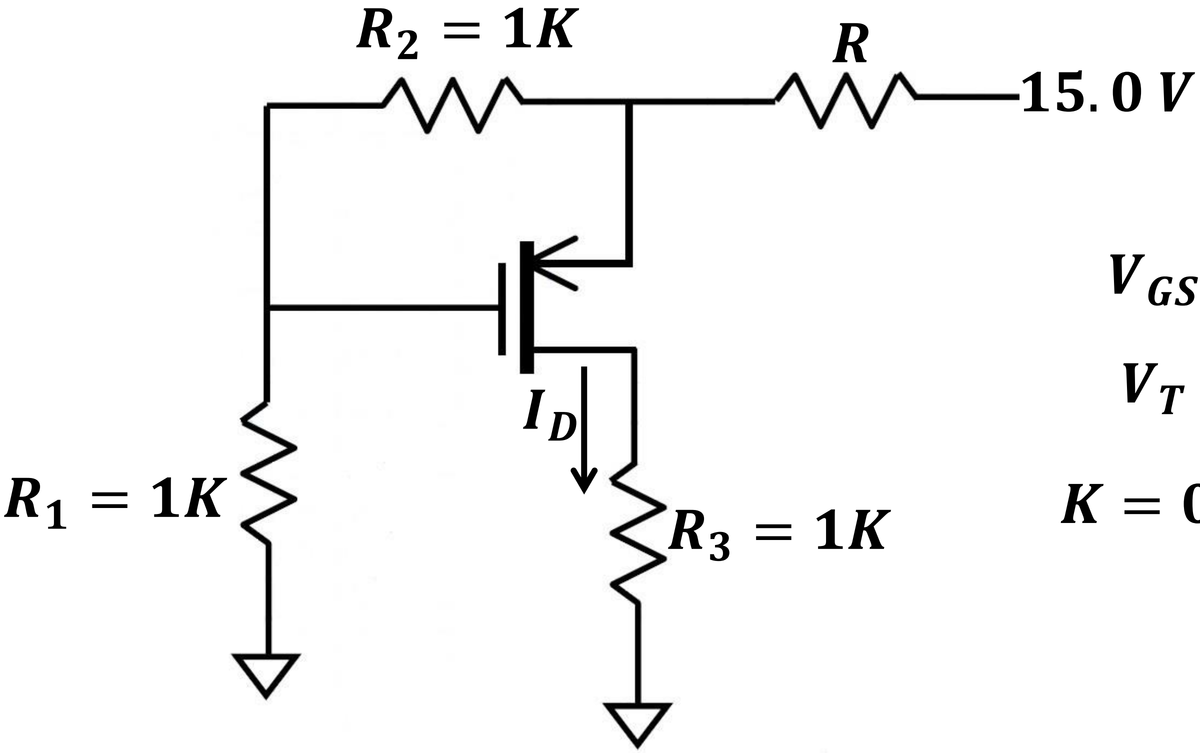
ECE315 / ECE515

Lecture – 4 **Date: 13.08.2015**

- The MOSFET as a Switch and Amplifier
- MOSFET Small Signal Operation, Models, Analysis

Example – 3

- Consider the **PMOS** circuit below, find the value of unknown of resistor R.



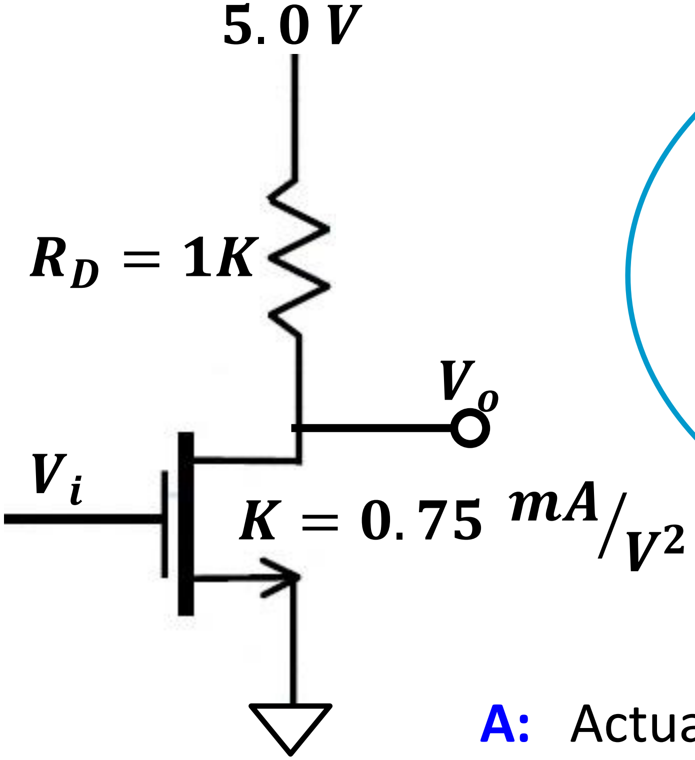
$V_{GS} = -4.0\,V$

$V_T = -2.0\,V$

$K = 0.75\ {}^{mA}/_{V^2}$

The MOSFET as a Switch and an Amplifier

- Consider this **simple** MOSFET circuit:



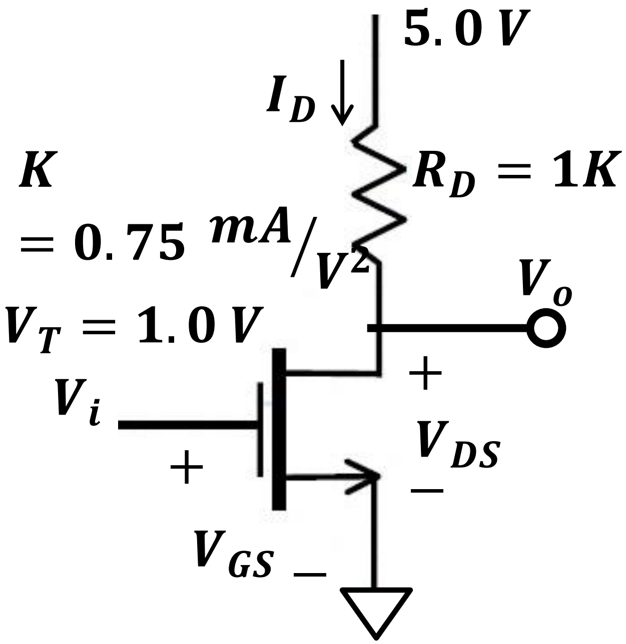
Q: Oh, goody—you're going to **waste** my time with another of these **pointless** academic problems. Why can't you discuss a circuit that actually **does** something?

A: Actually, this circuit **is** a fundamental electronic device! To see what this circuit does, we need to determine its **transfer function** $V_o = f(V_i)$.

The MOSFET as a Switch and an Amplifier (contd.)

Q: Transfer function! How can we determine the transfer function of a MOSFET circuit!?

A: Same as with junction diodes—we determine the output $\boldsymbol{V_o}$ for each device mode, and then determine **when** (i.e., for what values of $\boldsymbol{V_i}$) the device is in that mode!



- First, note that **regardless** of the MOSFET mode:

$$V_{GS} = V_i - 0.0 = V_i$$

$$V_{DS} = V_o - 0.0 = V_o$$

- From KVL, we can likewise conclude that:

$$V_{DS} = V_o = 5.0 - I_D R_D$$

The MOSFET as a Switch and an Amplifier (contd.)

- Now let’s ASSUME that the MOSFET is in **cutoff**, thus ENFORCING $I_D = 0.$

$$V_{DS} = V_o = 5.0 - I_D R_D \Rightarrow V_o = 5.0 - 0 \times (1 \times 10^3)$$

$$\Rightarrow \therefore V_o = 5.0\ \mathrm{V}$$

- Now, we know that MOSFET is in cutoff **when**: $V_{GS} = V_i < V_T = 1.0\ V$

- Thus, we conclude that:

$V_o = 5.0\ \mathrm{V}$ **when** $V_i < 1.0\ V$

The MOSFET as a Switch and an Amplifier (contd.)

- Now, let's ASSUME that the MOSFET is in **saturation**, thus ENFORCE:

$$I_D = K(V_{GS} - V_T)^2 \Longrightarrow I_D = K(V_i - 1.0)^2$$

- And thus the output voltage is:

$$V_o = 5.0 - I_D R_D = 5.0 - 0.75 \times 10^{-3} \times (V_i - 1.0)^2 \times 1 \times 10^3$$

$$\therefore V_o = 5.0 - 0.75 \times (V_i - 1.0)^2$$

- We know that MOSFET is in saturation **when**:

$$V_{GS} = V_i > V_T = 1.0\ V \quad \underline{\text{and}} \quad V_{DS} = V_o > V_{GS} - V_T = V_i - 1.0$$

- The second inequality means:

$$V_o > V_i - 1.0 \Longrightarrow 5.0 - 0.75 \times (V_i - 1.0)^2 > V_i - 1.0$$

The MOSFET as a Switch and an Amplifier (contd.)

$$0 > 0.75 \times (V_i - 1.0)^2 + (V_i - 1.0\,) - 5.0$$

- Solving this quadratic, we find that the **only** consistent solution is:

$$V_i - 1.0 < 2.0 \Rightarrow V_i < 3.0$$

- Thus we conclude that the MOSFET in saturation:

$$V_o = 5.0 - 0.75 \times (V_i - 1.0)^2 \quad \underline{\textbf{when}} \quad 1.0 < V_i < 3.0\,V$$

- Finally, let's ASSUME that the MOSFET is in **triode** mode, thus we ENFORCE:

$$I_D = K\left[2(V_{GS} - V_T)V_{DS} - (V_{DS})^2\right]$$

- And thus the output voltage is:

$$V_o = 5.0 - I_D R_D = 5.0 - 0.75 \times \left[2(V_i - 1.0)V_o - (V_o)^2\right]$$

The MOSFET as a Switch and an Amplifier (contd.)

- Rearranging the equation, we get the quadratic form:

$$\mathbf{0.75(V_o)^2 - (1.5V_i - 0.5)V_o + 5.0 = 0}$$

- The solutions of which are:

$$\boxed{V_o = \frac{(1.5V_i - 0.5) \pm \sqrt{(1.5V_i - 0.5)^2 - 15.0}}{1.5}}$$

- Note because of the ±, there are **two** possible solutions. However, to be in triode region, the MOSFET must not be in pinchoff, i.e.:

$$\mathbf{V_{DS} = V_o < V_{GS} - V_T = V_i - 1.0}$$

- This condition is satisfied with the **smaller** of the two solutions (i.e., the solution with the minus sign!):

$$\boxed{V_o = \frac{(1.5V_i - 0.5) - \sqrt{(1.5V_i - 0.5)^2 - 15.0}}{1.5}}$$

The MOSFET as a Switch and an Amplifier (contd.)

$$V_o = \frac{(1.5V_i - 0.5) - \sqrt{(1.5V_i - 0.5)^2 - 15.0}}{1.5}$$

This expression provides us with the output voltage **if** the MOSFET is in triode mode. The question remaining is thus **when** (i.e., for what values of V_i) is the MOSFET in triode mode?

We could do a lot more math to find this answer, but this answer is actually quite **obvious**!

- Recall that we have already determined that:

a) The MOSFET is in cutoff **when** $\boldsymbol{V_i < 1.0V}$.

b) The MOSFET is in saturation **when** $\boldsymbol{1.0V < V_i < 3.0V}$.

The MOSFET as a Switch and an Amplifier (contd.)

- Since there are only **three** modes of a MOSFET device, and since the transfer **function** must—well—be a **function**, we can conclude (correctly) that the MOSFET will be in triode region when $\boldsymbol{V_i}$ is the value of the **only region that is left**: $\boldsymbol{V_i > 3.0\ V}$.

- Thus we can conclude that:

$$\boxed{V_o = \frac{\left(1.5V_i - 0.5\right) - \sqrt{\left(1.5V_i - 0.5\right)^2 - 15.0}}{1.5}}$$

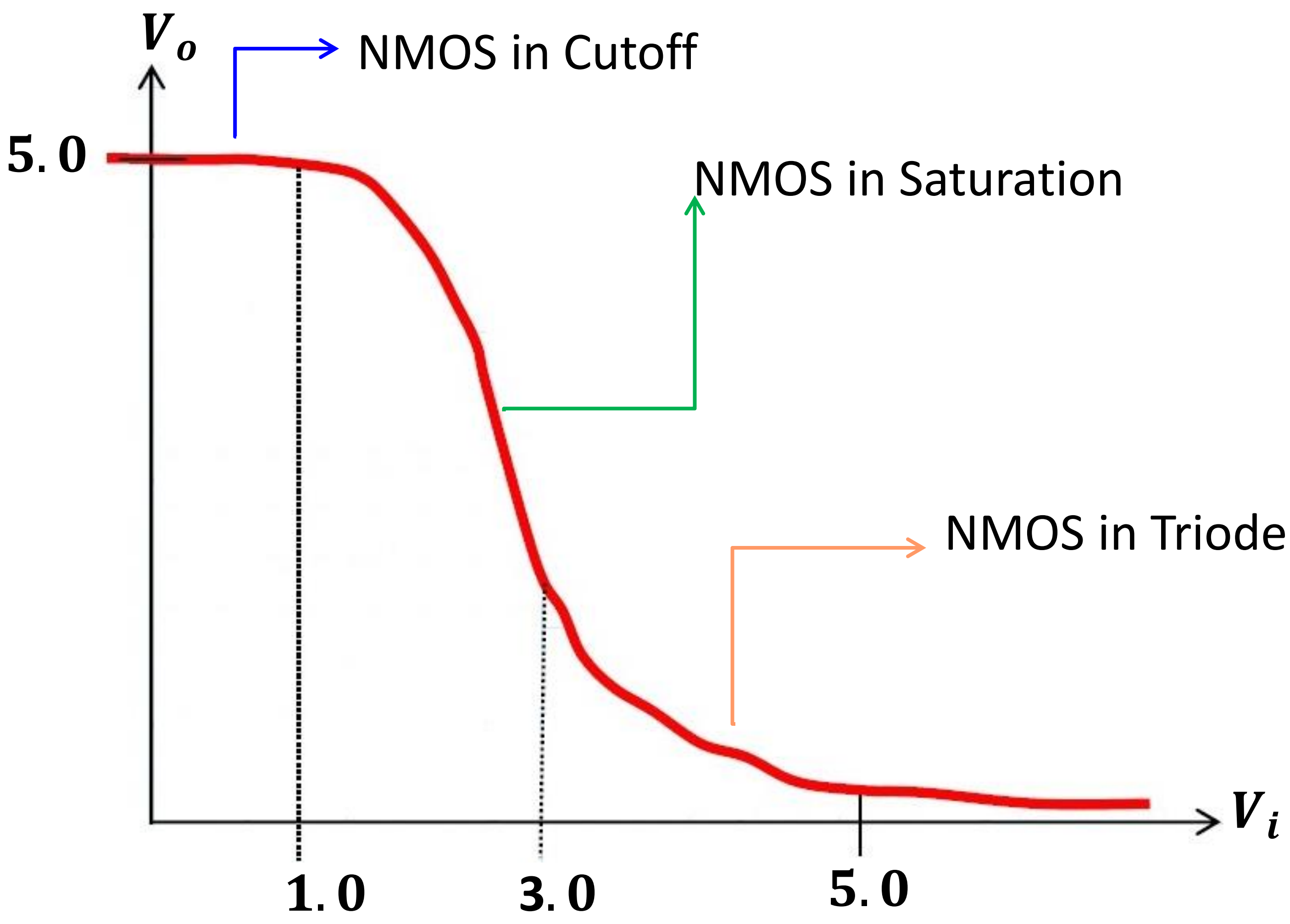
when $\boldsymbol{V_i > 3.0\ V}$

The MOSFET as a Switch and an Amplifier (contd.)

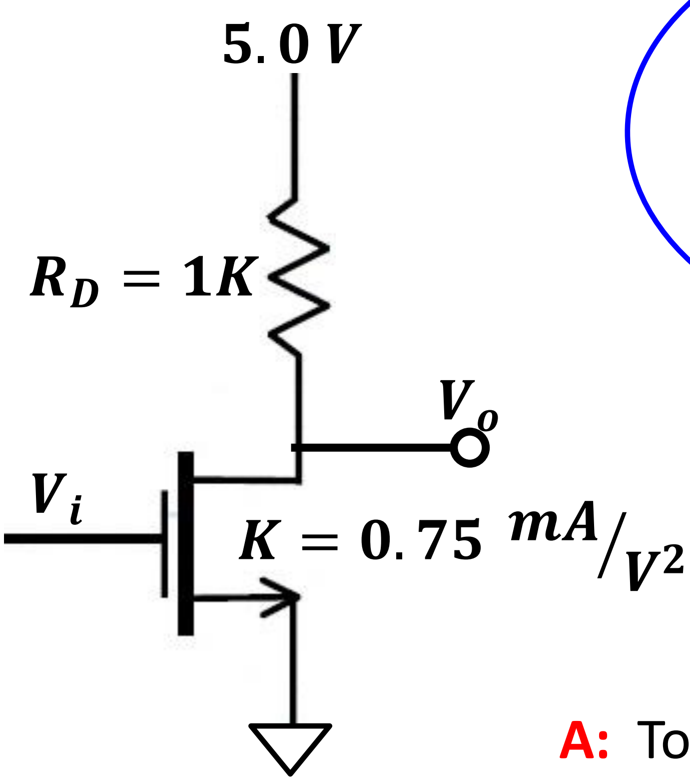
- We now have determined the complete, continuous **transfer function** of this circuit!

$$V_o = \begin{cases} 5.0\,V & \underline{\text{when}} \quad V_i < 1.0\,V \\ 5.0 - 0.75 \times (V_i - 1.0)^2 & \underline{\text{when}} \quad 1.0 < V_i < 3.0\,V \\ \boxed{V_o = \dfrac{(1.5V_i - 0.5) - \sqrt{(1.5V_i - 0.5)^2 - 15.0}}{1.5}} & \underline{\text{when}} \quad V_i > 3.0\,V \end{cases}$$

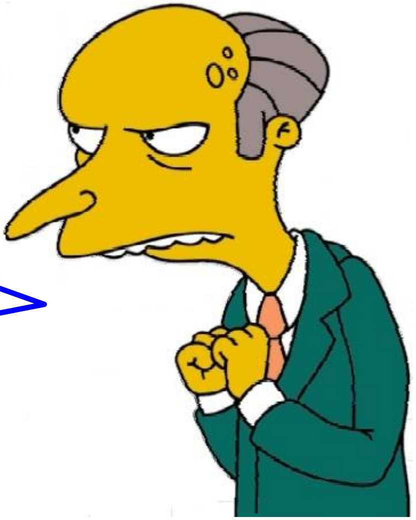
The MOSFET as a Switch and an Amplifier (contd.)



The MOSFET as a Switch and an Amplifier (contd.)



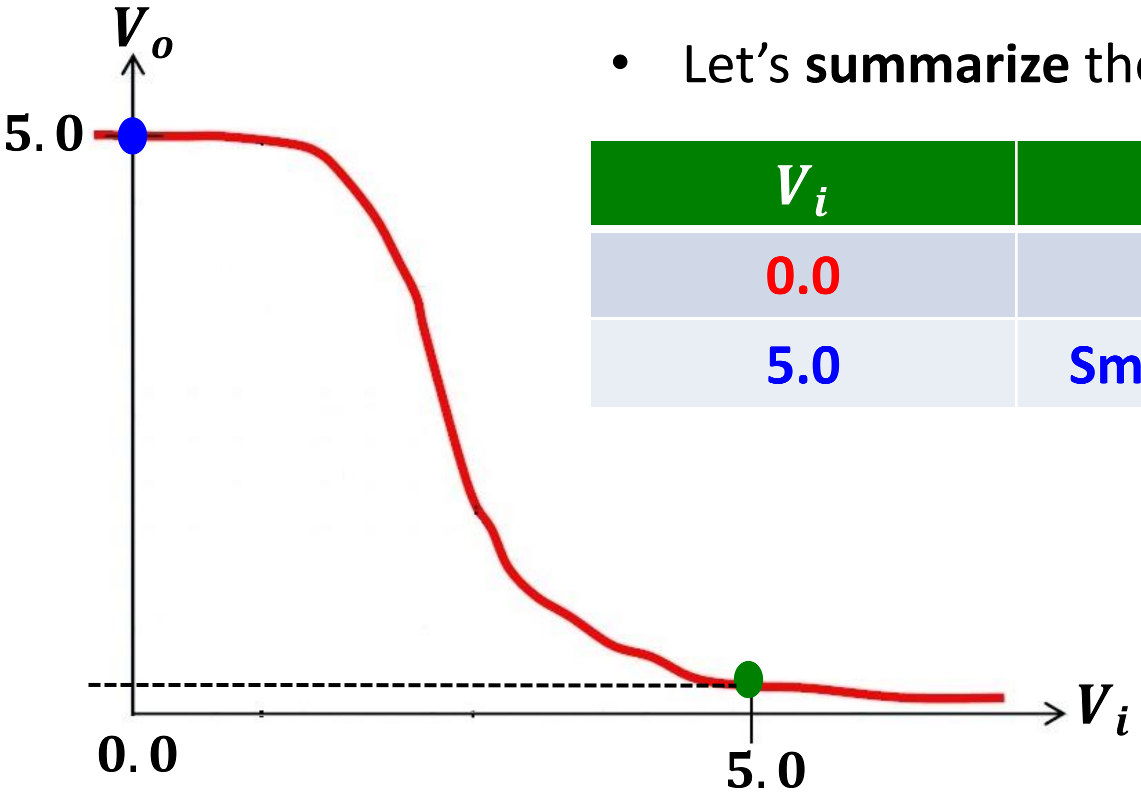
Q: I thought you said this circuit **did** something. It appears to be just as **pointless** as all the others!



A: To see how this circuit is **useful**, consider what happens when the **input** voltage V_i is 0 V and 5V.

The MOSFET as a Switch and an Amplifier (contd.)

- From the transfer function, we find that **if** $V_i = 0V$, the output voltage will be $V_o = 5V$. Likewise, **if** the input voltage is $V_i = 5V$, the output voltage will be **small.**



- Let's **summarize** these results in a table:

V_i	V_o	$Mode$
0.0	5.0	Cutoff
5.0	Small (~ 0.5)	Triode

The MOSFET as a Switch and an Amplifier (contd.)

Why, this device is not useless at all! It is **clearly** a:

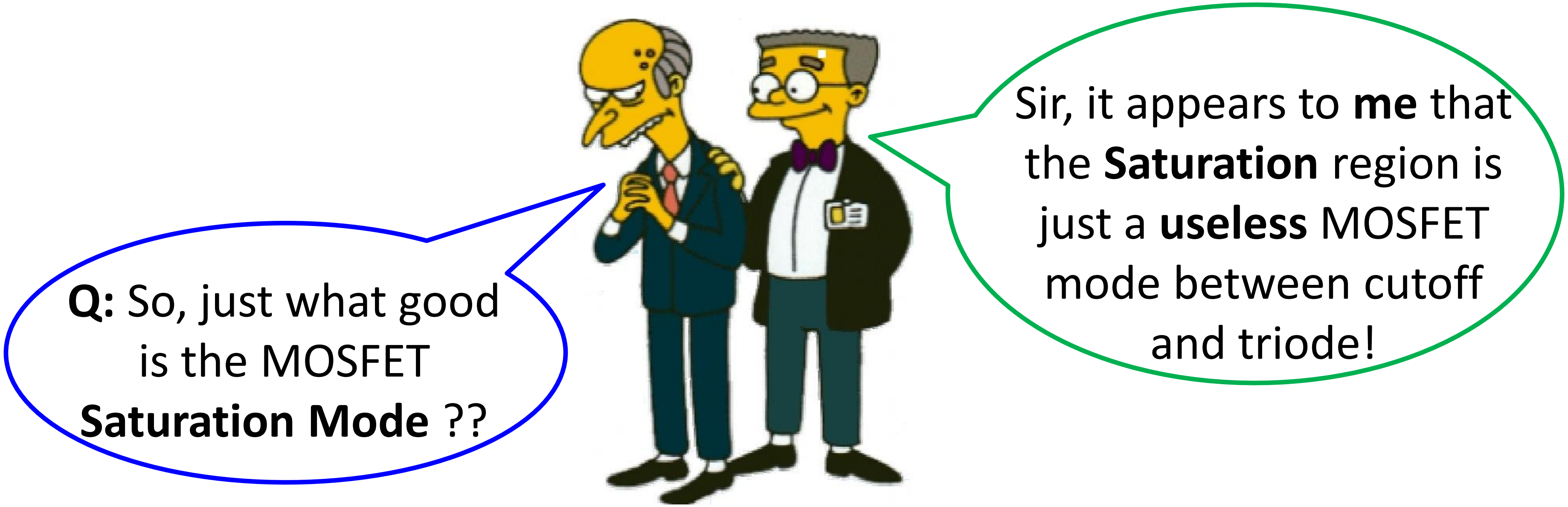
Switch

- This circuit provides a **simple** example of **one** of the primary **applications** of MOSFET devices—**digital circuit design**. We can use MOSFETs to make digital devices such as **logic gates** (AND, OR, NOR, etc.), **flip-flops**, and digital **memory**.

- We typically find that, just like this circuit, when a MOSFET digital circuit is in **either** of its **two** binary states (i.e., “0” or “1”), the MOSFETs in the circuit will either be in **cutoff** ($I_D = 0$) or in **triode** (V_{DS} small) modes.

→ **Cutoff** and **Triode** are the MOSFET modes associated with **digital** circuits and applications!

The MOSFET as a Switch and an Amplifier (contd.)



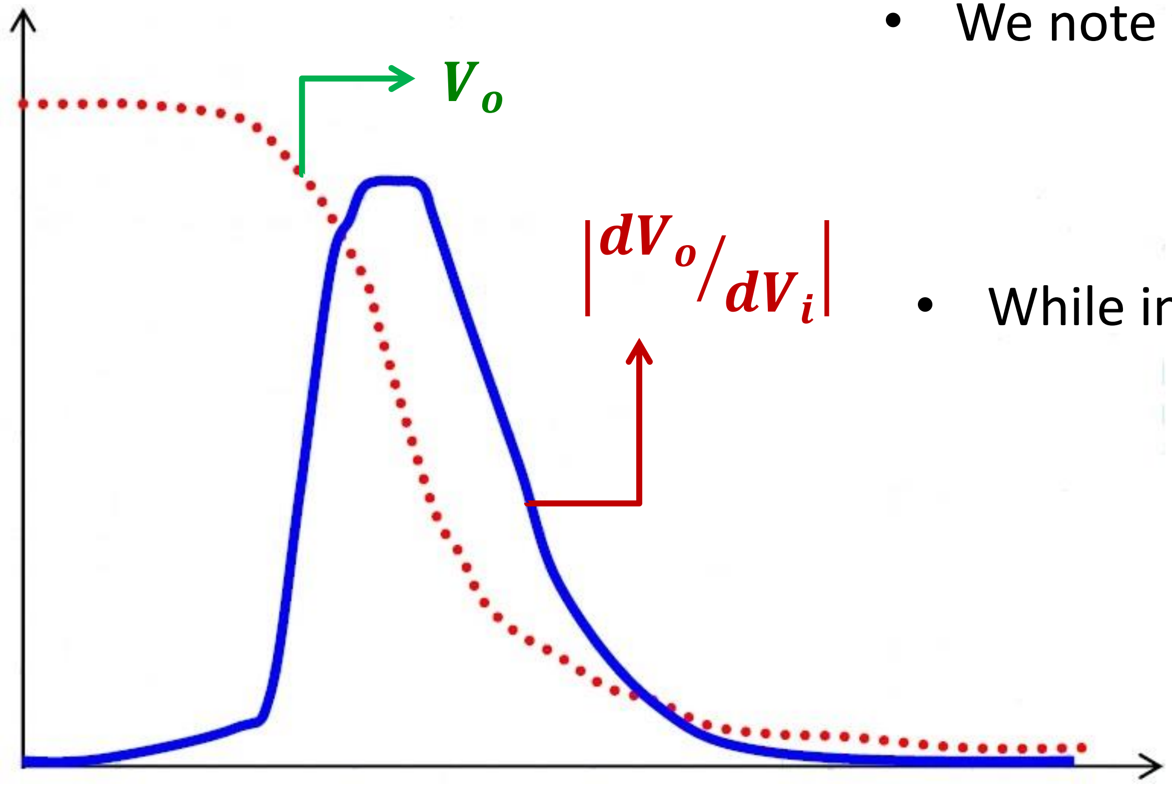
Q: So, just what good is the MOSFET **Saturation Mode** ??

Sir, it appears to **me** that the **Saturation** region is just a **useless** MOSFET mode between cutoff and triode!

A: Actually, we will find the MOSFET **saturation** mode to be **extremely useful**!

The MOSFET as a Switch and an Amplifier (contd.)

- To see why, let us take the **derivative** of the above circuit's transfer function (i.e., ${dV_o}/{dV_i}$).



- We note that in **cutoff** and **triode**:

$$\left|{dV_o}/{dV_i}\right| \approx 0$$

- While in the **saturation** mode:

$$\left|{dV_o}/{dV_i}\right| \gg 1$$

The MOSFET as a Switch and an Amplifier (contd.)

Q: Oh goody. The **slope** of the transfer function is **large** when the MOSFET is in **saturation**. Am I supposed to be impressed by that?! How are these results even **remotely** important!?

A: Since in **cutoff** and **triode** $\left|{}^{dV_o}/_{dV_i}\right| \approx \mathbf{0}$, a **small change** in input voltage V_i will result in almost **no change** in output voltage V_o.

Contrast this with the **saturation** region, where $\left|{}^{dV_o}/_{dV_i}\right| \gg \mathbf{1}$. This means that a **small change** in input voltage V_i results in a **large change** in the **output** voltage V_o!

The MOSFET as a Switch and an Amplifier (contd.)

- To see how this is important, consider the case where the input signal has both a **DC** and a **small-signal** (AC) component:

$$\boldsymbol{v_I}(\mathrm{t}) = V_i + v_i(t)$$

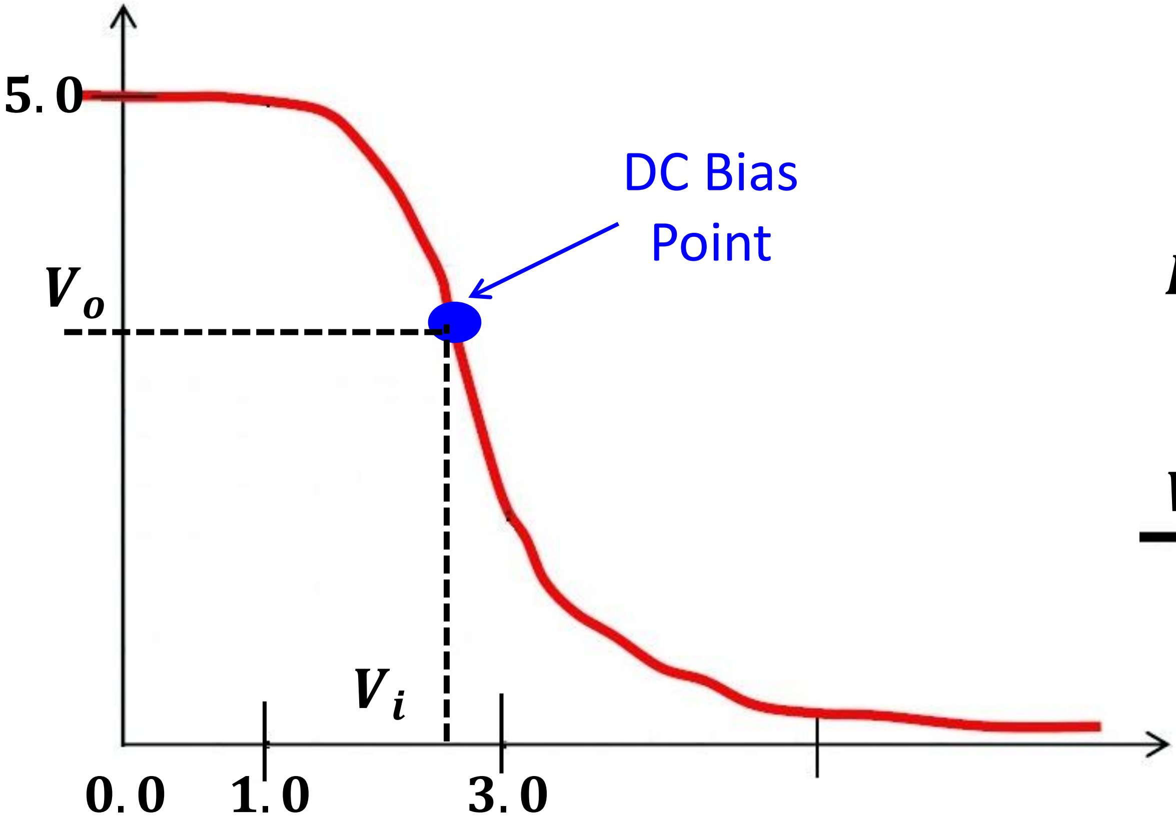
- As a result, the **output** voltage likewise has both a DC and small signal component:

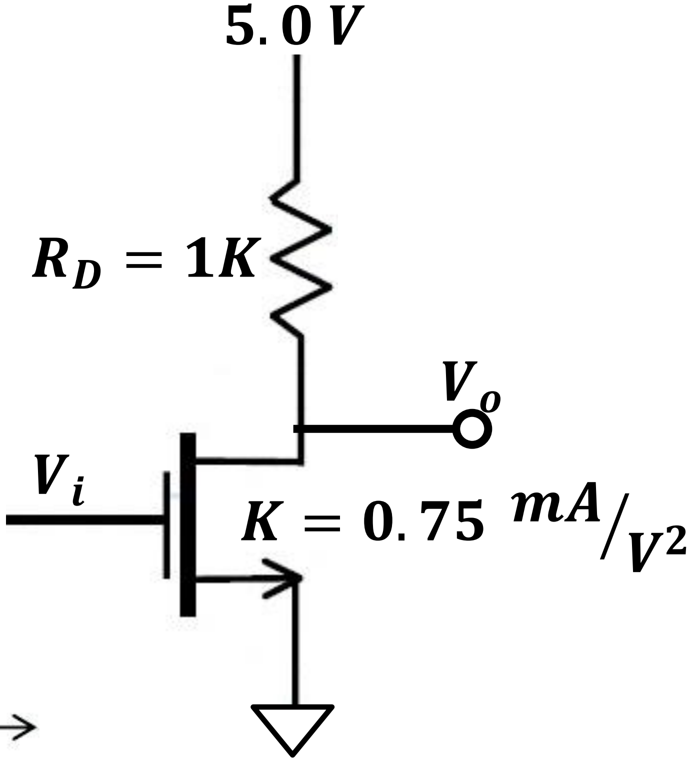
$$\boldsymbol{v_O}(\mathrm{t}) = V_o + v_o(t)$$

Now, let's consider **only** the **DC** components. We can select the DC input V_i such that the MOSFET is placed in **saturation**. The value V_i, along with the resulting DC output V_o, sets a **DC bias point** for this circuit.

By selecting the right value of V_i we **could** set this DC bias point to where the transfer function **slope** is the greatest.

The MOSFET as a Switch and an Amplifier (contd.)





Now, say we **add** a **small-signal** $\boldsymbol{v_i}$ to this input DC voltage (i.e., $\boldsymbol{v_I}(\mathrm{t}) = V_i + v_i(t)$). This small signal simply represents a small change in the input voltage from its **average** (i.e., DC) value. The result is of course as **small change** in the **output** voltage— the **small-signal output voltage** $v_o(t)$!

The MOSFET as a Switch and an Amplifier (contd.)

- Now for the **interesting** part (I bet you were wondering when I would get around to it)! The small change in the output voltage will have a much **larger** magnitude than the small change in the input!

For example, if the input voltage changes by **1 mV** (i.e., $v_i = 1\text{mV}$), the output **might** change by, say, **5 mV** (i.e., $v_o = 5mV$).

Q: Goodness! By how much would the **output** change in our example circuit? How can we **determine** the small signal **output** v_o ??

The MOSFET as a Switch and an Amplifier (contd.)

- Determining how much the output voltage of our circuit will change when we change the input voltage by a small amount is very **straightforward**—we simply take the **derivative** of the output voltage v_O with respect to input voltage v_I.

  - By taking the **derivative** of v_O with respect to v_I (when the MOSFET is in saturation, we find:

$$\frac{dv_O}{dv_I} = \frac{d\left(5.0 - 0.75(v_I - 1.0)^2\right)}{dv_I} \Longrightarrow = -1.50(v_I - 1.0)$$

<u>when</u> $\mathbf{1.0 < v_I < 3.0\ V}$

The expression describes the **slope** of our circuit's transfer function (for $\mathbf{1.0 < v_I < 3.0\ V}$). Note the slope with the **largest magnitude** occurs when $v_I = 3.0V$, providing a **slope** of -3.0 mV/mV.

The MOSFET as a Switch and an Amplifier (contd.)

- Thus, **if** we DC bias this circuit with V_i = 3.0 V (resulting in V_o = 2.0 V), we find that the small signal output will be **3 times** the small signal input!
- **For example**, say that the **input** to our circuit is:

$$\boldsymbol{v_I}(\mathrm{t}) = 3.0 + 0.01\cos(\omega t)\ V$$

Here: $V_i = 3.0\ V$ $v_i = 0.01\cos(\omega t)$

- We would find that the **output voltage** would approximately be:

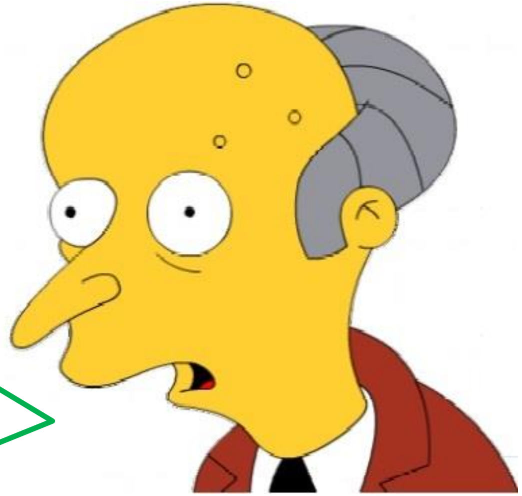
$$\boldsymbol{v_O}(\mathrm{t}) = 2.0 - 0.03\cos(\omega t)\ V$$

Here: $V_o = 2.0\ V$ $v_o = -0.03\, cos(\omega t)$

In other words, the magnitude of the small-signal output has a magnitude **three times larger** than the input magnitude.

We say then that our signal provides **small-signal gain**—our circuit is also a **small-signal amplifier**!

The MOSFET as a Switch and an Amplifier (contd.)



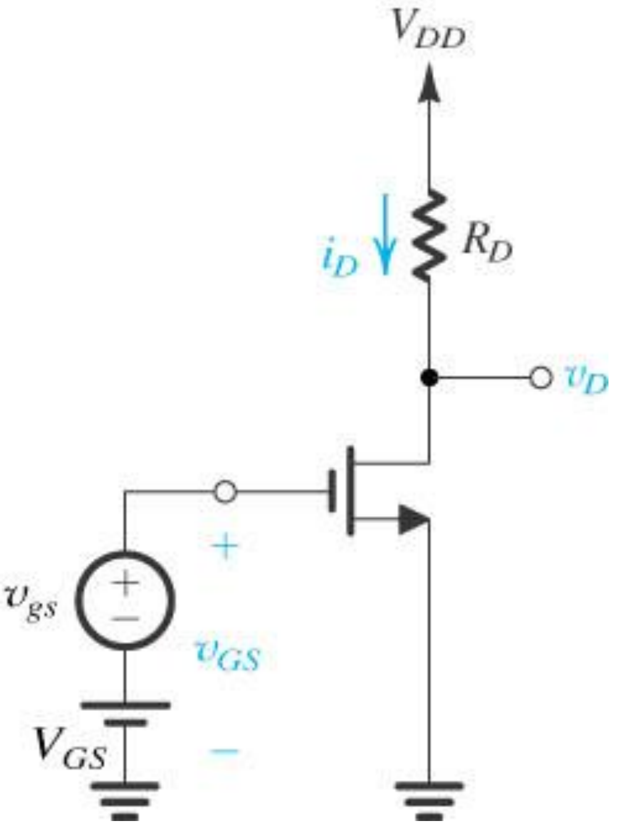
I see. A **small** voltage change results in a **big** voltage change—it's **voltage gain**!

The **MOSFET saturation** mode turns out to be—**excellent**.

- Even the simple circuit of this example is sufficient to demonstrate the **two primary applications** of MOSFET transistors--**digital** circuits and signal **amplification**.
- Whereas the important MOSFET regions for **digital** devices are **triode** and **cutoff**, MOSFETs in amplifier circuits are typically biased into the **saturation** mode!

MOSFET – Small Signal Operation

- Consider this circuit, which has both a **DC** and an AC **small signal** source. As a result, each voltage and current in the circuit has **both** a DC and small-signal component.



- If the MOSFET is in **saturation**, then the **total** drain current is:

$$i_D = K\left[V_{GS} + v_{gs} - V_T\right]^2$$

$$\Rightarrow i_D = \underbrace{K[V_{GS} - V_T]^2}_{I_D} + \underbrace{2K[V_{GS} - V_T]v_{gs}}_{i_d} + Kv_{gs}^2$$

Very Small If: $v_{gs} \ll 2[V_{GS} - V_T]$

We call this equation the **small-signal** condition.

MOSFET – Small Signal Operation (contd.)

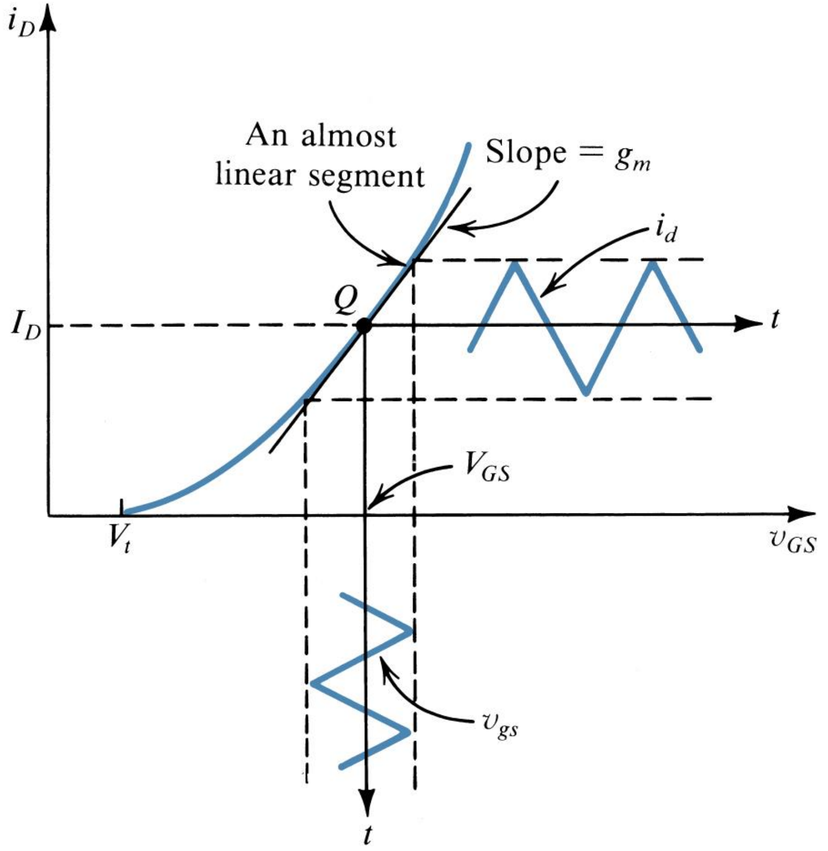
Now, $i_d = 2K[V_{GS} - V_T]v_{gs}$ $\quad \Rightarrow \dfrac{i_d}{v_{gs}} = \boxed{2K[V_{GS} - V_T]} \leftarrow \mathbf{g_m}$

Alternatively, $\Rightarrow g_m = 2KV_{OV}$

- The small-signal parameter **g_m** can likewise be **derived** from a small-signal analysis of the drain current:

$$i_d = \left.\frac{di_D}{dV_{GS}}\right|_{v_{GS}=V_{GS}} (v_{gs}) = 2K[v_{GS} - V_T]\Big|_{v_{GS}=V_{GS}} (v_{gs})$$

$$i_d = 2K[V_{GS} - V_T](v_{gs}) \qquad\qquad i_d = g_m(v_{gs})$$



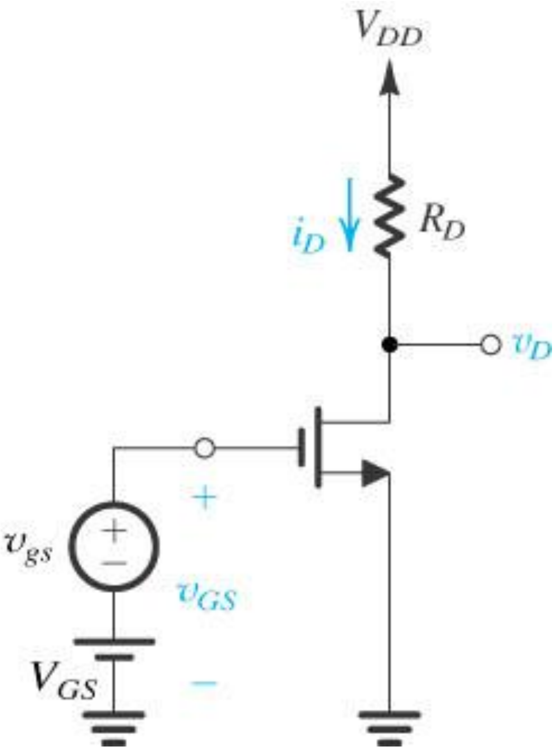
Physical meaning of the g_m

→ formal definition

MOSFET – Small Signal Operation (contd.)

The MOSFET transconductance relates a small **change** in v_{GS} to a small **change** in drain current i_D. This change is completely dependent on the **DC bias** point of the MOSFET, V_{GS} and I_D.

MOSFET – Small Signal Operation (contd.)



- The total instantaneous drain voltage $\boldsymbol{v_D}$ is given by:

$$v_{DS} = v_D = V_{DD} - i_D R_D$$

- Under small signal condition it changes to:

$$v_D = V_{DD} - R_D\left(I_D + i_d\right)$$

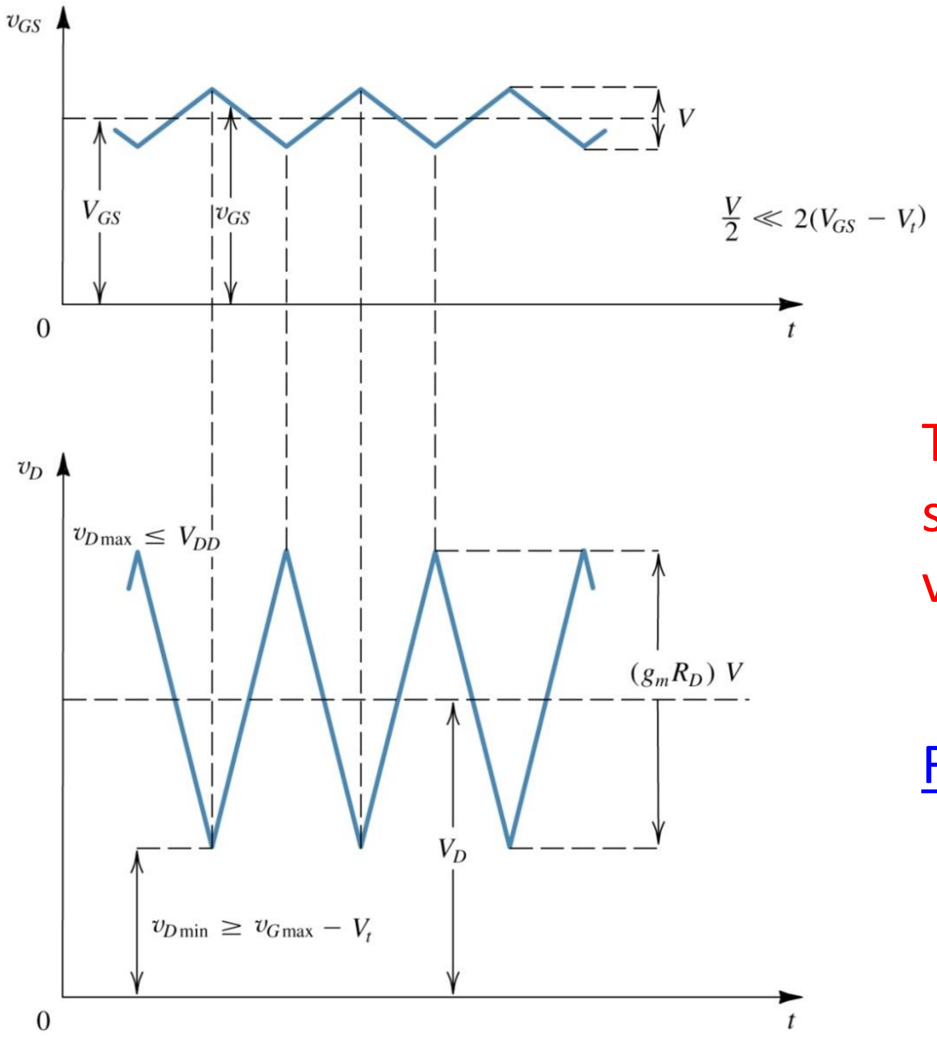
$$\Rightarrow V_D + v_d = V_{DD} - I_D R_D - i_d R_D$$

Signal component of drain voltage (v_d)

$$v_d = -i_d R_D = -g_m v_{gs} R_D \qquad \Rightarrow A_v = \frac{v_d}{v_{gs}} = -g_m R_D$$

Thus, if $g_m R_D \gg 1$, we have small-signal **voltage gain**.

MOSFET – Small Signal Operation (contd.)



$$A_v = \frac{v_d}{v_{gs}} = -g_m R_D$$

It indicates that v_d is 180° out of phase with respect to v_{gs}

The input has been assumed very small as compared to overdrive voltage [$v_{gs} << 2(V_{GS}-V_T)$]

<u>For saturation:</u> $v_{D\min} \geq v_{G\max} - V_T$

$$v_{D\max} \leq V_{DD}$$