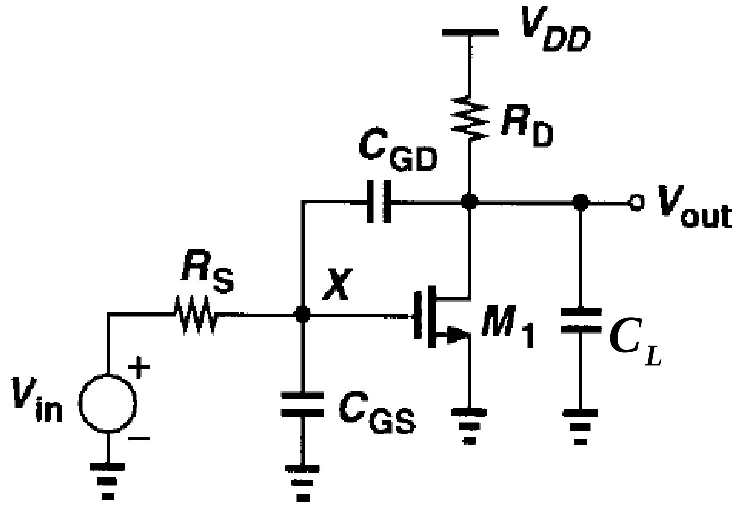
Lecture – 19 Date: 31.10.2015

- CS Stage (contd.)
- CS stage with Degeneration Resistor R_{deg}
- Common Drain (CD) Stage
- Common Gate Topology
- Cascode Configuration

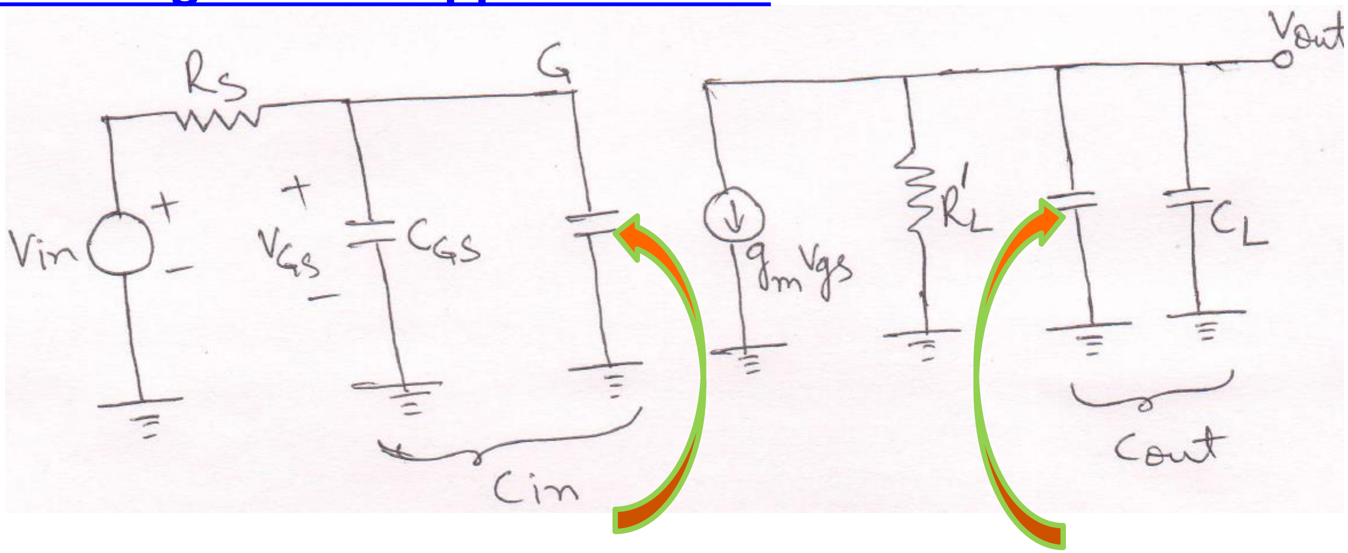
Quiz – 6

Q: For the CS Topology, find the poles and zeros using Miller's Approximation, OCTC, and Exact Analysis.



Common Source Amplifier (contd.)

Analysis using Miller's Approximation



$$C_A = (1 - A_v)C_{GD} = (1 + g_m R_L^{'})C_{GD} \qquad C_B = (1 - A_v^{-1})C_{GD} \approx C_{GD}$$

Therefore the poles are:

$$\omega_{in} = \frac{1}{R_S C_{in}} = \frac{1}{R_S (C_{GS} + C_A)} = \frac{1}{R_S \left(C_{GS} + (1 + g_m R_L^{'})C_{GD}\right)}$$

$$\omega_{out} = \frac{1}{R_L^{'} C_{out}} = \frac{1}{R_L^{'} (C_L + C_B)} = \frac{1}{R_L^{'} (C_L + C_{GD})}$$

Common Source Amplifier (contd.)

Then the transfer function is given by:

$$H(s) = \frac{V_{out}}{V_{in}}(s) = \frac{-g_m R_D}{\left(1+\frac{s}{\omega_{in}}\right)\left(1+\frac{s}{\omega_{out}}\right)}$$

When R_S is large and C_L is small

ω_{in} dominates, and the transfer function becomes:

$$H(s) = \frac{-g_m R_D}{\left(1+\frac{s}{\omega_{in}}\right)} \Rightarrow H(s) = \frac{A_M}{1+\frac{s}{\omega_H}}$$

Dominant Pole

3-dB Frequency: $f_H = \frac{1}{2\pi C_{in} R_S}$

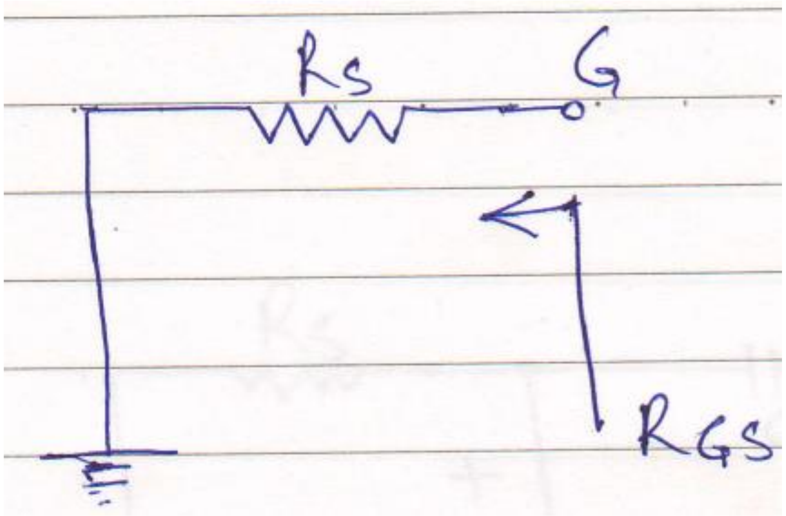
Where, $C_{in} = C_{GS} + C_{GD}(1+g_m R_L^{'})$

The main error in this expression is that the presence of zero has not been considered

Common Source Amplifier (contd.)

Analysis using OCTC Method

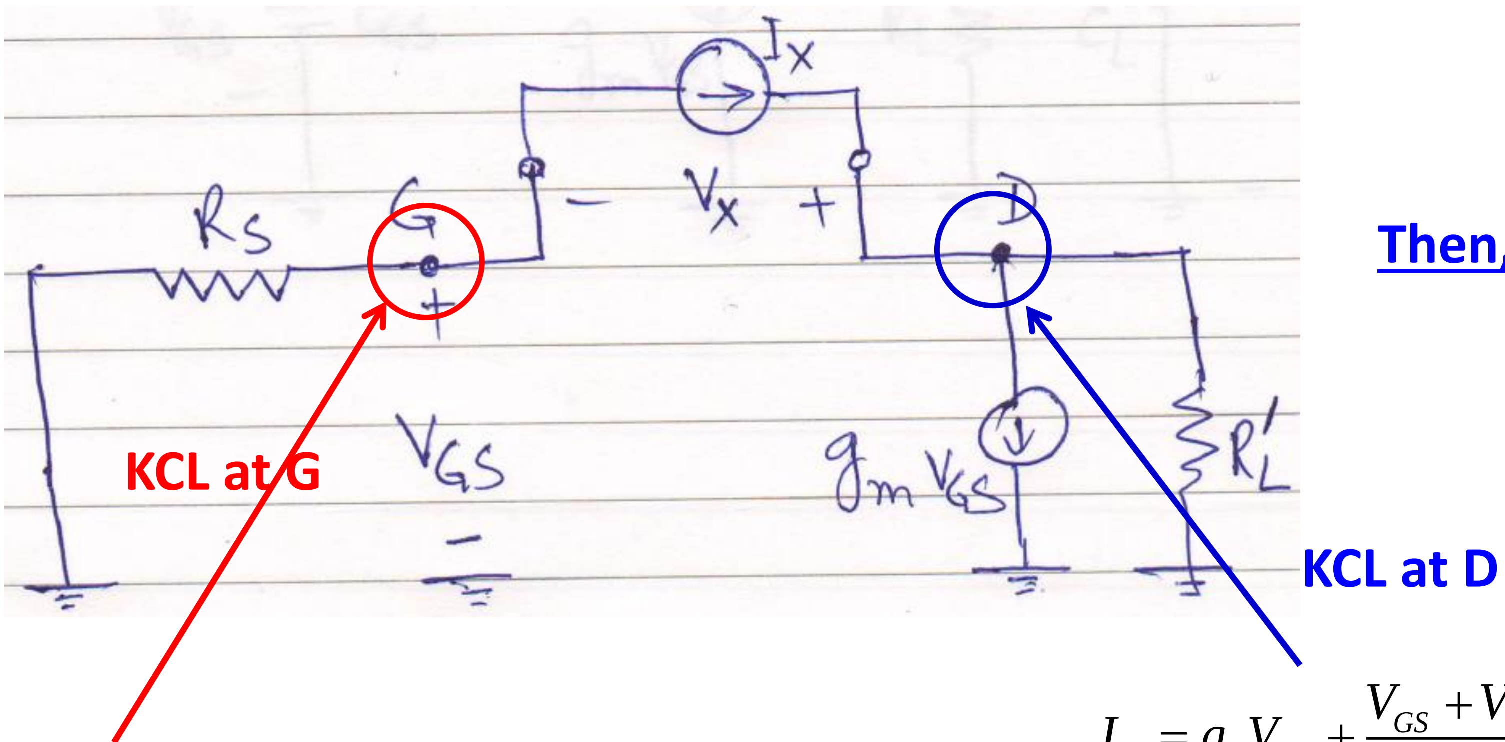
- Considering only C_{GS} → open other capacitances and short the voltage sources and open the current sources
- For R_{GS} we get:



$$R_{GS} = R_S$$

Common Source Amplifier (contd.)

- **Considering only C_{GD} → open C_{GS} and C_L**



Then, $R_{GD} = \dfrac{V_X}{I_X}$

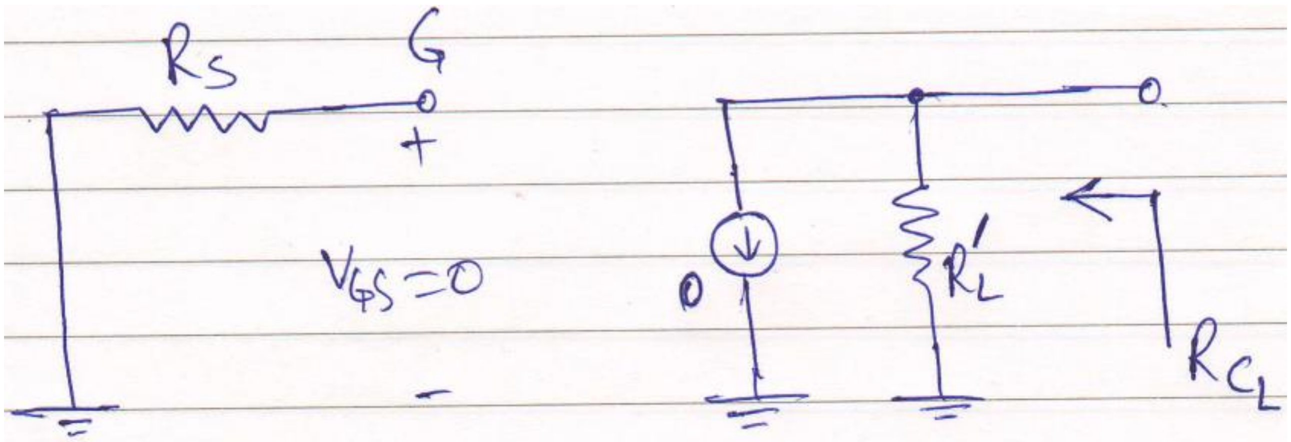
$$I_X = g_m V_{GS} + \frac{V_{GS} + V_X}{R_L^{'}}$$

$$\frac{V_{GS}}{R_S} + I_X = 0$$

$$\Rightarrow R_{GD} = \frac{V_X}{I_X} = R_S + (1 + g_m R_S) R_L^{'}$$

Common Source Amplifier (contd.)

- Considering only C_L → open C_{GS} and C_{GD}



$$R_{C_L} = R_L^{'}$$

Thus, the effective time constant: $\tau_H = C_{GS}R_{GS} + C_{GD}R_{GD} + C_L R_{C_L}$

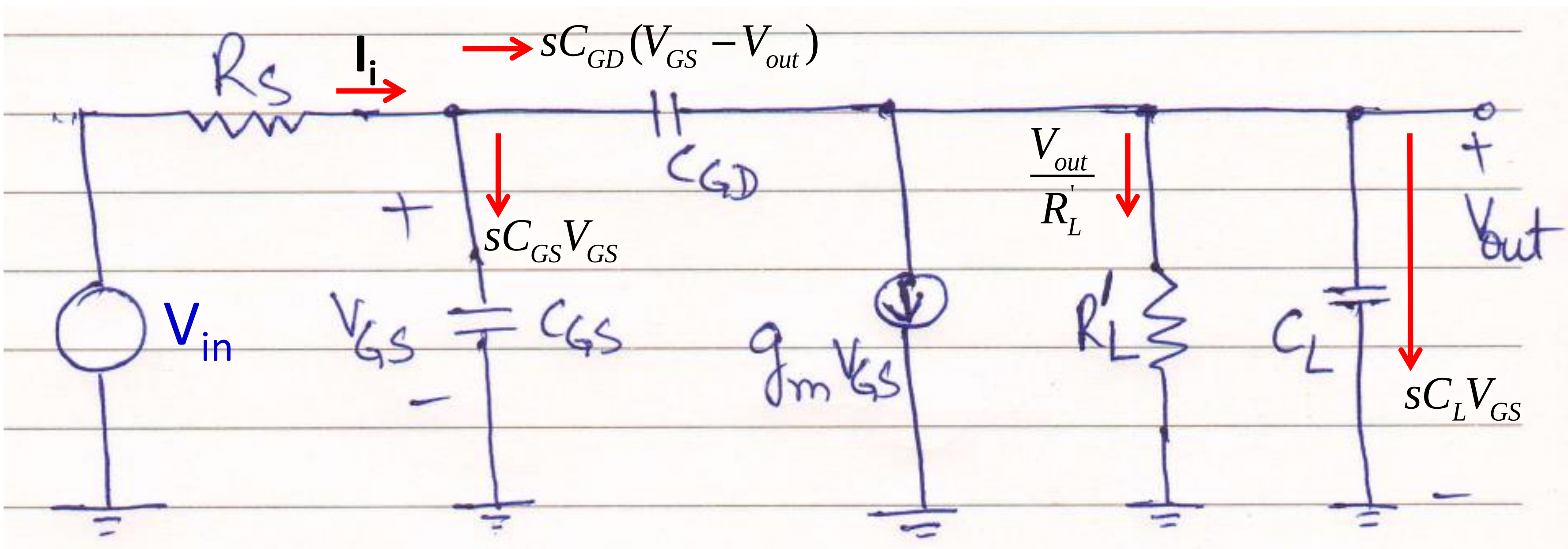
Therefore the 3-dB roll-off frequency is: $f_H = \dfrac{1}{2\pi\tau_H}$

Provides a better estimate than Miller's approximation

Common Source Amplifier (contd.)

Exact Analysis

- Miller's Approximation and OCTC Technique provides insight about the impact of various capacitances on the high frequency response of amplifier
- However, for simple circuits its imperative to carry out exact analysis



KCL at the drain: $sC_{GD}\left(V_{GS}-V_{out}\right)=g_mV_{GS}+\frac{V_{out}}{R_L^{'}}+sC_LV_{out}$

Common Source Amplifier (contd.)

KCL at the drain: $$sC_{GD}\left(V_{GS}-V_{out}\right)=g_mV_{GS}+\frac{V_{out}}{R_L^{'}}+sC_LV_{out}$$

$$V_{GS}=\frac{-V_{out}}{g_mR_L^{'}}\frac{1+s\left(C_L+C_{GD}\right)R_L^{'}}{1-\left(sC_{GD}/g_m\right)}$$

KVL at the gate: $$V_{in}=I_iR_S+V_{GS}$$

KCL at the gate: $$I_i=sC_{GS}V_{GS}+sC_{GD}\left(V_{GS}-V_{out}\right)$$

$$V_{in}=V_{GS}\left[1+s(C_{GS}+C_{GD})R_S\right]-sC_{GD}R_SV_{out}$$

$$\Rightarrow\frac{V_{out}}{V_{in}}=\frac{-\left(g_mR_L^{'}\right)\left[1-s\left(C_{GD}/g_m\right)\right]}{1+sA+s^2B}$$

Where,

$$A=\left[C_{GS}+C_{GD}\left(1+g_mR_L^{'}\right)\right]R_S+\left(C_L+C_{GD}\right)R_L^{'}$$

$$B=\left[\left(C_L+C_{GD}\right)C_{GS}+C_LC_{GD}\right]R_SR_L^{'}$$

Common Source Amplifier (contd.)

Observations

- There exists one zero → not known through the approximate analysis
- 2nd order denominator [D(s)] → presence of two poles
- There are three capacitances → why only two poles and one zero

Poles Determination

- As s → 0, the transfer function approaches:

$$\Rightarrow \frac{V_{out}}{V_{in}} = -\left(g_m R_L^{'}\right)$$ DC Gain

- Let ω_{p1} and ω_{p2} be the two poles then:

$$D(s) = \left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right) = 1 + s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

- If ω_{p1} is dominant then:

$$D(s) \cong 1 + \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

Common Source Amplifier (contd.)

- Now, equating the coefficients:

$$\omega_{p1} = \frac{1}{\left[C_{GS} + C_{GD}\left(1 + g_m R_L^{'}\right)\right]R_S + \left(C_L + C_{GD}\right)R_L^{'}}$$

$$\omega_{p1}\omega_{p2} = \frac{1}{\left[\left(C_L + C_{GD}\right)C_{GS} + C_L C_{GD}\right]R_S R_L^{'}}$$

$$\Rightarrow \omega_{p2} = \frac{\left[C_{GS} + C_{GD}\left(1 + g_m R_L^{'}\right)\right]R_S + \left(C_L + C_{GD}\right)R_L^{'}}{\left[\left(C_L + C_{GD}\right)C_{GS} + C_L C_{GD}\right]R_S R_L^{'}}$$

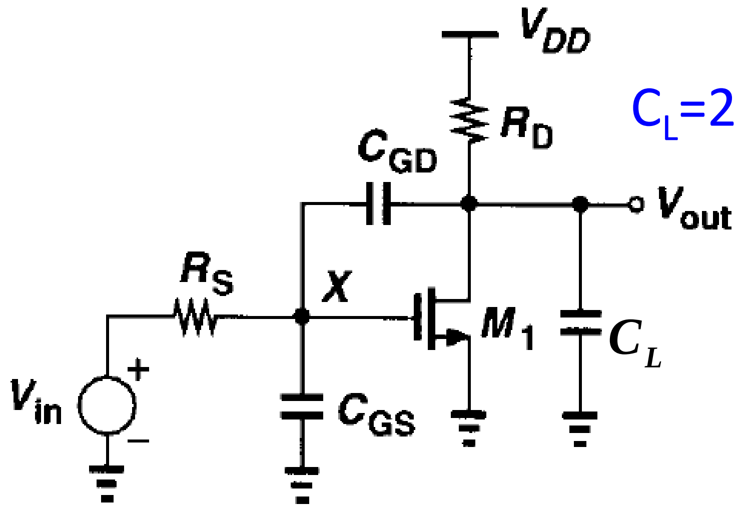
Very similar to the pole determined using OCTC method with the only addition being $R_L^{'}(C_{GD} + C_L)$

Common Source Amplifier (contd.)

Example:

R_S=50 Ohms
L=2.0 um
A_V=15

f_{in}=4.65 GHz
f_{out}=69.9 MHz

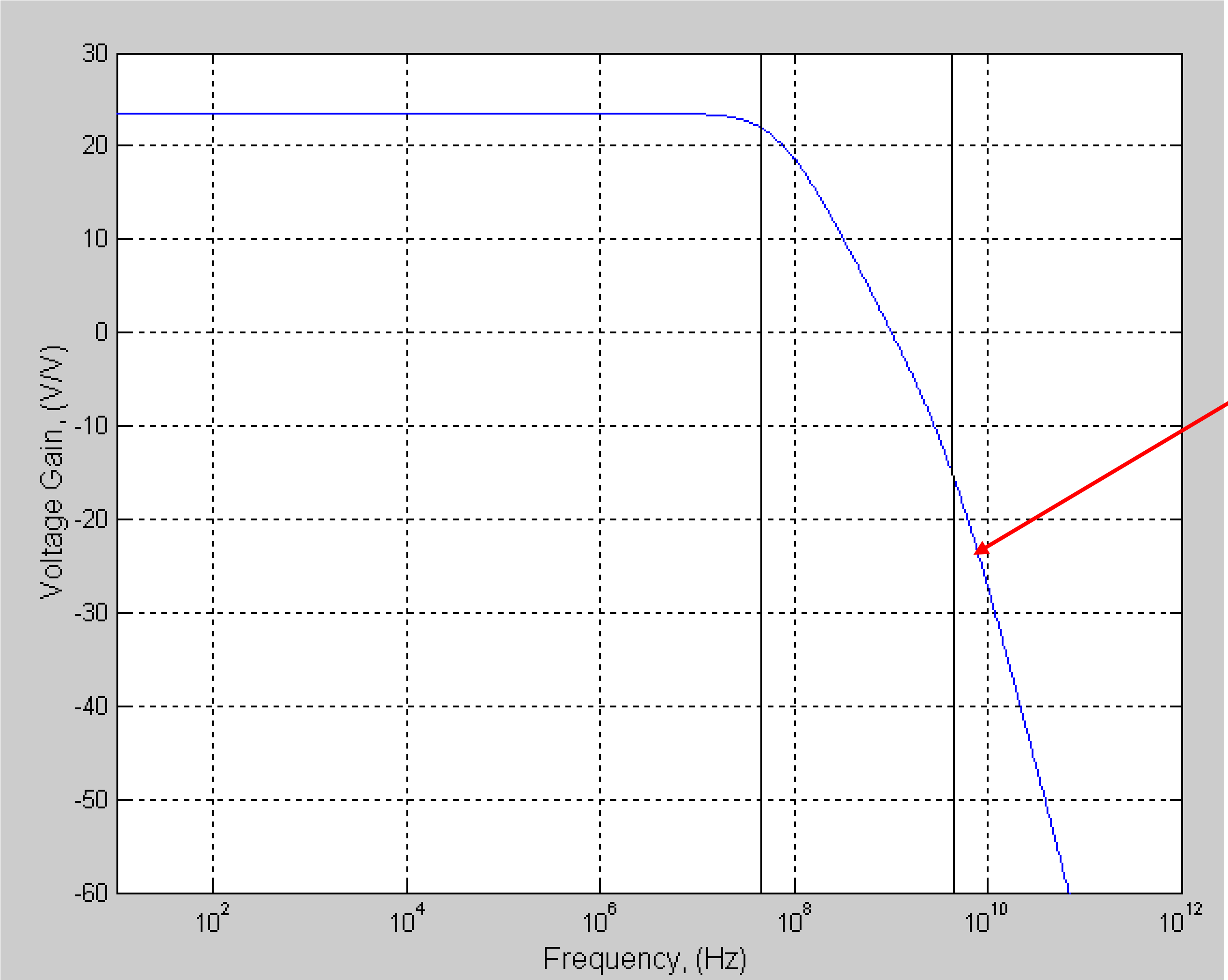


C_L=27.51 fF, R_L'=60 KOhm

$$\omega_{in} = \frac{1}{R_S\left(C_{GS} + \left(1 + g_m R_L^{'}\right)C_{GD}\right)}$$

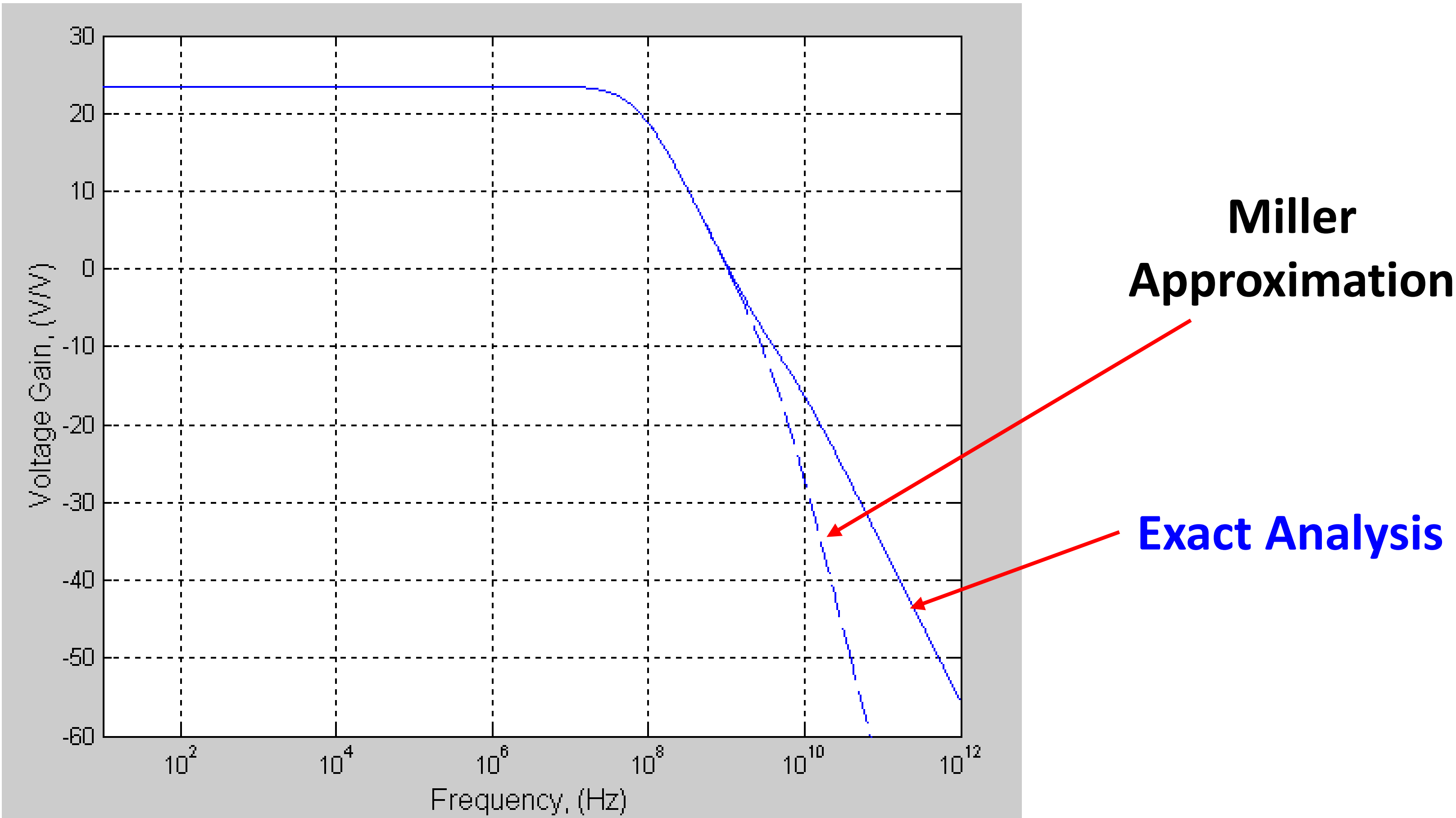
$$\omega_{out} = \frac{1}{R_L^{'}\left(C_L + C_{GD}\right)}$$

Transfer Function



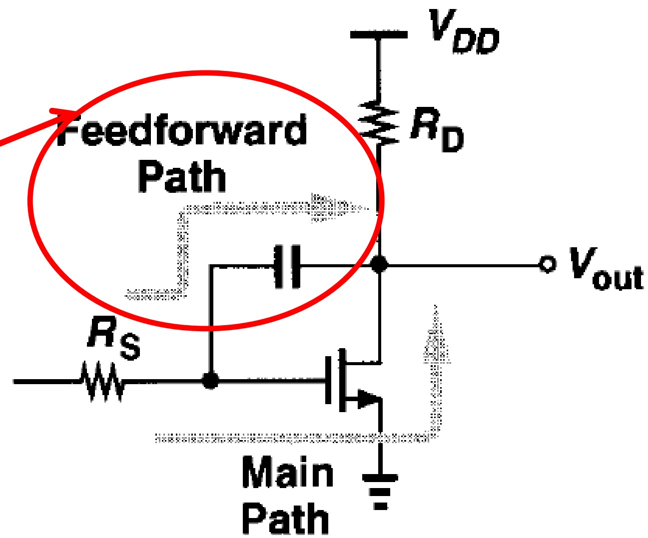
Miller Approximation

Transfer Function

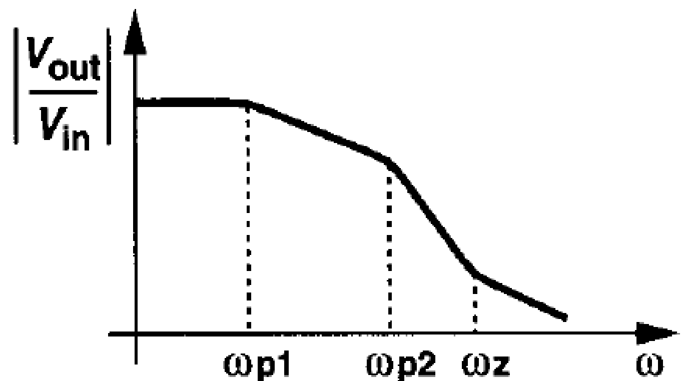


Common Source Amplifier (contd.)

- There exists one zero given by: $$\omega_{z1} = \frac{g_m}{C_{GD}}$$
- This zero results from the direct coupling of the input and output through C_{GD} at high frequencies
- the capacitor provides a feed-forward path



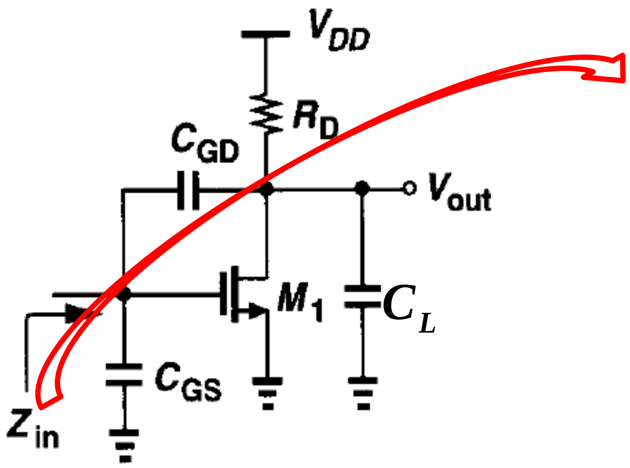
Only at high frequencies



It results in a slope in the frequency response that is less negative than -20dB/dec

Common Source Amplifier (contd.)

- In high speed applications, the input impedance of the common source stage is extremely important

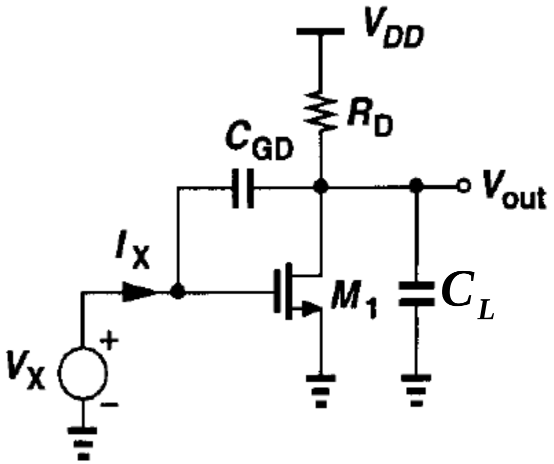


At high enough frequency, we can derive

$$Z_{in} = \frac{1}{\left[C_{GS} + \left(1 + g_m R_D\right) C_{GD}\right] s}$$

But at extremely high frequencies where Miller's approximation doesn't give appropriate performance, it's a must to take into account the contribution of output node

Common Source Amplifier (contd.)



For simplification, C_{GS} has been ignored

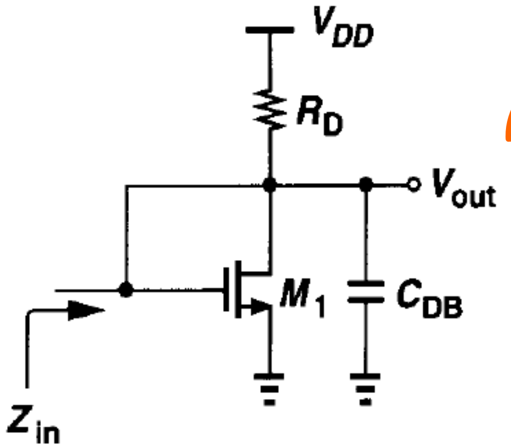
Using small signal model:

$$\frac{V_X}{I_X} = \frac{1 + R_D\left(C_{GD} + C_{DB}\right)s}{C_{GD}s\left[\left(1 + g_m R_D + R_D C_L s\right)\right]}$$

Therefore:

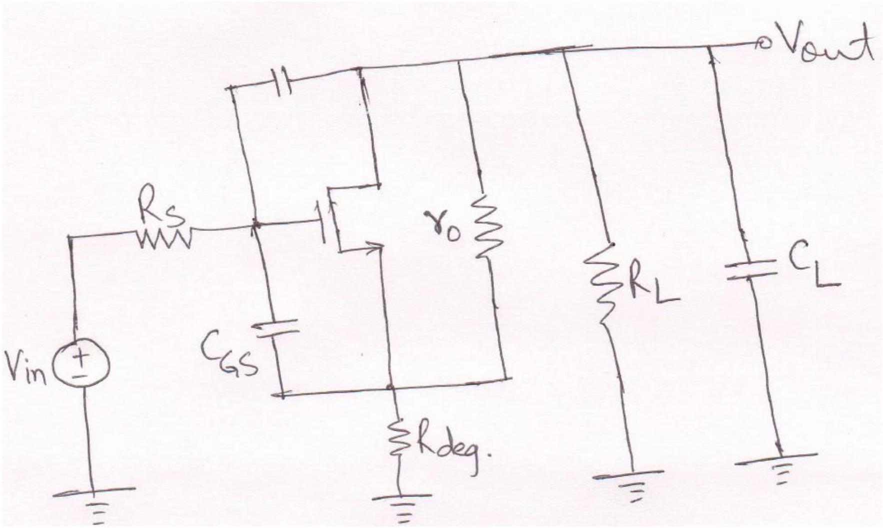
$$Z_{in} = X_{C_{GS}} \parallel \frac{V_X}{I_X}$$

At extremely high frequency



This is the case when C_{GD} is very high → provides a low impedance path between G and D

CS Amplifier with Source Degeneration



We know,

$$R_{out} = r_o\left[1+\left(g_m + g_{mb}\right)R_{\deg}\right]$$

$$G_m = \frac{g_m}{1+(g_m + g_{mb})R_{\deg}}$$

- **To determine the effective time constant, use OCTC by considering one capacitor at a time.**
- **Consider C_{GD} first:** $R_{GD} = R_S\left(1+G_m R_L^{'}\right) + R_L^{'}$ **Where,** $R_L^{'} = R_L \,||\, R_{out}$
- **Then Consider C_L:** $R_{C_L} = R_L \,||\, R_{out} = R_L^{'}$

CS Amplifier with Source Degeneration (contd.)

- **Finally Consider C_{GS}:** $$R_{GS} = \frac{R_S + R_{\deg}}{1 + (g_m + g_{mb})R_{\deg}\left(\frac{r_o}{r_o + R_L}\right)}$$

- **Now, the effective time constant:** $$\tau_H = C_{GS}R_{GS} + C_{GD}R_{GD} + C_L R_{C_L}$$

- **For relatively large R_S the contribution of $C_{GD}R_{GD}$ in open circuit time constants (τ_H) will be largest.**

$$\Rightarrow \tau_H = C_{GD}R_{GD} \qquad \therefore f_H \cong \frac{1}{2\pi C_{GD}R_{GD}}$$

Comment

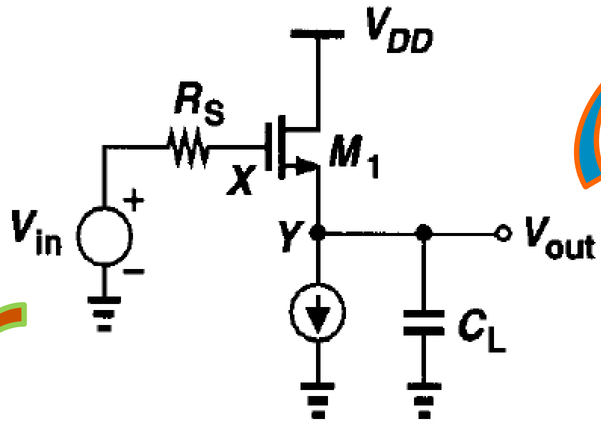
- **If R_{deg} is increased → the mid-band gain A_M will decrease → this causes reduction in R_{GD} → as a result f_H increases**
- **As G_mR_L' >>1 and G_mR_S>>1 the term R_{GD} can be approximated as:** $$R_{GD} \cong G_m R_L' R_S = |A_M| R_S$$

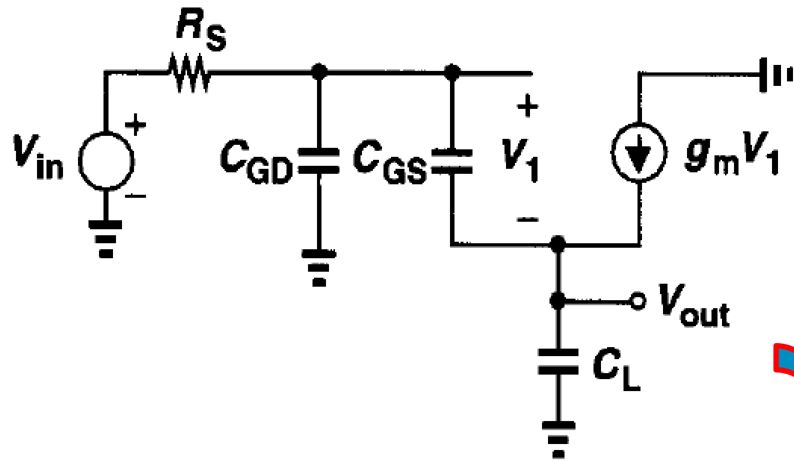
CS Amplifier with Source Degeneration (contd.)

$$\Rightarrow f_H = \frac{1}{2\pi C_{GD}\left|A_M\right|R_S}$$

Gain bandwidth product ($f_H.|A_M|$) remains constant for fixed R_S → however other capacitances make it variable

Common Drain





Strong interaction between XY, making it difficult to associate each pole with each node

$$H(s)=\frac{g_m+C_{GS}s}{R_S\left(C_{GS}C_L+C_{GS}C_{GD}+C_{GD}C_L\right)s^2+\left(g_mR_SC_{GD}+C_L+C_{GS}\right)s+g_m}$$

Common Drain

$$H(s)=\frac{g_m+C_{GS}s}{R_S\left(C_{GS}C_L+C_{GS}C_{GD}+C_{GD}C_L\right)s^2+\left(g_mR_SC_{GD}+C_L+C_{GS}\right)s+g_m}$$

$$D=\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)=\frac{s^2}{\omega_{p1}\omega_{p2}}+\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)s+1$$

Dominant Pole

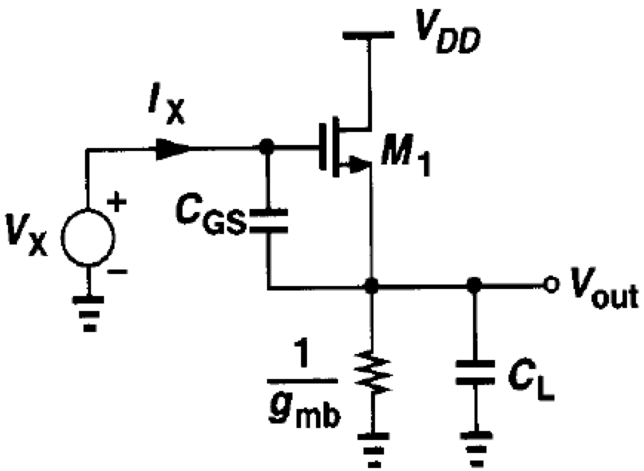
$$\omega_{p1}\approx\frac{g_m}{g_mR_SC_{GD}+C_L+C_{GS}}=\frac{1}{R_SC_{GD}+\dfrac{C_L+C_{GS}}{g_m}}$$

If, R_S = 0: then

$$\omega_{p1}=\frac{g_m}{C_L+C_{GS}}$$

Common Drain Amplifier (contd.)

Input Impedance



$$V_X = \frac{I_X}{C_{GS}s} + \left(I_X + \frac{g_m I_X}{C_{GS}s}\right)\left(\frac{1}{g_{mb}} \| \frac{1}{C_L s}\right)$$

$$Z_{in} = \frac{V_X}{I_X} = \frac{1}{C_{GS}s} + \left(1 + \frac{g_m}{C_{GS}s}\right)\frac{1}{g_{mb} + C_L s}$$

At relatively low frequencies:

$$Z_{in} \approx \frac{V_X}{I_X} = \frac{1}{C_{GS}s}\left(1 + \frac{g_m}{g_{mb}}\right) + \frac{1}{g_{mb}}$$

$$g_{mb} \gg |C_L s|$$

Equivalent Input Capacitance :

$$C_{in\_eq} = \frac{g_{mb}C_{GS}}{g_m + g_{mb}}$$

Can also be obtained using Miller's Theorem

Common Drain Amplifier (contd.)

At high frequencies:

$$Z_{in} \approx \frac{1}{C_{GS}s} + \frac{1}{C_L s} + \frac{g_m}{C_{GS}C_L s^2}$$

$$g_{mb} \ll |C_L s|$$

For a given, s=jω

$$Z_{in} = \frac{1}{j\omega C_{GS}} + \frac{1}{j\omega C_L} - \frac{g_m}{\omega^2 C_{GS} C_L}$$

Negative Resistance

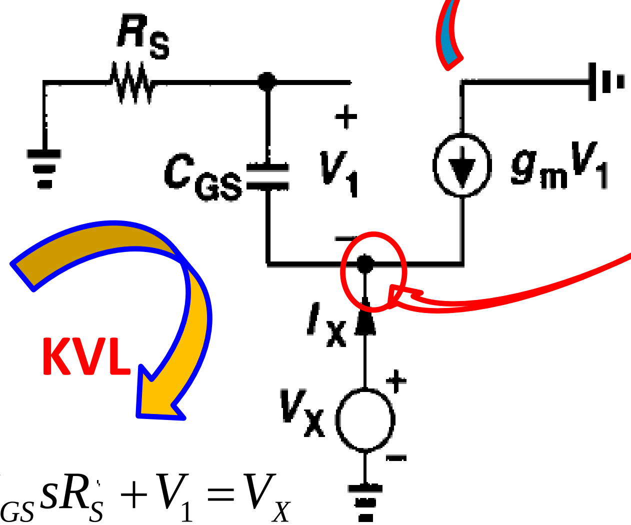
must be some positive feedback at the input node

could be useful in oscillator design

Common Drain Amplifier (contd.)

Output Impedance

The body effect and C_{SB} yields an impedance in parallel to the output → lets ignore this for now



$$-I_X = V_1 C_{GS} s + g_m V_1$$

KCL

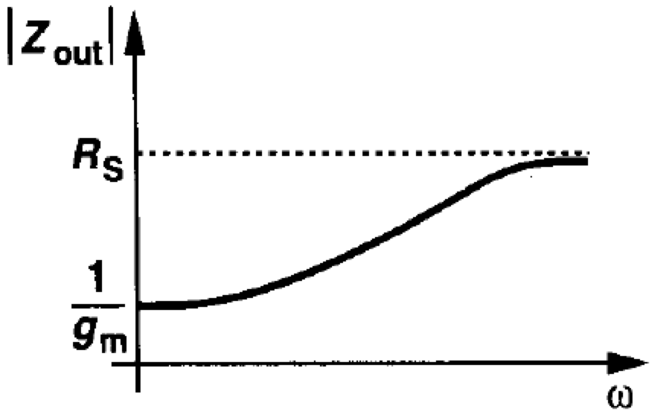
$$V_1 C_{GS} s R_S + V_1 = V_X$$

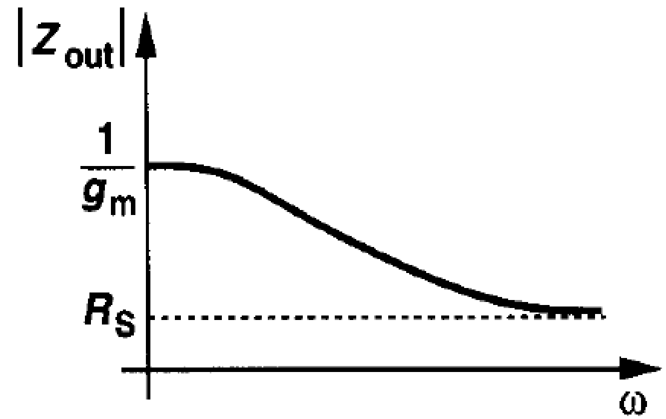
$$\therefore Z_{out} = \frac{Z_X}{I_X} = \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s}$$

At low frequencies: $Z_{out} = \frac{1}{g_m}$ **Expected**

At very high frequencies: $Z_{out} = R_S$ ← Due to the fact that C_{GS} shorts the G and S

Common Drain Amplifier (contd.)



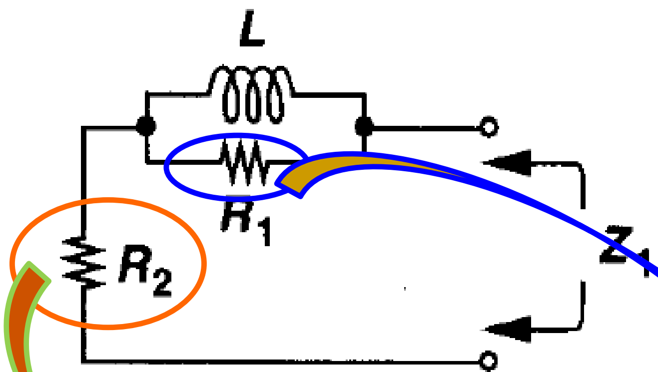


This is required considering that CD works as buffer and its purpose is to lower the output impedance i.e., $(1/g_m) < R_S$

The output impedance increases with frequency ↔ clear indication of the presence of inductive element

Common Drain Amplifier (contd.)

Output Network – first order model



This Ensures

$$Z_1 = \frac{1}{g_m}$$ At low frequencies

$$Z_1 = R_S$$ At very high frequencies

$$R_2 = \frac{1}{g_m}$$

$$R_1 = R_S - \frac{1}{g_m}$$

- For the determination of L, equate the Z_1 expression to that of Z_{out}

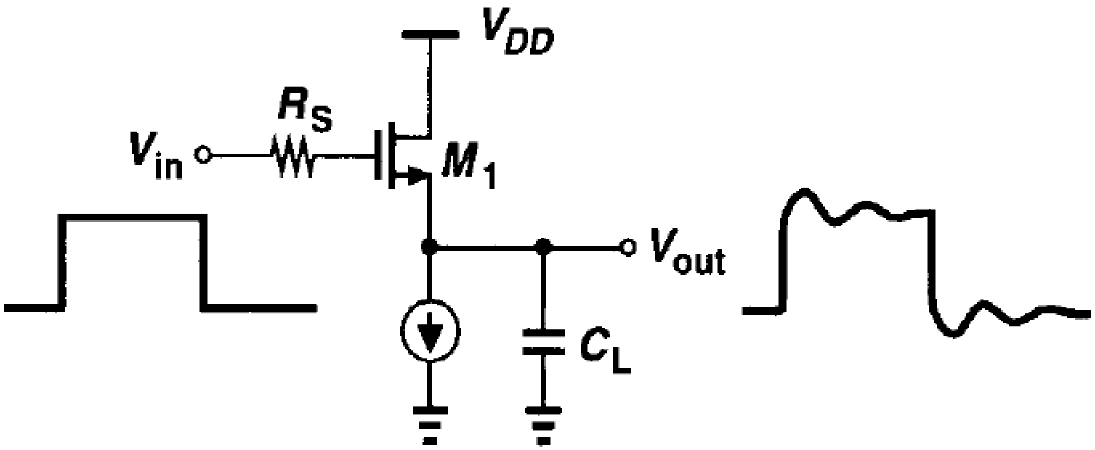
$$L = \frac{C_{GS}}{g_m}\left(R_S - \frac{1}{g_m}\right)$$

Common Drain Amplifier (contd.)

$$L = \frac{C_{GS}}{g_m}\left(R_S - \frac{1}{g_m}\right)$$

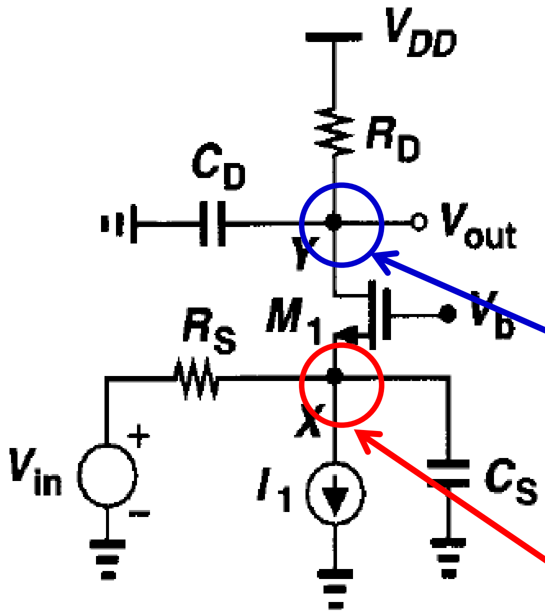
Its apparent that if a CD stage is driven by a R_S, then it exhibits inductive behavior at the output

It has detrimental effect on the output characteristics



For example, the ringing problem in step response

Common Gate Amplifier



Very important: no Miller multiplication of capacitances

Primary factor for providing wide-band performance

$$\omega_{out} = \left[C_D R_D\right]^{-1} \leftarrow C_D = C_{GD} \parallel C_{DB}$$

$$\omega_{in} = \left[C_S\left(R_S \parallel \frac{1}{g_m + g_{mb}}\right)\right]^{-1} \leftarrow C_S = C_{GS} \parallel C_{SB}$$

Therefore the transfer function:

$$\frac{V_{out}}{V_{in}} = \frac{(g_m + g_{mb})R_D}{1+(g_m+g_{mb})R_S} \frac{1}{\left(1+\frac{C_S}{g_m + g_{mb} + R_S^{-1}}s\right)(1+R_D C_D s)}$$

A_v

Common Gate Amplifier (contd.)

- If we consider channel length modulation → not easy to associate a pole to the input node → direct analysis of the network is needed to determine the transfer function

- The low input impedance of may load the preceding stage

- The voltage drop across RD is typically maximized to obtain a reasonable gain, therefore the dc level of input signal must be low

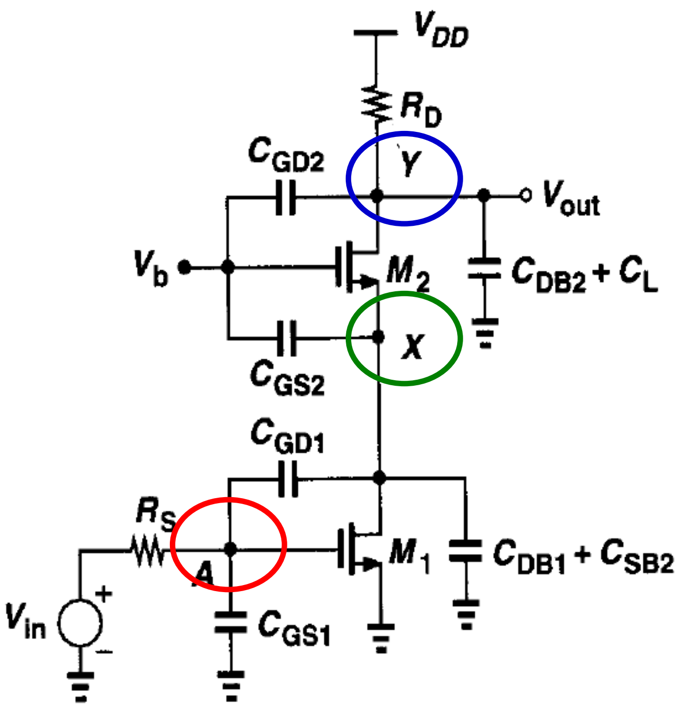
- CG stage with relatively large capacitance at the input → possesses low output impedance → good for cascode configuration

Cascode Stage

Why do we need cascode stage?

- High input impedance – good in a sense that it doesn't disturb the previous stage and doesn't get affected by the previous stage
- **High gain – important any way!**
- Relatively higher output impedance – doesn't disturb the succeeding stages
- **How about freq response?**
  - Provides a broader bandwidth of operation →due to CG stage (no Miller approximation of intrinsic capacitor) → it was the initial motivating factor for cascode stage!!!

Cascode Stage (contd.)



Capacitance at node A:

C_{GS1} in parallel with $(1-A_M)C_{GD1}$

Where A_M is given by: $-\frac{g_{m1}}{g_{m2}+g_{mb2}}$

With assumption that R_D is small and the channel length modulation is negligible

For equal dimensions of M_1 and M_2: A_M approximately equals 1 → therefore C_{GD} gets multiplied by a factor of roughly 2 both at node A as well as at node X → much smaller multiplication factor as compared to a single CS stage

Cascode Stage (contd.)

Therefore the pole associated with node A is:

$$\omega_{p,A} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2} + g_{mb2}}\right)C_{GD1}\right]}$$

Capacitance at node X:

$$2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}$$

Therefore the pole associated with node X is:

$$\omega_{p,X} = \frac{g_{m2} + g_{mb2}}{2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}}$$

Capacitance at node Y:

$$C_{DB2} + C_L + C_{GD2}$$

Therefore the pole associated with node Y is:

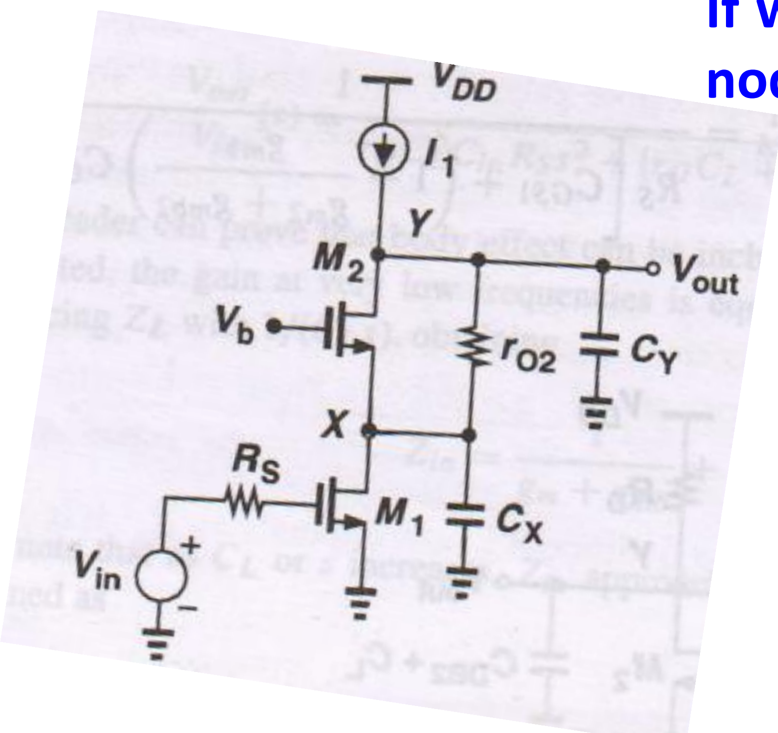
$$\omega_{p,Y} = \frac{1}{R_D\left(C_{DB2} + C_L + C_{GD2}\right)}$$

- Do you have any control on the choice of poles?
- Yes → through selection of appropriate devices
- Usually $\omega_{p,X}$ is chosen very high → to obtain better stability

Cascode Stage (contd.)

- Instead of R_D → if a constant current source is used → what happens? → do the designer have control on the choice of poles?

How about output impedance?



If we ignore C_{GD1} and the capacitance at the node Y then:

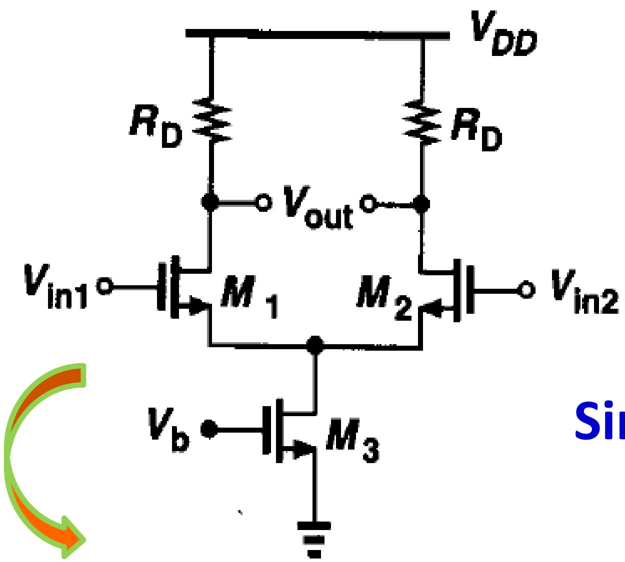
$$Z_{out} = \left(1 + g_{m2} r_{o2}\right) Z_X + r_{o2}$$

$$= r_{o1} \,||\, (C_X s)^{-1}$$

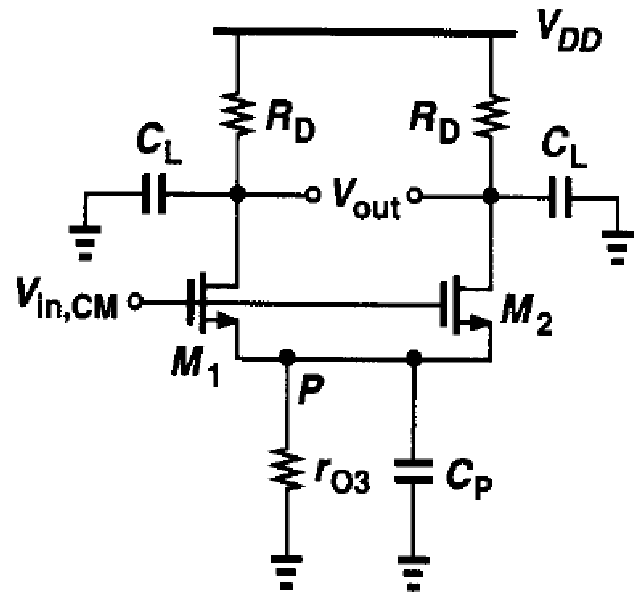
Z_{out} contains a pole at $(r_{o1} C_X)^{-1}$

Z_{out} falls above this frequency

Differential Pair



Equivalent Half Circuit



Similar to CS Stage

Problem with CMRR

To minimize voltage headroom consumed by M_3 → its width is maximized → increases the capacitance contribution by M_3 → higher capacitance at the source of M_1 and M_2 → degrades the differential gain of M_1 and M_2 at high frequencies → reduces the CMRR at high frequencies