ECE315 / ECE515

Lecture – 3

Date: 08.08.2016

- Steps for DC Analysis of MOSFET Circuits
- MOSFET as a Switch and Amplifier

DC Analysis of MOSFET Circuits

- To analyze MOSFET circuit with D.C. sources, we **must** follow these **five steps**:

1. ASSUME an operating mode
2. ENFORCE the equality conditions of that mode.
3. ANALYZE the circuit with the enforced conditions.
4. CHECK the inequality conditions of the mode for consistency with original assumption. If consistent, the analysis is complete; if inconsistent, go to step 5.
5. MODIFY your original assumption and repeat all steps

1. ASSUME

Here we have **three** choices—cutoff, triode, or saturation. You can make an "**educated guess**" here, but remember, until you CHECK, it's just a guess!

2. ENFORCE

For all three operating regions, we must ENFORCE just **one equality**.

- **Cutoff:** Since **no** channel is induced, we **ENFORCE**: $I_D = 0$
- **Triode:** Since the conducting channel **is** induced but **not** in pinch-off, we **ENFORCE**: $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2\left(V_{GS} - V_T\right)V_{DS} - V_{DS}^2\right]$
- **Saturation:** Since the conducting channel **is** induced but **is** in pinch-off, we **ENFORCE**: $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS} - V_T\right)^2$

3. ANALYZE

The task in D.C. analysis of a MOSFET circuit is to find **one current** and **two voltages**!

a) The gate current I_G is zero $(I_G = 0)$ for MOSFETS in all modes, we need **only** to find the **drain current** $\boldsymbol{I_D}$ - this current value must be **positive** (or zero).

b) We also need to find **two** of the three **voltages** associated with the MOSFET. Typically, these two voltages are V_{GS} and V_{DS}, but given any two voltages, we can find the third using KVL:

$$\boldsymbol{V_{DS} = V_{DG} + V_{GS}}$$

Some hints for MOSFET DC analysis:

1. Gate current $I_G = 0$ **always** !!!
2. Equations sometimes have **two** solutions! Choose solution that is **consistent** with the original ASSUMPTION.

4. CHECK

You do not know if your D.C. analysis is correct unless you CHECK to see if it is consistent with your original assumption!

Q: What exactly do we CHECK?

A: We ENFORCED the mode **equalities**, we CHECK the mode **inequalities.**

We must CHECK **two** separate inequalities after analyzing a MOSFET circuit. Essentially, we check if we have/have not induced a conducting channel, and then we check if we have/have not pinched-off the channel (if it is conducting).

Cutoff

We must only CHECK to see if the MOSFET has a **conducting channel**. If **not**, the MOSFET is indeed in **cutoff**. We therefore CHECK to see if:

$V_{GS} < V_T$ (NMOS) $V_{GS} > V_T$ (PMOS)

Triode

- Here we must first CHECK to see **if** a channel has been induced:

$V_{GS} > V_T$ (NMOS) $V_{GS} < V_T$ (PMOS)

- Likewise, we must CHECK to see if the channel has reached **pinch off**. If **not**, the MOSFET is indeed in the **triode** region. We therefore CHECK to see if:

$V_{DS} < V_{GS} - V_T$ (NMOS) $V_{DS} > V_{GS} - V_T$ (PMOS)

Saturation

- Here we must first CHECK to see **if** a channel has been induced:

$V_{GS} > V_T$ (NMOS) $V_{GS} < V_T$ (PMOS)

- Likewise, we must CHECK to see if the channel has reached **pinch off**. If it **has**, the MOSFET is indeed in the **saturation** region and we need to CHECK:

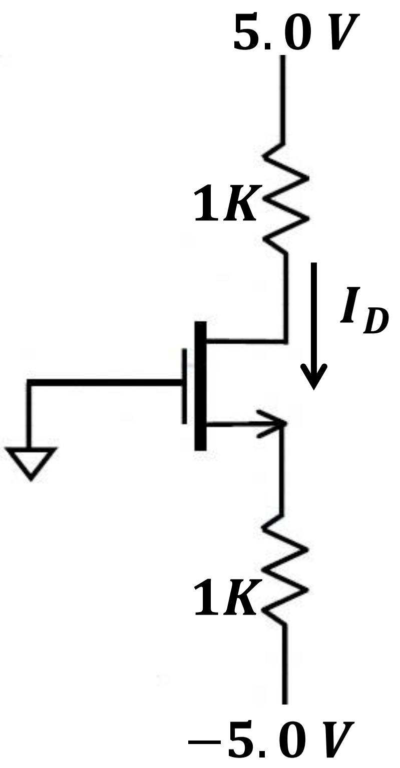
$V_{DS} > V_{GS} - V_T$ (NMOS) $V_{DS} < V_{GS} - V_T$ (PMOS)

- If the results of our analysis are consistent with **each** of these inequalities, then we have made the **correct** assumption! The **numeric** results of our analysis are then likewise **correct**. We can **stop** working!
- However, if **even one** of the results of our analysis is **inconsistent** with our ASSUMPTION, then we have made the **wrong** assumption → move to step 5.

5. MODIFY

- If **one or more** of the circuit MOSFETSs are **not** in their ASSUMED mode, we must change our assumptions and start **completely** over!
- In general, **all** of the results of our previous analysis are incorrect, and thus must be **completely** scraped!

Example – 1



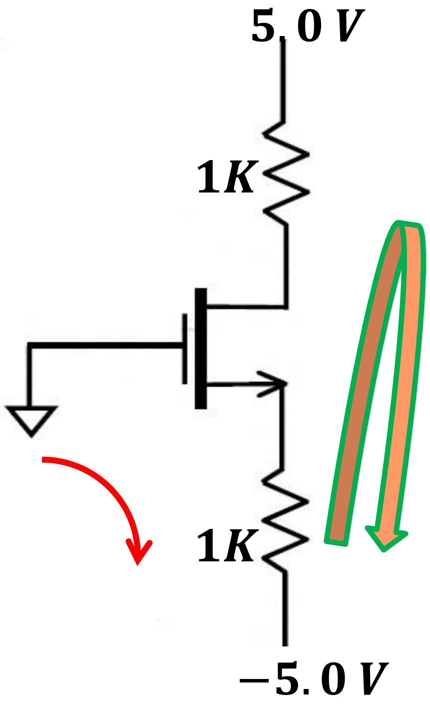
$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right) = K = 0.4mA/V^2 \qquad V_T = 2.0V$$

ASSUME: the NMOS device is in saturation.

- Thus, we must **ENFORCE** the condition that: $$I_D = K\left(V_{GS} - V_T\right)^2$$
- Now we must **ANALYZE** the circuit.

Q: What now? How do we proceed with this analysis?

Example – 1 (contd.)



A: It's certainly not clear. Let's write the circuit equations and see what happens.

- **From the Gate-Source loop KVL:** $0 - V_{GS} - 1 \times 10^3(I_D) = -5.0$
- **Therefore, rearranging:** $I_D = (5.0 - V_{GS})/10^3$

- **And from the Drain-Source loop KVL:**

$$5 - 1 \times 10^3(I_D) - V_{DS} - 1 \times 10^3(I_D) = -5.0$$

- **Therefore, rearranging:** $V_{DS} = 10.0 - 2 \times 10^3(I_D)$

$$I_D = (5.0 - V_{GS})/10^3 \quad = \boldsymbol{K(V_{GS} - V_T)^2}$$

$$0.4mA/V^2\left[V_{GS} - 2\right]^2 = \frac{5 - V_{GS}}{10^3}$$

$$\boldsymbol{V_{GS} = 3.76V}$$

$$\boldsymbol{V_{GS} = -2.26V}$$

Example – 1 (contd.)

- We **assumed** saturation. If the NMOS is in saturation, we know that:

$$V_{GS} > V_T = 2.0V$$

$$\boxed{\therefore \boldsymbol{V_{GS} = 3.76V}}$$

- Inserting this voltage into the Gate-Source KVL equation, we find that the drain current is:

$$I_D = (5.0 - 3.76)/10^3 \longrightarrow \therefore \boldsymbol{I_D = 1.24\,mA}$$

- And using the Drain-Source KVL, we find the remaining voltage:

$$V_{DS} = 10.0 - 2 \times 10^3 \times 1.24 \times 10^{-3} \longrightarrow \therefore \boldsymbol{V_{DS} = 7.52\,V}$$

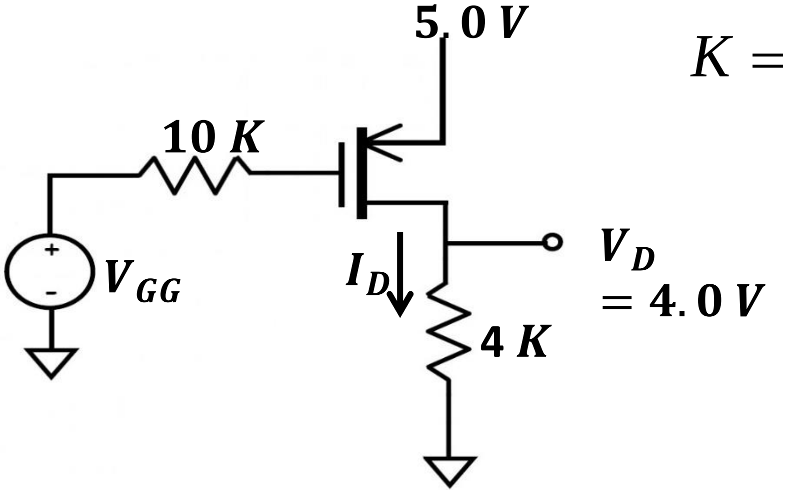
- Even though we have answers (one current and two voltages), we still are not finished, as we now must **CHECK** our solution to see if it is consistent with the saturation mode inequalities.

$$\boldsymbol{3.76 = V_{GS} > V_T = 2.0} \quad ✓$$

$$\boldsymbol{7.52 = V_{DS} > V_{GS} - V_T = 1.76} \quad ✓$$

Both answers are consistent! Our solutions are correct!

Example – 2



$$K = 0.2mA/V^2 \qquad V_T = -2.0V$$

Find V_{GG}

- let's **ASSUME** that the PMOS is in **saturation** mode.
- Therefore, we **ENFORCE** the **saturation** drain current equation: $I_D = K\left(V_{GS} - V_T\right)^2$

Q: Yikes! Where do we start ?

A: The best way to start is by "**picking the low-hanging fruit**". In other words, determine the **obvious** and easy values. **Don't** ask, "What is $\mathbf{V_{GG}}$?", but **instead** ask, "What **do** I know?" !

$I_G = 0.0\ mA \qquad V_S = 5.0\ V$

$$I_D = \frac{V_D - 0.0}{4\times 10^3} = \frac{4}{4\times 10^3} = 1mA$$

$$V_G = V_{GG} - 10\times 10^3 \times I_G = V_{GG}$$

$$V_{DS} = V_D - V_S = 4 - 5 = -1\text{V}$$

$$V_{GS} = V_G - V_S = V_{GG} - 5 \quad \text{V}$$

Example – 2 (contd.)

- We can now relate these values using PMOS **drain current equation**.

$$I_D = K\left(V_{GS} - V_T\right)^2 \quad \Rightarrow \quad 1\times10^{-3} = 0.2\times10^{-3}\times\left(V_{GS} - (-2.0)\right)^2$$

$$\boxed{V_{GS} = 0.24\ \text{V} \qquad V_{GS} = -4.23\ \text{V}}$$

- For this example, we have **ASSUMED** that the PMOS device is in **saturation.** Therefore, the gate-to-source voltage must be **less** (remember, it's a **PMOS** device!) than the **threshold voltage**:

$$V_{GS} < V_T$$

$$\therefore V_{GS} = -4.23\ V$$

Q: Does this mean our saturation **ASSUMPTION** is **correct**?

A: NO! It merely means that our saturation ASSUMPTION **might** be correct! We need to CHECK the other inequalities to know for **sure**.

- Now, returning to our circuit **analysis**, we can quickly determine the **unknown** value of $\mathbf{V_{GG}}$. Recall that we **earlier** determined that:

$$V_{GS} = V_G - V_S = V_{GG} - 5 \quad \Rightarrow \quad -4.23 = V_{GG} - 5 \quad \Rightarrow \quad \therefore V_{GG} = 0.77\ \text{V}$$

Example – 2 (contd.)

This solution ($V_{GG} = 0.77V$) is of course true **only if** our original ASSUMPTION was correct. Thus, we must CHECK to see if our **inequalities** are valid.

- We of course already know that the **first** inequality is true—a p-type channel is induced:

$$\boldsymbol{V_{GS} = -4.23V < V_T = -2.0V}$$ ✓

- However:

$$V_{DS} = -1.0V \not< V_{GS} - V_T = -2.23V$$ ✗

shows us that our ASSUMPTION was **incorrect**!

→ Time to make a **new ASSUMPTION** and **start over**!

- let's **now** ASSUME the PMOS device is in **triode** region. Then ENFORCE:

$$I_D = K\left[2\left(V_{GS} - V_T\right)V_{DS} - V_{DS}^2\right]$$

- Note that most of our **original** analysis was **independent** of our PMOS mode ASSUMPTION. Thus, we **again** conclude that:

$$I_G = 0.0\,mA \qquad V_S = 5.0\,V \qquad I_D = \frac{V_D - 0.0}{4\times10^3} = \frac{4}{4\times10^3} = 1mA$$

Example – 2 (contd.)

$$I_D = \frac{V_D - 0.0}{4\times10^3} = \frac{4}{4\times10^3} = 1mA \qquad V_G = V_{GG} - 10\times10^3 \times I_G = V_{GG}$$

$$V_{DS} = V_D - V_S = 4 - 5 = -1 \text{ V} \qquad V_{GS} = V_G - V_S = V_{GG} - 5 \quad \text{V}$$

- Now, inserting these values in the **triode drain current equation**:

$$1\times10^{-3} = 0.2\times10^{-3}\times\left[2\left(V_{GS} - (-2.0)\right)(-1.0) - (-1.0)^2\right]$$

- Solving for V_{GS} we find: $\boxed{V_{GS} = -5.0V}$

- Therefore: $\boxed{V_{GG} = V_{GS} + 5.0 = -5.0 + 5.0 = 0}$

The voltage source $\boldsymbol{V_{GG}}$ is equal to **zero**—provided that our triode ASSUMPTION is **correct**.

Example – 2 (contd.)

- First, we CHECK to see if a channel has indeed been **induced.**

$$\boldsymbol{V_{GS} = -5.0V < V_T = -2.0V}$$ ✓

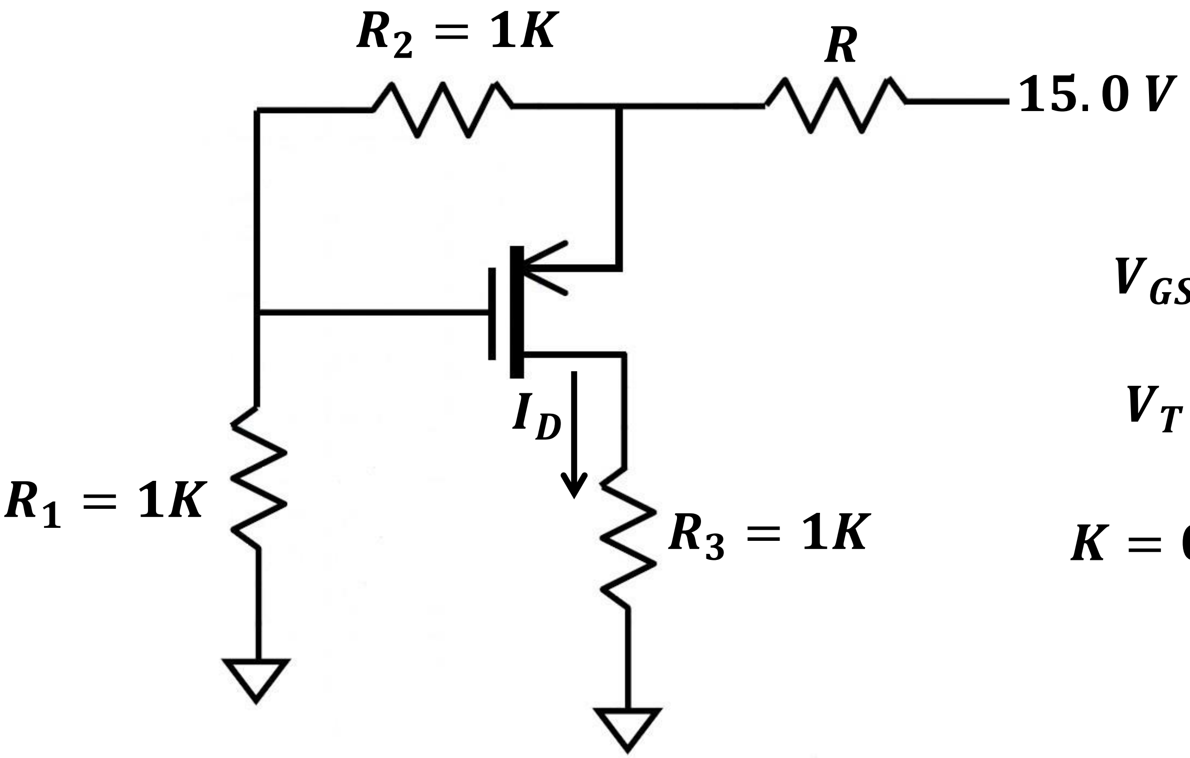
- Next, we CHECK to make sure that our channel is **not** in pinch off.

$$V_{DS} = -1.0V > V_{GS} - V_T = -3.0V$$ ✓

Our **triode** ASSUMPTION is **correct**! Thus, the voltage source $\boldsymbol{V_{GG} = 0.0\,V}$.

Example – 3

- Consider the **PMOS** circuit below, find the value of unknown of resistor R.



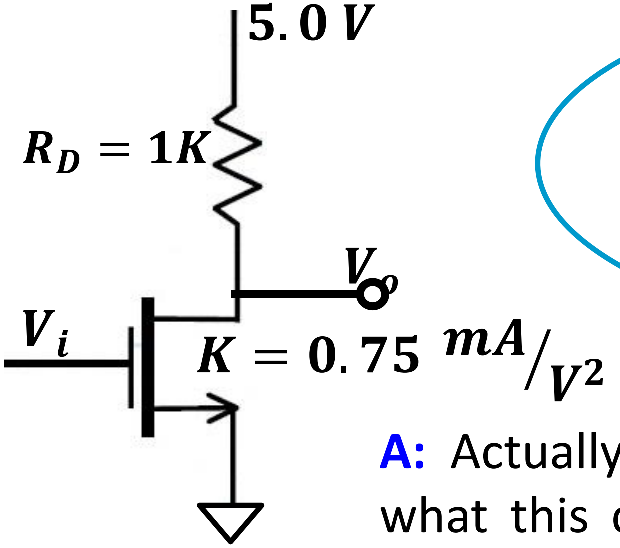
$V_{GS} = -4.0\,V$

$V_T = -2.0\,V$

$K = 0.75\,{}^{mA}/_{V^2}$

The MOSFET as a Switch and an Amplifier

- Consider this **simple** MOSFET circuit:



Q: Oh, goody—you're going to **waste** my time with another of these **pointless** academic problems. Why can't you discuss a circuit that actually **does** something?

A: Actually, this circuit **is** a fundamental electronic device! To see what this circuit does, we need to determine its **transfer function** $V_o = f(V_i)$.

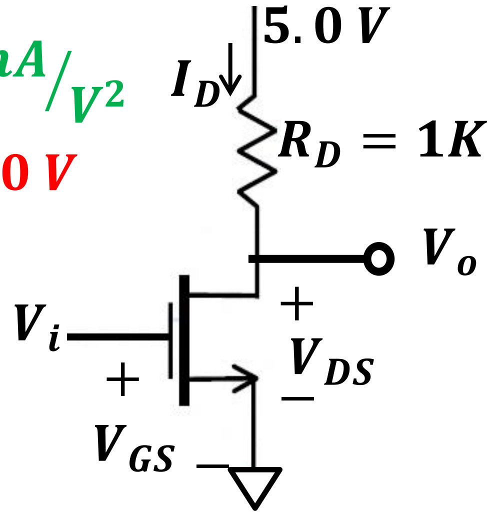
Q: Transfer function! How can we determine the transfer function of a MOSFET circuit!?

A: Same as junction diodes—determine the output V_o for each device mode, and then determine **when** (i.e., for what values of V_i) the device is in that mode!

The MOSFET as a Switch and an Amplifier (contd.)

$K = 0.75\ {}^{mA}/_{V^2}$

$V_T = 1.0\ V$



- First, note that **regardless** of the MOSFET mode:

$$V_{GS} = V_i - 0.0 = V_i \qquad V_{DS} = V_o - 0.0 = V_o$$

- From KVL, we can likewise conclude that:

$$V_{DS} = V_o = 5.0 - I_D R_D$$

- Now let's ASSUME that the MOSFET is in **cutoff**, thus ENFORCING $I_D = 0$.

$$V_{DS} = V_o = 5.0 - I_D R_D \Rightarrow V_o = 5.0 - 0 \times (1 \times 10^3) \Rightarrow \therefore V_o = 5.0\ \text{V}$$

- Now, we know that MOSFET is in cutoff **when**: $V_{GS} = V_i < V_T = 1.0\ V$

- Thus, we conclude that: $V_o = 5.0\ \text{V}$ **when** $V_i < 1.0\ V$

The MOSFET as a Switch and an Amplifier (contd.)

- Now, let’s ASSUME that the MOSFET is in **saturation**, thus ENFORCE:

$$I_D = K(V_{GS} - V_T)^2 \Longrightarrow I_D = K(V_i - 1.0)^2$$

- And thus the output voltage is:

$$V_o = 5.0 - I_D R_D = 5.0 - 0.75 \times 10^{-3} \times (V_i - 1.0)^2 \times 1 \times 10^3$$

$$\therefore V_o = 5.0 - 0.75 \times (V_i - 1.0)^2$$

- We know that MOSFET is in saturation **when**:

$$V_{GS} = V_i > V_T = 1.0\ V \qquad \underline{\text{and}} \qquad V_{DS} = V_o > V_{GS} - V_T = V_i - 1.0$$

- The second inequality means:

$$V_o > V_i - 1.0 \Longrightarrow 5.0 - 0.75 \times (V_i - 1.0)^2 > V_i - 1.0$$

- Solving this quadratic, we find that the **only** consistent solution is:

$$V_i - 1.0 < 2.0 \Longrightarrow V_i < 3.0$$

- Thus we conclude that the MOSFET in saturation:

$$V_o = 5.0 - 0.75 \times (V_i - 1.0)^2 \qquad \underline{\text{when}} \qquad 1.0 < V_i < 3.0\ V$$

The MOSFET as a Switch and an Amplifier (contd.)

- Finally, let's ASSUME that the MOSFET is in **triode** mode, thus we ENFORCE:

$$I_D = K\left[2(V_{GS} - V_T)V_{DS} - (V_{DS})^2\right]$$

- And thus the output voltage is:

$$V_o = 5.0 - I_D R_D = 5.0 - 0.75 \times \left[2(V_i - 1.0)V_o - (V_o)^2\right]$$

- Rearranging the equation, we get the quadratic form:

$$0.75(V_o)^2 - (1.5V_i - 0.5)V_o + 5.0 = 0$$

- The solutions of which are:

$$V_o = \frac{(1.5V_i - 0.5) \pm \sqrt{(1.5V_i - 0.5)^2 - 15.0}}{1.5}$$

- Note due to ±, there are **two** possible solutions. However, for triode region, the MOSFET must not be in pinchoff, i.e.:

$$V_{DS} = V_o < V_{GS} - V_T = V_i - 1.0$$

- This is satisfied with the **smaller** of the two solutions (i.e., the solution with the minus sign!):

$$V_o = \frac{(1.5V_i - 0.5) - \sqrt{(1.5V_i - 0.5)^2 - 15.0}}{1.5}$$

The MOSFET as a Switch and an Amplifier (contd.)

$$V_o = \frac{(1.5V_i - 0.5) - \sqrt{(1.5V_i - 0.5)^2 - 15.0}}{1.5}$$

This provides us with the output voltage **if** the MOSFET is in triode. The question remaining is thus **when** (i.e., for what values of V_i) is the MOSFET in triode mode?

We could do a lot more math to find this answer, but this answer is actually quite **obvious**!

- Recall that we have already determined that:

a) The MOSFET is in cutoff **when** $\boldsymbol{V_i < 1.0V}$.

b) The MOSFET is in saturation **when** $\boldsymbol{1.0V < V_i < 3.0V}$.

- Since there are only **three** modes of a MOSFET device, and since the transfer **function** must—well—be a **function**, we can conclude (correctly) that the MOSFET will be in triode region when $\boldsymbol{V_i}$ is the value of the **only region that is left**: $\boldsymbol{V_i > 3.0\ V}$.

The MOSFET as a Switch and an Amplifier (contd.)

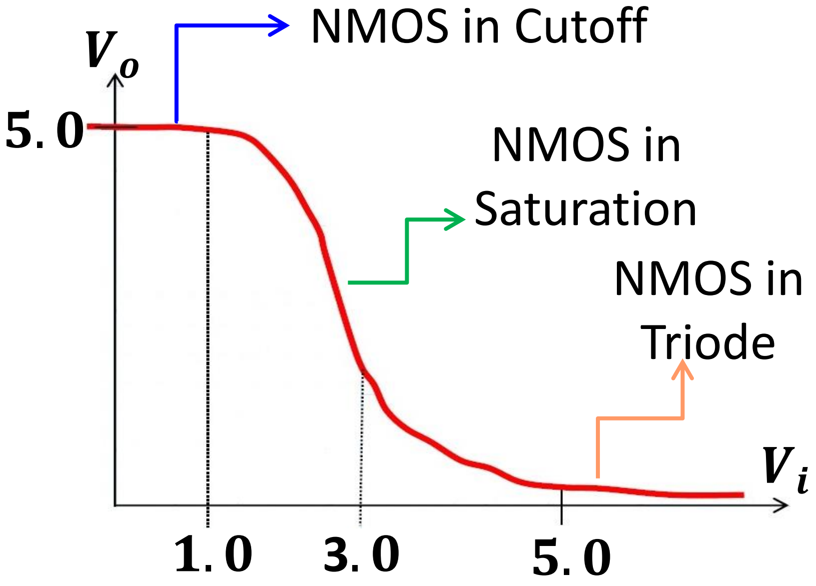
- Thus we can conclude that:

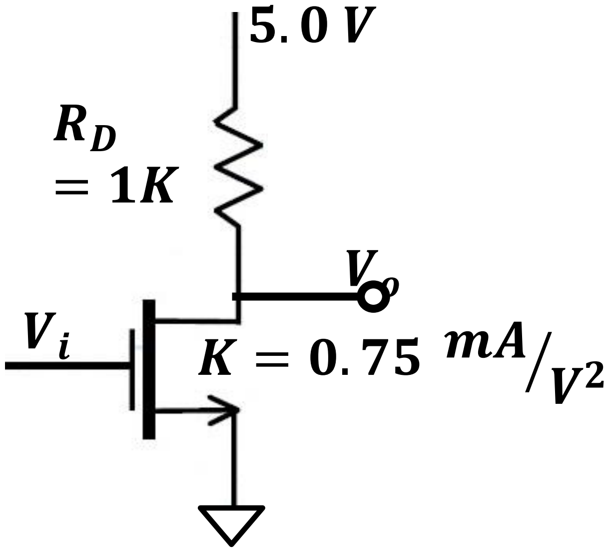
$$\boxed{V_o = \frac{\left(1.5V_i - 0.5\right) - \sqrt{\left(1.5V_i - 0.5\right)^2 - 15.0}}{1.5}} \qquad \underline{\textbf{when}} \quad V_i > 3.0\ V$$

- We now have determined the complete, continuous **transfer function** of this circuit!

$$V_o = \begin{cases} 5.0\ V & \underline{\textbf{when}} \quad V_i < 1.0\ V \\ 5.0 - 0.75 \times (V_i - 1.0)^2 & \underline{\textbf{when}} \quad 1.0 < V_i < 3.0\ V \\ \boxed{V_o = \dfrac{\left(1.5V_i - 0.5\right) - \sqrt{\left(1.5V_i - 0.5\right)^2 - 15.0}}{1.5}} & \underline{\textbf{when}} \quad V_i > 3.0\ V \end{cases}$$

The MOSFET as a Switch and an Amplifier (contd.)



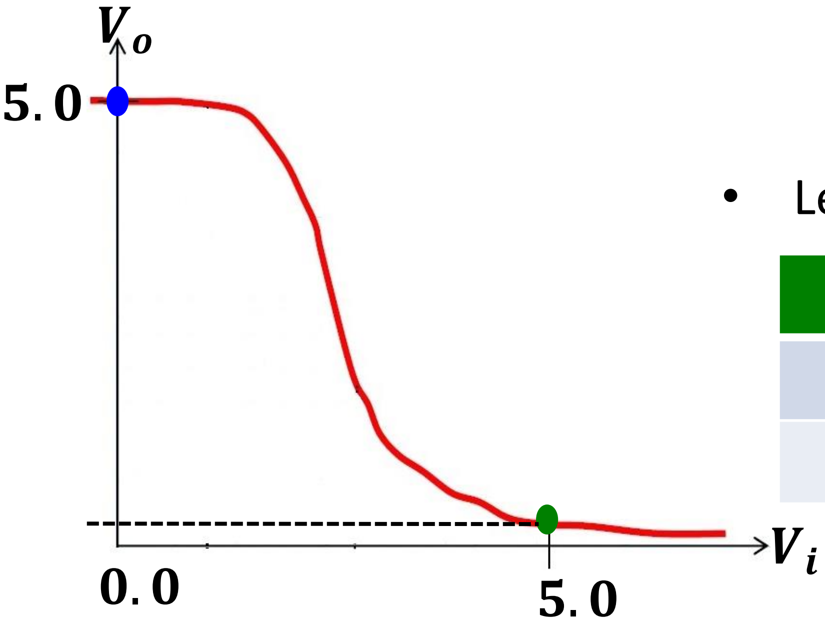


Q: I thought you said this circuit **did** something. It appears to be just as **pointless** as all the others!

A: To see how this circuit is **useful**, consider what happens when the **input** voltage V_i is 0 V and 5V.

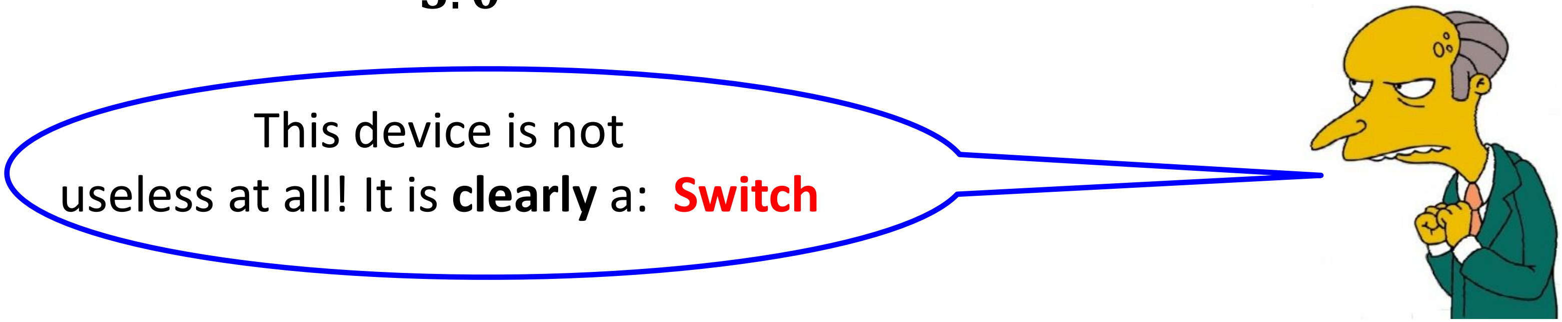
The MOSFET as a Switch and an Amplifier (contd.)

- From the transfer function, we find **if** $V_i = 0V$, the output voltage will be $V_o = 5V$. Likewise, **if** the input voltage is $V_i = 5V$, the output voltage will be **small.**



- Let's **summarize** these results in a table:

V_i	V_o	$Mode$
0.0	5.0	Cutoff
5.0	Small (~ 0.5)	Triode

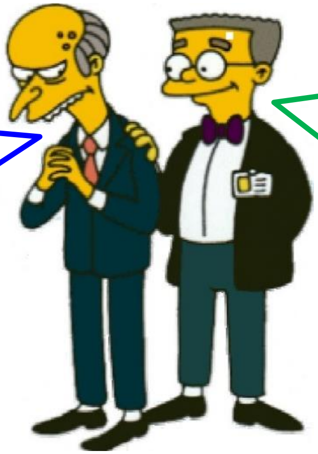


The MOSFET as a Switch and an Amplifier (contd.)

- This circuit provides a **simple** example of **one** of the primary **applications** of MOSFET devices—**digital circuit design**. We can use MOSFETs to make digital devices such as **logic gates** (AND, OR, NOR, etc.), **flip-flops**, and digital **memory**.
- We typically find that, just like this circuit, when a MOSFET digital circuit is in **either** of its **two** binary states (i.e., “0” or “1”), the MOSFETs in the circuit will either be in **cutoff** ($I_D = 0$) or in **triode** (V_{DS} small) modes.

→ **Cutoff** and **Triode** are the MOSFET modes associated with **digital** circuits and applications!

Q: So, just what good is the MOSFET **Saturation Mode** ??

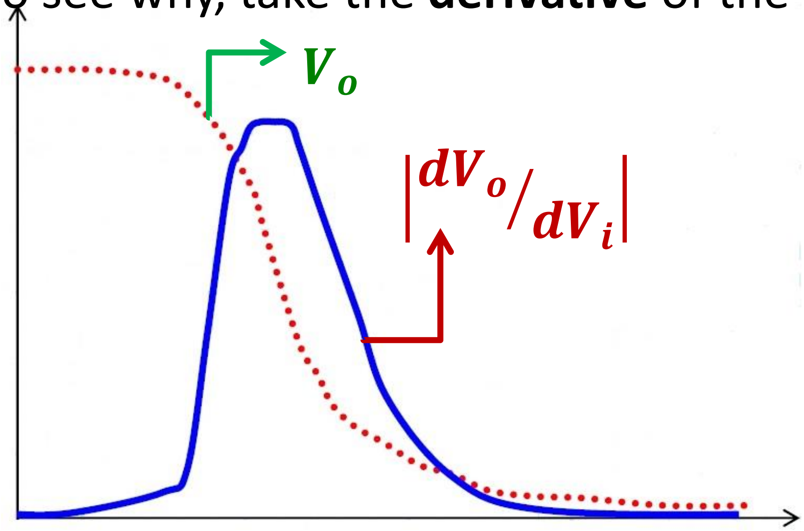


Sir, it appears to **me** that the **Saturation** region is just a **useless** MOSFET mode between cutoff and triode!

A: Actually, we will find the MOSFET **saturation** mode to be **extremely useful**!

The MOSFET as a Switch and an Amplifier (contd.)

- To see why, take the **derivative** of the above circuit's transfer function $\left({}^{dV_o}/_{dV_i}\right)$.



- We note that in **cutoff** and **triode**: $\left|{}^{dV_o}/_{dV_i}\right| \approx 0$
- While in the **saturation** mode: $\left|{}^{dV_o}/_{dV_i}\right| \gg 1$

Q: Oh goody. The **slope** of the transfer function is **large** when the MOSFET is in **saturation**. Am I supposed to be impressed by that?! How are these results even **remotely** important!?

A: Since in **cutoff** and **triode** $\left|{}^{dV_o}/_{dV_i}\right| \approx 0$, a **small change** in input voltage V_i will result in almost **no change** in output voltage V_o.

Contrast this with the **saturation** region, where $\left|{}^{dV_o}/_{dV_i}\right| \gg 1$. This means that a **small change** in input voltage V_i results in a **large change** in the **output** voltage V_o!

The MOSFET as a Switch and an Amplifier (contd.)

- To see how this is important, consider the case where the input signal has both a **DC** and a **small-signal** (AC) component:

$$\boldsymbol{v_I}(\mathrm{t}) = V_i + v_i(t)$$

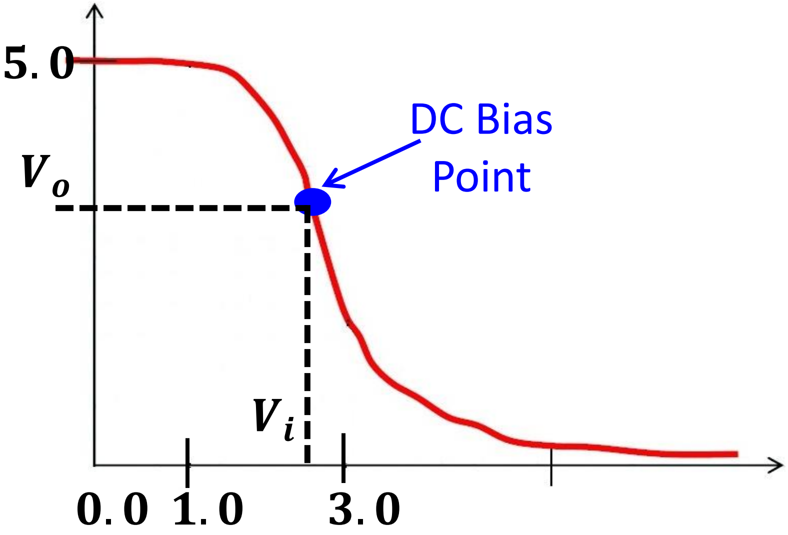
- As a result, the **output** voltage likewise has both a DC and small signal component:

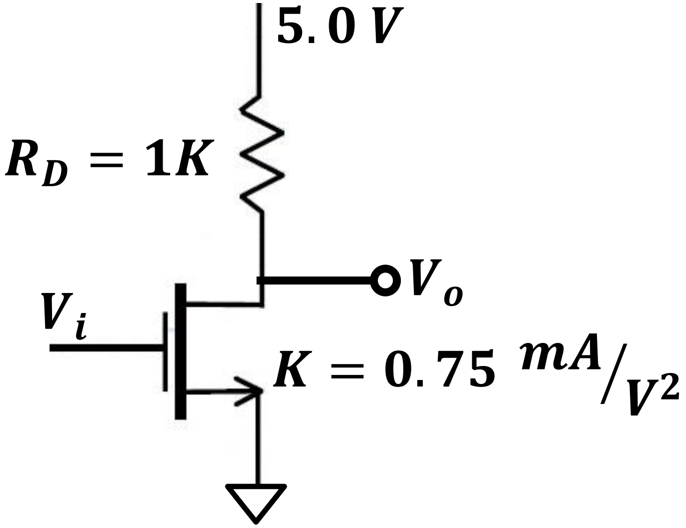
$$\boldsymbol{v_O}(\mathrm{t}) = V_o + v_o(t)$$

Now, let's consider **only** the **DC** components. We can select the DC input V_i such that the MOSFET is placed in **saturation**. The value V_i, along with the resulting DC output V_o, sets a **DC bias point** for this circuit.

By selecting the right value of V_i we **could** set this DC bias point to where the transfer function **slope** is the greatest.

The MOSFET as a Switch and an Amplifier (contd.)





Now, say we **add** a **small-signal** $\boldsymbol{v_i}$ to this input DC voltage (i.e., $\boldsymbol{v_I}(\mathrm{t}) = V_i + v_i(t)$). This small signal simply represents a small change in the input voltage from its **average** (i.e., DC) value. The result is of course a **small change** in the **output** voltage— the **small-signal output voltage** $v_o(t)$!

- Now for the **interesting** part (I bet you were wondering when I would get around to it)! The small change in the output voltage will have a much **larger** magnitude than the small change in the input!

  For example, if the input voltage changes by **1 mV** (i.e., $v_i = 1\mathrm{mV}$), the output **might** change by, say, **5 mV** (i.e., $v_o = 5mV$).

The MOSFET as a Switch and an Amplifier (contd.)

Q: Goodness! By how much would the **output** change in our example circuit? How can we **determine** the small signal **output** v_o ??

- Determining how much the output voltage of our circuit will change when we change the input voltage by a small amount is very **straightforward**—we simply take the **derivative** of the output voltage v_O with respect to input voltage v_I.
  - By taking the **derivative** of v_O with respect to v_I (when the MOSFET is in saturation, we find:

$$\frac{dv_O}{dv_I} = \frac{d\left(5.0 - 0.75(v_I - 1.0)^2\right)}{dv_I} \Rightarrow = -1.50(v_I - 1.0)$$

when

$$\mathbf{1.0 < v_I < 3.0\ V}$$

The expression describes the **slope** of our circuit's transfer function (for $\mathbf{1.0 < v_I < 3.0\ V}$). Note the slope with the **largest magnitude** occurs when $v_I = 3.0V$, providing a **slope** of -3.0 mV/mV.

The MOSFET as a Switch and an Amplifier (contd.)

- Thus, **if** we DC bias this circuit with V_i = 3.0 V (resulting in V_o = 2.0 V), we find that the small signal output will be **3 times** the small signal input!
- **For example**, say that the **input** to our circuit is:

$$\boldsymbol{v_I}(\mathrm{t}) = 3.0 + 0.01\cos(\omega t)\ V$$

Here: $V_i = 3.0\,V$ $v_i = 0.01\cos(\omega t)$

- We would find that the **output voltage** would approximately be:

$$\boldsymbol{v_O}(\mathrm{t}) = 2.0 - 0.03\cos(\omega t)\ V$$

Here: $V_o = 2.0\,V$ $v_o = -0.03\,cos(\omega t)$

In other words, the magnitude of the small-signal output has a magnitude **three times larger** than the input magnitude.

We say then that our signal provides **small-signal gain**—our circuit is also a **small-signal amplifier**!

The MOSFET as a Switch and an Amplifier (contd.)

I see. A **small** voltage change results in a **big** voltage change—it's **voltage gain**!

The **MOSFET saturation** mode turns out to be—**excellent**.

- Even the simple circuit of this example is sufficient to demonstrate the **two primary applications** of MOSFET transistors--**digital** circuits and signal **amplification**.
- The important MOSFET regions for **digital** devices are **triode** and **cutoff**, MOSFETs in amplifier circuits are typically biased into the **saturation** mode!