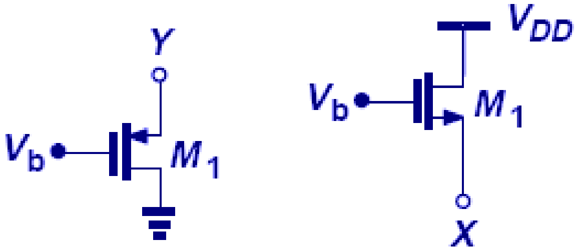
NFET ideal current source

PFET ideal current source

- As long as a MOS transistor is in saturation region and λ=0, the current is independent of the drain voltage and it behaves as an ideal current source seen from the drain terminal.

Constant Current Source (contd.)

Example of poor current source



- Since the variation of the source voltage directly affects the current of a MOS transistor, it does not operate as a good current source if seen from the source terminal