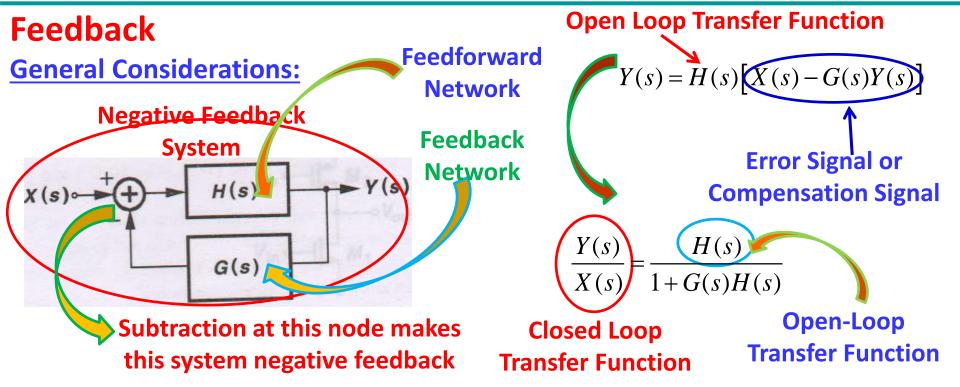
Lecture – 17

Date: 27.10.2016

- Feedback and Properties, Types of Feedback
- Amplifier Stability
- Gain and Phase Margin Modification





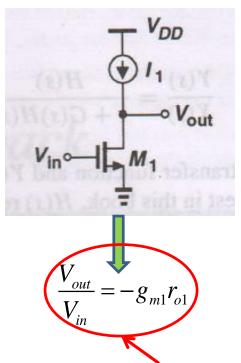
In our discussion: H(s) represents an amplifier and G(s) is a frequencyindependent quantity representing the feedback network

Elements of Feedback System:

(a) The feed forward amplifier [H(s)]; (b) A means of sensing the output; (c) The feedback network [G(s)]; (d) A means of generating the feedback error [X(s) - G(s)Y(s)]

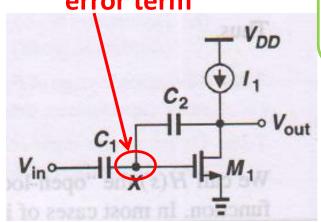
Properties of Feedback Circuits

1. Gain Desensitization



Dependent on process parameters, temperature, bias etc.

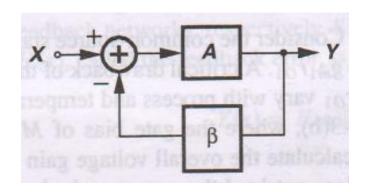
Node generating error term



$$\frac{V_{out}}{V_{in}} = -\frac{1}{\left(1 + \frac{1}{g_{m1}r_{o1}}\right)\frac{C_2}{C_1} + \frac{1}{g_{m1}r_{o1}}}$$
For large
$$g_{m1}r_{o1}$$

The closed-loop gain is much less sensitive to device parameters

Feedback makes gain of this CS stage independent of process and temperature



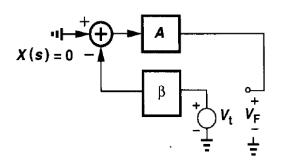
$$\frac{Y}{X} = \frac{A}{1 + A\beta} = \frac{1}{\beta} \frac{1}{\left(1 + \frac{1}{A\beta}\right)}$$

$$\frac{Y}{X} \cong \frac{1}{\beta} \left(1 - \frac{1}{A\beta} \right)$$
 variable of α

The impact of variations in A on the closed loop gain is insignificant

- The quantity βA (loop gain) is critical for any feedback system \rightarrow higher the βA , less sensitive is the closed loop gain to the variations of A
- Accuracy of the closed loop gain improves by maximizing either A or β
- β increases → closed loop gain decreases → means a trade-off exist between precision and the closed loop gain

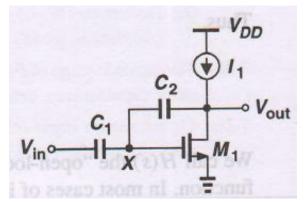
Loop Gain Calculation

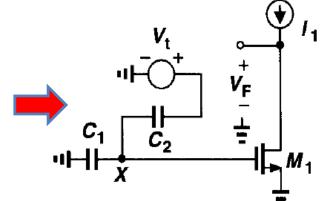


- Set the main input to zero
- Break the loop at some point
- Inject a test signal 'while maintaining the direction and polarity'
- Follow the signal around the loop and obtain the expression/value

$$V_t \beta(-1)A = V_F \qquad \qquad \therefore \frac{V_F}{V_L} = -\beta A$$

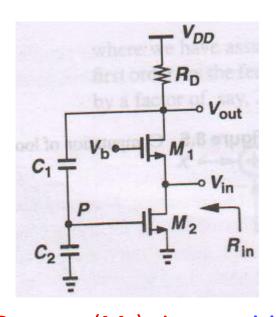






$$\therefore \frac{\mathbf{V}_{\mathrm{F}}}{\mathbf{V}_{\mathrm{t}}} = -\frac{C_2}{C_1 + C_2} g_{m1} r_{o1}$$

2. <u>Input Impedance Modification</u>



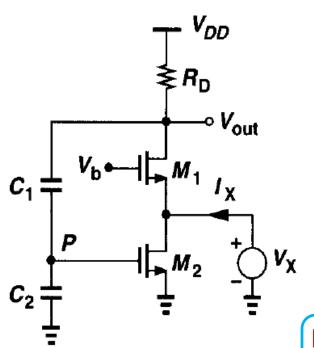
Lets check input impedance with and without feedback

 V_{DD} R_{D} V_{Out} V_{b} M_{1} C_{2} M_{2} R_{in}

CG stage $(M_1) \rightarrow$ capacitive divider senses V_{out} and applies it to gate of current source $(M_2) \rightarrow M_2$ returns a current feedback signal to the input of M_1

$$R_{in,open} = \frac{1}{g_{m1} + g_{mb1}}$$

Assumption: no channellength modulation present



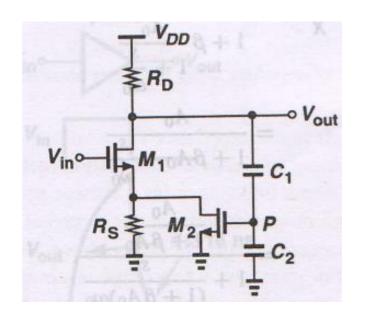
$$\frac{V_X}{I_X} = R_{in,closed} = \frac{1}{g_{m1} + g_{mb1}} \frac{1}{1 + g_{m2}R_D \frac{X_{C1}}{X_{C1} + X_{C2}}}$$

$$R_{in,closed} = R_{in,open} \frac{1}{1 + \left(g_{m2}R_D \frac{X_{C1}}{X_{C1} + X_{C2}}\right)}$$
 Loop Gair

Feedback reduces the input impedance in this instance → quite useful circuit topology

Four Elements of Feedback: feed-forward amplifier consists of M₁ and R_D, the output is sensed by C₁ and C₂, the feedback network comprise of C₁, C₂, and M₂, subtraction occurs in current domain at the input

3. Output Impedance Modification



$$R_{out,open} = R_D$$

$$R_{out,closed} = \frac{R_{D}}{1 + \frac{g_{m2}R_{S}(g_{m1} + g_{mb1})R_{D}}{(g_{m1} + g_{mb1})R_{S}} \frac{X_{C1}}{X_{C1} + X_{C2}}}$$

Loop Gain

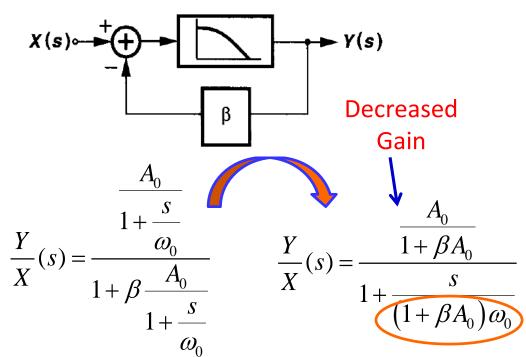
Can you identify if this is a positive feedback or negative feedback circuit? Why?

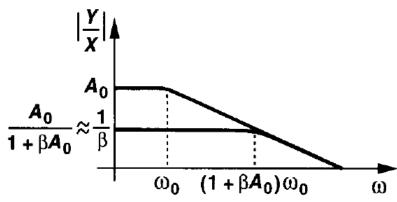
4. Bandwidth Modification

One pole transfer function:

$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_0}}$$

One pole transfer function of a closed-loop system:







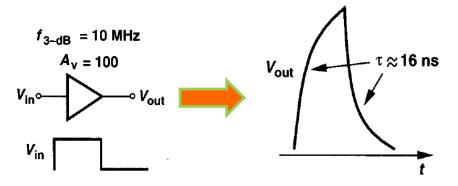
Constant GBW



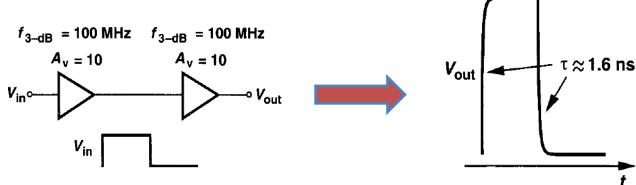
High pole frequency

←>Increased Bandwidth

- 4. Bandwidth Modification
- GBW of a single pole system doesn't change with feedback. But how to improve the speed of the system with high gain?
- Suppose we need to amplify a 20 MHz square wave by a factor of 100 and maximum bandwidth but we only have single pole amplifier with an open-loop gain of 100 and 3-dB bandwidth of 10 MHz.



Apply feedback in such a way that the gain and bandwidth are modified to 10 and 100 MHz. Then use two stage amplification to achieve the desired.

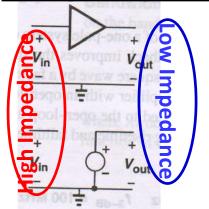


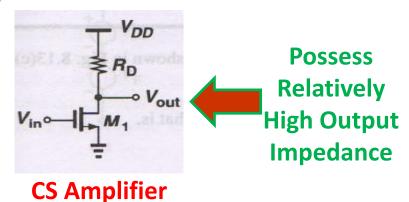
Types of Amplifiers

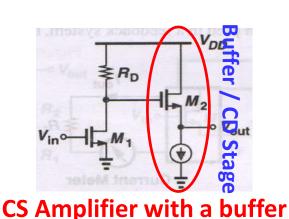
Type: Based on the type of parameters (current or voltage) they sense at the input and the type of parameters (current or voltage) they produce at the output

- Amplifier sensing voltage at the input: exhibit high input impedance (as a voltmeter)
- Amplifier sensing current at the input: exhibit low input impedance (as an ammeter)
- Amplifier sensing voltage at the output: exhibit low output impedance (as a voltage source)
- Amplifier sensing current at the output: exhibit high output impedance (as a current source)

Voltage Amplifier

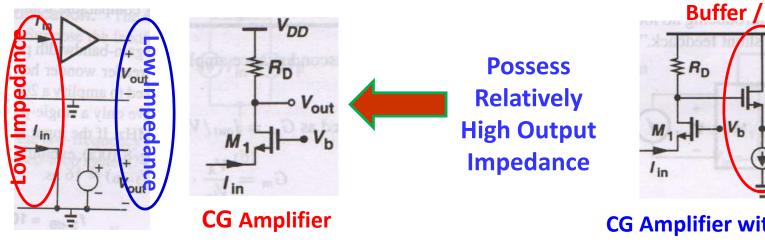


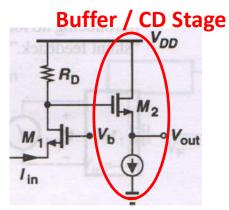




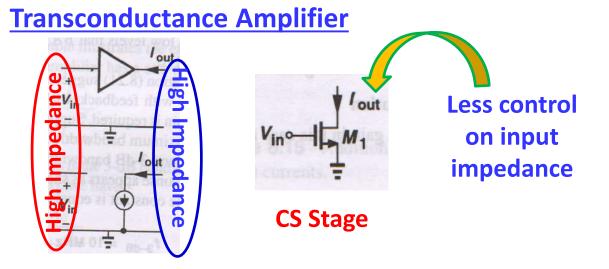
Types of Amplifiers (contd.)

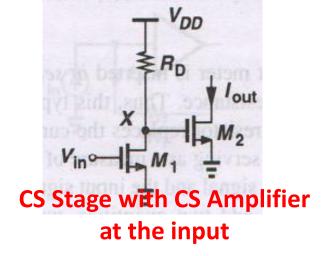
Transimpedance Amplifier





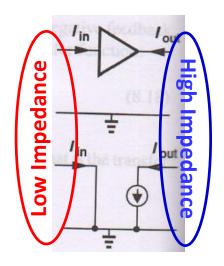
CG Amplifier with a buffer

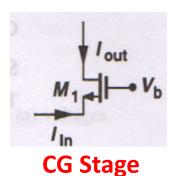




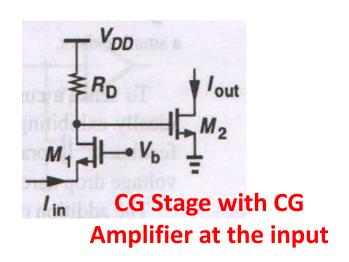
Types of Amplifiers (contd.)

Current Amplifier





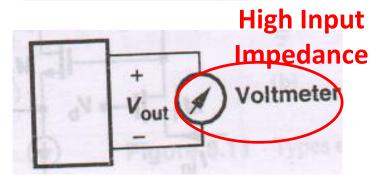
Less control on input impedance



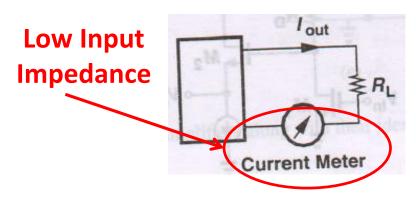
Sense and Return Mechanism

- Placing a circuit in the feedback requires sensing the output signal and then returning a fraction to the input
- Voltage and Current as input and output quantities provide 4 different possibilities for feedback circuit (sense and return circuit)
- Voltage-Voltage: both the input and output of the feedback circuit is voltage
- Voltage-Current: input of feedback is voltage and output is current
- Current-Voltage: input of feedback is current and output is voltage
- Current-Current: both the input and output of feedback circuit is current

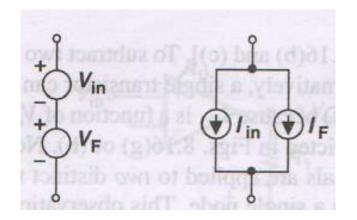
To sense a voltage:



To sense a current:

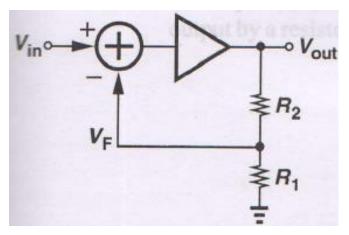


 The addition/subtraction at the input can be done in current or voltage domain: (a) currents are added by placing them in parallel; (b) voltages are added by placing them in series

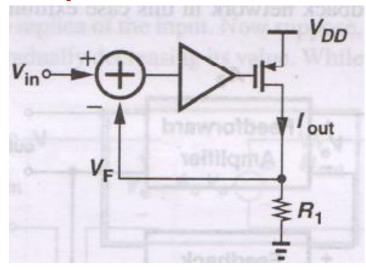


The sense and return mechanism ideally do not affect the operation of feed-forward amplifier → in practical circuits they do introduce loading effects

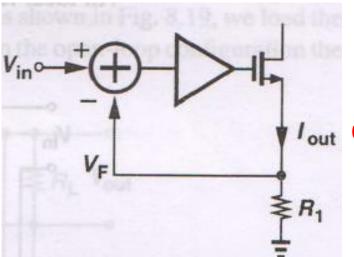
Practical Implementations of Sensing:



Voltage Sensing

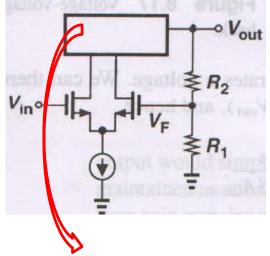


Current Sensing

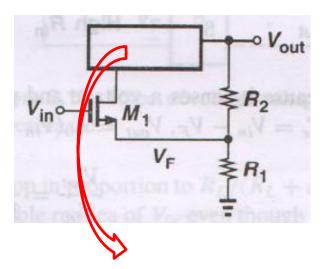


lout Current Sensing

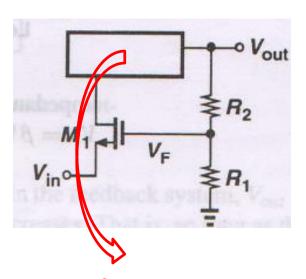
Practical Implementations of Voltage Subtraction:



Provides the amplified version of difference between V_{in} and the portion of V_{out}

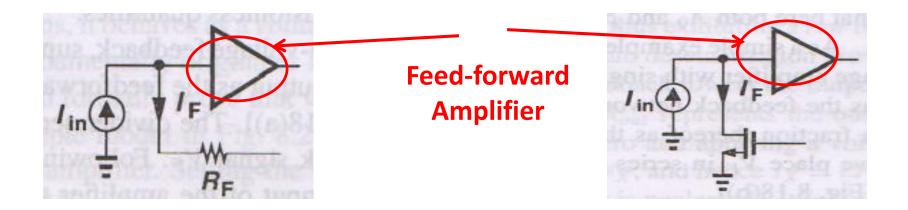


This CS stage provides output in terms of voltage difference V_{in} - V_F



This CG stage provides output in terms of voltage difference V_{in} - V_F

Practical Implementations of Current Subtraction:



Important: voltage subtraction happens when they are applied to two distinct nodes whereas current subtraction happens when they are applied to a single node → a precursor to feedback topologies

Feedback Topologies

- Voltage-Voltage Feedback (also called Shunt-Series Feedback): both the input and output of the feedback circuit is voltage
- Voltage-Current Feedback (also called Shunt-Shunt Feedback): input of feedback is voltage and output is current
- Current-Voltage (also called Series-Series Feedback): input of feedback is current and output is voltage
- Current-Current (also called Series-Shunt Feedback): both the input and output of feedback circuit is current

Sensed Quantity is Voltage





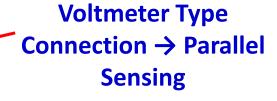


Voltmeter Type Characteristics

Should be added/subtracted in series

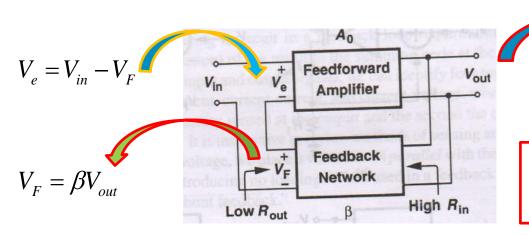
Voltage-Voltage Feedback Increased Input Impedance Vin Ve Reedforward Amplifier

Subtracted in series



Reduced Output Impedance

This high impedance is in parallel to the feedforward amplifier



Low Rout

Feedback

Network

High Rin

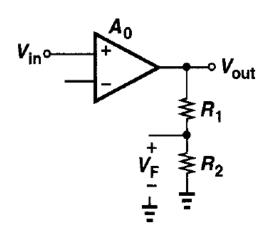
$$V_{out} = A_0 V_e = A_0 (V_{in} - \beta V_{out})$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{A_0}{1 + \beta A_0}$$

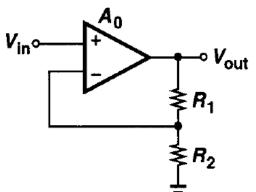
Closed loop gain

- → modified gain
 - → smaller !!!

Example: Voltage-Voltage Feedback



For voltage V_{out} sensing – parallel to the output node of this differential input but single ended output amplifier

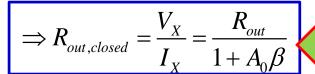


The voltage signal from feedback network is fed to the other input node of the differential amplifier

$$V_e = -V_F = -\beta V_X$$

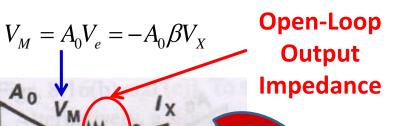
Output Impedance:

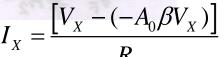
$$V_F = \beta V_X$$

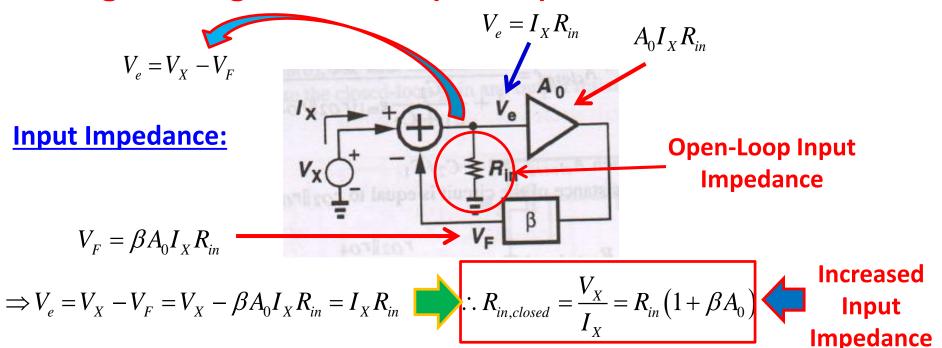


—Reduced Closed-Loop **Output Impedance**

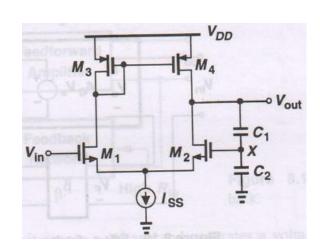
Ao



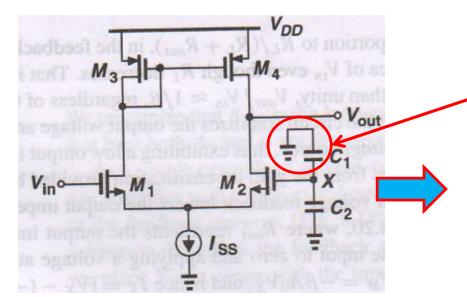




Example: calculate gain and output impedance of this circuit:



Step-1:
determine
open-loop
voltage gain

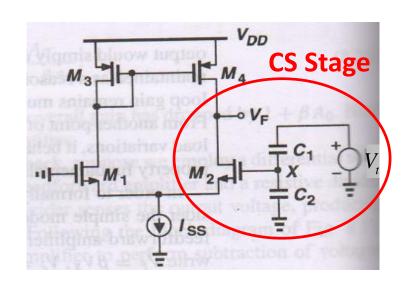


Grounding ensures there is no voltage feedback

Open-loop gain is:

$$A_0 = g_{m1}(r_{o2} || r_{o4})$$

Step-2: determine the loop gain



Drain Current

Output impedance

$$V_{F} = V_{t} \frac{C_{1}}{C_{1} + C_{2}} g_{m1} (r_{o2} || r_{o4})$$

Therefore,

$$\beta A_0 = \frac{C_1}{C_1 + C_2} g_{m1} (r_{o2} || r_{o4})$$

$$\Rightarrow A_{closed} = \frac{A_0}{1 + \beta A_0} = \frac{g_{m1}(r_{o2} || r_{o4})}{1 + \frac{C_1}{C_1 + C_2} g_{m1}(r_{o2} || r_{o4})}$$

• For
$$\beta A_0 >> 1$$
,

For
$$\beta A_0 >>1$$
, $A_{closed} \simeq \frac{g_{m1}(r_{o2} || r_{o4})}{\frac{C_1}{C_1 + C_2} g_{m1}(r_{o2} || r_{o4})} = 1 + \frac{C_2}{C_1}$

The closed-loop output impedance,

$$R_{out,closed} = \frac{R_{out,open}}{1 + \beta A_0} = \frac{(r_{o2} || r_{o4})}{1 + \frac{C_1}{C_1 + C_2} g_{m1}(r_{o2} || r_{o4})}$$

• For
$$\beta A_0 >> 1$$
,

$$R_{out,closed} \simeq \left(1 + \frac{C_2}{C_1}\right) \frac{1}{g_{m1}}$$

Relatively Smaller Value

Stability Issues in Feedback Amplifiers

The generic closed-loop transfer function:

$$A_{closed}(j\omega) = \underbrace{\frac{A_0(s)}{1 + A_0(s)\beta(s)}}$$

It is assumed that both the open-loop gain and the feedback gain is frequency dependent

At low frequencies:

 $\beta(s)$ is assumed as a constant value and $A_0(s)$ is also assumed as a constant value \rightarrow the loop gain becomes constant \rightarrow obviously this happens for any direct-coupled amplifier with poles and zeros present at high frequency \rightarrow the loop gain (Aβ) should be positive value for negative feedback

At high frequencies: $A_{closed}(j\omega) = \frac{A_0(j\omega)}{1 + A_0(i\omega)B(i\omega)}$

Therefore it is apparent that the loop gain is:

$$L(j\omega) = A_0(j\omega)\beta(j\omega) = A_0(j\omega)\beta(j\omega)e^{j\varphi(\omega)}$$

Phase Angle

Magnitude

It is real with negative sign at the frequency when $\varphi(\omega)$ is 180°



If for $\omega = \omega_1$, the loop \longrightarrow What happens gain is less than unity

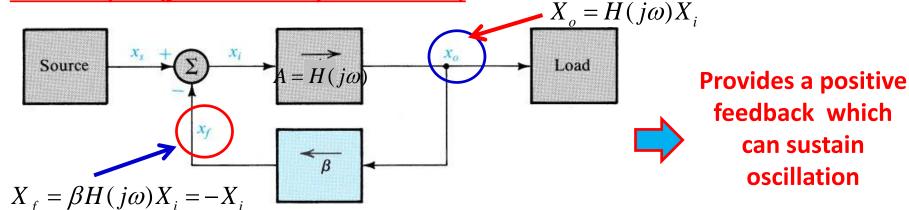


to stability?

At high frequencies:

If at $\omega = \omega_1$, the loop gain (L) is equal to unity with negative sign \rightarrow Closed-loop gain will be infinite \rightarrow even for zero input there will be some output \rightarrow an oscillation condition!!!





Summary:

- The phase angle of the loop gain equals -180°
- The magnitude of the loop gain is either unity or greater than unity

Notice that the total phase shift around the loop at ω_1 is 360° because negative feedback itself introduces 180°

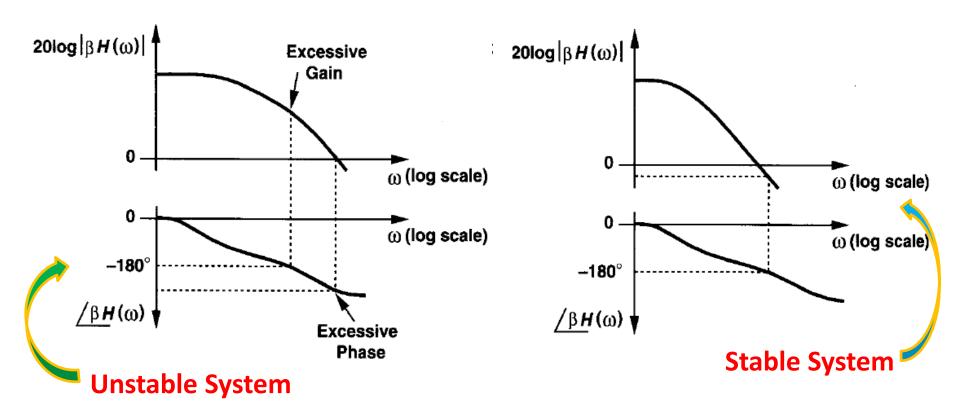
360° phase shift is necessary for oscillation since the feedback signal must add in phase to the original noise to allow oscillation

Similarly, a loop gain of unity (or greater) is also required to enable growth of the oscillation amplitude

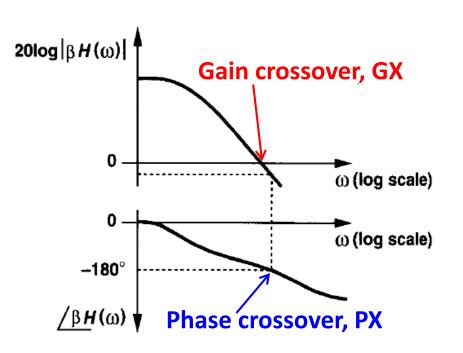
Summary

A negative feedback system may oscillate at ω_1 if:

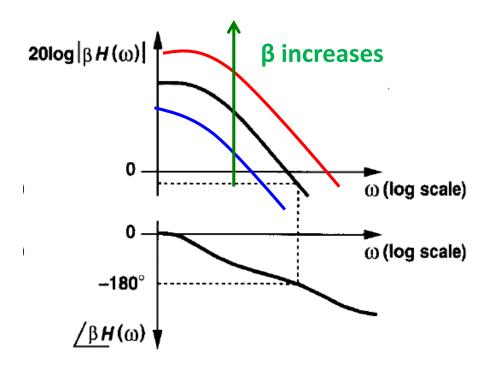
- the phase shift around the loop at this frequency is so much that the feedback becomes positive, and
- the loop gain is still enough to allow signal buildup



To make the system stable, the idea is to minimize the total phase shift so that for $|\beta H|=1$, $<\beta H$ is still more positive than -180°

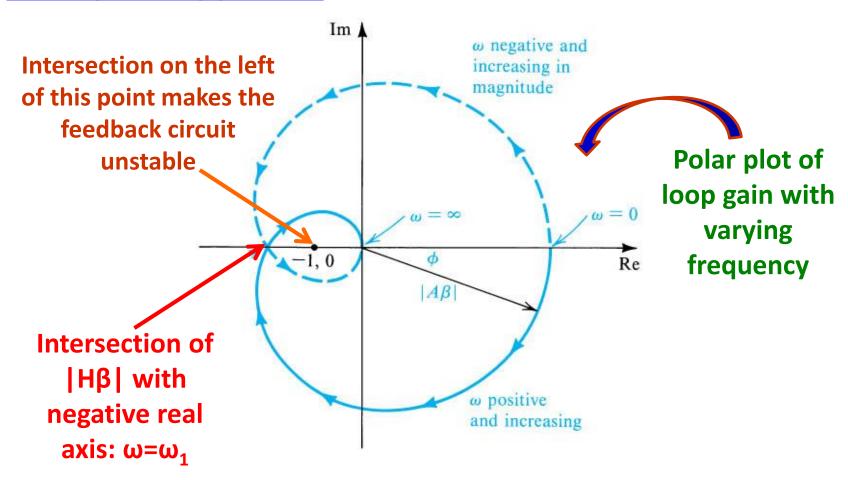


For a unity gain (β=1) Feedback

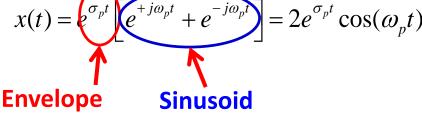


If β is reduced (i.e., less feedback is applied), the magnitude plot will shift down \rightarrow essentially moves GX closer to origin \rightarrow in turn makes the system more stable

Stability Test: Nyquist Plot

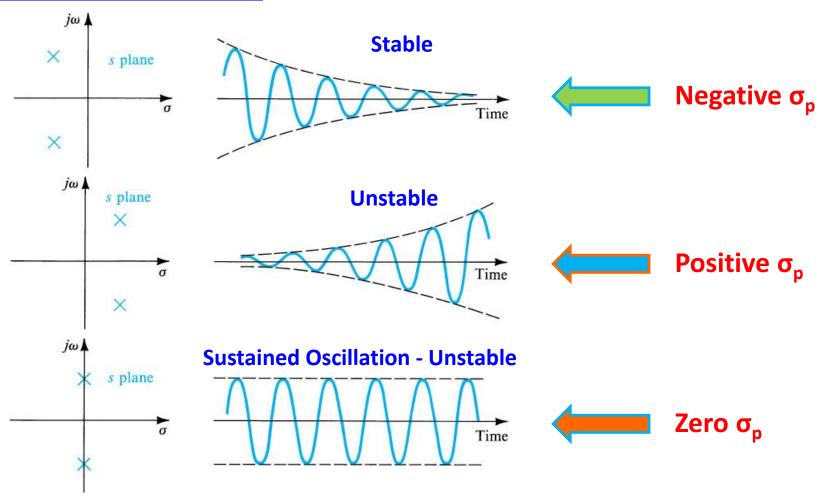


Stability and Pole Location \rightarrow the transient response of an amplifier with a pole pair $s_p = \sigma_p \pm j\omega_p$ subjected to disturbance will show a transient response:



- For poles in right half of the s-plane the oscillations will grow exponentially considering that σ_p will be positive
- For poles with $\sigma_p = 0$, the oscillation will be sustained
- For poles in the left half of the s-plane, the term σ_p will be negative and therefore the oscillation will decay exponentially towards zero

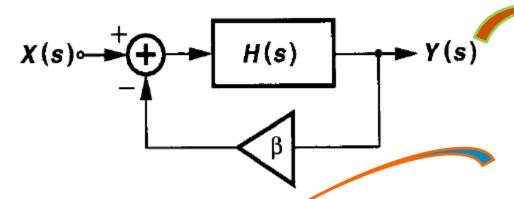
Stability and Pole Location



Obviously, the presence of zeros have been ignored

Poles of the Feedback Amplifier

Study of single-pole feedforward amplifier



Closed-loop transfer function

$$A_{closed}(s) = \frac{Y}{X}(s) = \frac{H(s)}{1 + \beta H(s)}$$

Where,
$$H(s) = \frac{A_0}{(1+s/\omega_p)}$$

Then the closed-loop transfer function becomes

$$A_{closed}(s) = \frac{A_0 / (1 + A_0 \beta)}{1 + s / \omega_P (1 + A_0 \beta)}$$

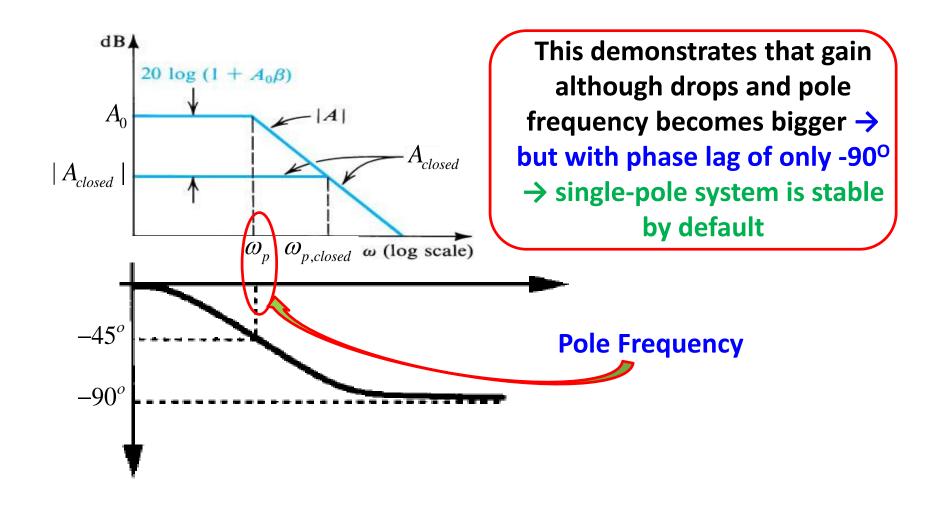
It is apparent that the feedback moves the pole

frequency from ω_p to:

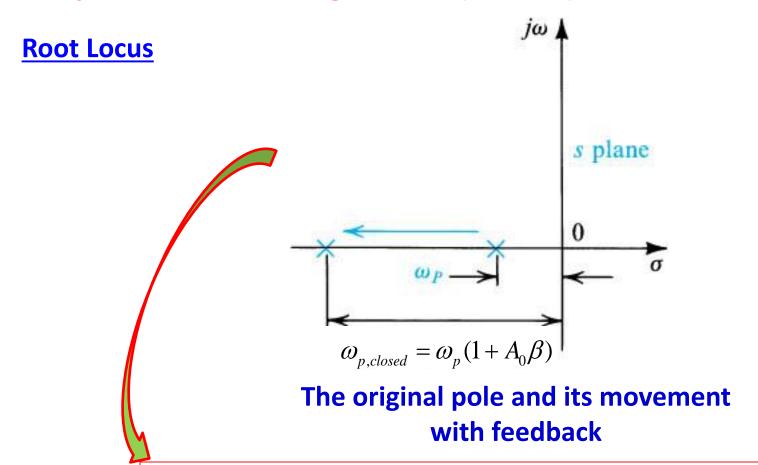
$$\omega_{p,closed} = \omega_p (1 + A_0 \beta)$$

Amplifier with a Single Pole (contd.)

The frequency response of amplifier with and without feedback



Amplifier with a Single Pole (contd.)



It is apparent that the pole never enters the right half of the s-plane → unconditionally stable scenario!

Amplifier with Two Poles

 Open-loop transfer function of an amplifier with two pole is given as:

$$A(s) = \frac{A_0}{(1 + s / \omega_{P1})(1 + s / \omega_{P2})}$$

 The closed-loop poles are obtained from:

$$1 + A(s)\beta = 0$$
 $\Rightarrow s^2 + s(\omega_{P1} + \omega_{P2}) + (1 + A_0\beta)\omega_{P1}\omega_{P2} = 0$

 Therefore the closedloop poles are:

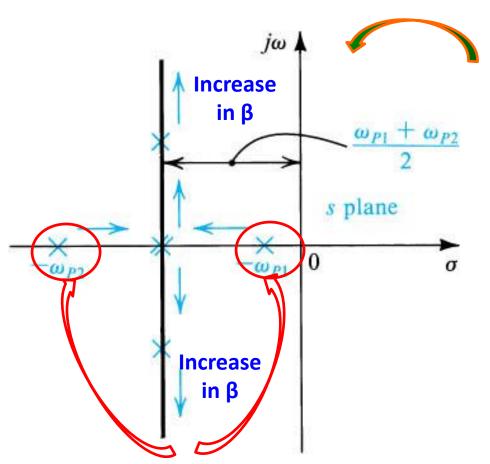
$$s = -\frac{1}{2}(\omega_{P1} + \omega_{P2}) \pm \frac{1}{2}\sqrt{(\omega_{P1} + \omega_{P2})^2 - 4(1 + A_0\beta)\omega_{P1}\omega_{P2}}$$

- As the loop gain $A_0\beta$ is increased from zero, the poles come closer
- At certain $A_0\beta$ the poles will coincide
- Further increase in $A_0\beta$ make poles complex conjugate which move along a vertical line

Amplifier with Two Poles (contd.)

Root-locus Diagram

$$s = -\frac{1}{2}(\omega_{P1} + \omega_{P2}) \pm \frac{1}{2}\sqrt{(\omega_{P1} + \omega_{P2})^2 - 4(1 + A_0\beta)\omega_{P1}\omega_{P2}}$$

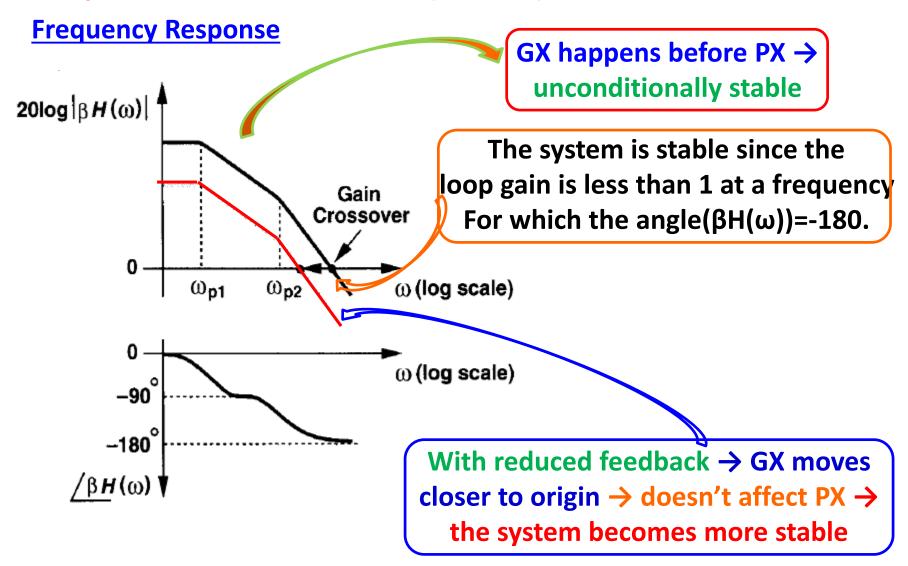


Root-locus shows that the poles never enter the right half of splane

- Unconditionally stable !!!
- Reason is simple: the maximum phase shift of A(s) is -180° (-90° per pole) [that too when $\omega_p \rightarrow \infty$]
- There is no finite frequency at which the phase shift reaches 180° → therefore no polarity reversal of feedback

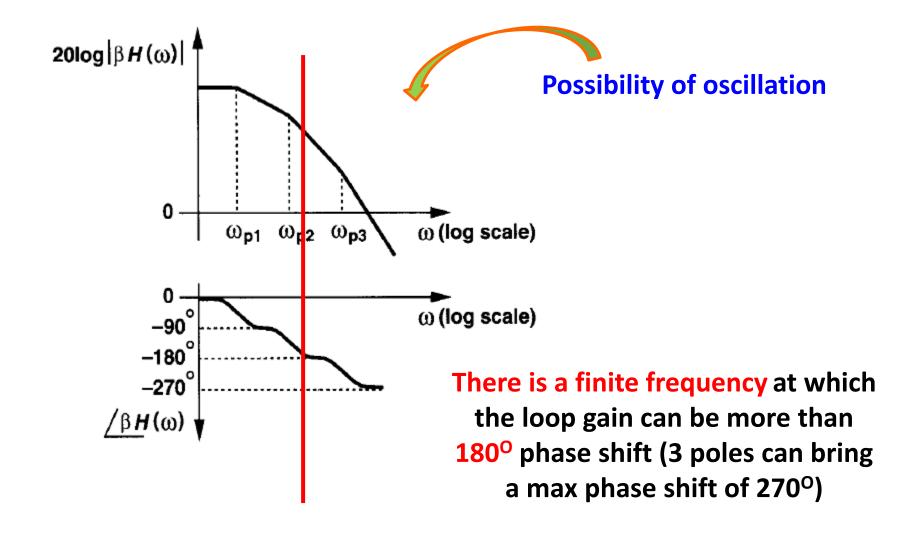
Poles when no feedback (i.e., $\beta = 0$)

Amplifier with Two Poles (contd.)

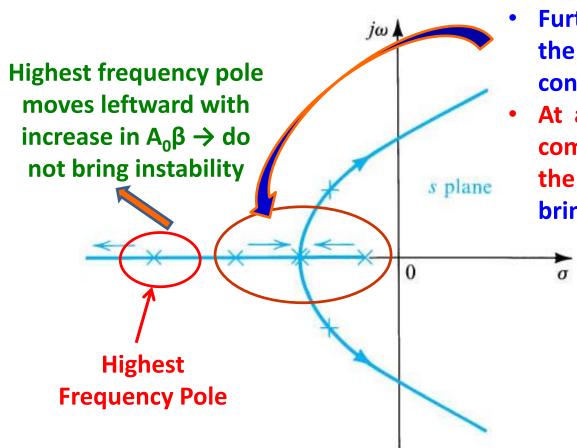


Amplifier with Three Poles

Frequency Response



Amplifier with Three Poles



- Increase in A₀β bring the other two poles together
- Further increase in A₀β make the poles complex and then conjugate
- At a definite A₀β the pair of complex-conjugate poles enter the right half of s-plane → bring instability!!!

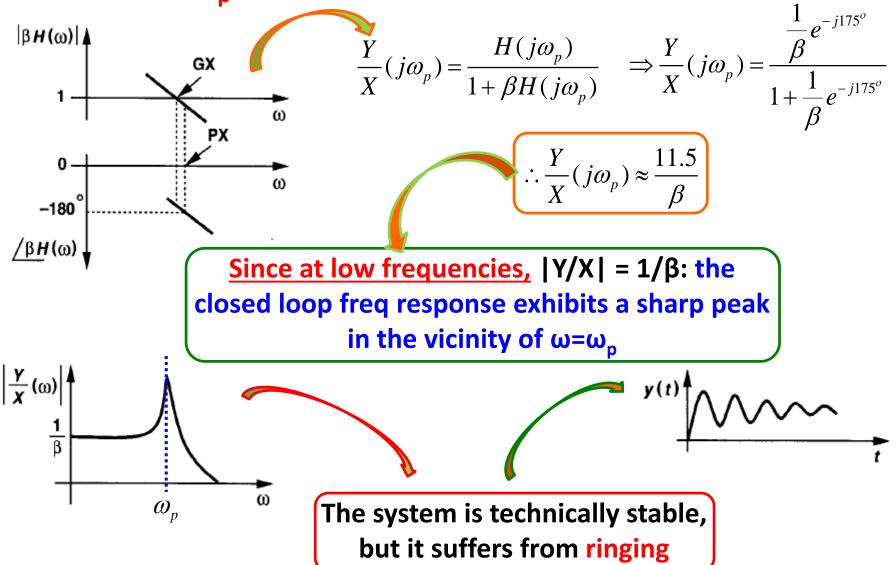
Amplifier with Three Poles (contd.)

- In order to maintain the stability of amplifiers it is imperative to keep loop gain $A_0\beta$ smaller than the value corresponding to the poles entering right half splane
- In terms of Nyquist diagram, the critical value of $A_0\beta$ is that for which the diagram passes through the (-1, 0) point
- Reducing $A_0\beta$ below this value causes the Nyquist plot to shrink \rightarrow the plot intersects the negative real axis to the right of (-1, 0) point \rightarrow indicates stable amplifier
- Increasing $A_0\beta$ above this value causes expansion of Nyquist plot \rightarrow plot encircles the (-1, 0) point \rightarrow unstable performance

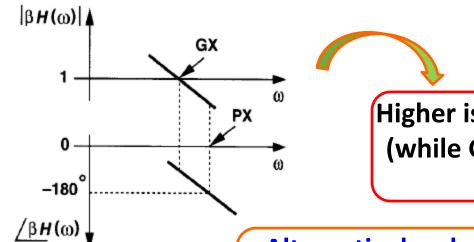
Case Study: Relative Location of GX and PX

- Case 1: $<\beta H(j\omega_p)=-175^\circ$
- Case 2: $<\beta H(j\omega_p)$ such that GX<<PX
- Case 3: $<\beta H(j\omega_p)=-135^\circ$

Case 1: $<\beta H(j\omega_p)=-175^\circ$



Case 2: $<\beta H(j\omega_p)$ such that GX<<PX



Higher is the spacing between GX and PX (while GX remains below PX), the more stable is the system

Alternatively, phase of βH at the GX frequency can serve as the measure of stability: the smaller <βH at GX, the more stable the system

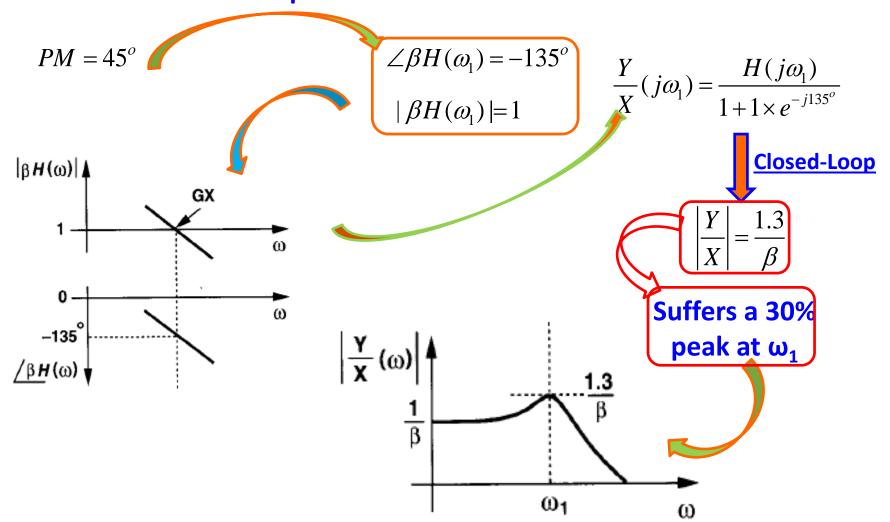
Leads to the concept of phase margin (PM)

$$PM = 180^{\circ} + \angle \beta H(\omega_1)$$

Where, ω_1 is the GX frequency

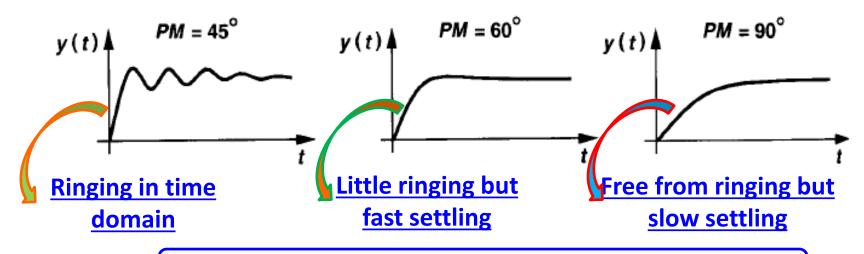
Case 3: $<\beta H(j\omega 1)=-135$

How much PM is adequate?



Case 3: $<\beta H(j\omega 1)=-135$

Peaking is associated with ringing in time domain



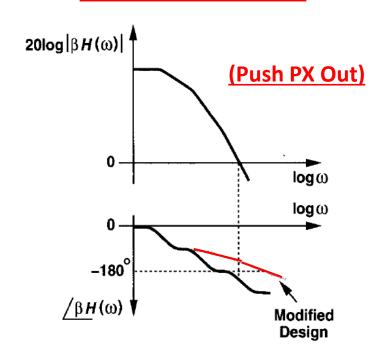
You design your system to achieve PM of around 60°

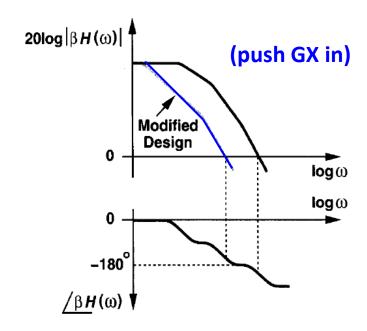
Caution

- PM is useful for small signal analysis.
- For large signal step response of a feedback system, the nonlinear behavior is usually such that a system with satisfactory PM may still exhibit excessive ringing.
- <u>Transient analysis</u> should be used to analyze large signal response.

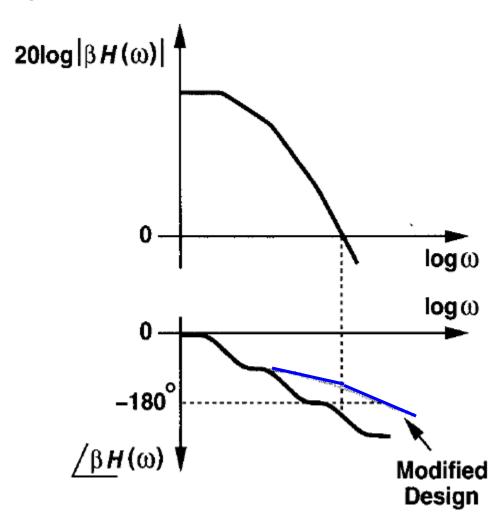
Frequency Compensation

- Open loop transfer function is modified <u>such that</u> the closed-loop circuit is stable and the time response is well behaved
- Reason for frequency compensation:
 - $|\beta H(\omega)|$ does not drop to unity when $<\beta H(\omega)$ reaches -180°.
- Possible Solutions:



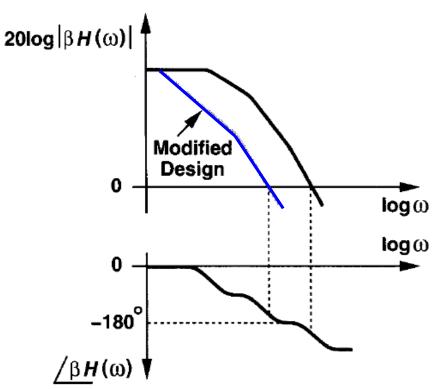


Option 1: Push PX OUT



- Minimize the # of poles
- What's the problem?
 - Each stage contributes a pole.
- Reduction in # of stages implies difficult trade-off of gain versus output swings.

Option 2: Push GX In



Problem:

Bandwidth is sacrificed for stability

Typical Approach

- Minimize the number of poles first to push PX out
- Use compensation to move the GX towards the origin next