# Lecture – 22

# Date: 16.11.2015

- Feedback Topologies
- Stability
- Intro to Frequency Compensation

# Feedback Topologies

- **Voltage-Voltage Feedback (also called Shunt-Series Feedback):** both the input and output of the feedback circuit is voltage
- **Voltage-Current Feedback (also called Shunt-Shunt Feedback):** input of feedback is voltage and output is current
- **Current-Voltage (also called Series-Series Feedback):** input of feedback is current and output is voltage
- **Current-Current (also called Series-Shunt Feedback):** both the input and output of feedback circuit is current

# Feedback Topologies (contd.)

**Voltage – Voltage Feedback**

**Sensed Quantity is Voltage**

**Returned Quantity is Voltage**

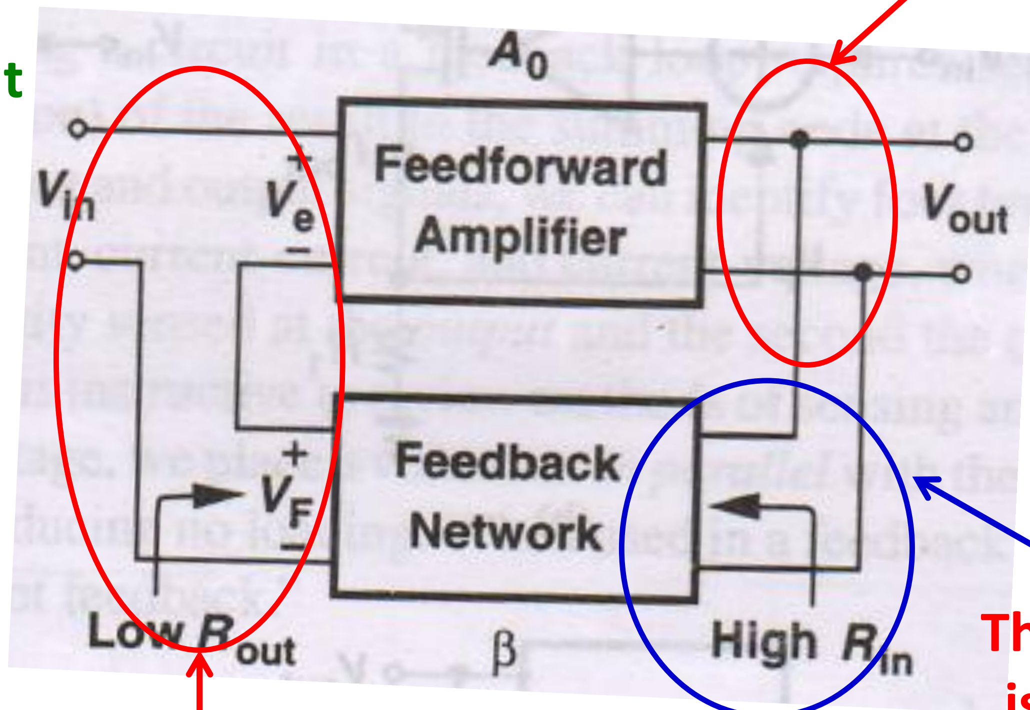
**Voltmeter Type Characteristics**

**Should be added/subtracted in series**

# Voltage-Voltage Feedback

Voltmeter Type Connection → Parallel Sensing

Increased Input Impedance

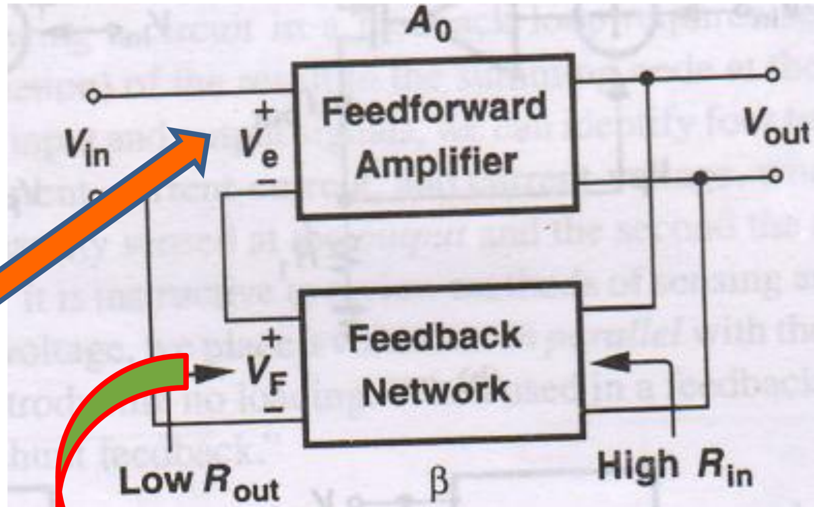



Reduced Output Impedance

This high impedance is in parallel to the feedforward amplifier

Subtracted in series

# Voltage-Voltage Feedback (contd.)




$$V_e = V_{in} - V_F$$

$$V_F = \beta V_{out}$$

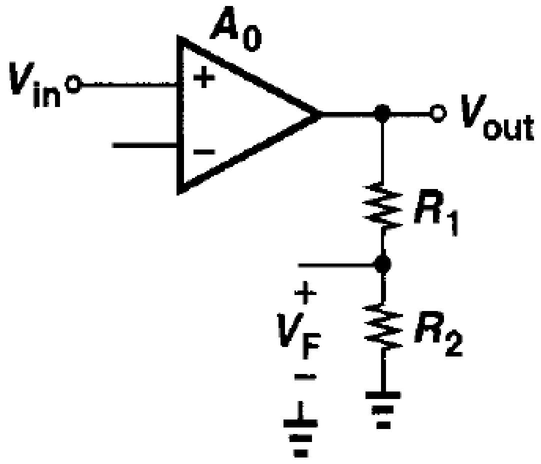
$$V_{out} = A_0 V_e = A_0 \left( V_{in} - \beta V_{out} \right)$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{A_0}{1 + \beta A_0}$$

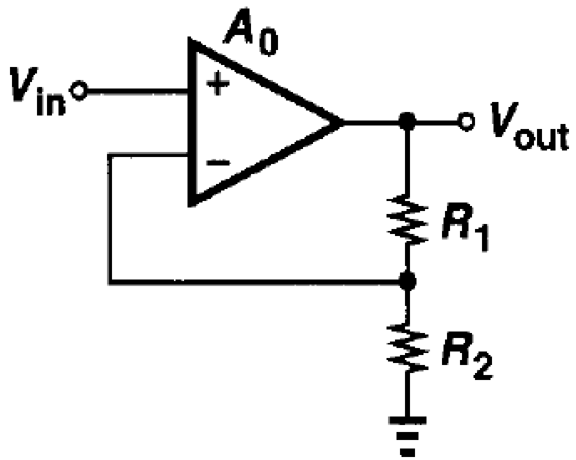
**Closed loop gain → modified gain → smaller !!!**

# Voltage-Voltage Feedback (contd.)

## Example: Voltage-Voltage Feedback




For voltage sensing – parallel to the output node of this differential input but single ended output amplifier



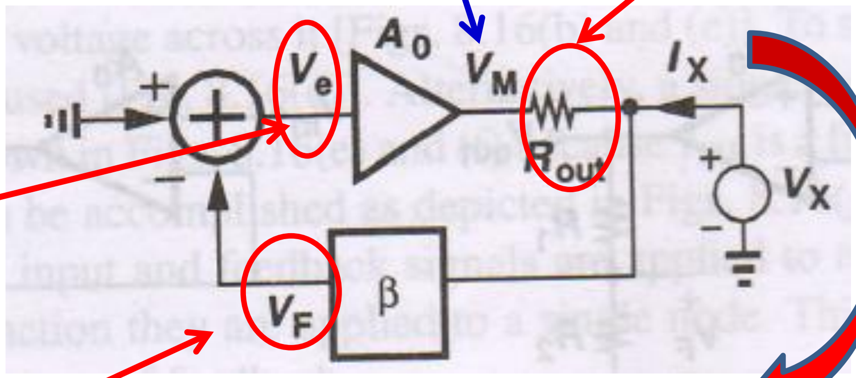

The voltage signal from feedback network is fed to the other input node of the differential amplifier

# Voltage-Voltage Feedback (contd.)

## Output Impedance

$$V_M = A_0 V_e = -A_0 \beta V_X$$

Open-Loop Output Impedance




$$V_e = -V_F = -\beta V_X$$

$$V_F = \beta V_X$$

$$I_X = \frac{[V_X - (-A_0 \beta V_X)]}{R_{out}}$$

$$\Rightarrow R_{out,closed} = \frac{V_X}{I_X} = \frac{R_{out}}{1 + A_0 \beta}$$

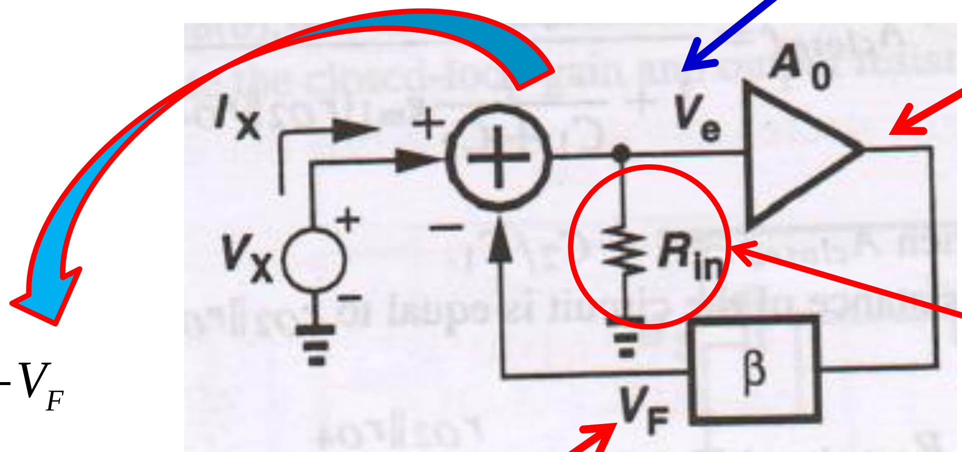
Reduced Closed-Loop Output Impedance

# Voltage-Voltage Feedback (contd.)

## Input Impedance

$$V_e = I_X R_{in}$$

$$A_0 I_X R_{in}$$




Open-Loop Input Impedance

$$V_e = V_X - V_F$$

$$V_F = \beta A_0 I_X R_{in}$$

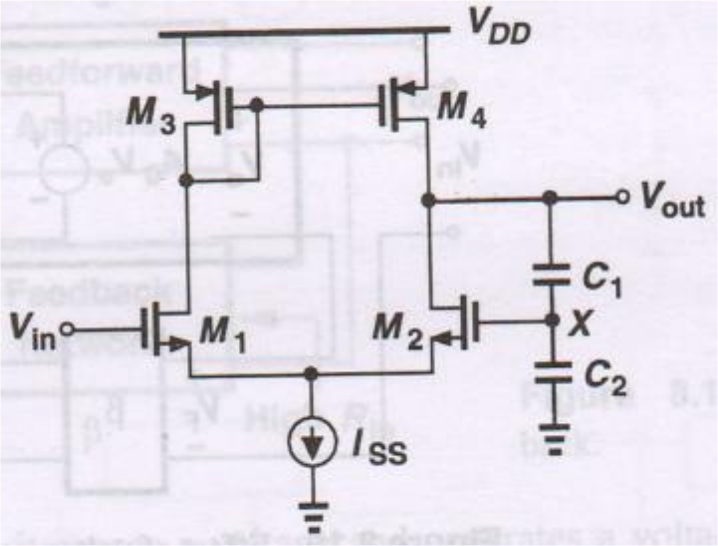
$$\Rightarrow V_e = V_X - V_F = V_X - \beta A_0 I_X R_{in} = I_X R_{in}$$

$$\therefore R_{in,closed} = \frac{V_X}{I_X} = R_{in}\left(1 + \beta A_0\right)$$

Increased Input Impedance

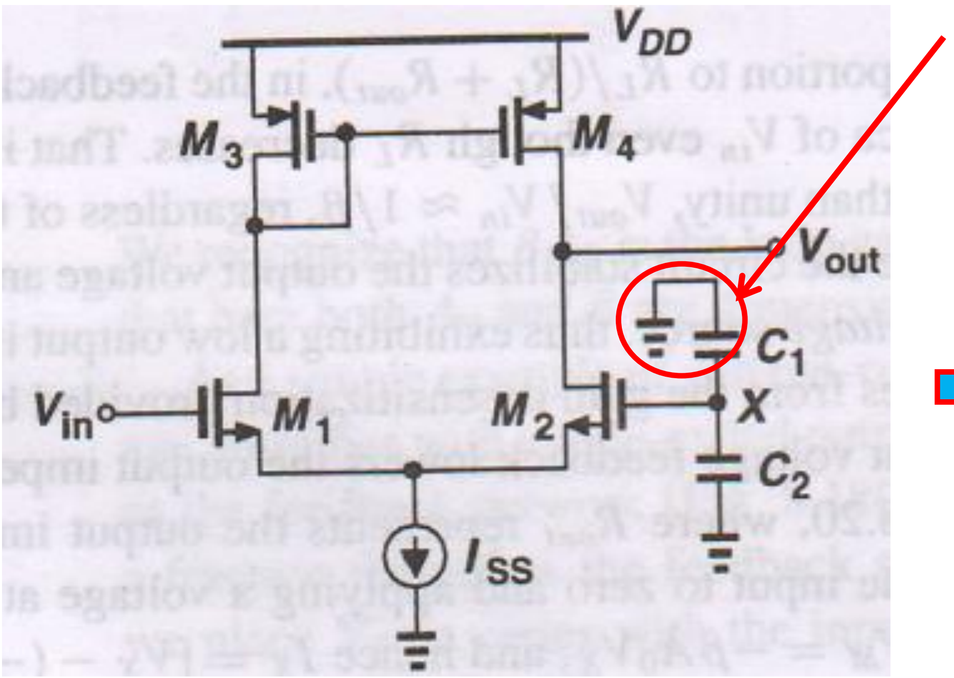
# Voltage-Voltage Feedback (contd.)

**Example: calculate gain and output impedance of the following circuit**



# Voltage-Voltage Feedback (contd.)

## Step-1: determine open-loop voltage gain



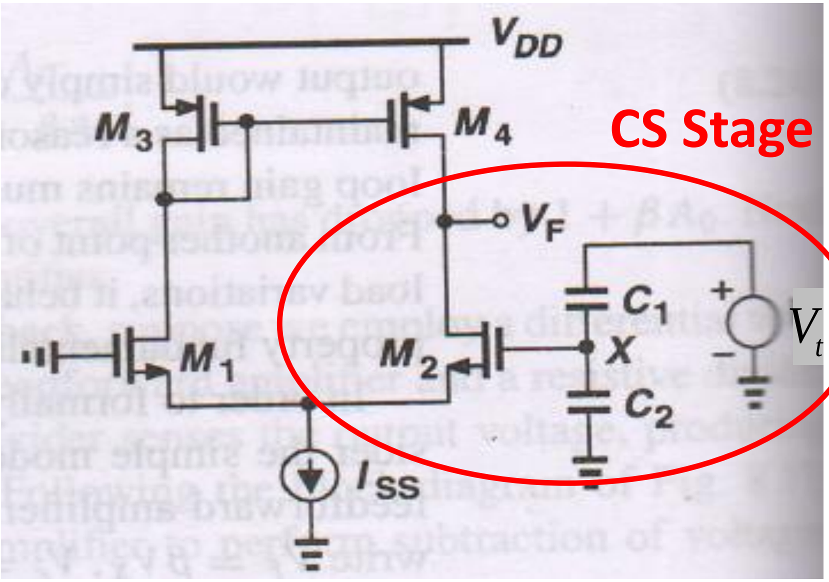

Grounding ensures there is no voltage feedback

Open-loop gain is:

$$A_0 = g_{m1}\left(r_{o2} \parallel r_{o4}\right)$$

# Voltage-Voltage Feedback (contd.)

## Step-2: determine the loop gain




$$V_F = -V_t \frac{C_1}{C_1 + C_2} g_{m1} (r_{o2} \| r_{o4})$$

Drain Current

Output impedance

**Therefore,**

$$\beta A_0 = \frac{C_1}{C_1 + C_2} g_{m1} (r_{o2} \| r_{o4})$$

$$\Rightarrow A_{closed} = \frac{A_0}{1 + \beta A_0} = \frac{g_{m1}(r_{o2} \| r_{o4})}{1 + \frac{C_1}{C_1 + C_2} g_{m1}(r_{o2} \| r_{o4})}$$

# Voltage-Voltage Feedback (contd.)

- **For $\beta A_0 >> 1$,**

$$A_{closed} \simeq \frac{g_{m1}(r_{o2} \| r_{o4})}{\frac{C_1}{C_1+C_2} g_{m1}(r_{o2} \| r_{o4})} = 1 + \frac{C_2}{C_1}$$

- **The closed-loop output impedance,**

$$R_{out,closed} = \frac{R_{out,open}}{1+\beta A_0} = \frac{(r_{o2} \| r_{o4})}{1 + \frac{C_1}{C_1+C_2} g_{m1}(r_{o2} \| r_{o4})}$$

- **For $\beta A_0 >> 1$,**

$$R_{out,closed} \simeq \left(1 + \frac{C_2}{C_1}\right)\frac{1}{g_{m1}}$$

Relatively Smaller Value

# Stability Issues in Feedback Amplifiers

- **The generic closed-loop transfer function:**

$$A_{closed}(j\omega) = \frac{A_0(s)}{1 + A_0(s)\beta(s)}$$

**It is assumed that both the open-loop gain and the feedback gain is frequency dependent**

**At low frequencies:**

- **β(s) is assumed as a constant value and $A_0$(s) is also assumed as a constant value → the loop gain becomes constant → obviously this happens for any direct-coupled amplifier with poles and zeros present at high frequency → the loop gain (Aβ) should be positive value for negative feedback**

# Stability Issues in Feedback Amplifiers (contd.)

## At high frequencies:

$$A_{closed}(j\omega) = \frac{A_0(j\omega)}{1 + A_0(j\omega)\beta(j\omega)}$$

- Therefore it is apparent that the loop gain is:

$$L(j\omega) = A_0(j\omega)\beta(j\omega) = |A_0(j\omega)\beta(j\omega)| e^{j\varphi(\omega)}$$

Phase Angle

Magnitude

It is real with negative sign at the frequency when $\varphi(\omega)$ is $180^{O}$

If for $\omega=\omega_1$, the loop gain is less than unity

What happens to stability?

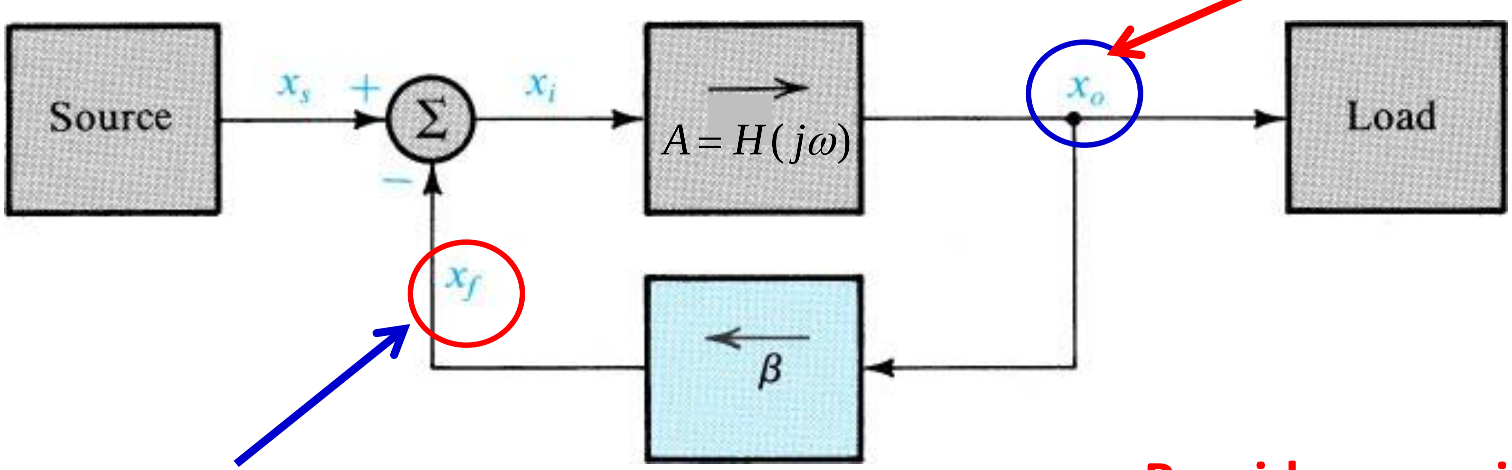
# Stability Issues in Feedback Amplifiers (contd.)

## At high frequencies:

If at $\omega=\omega_1$, the loop gain (L) is equal to unity with negative sign → Closed-loop gain will be infinite → even for zero input there will be some output → an oscillation condition!!!

## For no input: (just excited by some noise)

$$X_o = H(j\omega)X_i$$




$$X_f = \beta H(j\omega)X_i = -X_i$$

→ Provides a positive feedback which can sustain oscillation

# Stability Issues in Feedback Amplifiers (contd.)

## At high frequencies:

If at $\omega=\omega_1$, the loop gain (L) is higher than unity → the circuits (specially the amplifiers) undergo sustained damping until the loop gain (L) reaches unity → this will then provide oscillation condition!!!

## Summary:

- The phase angle of the loop gain equals $-180^{O}$
- The magnitude of the loop gain is either unity or greater than unity

Both exist together to cause oscillation

Barkhausen Criterion

**Boundary Condition**

$$|\beta H(j\omega_1)|=1$$

$$\angle\beta H(j\omega_1)=-180^{\circ}$$

# Stability Issues in Feedback Amplifiers (contd.)

## Boundary Condition

$$|\beta H(j\omega_1)| = 1$$
$$\angle \beta H(j\omega_1) = -180°$$

Notice that the total phase shift around the loop at $\omega_1$ is $360^O$ because negative feedback itself introduces $180^O$

$360^O$ phase shift is necessary for oscillation since the feedback signal must add in phase to the original noise to allow oscillation

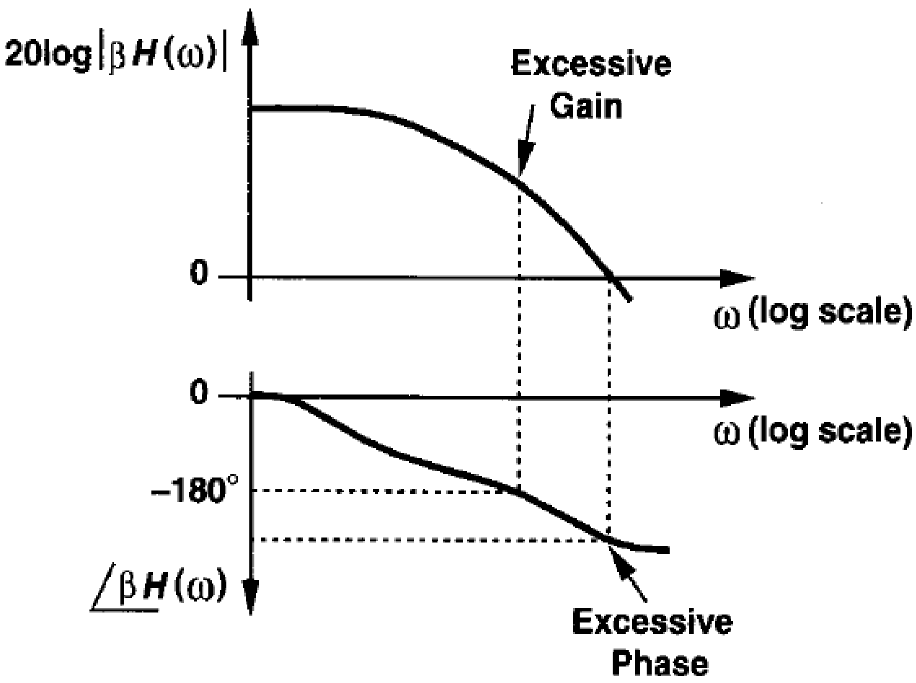
Similarly, a loop gain of unity (or greater) is also required to enable growth of the oscillation amplitude

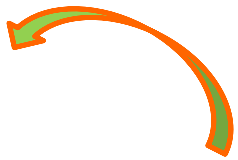
# Stability Issues in Feedback Amplifiers (contd.)

## Summary

**A negative feedback system may oscillate at $\omega_1$ if:**

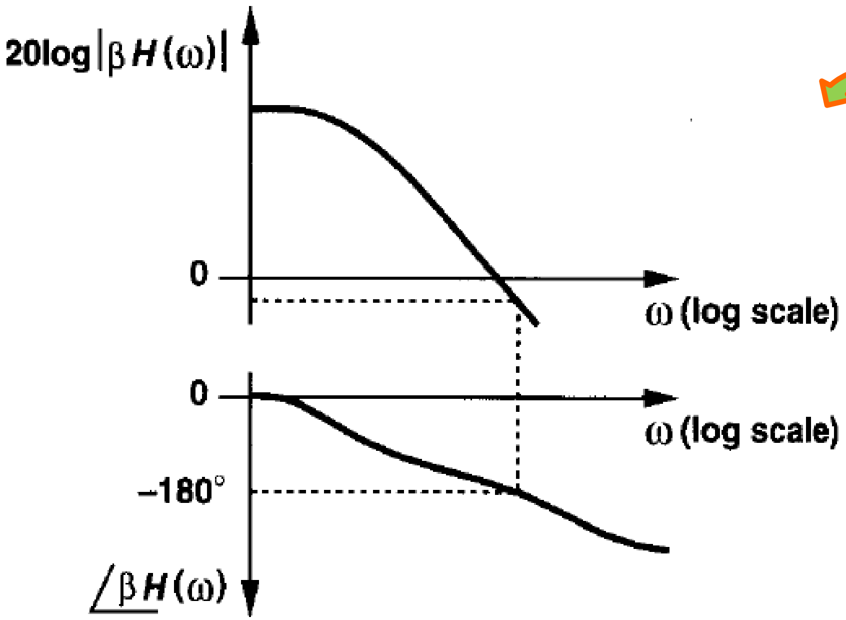
- **the phase shift around the loop at this frequency is so much that the feedback becomes positive, and**
- **the loop gain is still enough to allow signal buildup**



Unstable system

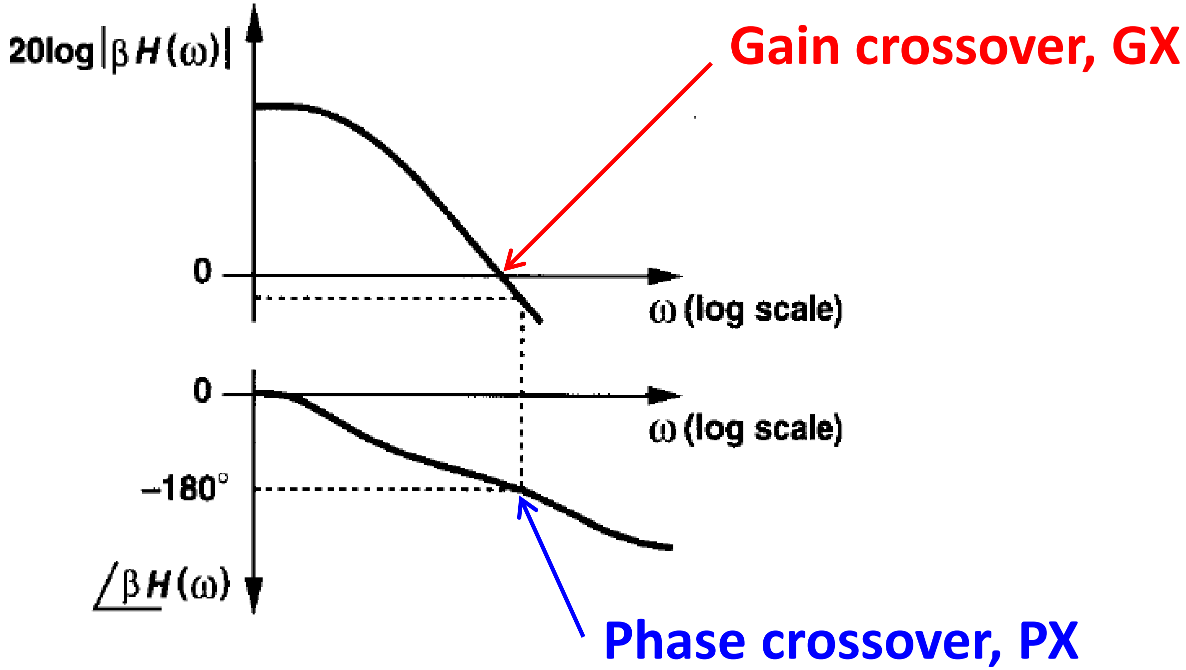
# Stability Issues in Feedback Amplifiers (contd.)




stable system

To make the system stable, the idea is to minimize the total phase shift so that for $|\beta H|=1$, $<\beta H$ is still more positive than $-180^{o}$

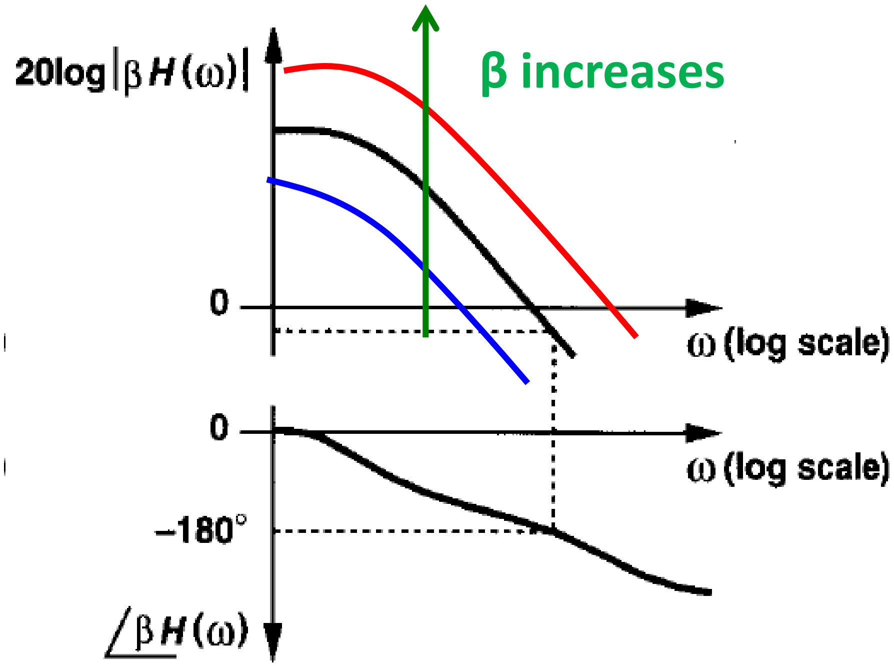
# Stability Issues in Feedback Amplifiers (contd.)




If β is reduced (i.e., less feedback is applied), the magnitude plot will shift down → essentially moves GX closer to origin → in turn makes the system more stable

# Stability Issues in Feedback Amplifiers (contd.)

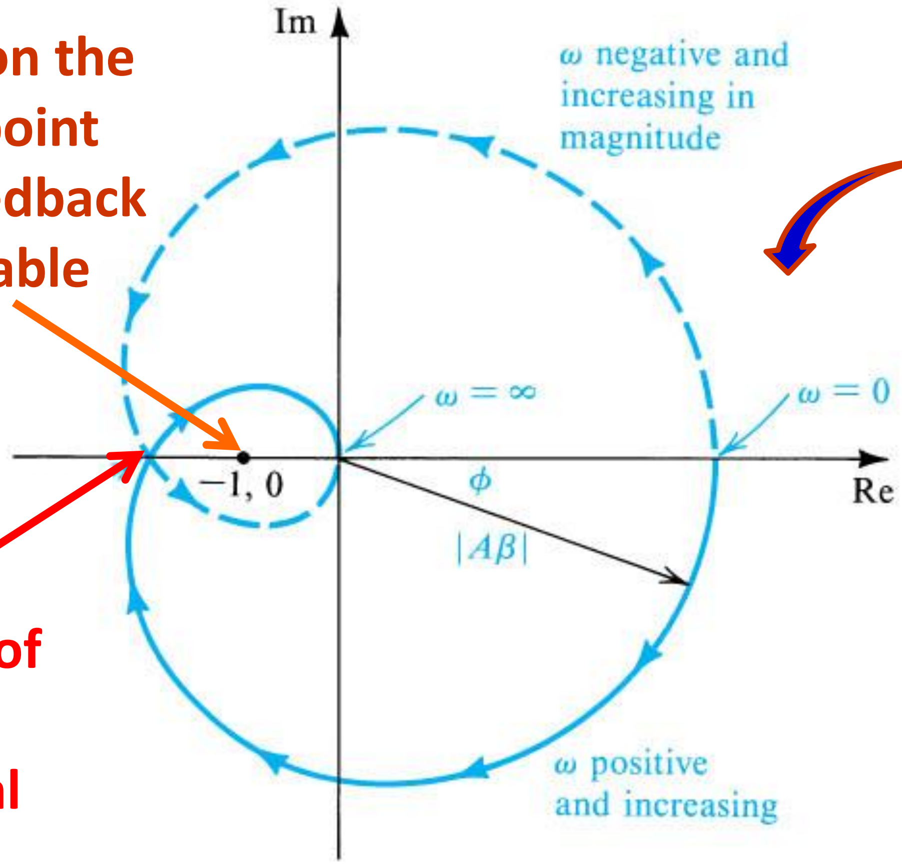
- For a unity gain (β=1) Feedback



# Stability Issues in Feedback Amplifiers (contd.)

## Stability Test: Nyquist Plot

Intersection on the left of this point makes the feedback circuit unstable




Polar plot of loop gain with varying frequency

Intersection of |Hβ| with negative real axis: $\omega = \omega_1$

# Stability Issues in Feedback Amplifiers (contd.)

**Stability and Pole Location** → the transient response of an amplifier with a pole pair $s_p = \sigma_p \pm j\omega_p$ subjected to disturbance will show a transient response:

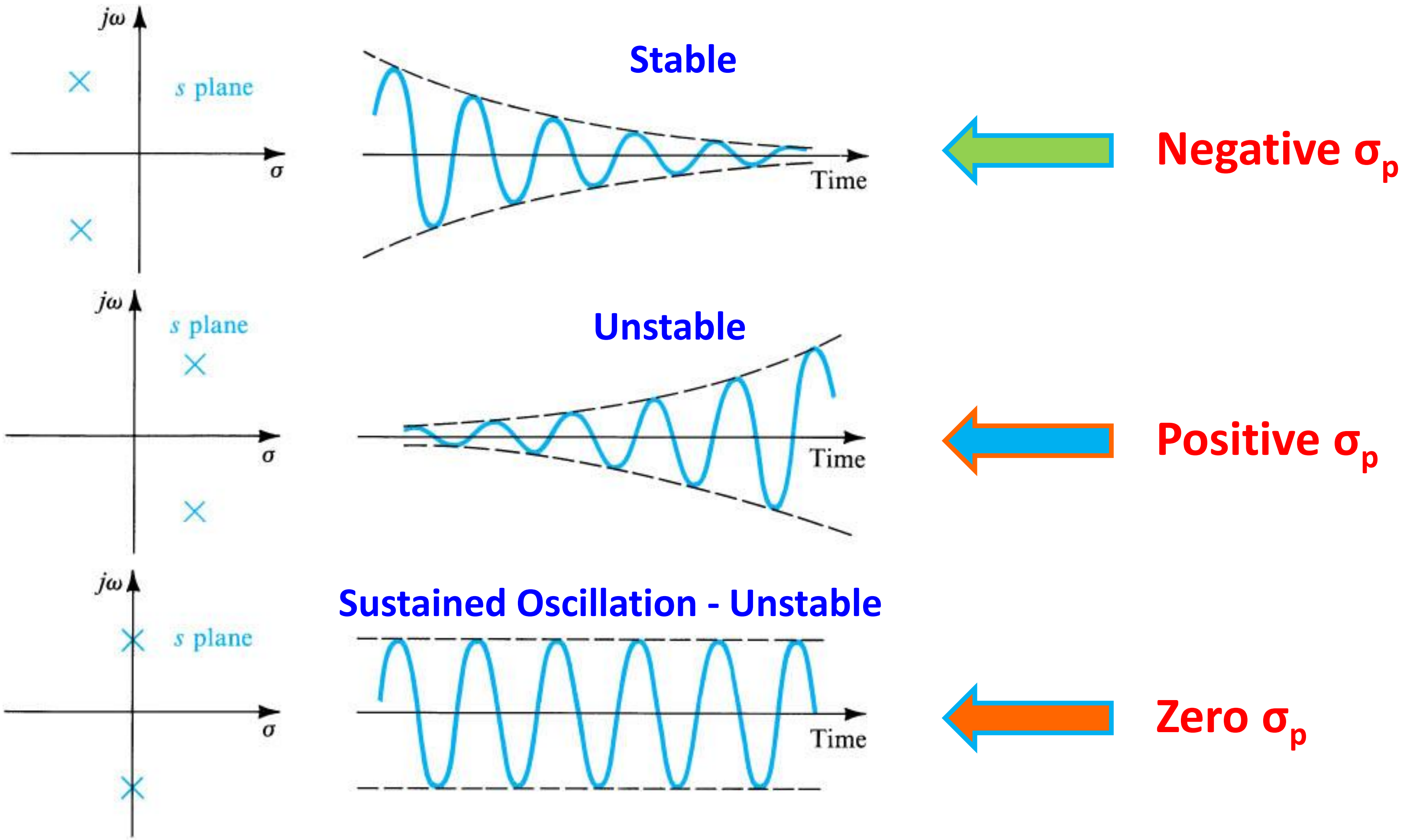
$$x(t) = e^{\sigma_p t}\left[e^{+j\omega_p t} + e^{-j\omega_p t}\right] = 2e^{\sigma_p t}\cos(\omega_p t)$$

**Envelope** ($e^{\sigma_p t}$)   **Sinusoid** ($e^{+j\omega_p t} + e^{-j\omega_p t}$)

- **For poles in right half of the s-plane the oscillations will grow exponentially considering that $\sigma_p$ will be positive**
- **For poles with $\sigma_p$ =0, the oscillation will be sustained**
- **For poles in the left half of the s-plane the term $\sigma_p$ will be negative and therefore the oscillation will decay exponentially towards zero**

# Stability Issues in Feedback Amplifiers (contd.)

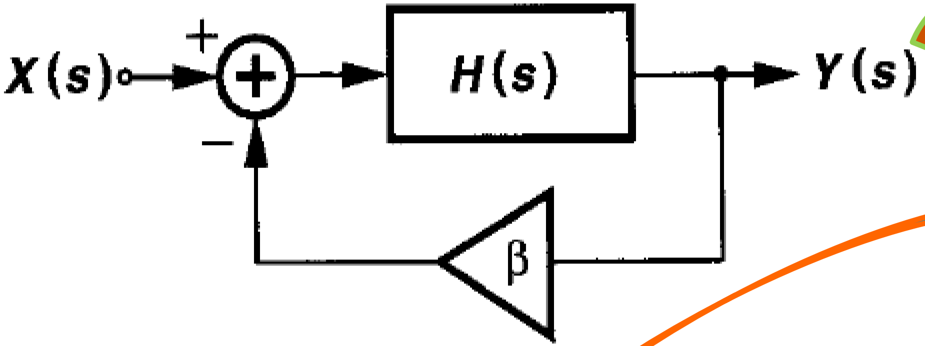
## Stability and Pole Location




**Obviously, the presence of zeros have been ignored**

# Poles of the Feedback Amplifier

- Study of single-pole feedforward amplifier




Closed-loop transfer function

$$A_{closed}(s) = \frac{Y}{X}(s) = \frac{H(s)}{1+\beta H(s)}$$

Where, $H(s) = \frac{A_0}{\left(1+s/\omega_p\right)}$

Then the closed-loop transfer function becomes

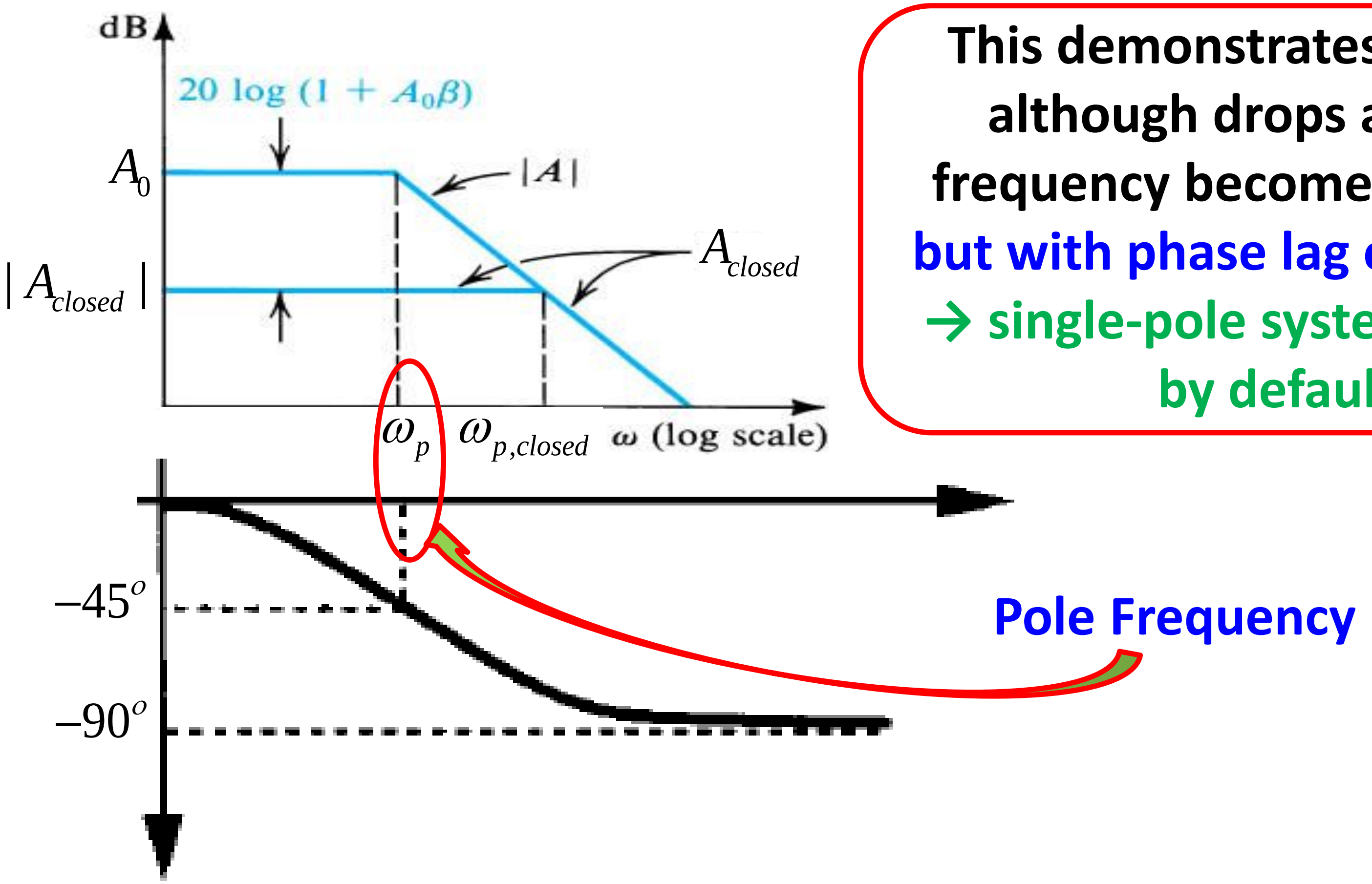
$$A_{closed}(s) = \frac{A_0/(1+A_0\beta)}{1+s/\omega_P(1+A_0\beta)}$$

It is apparent that the feedback moves the pole frequency from $\omega_p$ to:

$$\omega_{p,closed} = \omega_p(1+A_0\beta)$$

# Amplifier with a Single Pole (contd.)

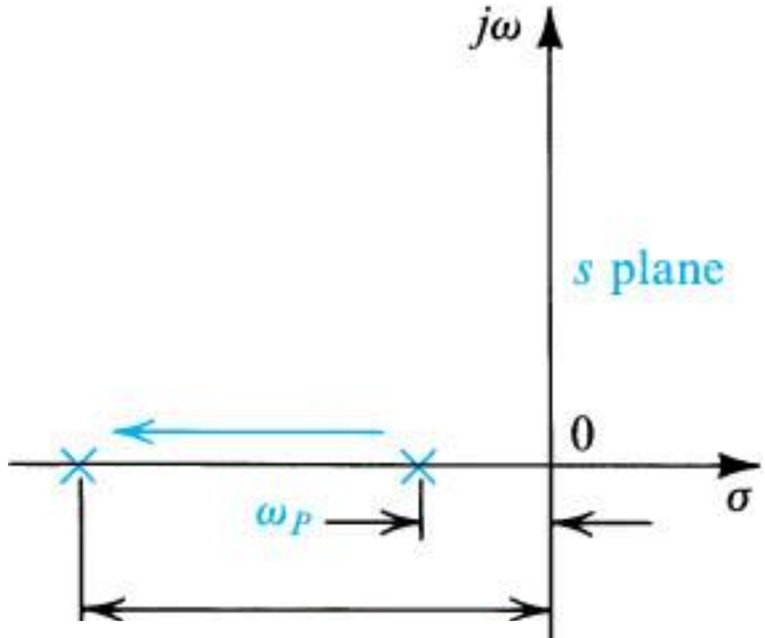
- The frequency response of amplifier with and without feedback




This demonstrates that gain although drops and pole frequency becomes bigger → but with phase lag of only -90$^o$ → single-pole system is stable by default

# Amplifier with a Single Pole (contd.)

Root Locus




$$\omega_{p,closed} = \omega_p(1 + A_0\beta)$$

**The original pole and its movement with feedback**

**It is apparent that the pole never enters the right half of the s-plane → unconditionally stable scenario!**

# Amplifier with Two Poles

- Open-loop transfer function of an amplifier with two pole is given as:

$$A(s) = \frac{A_0}{(1+s/\omega_{P1})(1+s/\omega_{P2})}$$

- The closed-loop poles are obtained from: $1 + A(s)\beta = 0$

$$\Rightarrow s^2 + s(\omega_{P1} + \omega_{P2}) + (1 + A_0\beta)\omega_{P1}\omega_{P2} = 0$$

- Therefore the closed-loop poles are:

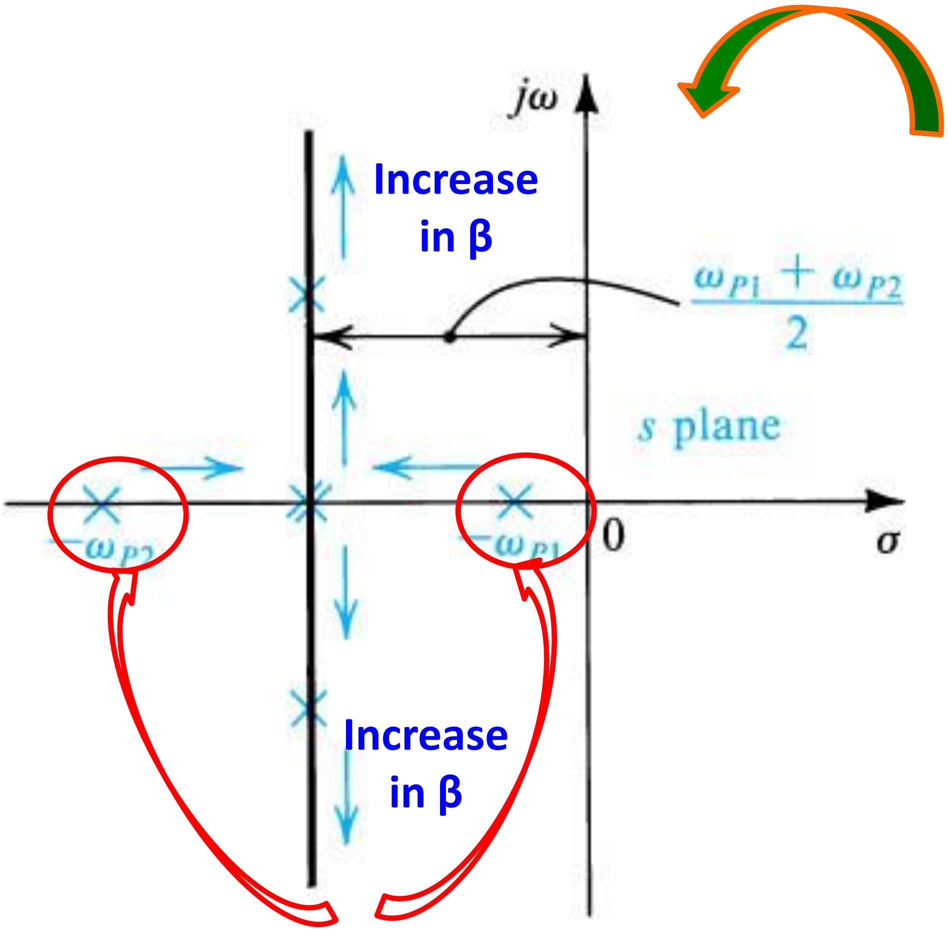
$$s = -\frac{1}{2}(\omega_{P1} + \omega_{P2}) \pm \frac{1}{2}\sqrt{(\omega_{P1} + \omega_{P2})^2 - 4(1 + A_0\beta)\omega_{P1}\omega_{P2}}$$

- As the loop gain $A_0\beta$ is increased from zero, the poles come closer
- At certain $A_0\beta$ the poles will coincide
- Further increase in $A_0\beta$ make poles complex conjugate which move along a vertical line

# Amplifier with Two Poles (contd.)

**Root-locus Diagram**

$$s = -\frac{1}{2}(\omega_{P1} + \omega_{P2}) \pm \frac{1}{2}\sqrt{(\omega_{P1} + \omega_{P2})^2 - 4(1 + A_0\beta)\omega_{P1}\omega_{P2}}$$

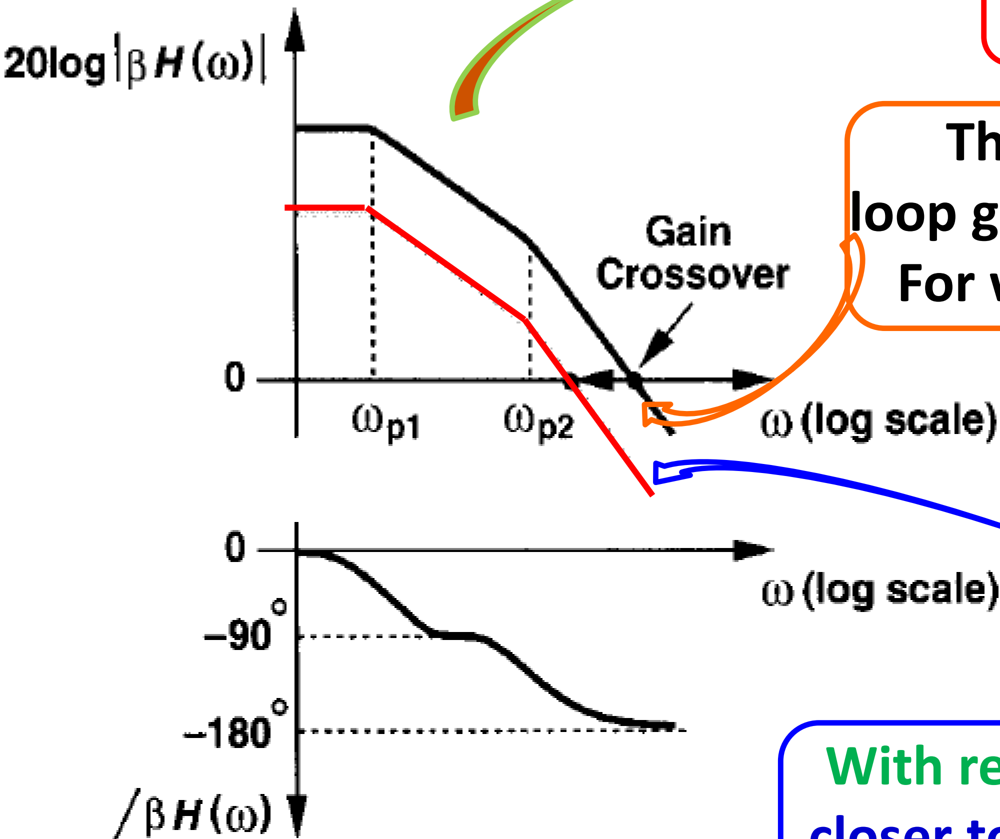



Poles when no feedback (i.e., β = 0)

- Root-locus shows that the poles never enter the right half of s-plane
  - Unconditionally stable !!!
- Reason is simple: the maximum phase shift of A(s) is -180$^{o}$ (-90$^{o}$ per pole) [that too when $\omega_p \rightarrow \infty$]
- There is no finite frequency at which the phase shift reaches -180$^{o}$ → therefore no polarity reversal of feedback

# Amplifier with Two Poles (contd.)

## Frequency Response



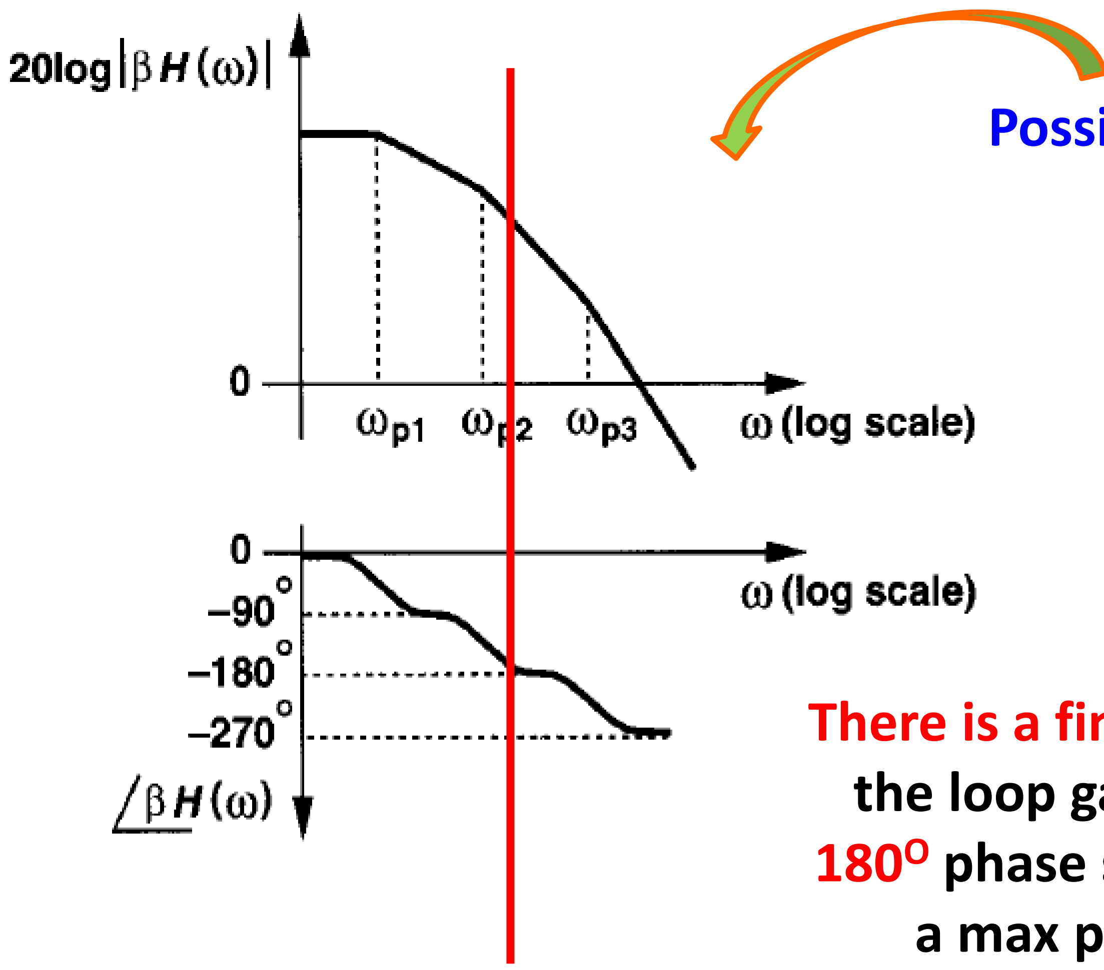

GX happens before PX → unconditionally stable

The system is stable since the loop gain is less than 1 at a frequency For which the angle(βH(ω))=-180.

With reduced feedback → GX moves closer to origin → doesn't affect PX → the system becomes more stable

# Amplifier with Three Poles

## Frequency Response

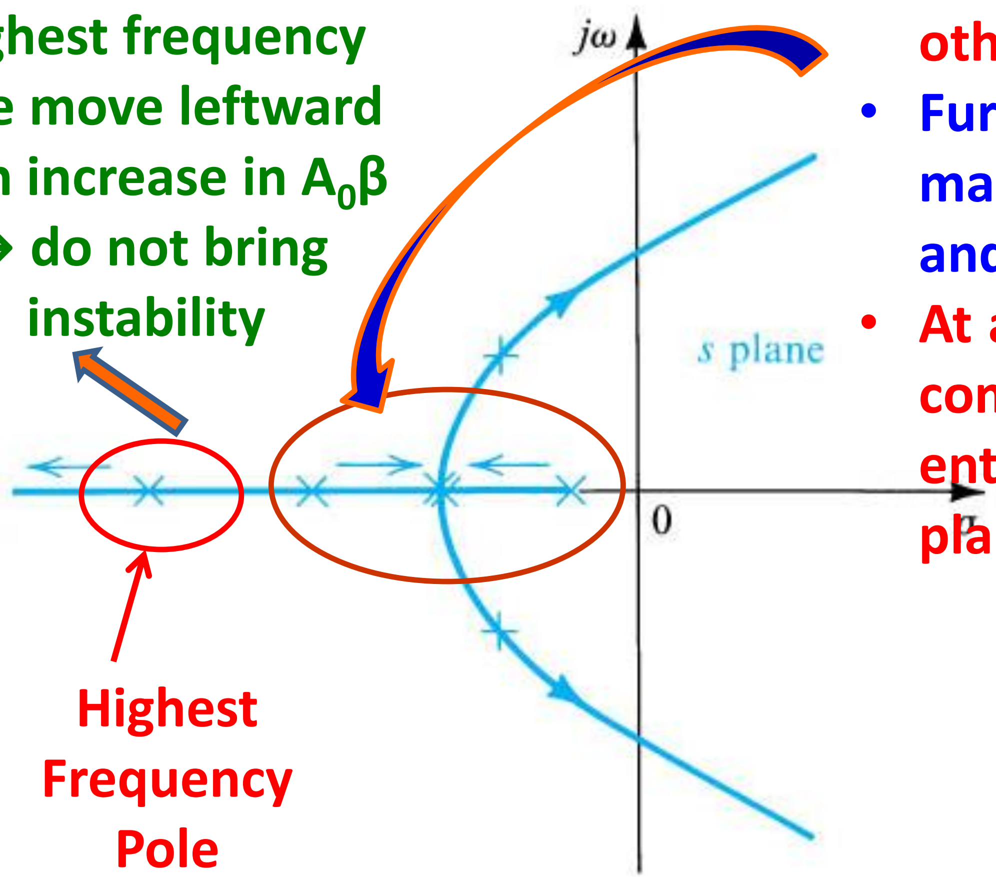



**Possibility of oscillation**

**There is a finite frequency at which the loop gain can be more than $180^{O}$ phase shift (3 poles can bring a max phase shift of $270^{O}$)**

# Amplifier with Three Poles

Highest frequency pole move leftward with increase in $A_0\beta$ → do not bring instability




Highest Frequency Pole

- Increase in $A_0\beta$ bring the other two poles together
- Further increase in $A_0\beta$ make the poles complex and then conjugate
- At a definite $A_0\beta$ the pair of complex-conjugate poles enter the right half of s-plane → bring instability!!!

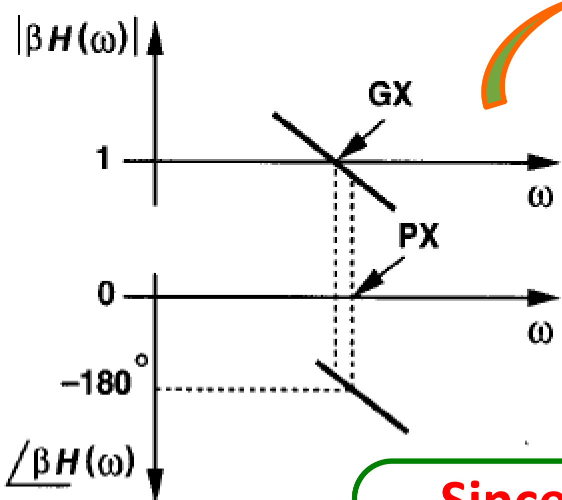
# Amplifier with Three Poles (contd.)

- In order to maintain the stability of amplifiers it is imperative to keep loop gain $A_0\beta$ smaller than the value corresponding to the poles entering right half s-plane
- In terms of Nyquist diagram, the critical value of $A_0\beta$ is that for which the diagram passes through the (-1, 0) point
- Reducing $A_0\beta$ below this value causes the Nyquist plot to shrink → the plot intersects the negative real axis to the right of (-1, 0) point → indicates stable amplifier
- Increasing $A_0\beta$ above this value causes expansion of Nyquist plot → plot encircles the (-1, 0) point → unstable performance

Case Study: Relative Location of GX and PX

- Case 1: $<\beta H(j\omega_p)=-175^o$
- Case 2: $<\beta H(j\omega_p)$ such that GX<<PX
- Case 3: $<\beta H(j\omega_p)=-135^o$

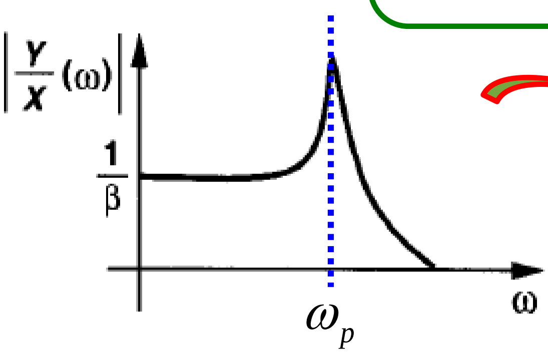
# Case 1: <βH(jω$_p$)=-175°

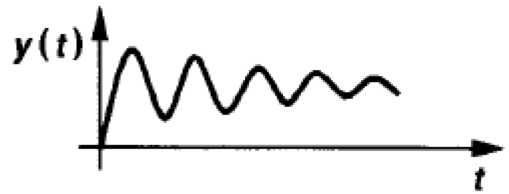



$$\frac{Y}{X}(j\omega_p)=\frac{H(j\omega_p)}{1+\beta H(j\omega_p)} \Rightarrow \frac{Y}{X}(j\omega_p)=\frac{\frac{1}{\beta}e^{-j175^o}}{1+\frac{1}{\beta}e^{-j175^o}}$$

$$\therefore \frac{Y}{X}(j\omega_p)\approx\frac{11.5}{\beta}$$

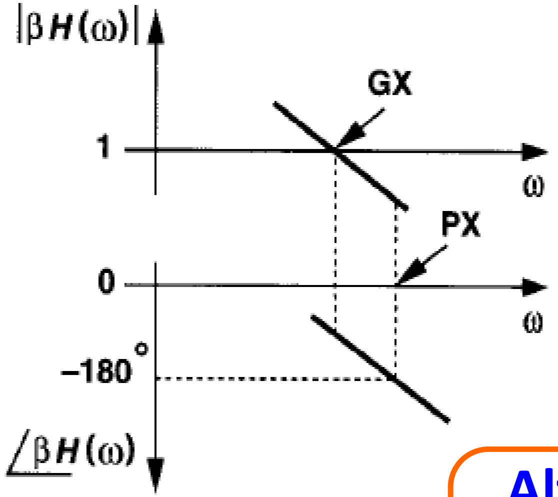
Since at low frequencies, |Y/X| = 1/β: the closed loop freq response exhibits a sharp peak in the vicinity of ω=ω$_p$




The system is technically stable, but it suffers from ringing

# Case 2: <βH(j$\omega_p$) such that GX<<PX




Higher is the spacing between GX and PX (while GX remains below PX), the more stable is the system

Alternatively, phase of βH at the GX frequency can serve as the measure of stability: the smaller <βH at GX, the more stable the system

Leads to the concept of phase margin (PM)

$$PM = 180^o + \angle \beta H(\omega_1)$$

Where, $\omega_1$ is the GX frequency

# Case 3: <βH(jω1)=-135

- **How much PM is adequate?**

$$PM = 45^o$$

$$\angle \beta H(\omega_1) = -135^o$$

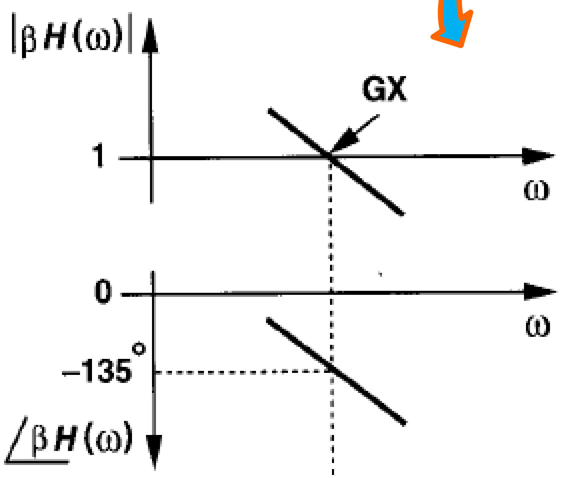
$$|\beta H(\omega_1)| = 1$$

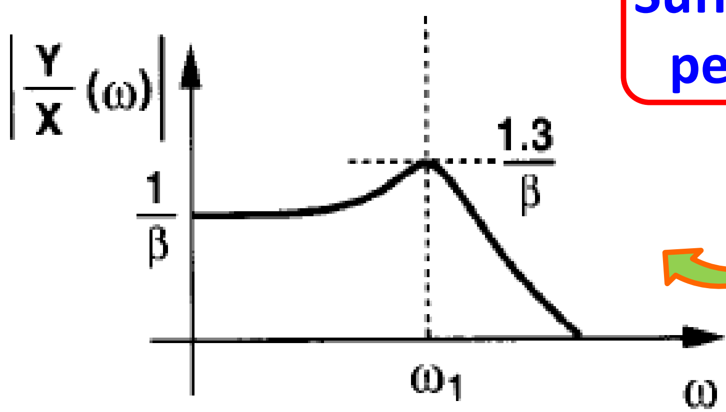
$$\frac{Y}{X}(j\omega_1) = \frac{H(j\omega_1)}{1 + 1 \times e^{-j135^o}}$$

Closed-Loop

$$\left|\frac{Y}{X}\right| = \frac{1.3}{\beta}$$

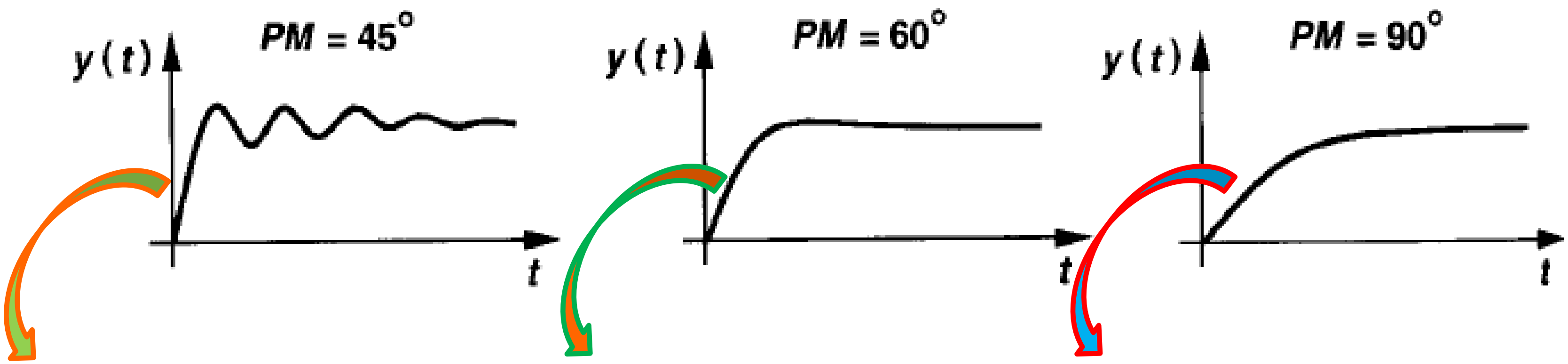
Suffers a 30% peak at $\omega_1$



# Case 3: <βH(jω1)=-135

- Peaking is associated with ringing in time domain




Ringing in time domain

Little ringing but fast settling

Free from ringing but slow settling

You design your system to achieve PM of around 60°

# Caution

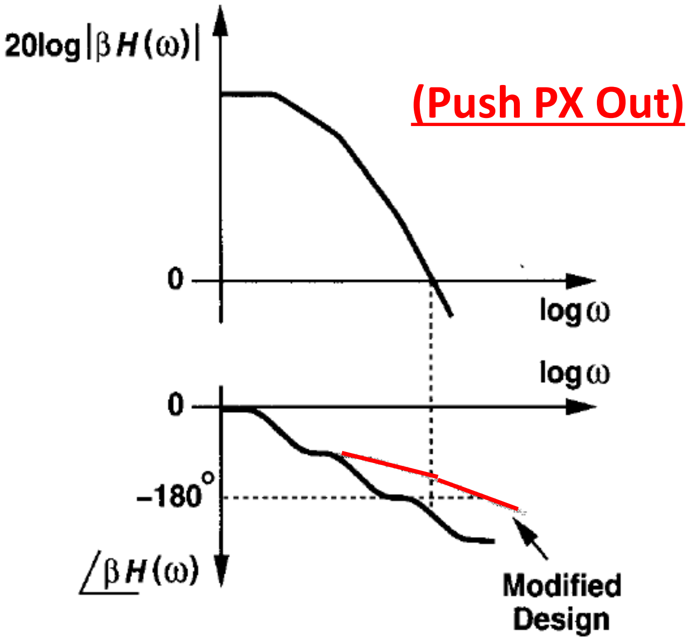
- PM is useful for small signal analysis.
- For large signal step response of a feedback system, the nonlinear behavior is usually such that a system with satisfactory PM may still exhibit excessive ringing.
- Transient analysis should be used to analyze large signal response.

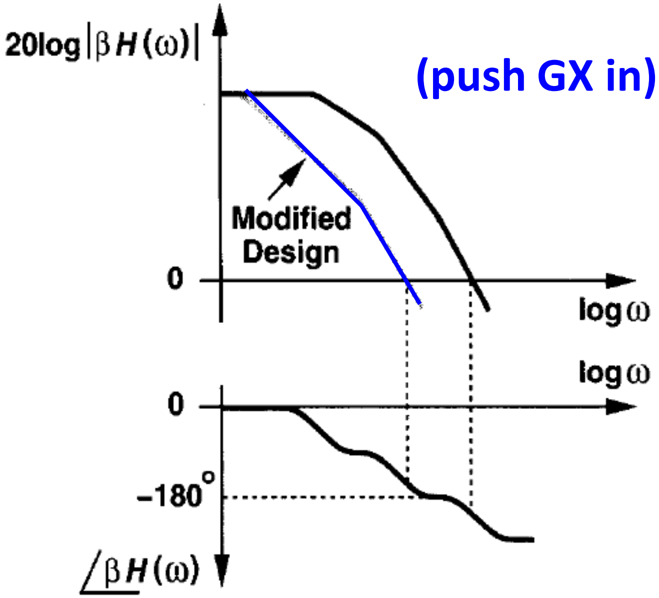
# Frequency Compensation

- Open loop transfer function is modified such that the closed-loop circuit is stable and the time response is well behaved

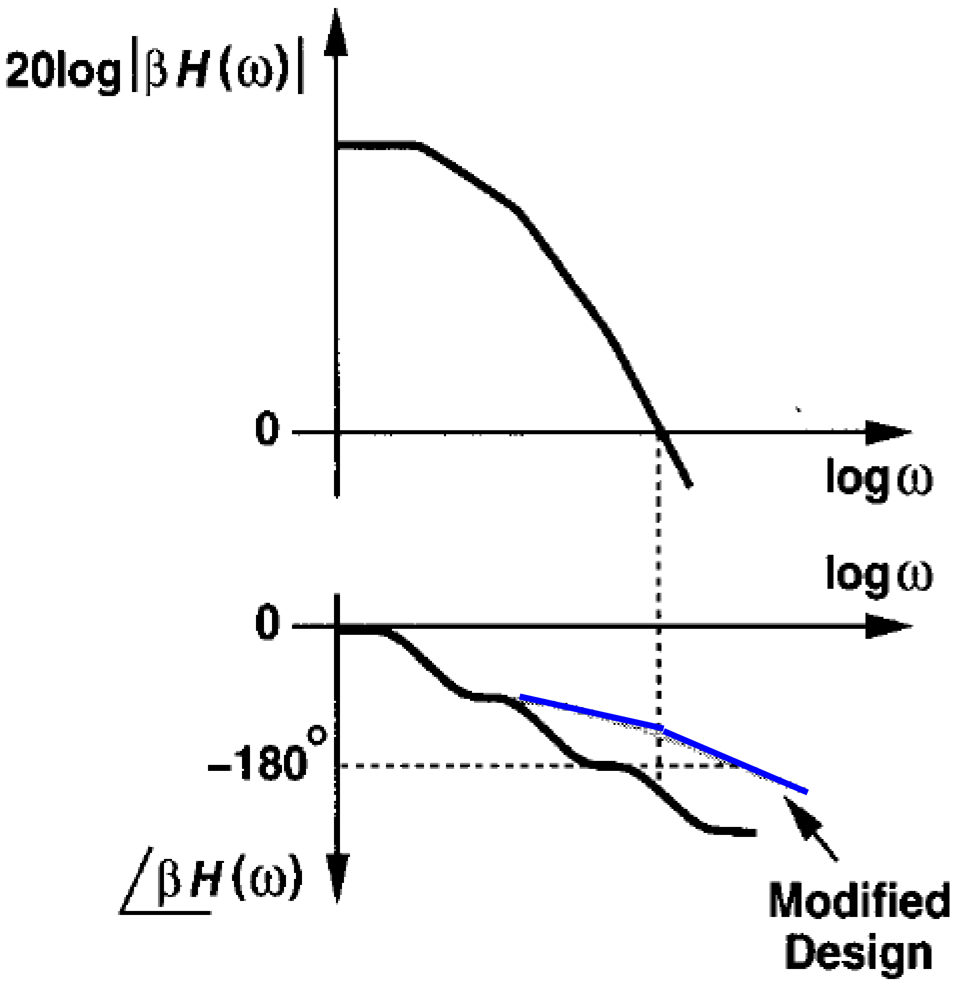
- **Reason for frequency compensation:**
  - |βH(ω)| does not drop to unity when <βH(ω) reaches -180°.

- **Possible Solutions:**



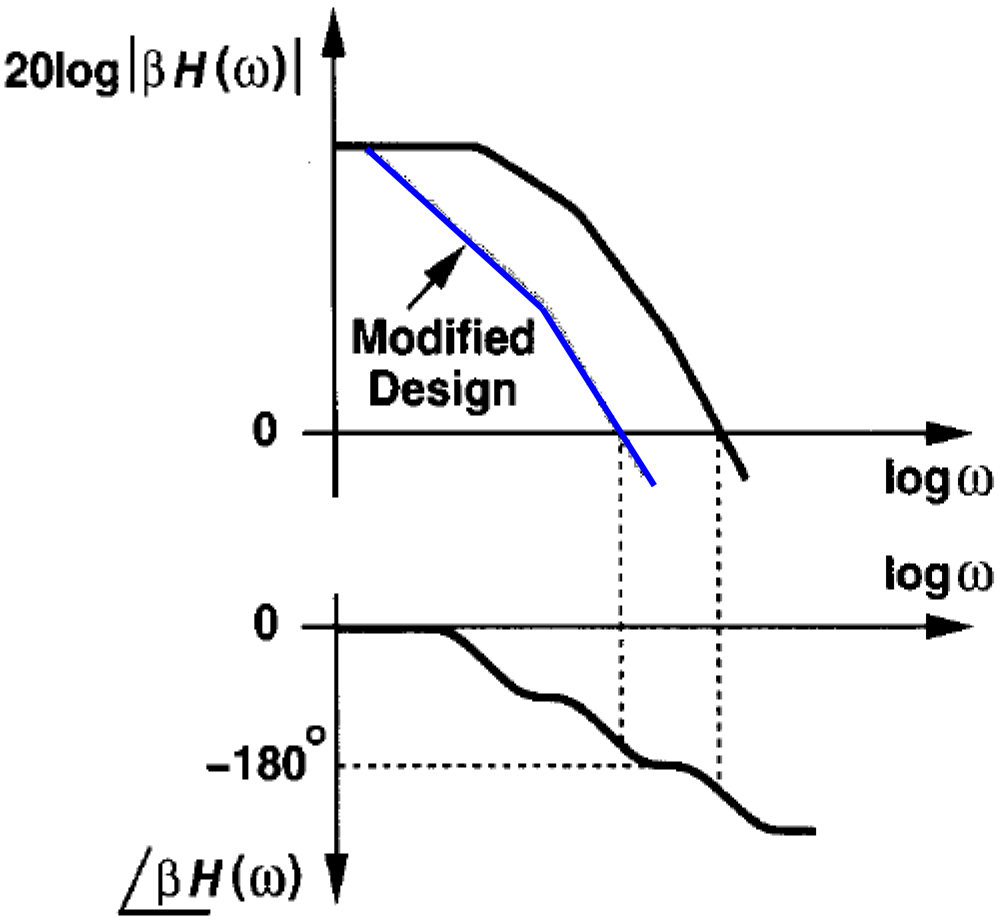



# Option 1: Push PX OUT




- Minimize the # of poles
- What's the problem?
  - Each stage contributes a pole.
- Reduction in # of stages implies difficult trade-off of gain versus output swings.

# Option 2: Push GX In




**Problem:**

Bandwidth is sacrificed for stability

# Frequency Compensation (contd.)

## Typical Approach

- Minimize the number of poles first to push PX out
- Use compensation to move the GX towards the origin next