Solution -1:

1) If M$_2$ (note I said M$_2$!) is in **saturation**, then:

$$I_{D2} = K\left(V_{GS2} - V_{t2}\right)^2 \quad \Rightarrow \quad V_{GS2} = \sqrt{\frac{I_{D2}}{K}} + V_{t2} = \sqrt{\frac{4}{1}} + 2 = 4.0V$$

Since the **source** terminal is at **ground** potential, we find that the **DC gate voltage** of M$_2$ is:

$$V_{G2} = V_{GS2} + V_{S2} = 4 + 0 = 4.0V$$

Note the gate terminals of each transistor are connected, so that:

$$V_{G1} = V_{G2} = 4.0V$$

Since the source terminal of M$_1$ is likewise at ground potential, we can conclude that:

$$V_{GS1} = V_{G1} - V_{S1} = 4 - 0 = 4.0V$$

So, if MOSFET M$_1$ is in saturation, then:

$$I_{D1} = K\left(V_{GS1} - V_{t1}\right)^2 = 1.0\left(4.0 - 1.0\right)^2 = 9.0\ mA$$

Note this is **not** (I said **not**!) equal to drain current $I_{D2} = 4\ mA$!!

Since the gate and source terminals are connected, we find:

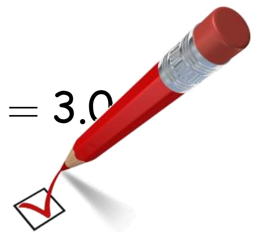
$$V_{S1} = V_{G1} = 4.0V$$

And so from Ohm's Law:

$$R_1 = \frac{13 - V_{S1}}{I_{D1}} = \frac{13 - 4}{9} = 1\ K\Omega$$

Finally, we verify that the MOSFET M_1 **is** in saturation:

$$V_{DS1} = V_{D1} - V_{S1} = 4 - 0 = 4 > V_{GS} - V_t = 3.0$$



2) MOSFET M_2 **is** in saturation if:

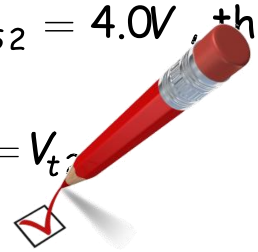
$$V_{GS2} > V_{t2}$$

And:

$$V_{DS2} > V_{GS2} - V_{t2}$$

For the **first** condition, we know that $V_{GS2} = 4.0V$, therefore:

$$V_{GS2} = 4.0 > 2.0 = V_{t2}$$



Thus:

$$V_{GS2} - V_{t2} = 4 - 2 = 2.0V$$

From **KVL**, we find:

$$V_{DS2} = 13.0 - 4R_2$$

So that for MOSFET M_2 to be in **saturation**:

$$V_{DS2} > V_{GS2} - V_{t2} \qquad \Rightarrow \qquad 13.0 - 4R_2 > 2.0$$

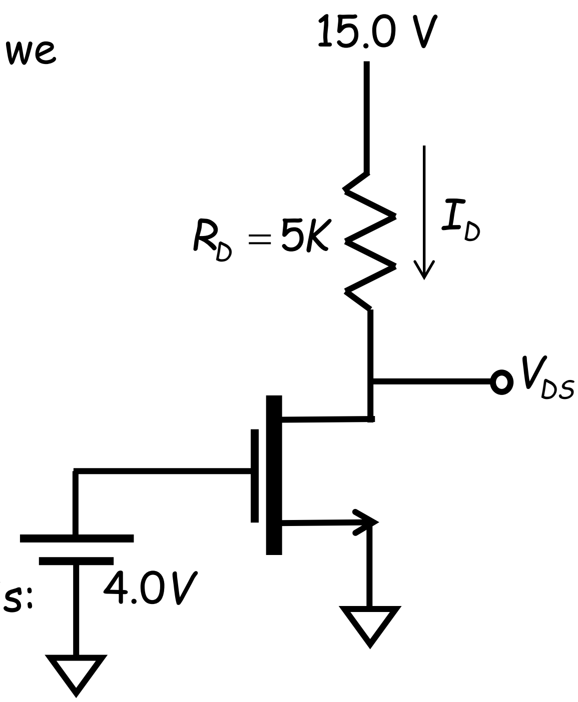
Meaning:

$$R_2 \leq \frac{13.0 - 2.0}{4} = \frac{11}{4} = \underline{\underline{2.75\ K\Omega}}$$

Solution -2:

Step 1: DC Analysis

Turning **off** the small signal source leaves a DC circuit of:



We **ASSUME** saturation, so that we **ENFORCE**:

$$I_D = K\left(V_{GS} - V_t\right)^2$$

It is evident that:

$$V_{GS} = 4.0 \text{ V}$$

Therefore the DC drain current is:

$$\begin{aligned} I_D &= K\left(V_{GS} - V_t\right)^2 \\ &= 0.25(4-2)^2 \\ &= 1.0 \text{ mA} \end{aligned}$$

Thus, the DC voltage V_{DS} can be determined from KVL as:

$$\begin{aligned} V_{DS} &= 15.0 - I_D R_D \\ &= 15.0 - 1(5) \\ &= 10.0 \text{ V} \end{aligned}$$

We **CHECK** our results and find:

$$V_{GS} = 4.0 > V_t = 2.0 \checkmark$$

and:

$$V_{DS} = 10.0 > V_{GS} - V_t = 2.0$$

Step 2: Determine the small-signal parameters

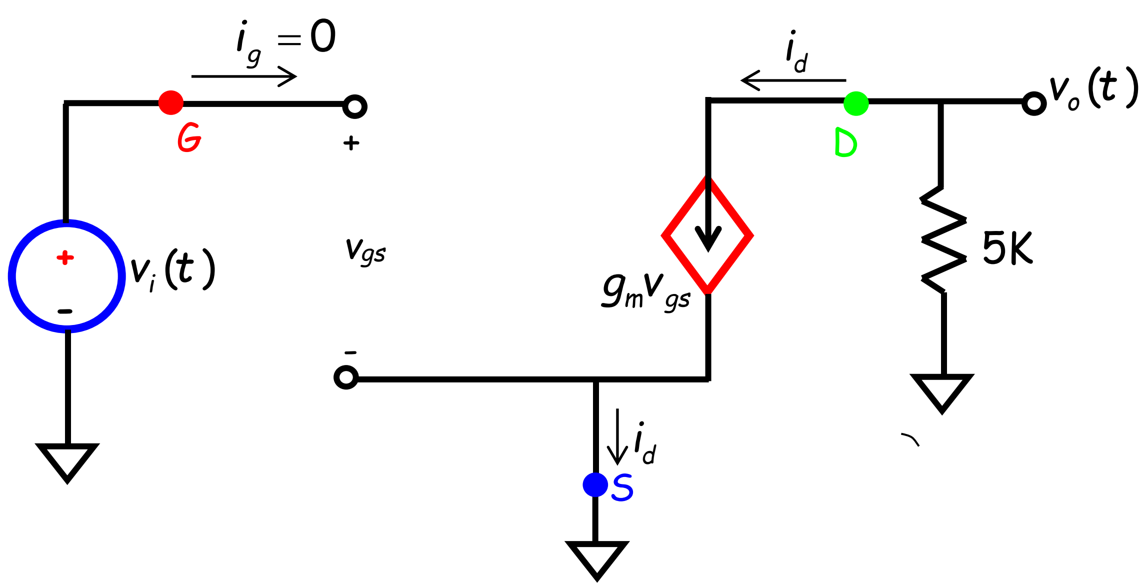
We find that the **transconductance** is:

$$\begin{aligned} g_m &= 2K\left(V_{GS} - V_t\right) \\ &= 2(0.25)(4.0 - 2.0) \\ &= 1 \ \mathrm{mA/V} \end{aligned}$$

Note that **no** value of λ was given, so we will assume $\lambda = 0$, and thus **output resistance** $r_o = \infty$.

Steps 3 and 4: Determine the small-signal circuit

We now turn off the **two** DC voltage source, and replace the MOSFET with its **small signal model**. The result is our **small-signal circuit**:



Step 5: Analyze the small-signal circuit

The analysis of this small-signal circuit is fairly **straightforward**. First, we note from KVL that:

$$v_{gs} = v_i$$

and that:

$$\begin{aligned} i_d &= g_m v_{gs} \\ &= 1.0 v_{gs} \\ &= v_{gs} \end{aligned}$$

and that from Ohm's Law:

$$v_o = -5 i_d$$

Combining these equations, we find that:

$$v_o = -5 v_i$$

And thus the **small-signal** voltage gain of this amplifier is:

$$A_v = \frac{v_o(t)}{v_i(t)} = -5.0$$