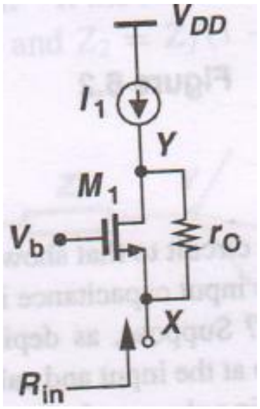
Lecture – 18 Date: 29.10.2015

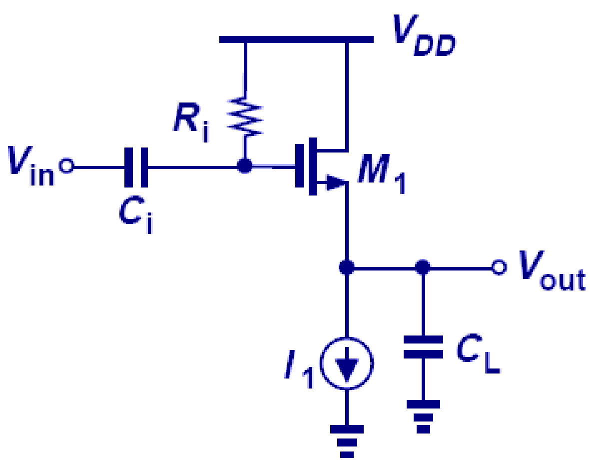
- General Frequency Response
- High Frequency MOSFET Model
- Transit Frequency
- Determination of 3-dB Frequency
- CS Stage - Analysis using Miller's Approximation, OCTC Method, Exact Technique

Quiz – 5

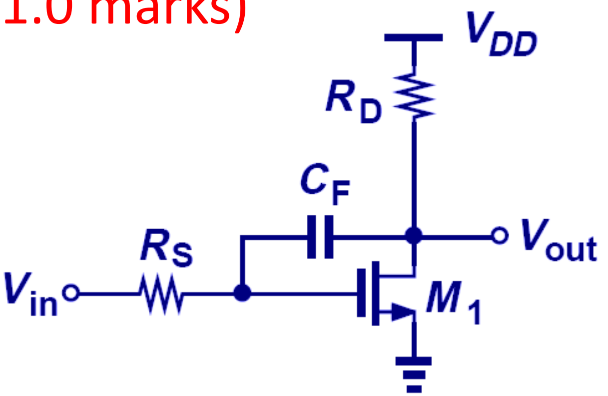
Q1: Through appropriate derivations, prove that the input resistance, R_{in}, of this circuit is extremely high. (0.75 marks)



Q2: what is the need of input capacitance C_i in the following circuit. Give appropriate examples to justify your answer. (0.75 marks)

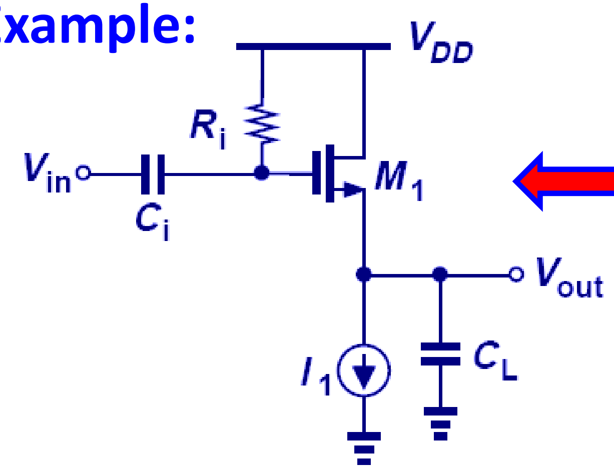


Q3: Use Miller's theorem to determine the poles and transfer function of the following network. (1.0 marks)



General Frequency Response

Example:



High quality audio amplifier: R_i establishes a gate bias voltage equal to V_{DD} for M_1, and I_1 defines the drain bias current. Assume $\lambda=0$, $g_m=1/(200\Omega)$, and $R_i=100k\Omega$. Determine the minimum required value of C_i and the maximum tolerable value of C_L

- The input network consisting of R_i and C_i attenuates the signal at low frequencies. The roll-off frequency for audio signal is given as:

$$2\pi*(20Hz)=\frac{1}{R_iC_i}=\frac{1}{100*10^3*C_i}$$

$$\therefore C_i=79.6nF$$ **Min. Value**

- The load capacitance creates a pole at the output node, lowering the gain at the high frequencies. Let us suppose pole frequency at 20kHz (upper end of audio):

Max. Value

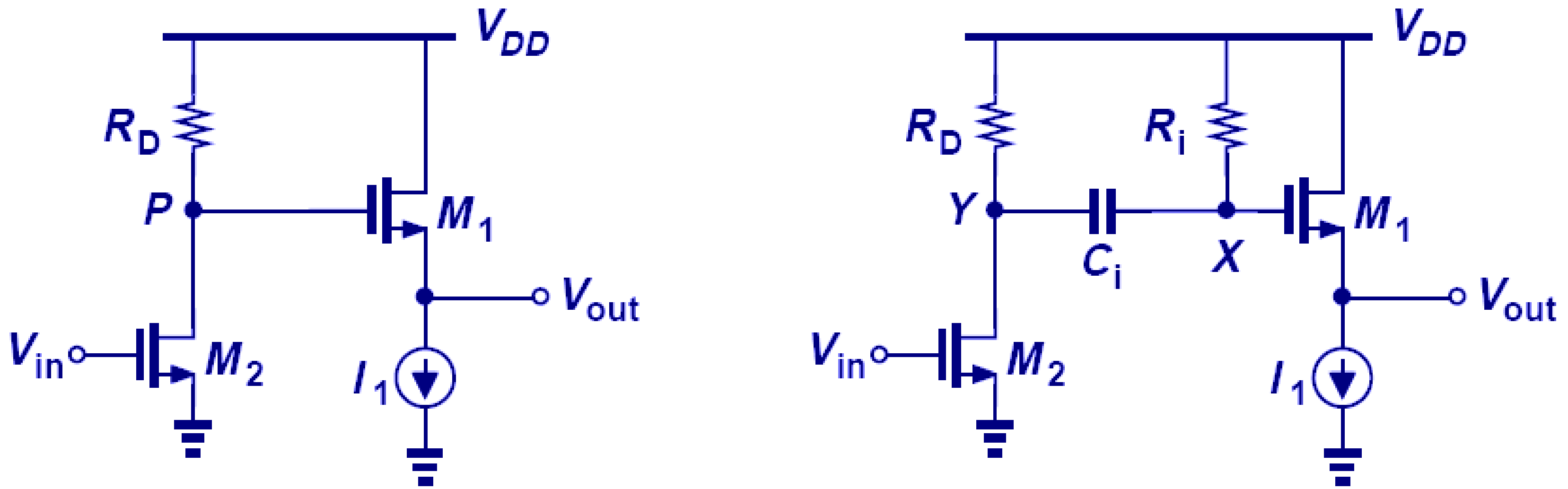
$$\omega_{p,out}=\frac{g_m}{C_L}=2\pi*20*10^3$$

$$\therefore C_L=39.8nF$$

General Frequency Response (contd.)

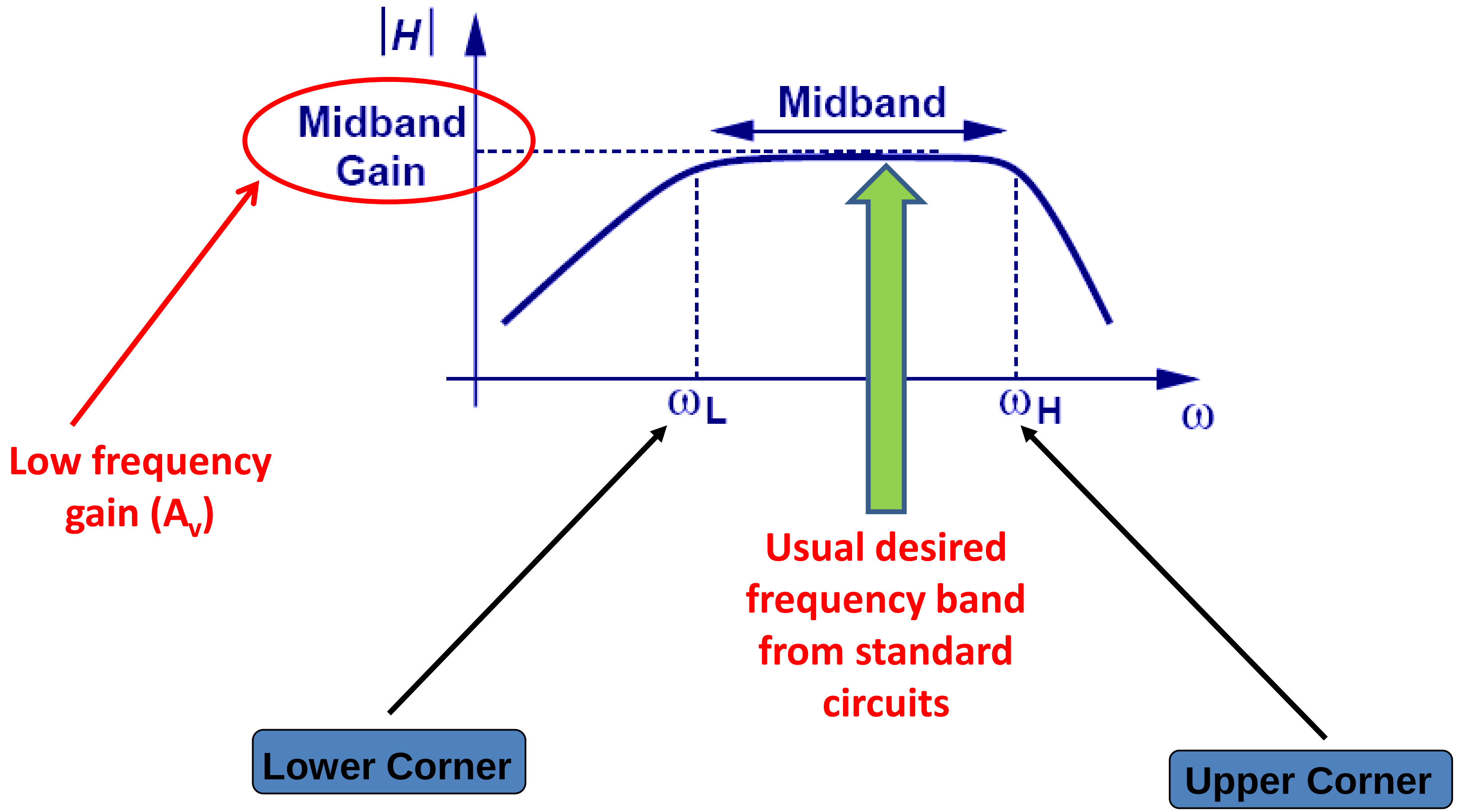
Why do we need capacitor C_i at the input in the previous example?

The absence of C_i could be blessing as it will not affect the performance at low frequencies → we would be saved from computing C_i as well

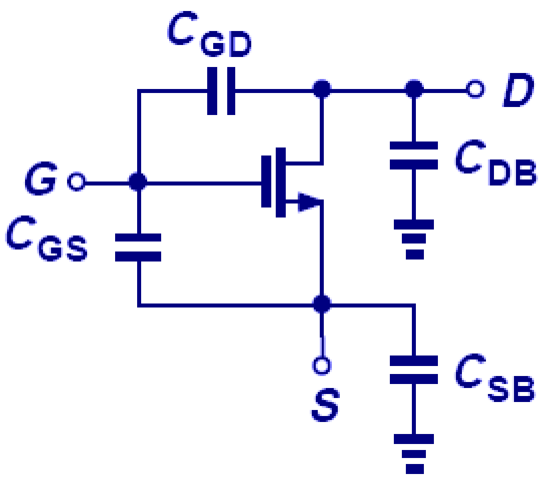


- Capacitive coupling, also known as AC coupling, passes AC signals from Y to X while blocking DC contents.
- This technique allows independent bias conditions between stages. Direct coupling does not.

General Frequency Response (contd.)



High Frequency MOSFET Model



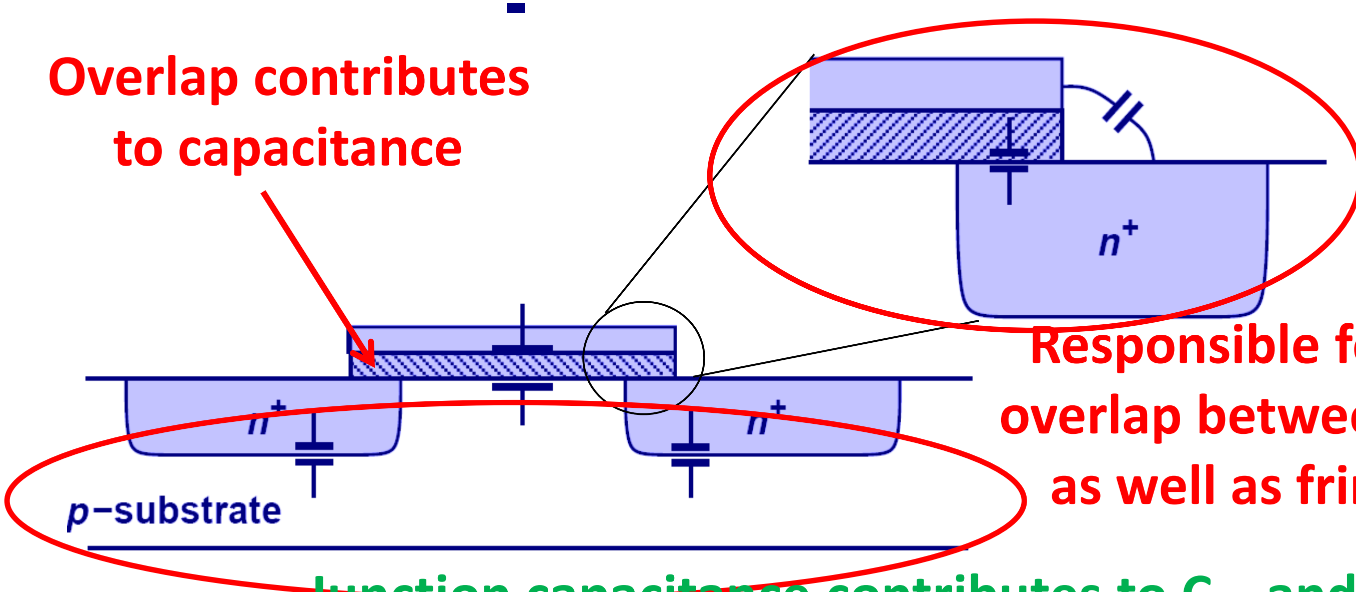
C_{GD}: gate-drain capacitance

C_{GS}: gate-source capacitance

C_{SB}: source-bulk capacitance

C_{DB}: drain-bulk capacitance

Overlap contributes to capacitance

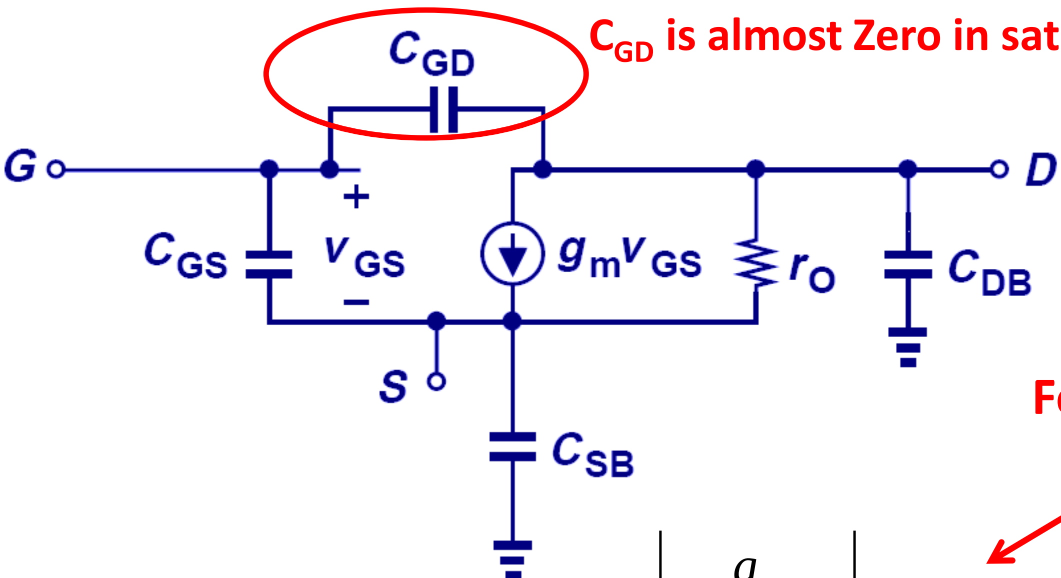


Responsible for C_{GD} and C_{GS}: overlap between gate and S/D as well as fringe field lines

Junction capacitance contributes to C_{SB} and C_{DB}

Cut-off Frequency or Transit Frequency

- So many capacitances in the MOSFET reduces the performance of amplifiers → cut-off or transit frequency, f_T, regulates the speed of MOSFET
- It is the frequency at which the small-signal current gain falls to unity



C_{GD} is almost Zero in saturation

Input Current $= j\omega C_{GS} V_{GS}$

Output Current $= g_m V_{GS}$

For f_T: $\left| \dfrac{g_m V_{GS}}{\omega_T C_{GS} V_{GS}} \right| = 1$

$$\Rightarrow \left| \frac{g_m}{2\pi f_T C_{GS}} \right| = 1$$

$$\therefore f_T = \left| \frac{g_m}{2\pi C_{GS}} \right|$$

The source-bulk and drain-bulk capacitance doesn't affect the speed of transistor

Determination of 3-dB frequency (f_H)

- As a designer it is important to understand the implications of various capacitive effects (present in the circuit) on the overall performance of the circuit
- In order to understand such implications there are three different techniques to determine f_H (a key parameter in high frequency performance estimation)
- **Miller's Approximation Technique:** It is useful for certain cases when the input resistance is relatively large and output capacitance (C_L) is relatively small → in such a case the high-frequency response is dominated by the pole formed at the input node
- **OCTC Method:** Its useful for circuits when its not easy to determine the poles and zeros by hand analysis → is an approximate method
- **Exact Analysis:** Involves full analysis of the circuit to find the transfer function

Determination of 3-dB frequency (f_H) – contd.

Physical Significance of Poles and Zeros in a Transfer Function:

- Think of Poles and Zeros as INFINITY's and ZEROs.
- At Zeros: the system produces ZERO output
- At Poles: the system produces INFINITE output
- Obviously, you cannot produce infinite voltage with any electronics

→ So, it means that, the output will be unbounded (in theory) and saturated at the highest possible value (in practice).

========================

Now, let's talk about a specific case: The TRANSFER FUNCTION can be the IMPEDANCE of a filter, it will be zero (short circuit) at zeros, and INFINITY (open circuit) at poles

Miller Approximation Technique

- **High Frequency Gain function of an amplifier can be given as:**

$$A(s) = A_M . F_H(s)$$

Transfer function of amp

Mid-band gain → small-signal gain

- **$F_H(s)$ can be represented in terms of poles and zeros as:**

$$F_H(s) = \frac{(1+s/\omega_{z1})(1+s/\omega_{z2})......(1+s/\omega_{zn})}{(1+s/\omega_{p1})(1+s/\omega_{p2})......(1+s/\omega_{pn})}$$

- **If a dominant pole (ω_{p1}) exists then:**

$$F_H(s) \cong \frac{1}{(1+s/\omega_{p1})}$$

Assuming that zeros are usually either at infinity or possess very high value

Miller Approximation Technique (contd.)

- Thus presence of a dominant pole provides 3-dB roll-off frequency as:

$$\omega_H \cong \omega_{p1}$$

- Condition for the existence of dominant pole: the lowest-frequency pole is at least two octave away from the nearest pole or zero.

- If a dominant pole doesn't exist then:

$$F_H(s) = \frac{(1+s/\omega_{z1})(1+s/\omega_{z2})}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$$

For 2-pole and 2-zero network

$$\Rightarrow F_H(j\omega) = \frac{(1+j\omega/\omega_{z1})(1+j\omega/\omega_{z2})}{(1+j\omega/\omega_{p1})(1+j\omega/\omega_{p2})}$$

Miller Approximation Technique (contd.)

$$\Rightarrow \left|F_H(j\omega)\right|^2 = \frac{\left(1+\omega^2/\omega_{z1}^2\right)\left(1+\omega^2/\omega_{z2}^2\right)}{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}$$

- **For $\omega=\omega_H \rightarrow |F_H|^2 = 1/2$ and therefore:**

$$\Rightarrow \frac{1}{2} = \frac{\left(1+\omega_H^2/\omega_{z1}^2\right)\left(1+\omega_H^2/\omega_{z2}^2\right)}{\left(1+\omega_H^2/\omega_{p1}^2\right)\left(1+\omega_H^2/\omega_{p2}^2\right)}$$

- **ω_H is smaller than all other poles and zeros and as a consequence terms with ω_H^4 could be neglected. Therefore simplification gives:**

$$\omega_H \cong \frac{1}{\sqrt{\left(\frac{1}{\omega_{p1}^2}+\frac{1}{\omega_{p2}^2}\right)-2\left(\frac{1}{\omega_{z1}^2}+\frac{1}{\omega_{z2}^2}\right)}}$$

Open Circuit Time Constant (OCTC) Method

- **Its not always straightforward to apply Miller technique and determine the poles and zeros**
- **In such cases OCTC method prove handy**
- **Alternate form of $F_H(s)$ for n-zero and n-pole network is:**

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \ldots\ldots.. + a_n s_n}{1 + b_1 s + b_2 s^2 + \ldots\ldots.. + b_n s_n}$$

Where, **a** and **b** are related to zeros and poles respectively. For example, **b_1** is given by:

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \ldots\ldots + \frac{1}{\omega_{pn}}$$

Ref: Paul E. Gray and Campbell L. Searle, Electronic Principles: Physics, Models, and Circuits (1969), John Wiley & Sons Inc., New York

Open Circuit Time Constant (OCTC) Method (contd.)

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \ldots\ldots + \frac{1}{\omega_{pn}}$$

- **b_1 can be determined by considering various capacitances in the network one at a time while reducing all other capacitors to zero i.e, replacing them with open circuits**
- **Determine C_iR_i for each capacitors and then compute:**

$$b_1 = \sum_{i=1}^{n} C_i R_i$$

- **If one of the poles is dominant (say P1) then:**

$$b_1 \cong \frac{1}{\omega_{p1}} \qquad \Rightarrow \omega_H \simeq \frac{1}{b_1} = \frac{1}{\sum_i C_i R_i}$$

Open Circuit Time Constant (OCTC) Method (contd.)

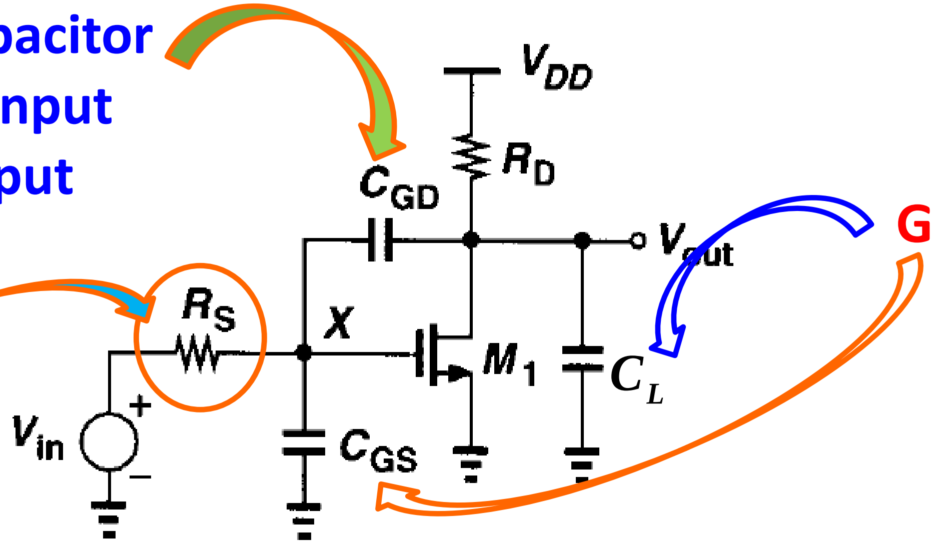
Advantage of OCTC method:

- It tells the circuit designer which of the various capacitances is significant in determining the network (amplifier) frequency response
- The relative contribution of the various capacitances to the effective time constant b_1 is immediately obvious
- For example, if in any amplifier the contribution of $C_{GD}R_{GD}$ in the overall time constant is maximum → then C_{GD} is dominant capacitor in determining f_H → to increase f_H, either use MOSFET with smaller C_{GD} or for a given MOSFET reduce R_{GD} by either reducing the load impedance or by employing smaller source impedance → furthermore, if source impedance is also fixed then the only way to increase f_H (and hence the bandwidth) is by reducing the load impedance
- Reduction in load impedance → leads to reduction in A_M

Common Source Amplifier

Floating Capacitor
between Input
and Output

Grounded Capacitor



Driven by finite
source
resistance

Common Source Amplifier Trade-Offs

A_V	I (μA)	L(μm)	W(μm)	g_m (uS)	C_{DB} (fF)	C_{GD}(fF)	C_{GS}(fF)
10	10	2	5.78	3.613	5.19	1.84	98.16
15	10	2	32.5	5.33	27.5	10.4	517.8
20	10	2	668.2	6.66	319.6	239.8	6,041.1

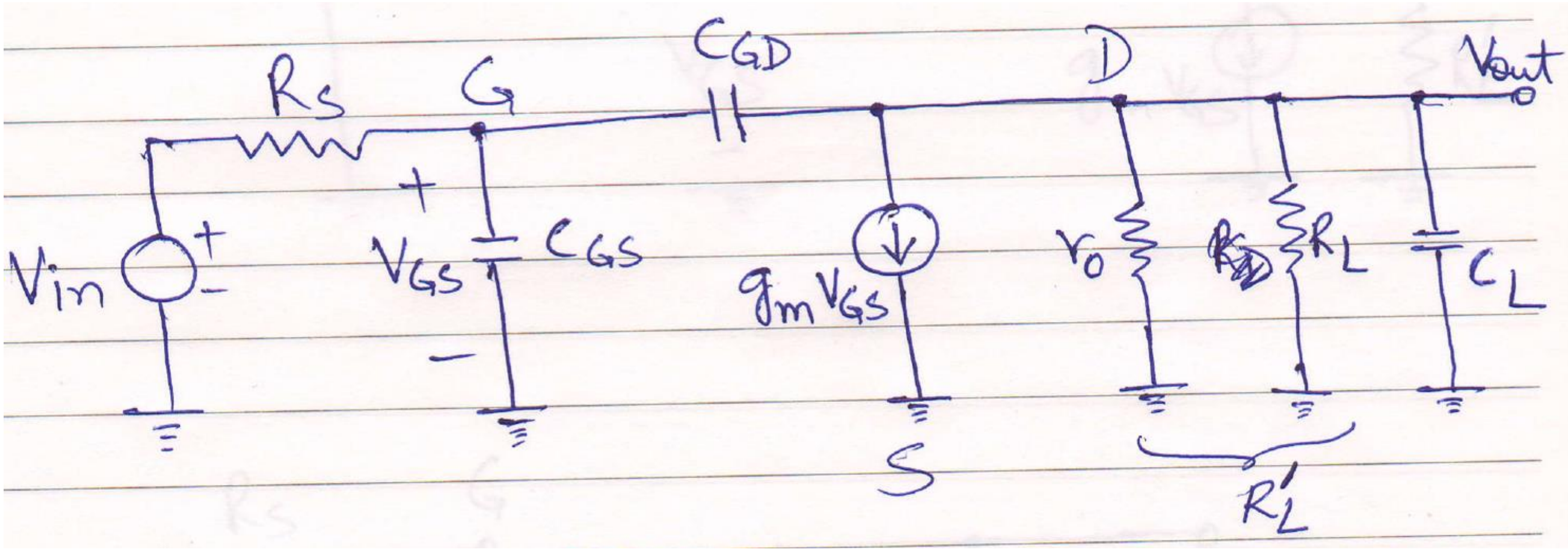
Increasing Gain

Constant Current

Increasing C_{GS} ↔ Reduced Speed

Difficult to achieve high gain and high speed at the same time!

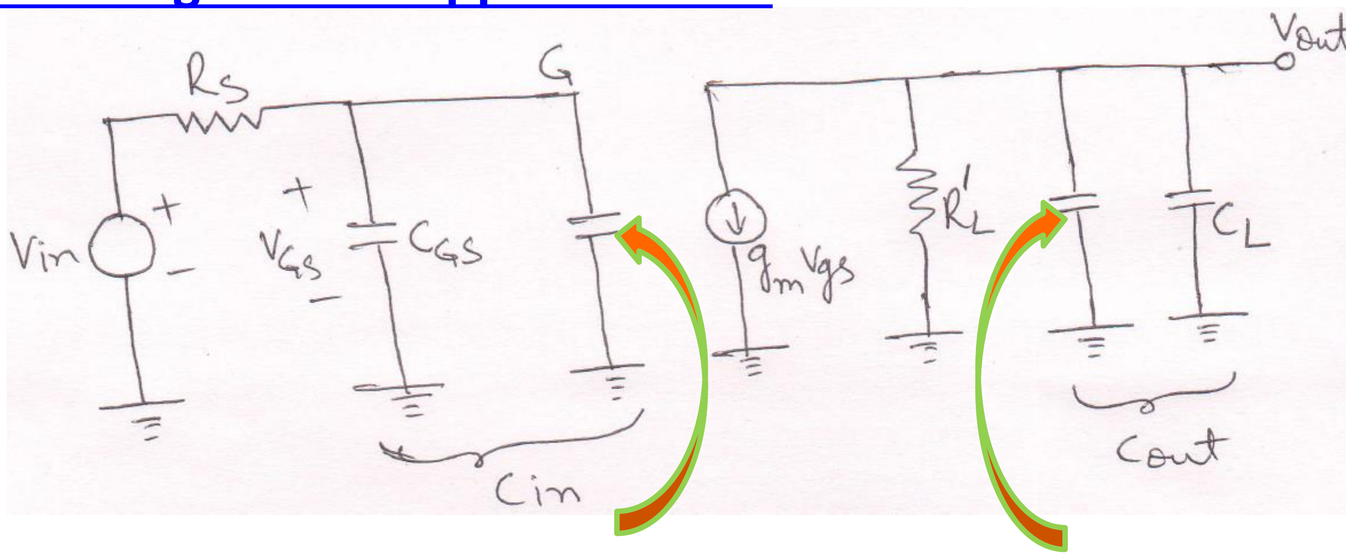
Common Source Amplifier (contd.)



- R_S: also includes the resistance due to the biasing network
- R_L: includes R_D → usually R_L is of the order of r_o
- C_L: represents the total capacitance between the drain and the ground → includes C_{DB} and input capacitance of succeeding amplifier stage → C_L in an IC is substantial

Common Source Amplifier (contd.)

Analysis using Miller's Approximation



$$C_A = (1 - A_v)C_{GD} = (1 + g_m R_L^{'})C_{GD} \qquad C_B = (1 - A_v^{-1})C_{GD} \approx C_{GD}$$

Therefore the poles are:

$$\omega_{in} = \frac{1}{R_S C_{in}} = \frac{1}{R_S (C_{GS} + C_A)} = \frac{1}{R_S \left(C_{GS} + (1 + g_m R_L^{'})C_{GD}\right)}$$

$$\omega_{out} = \frac{1}{R_L^{'} C_{out}} = \frac{1}{R_L^{'} (C_L + C_B)} = \frac{1}{R_L^{'} (C_L + C_{GD})}$$

Common Source Amplifier (contd.)

Then the transfer function is given by:

$$H(s) = \frac{V_{out}}{V_{in}}(s) = \frac{-g_m R_D}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

When R_S is large and C_L is small

ω_{in} dominates, and the transfer function becomes:

$$\boxed{H(s) = \frac{-g_m R_D}{\left(1 + \frac{s}{\omega_{in}}\right)}}$$

$$H(s) = \frac{A_M}{1 + \frac{s}{\omega_H}}$$

Dominant Pole

3-dB Frequency: $f_H = \dfrac{1}{2\pi C_{in} R_S}$

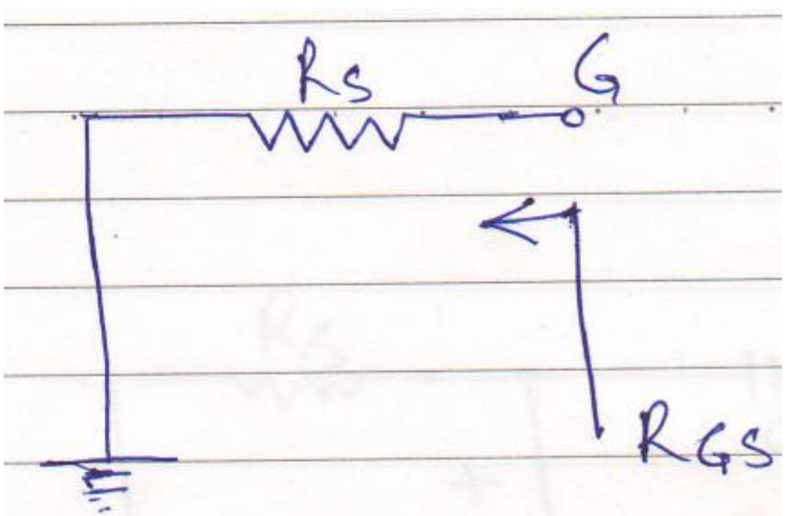
Where, $C_{in} = C_{GS} + C_{GD}(1 + g_m R_L^{'})$

The main error in this expression is that the presence of zero has not been considered

Common Source Amplifier (contd.)

Analysis using OCTC Method

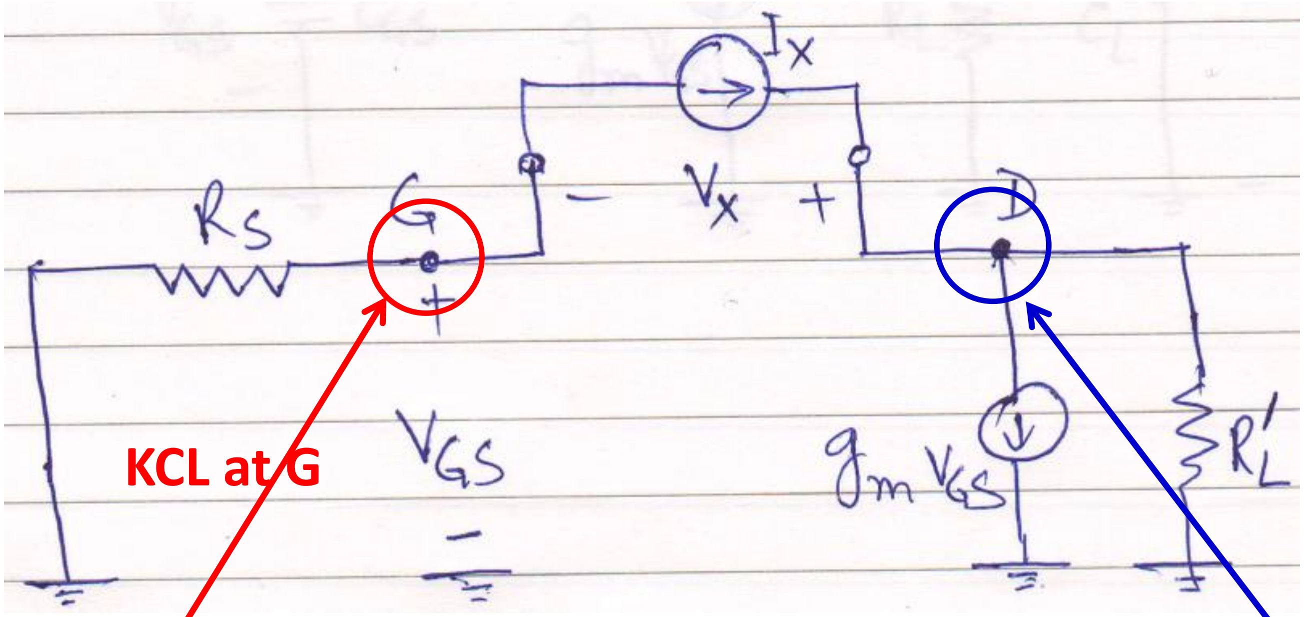
- Considering only C_{GS} → open other capacitances and short the voltage sources and open the current sources
- For R_{GS} we get:



$$R_{GS} = R_S$$

Common Source Amplifier (contd.)

- Considering only C_{GD} → open C_{GS} and C_L



KCL at G

KCL at D

Then, $R_{GD} = \dfrac{V_X}{I_X}$

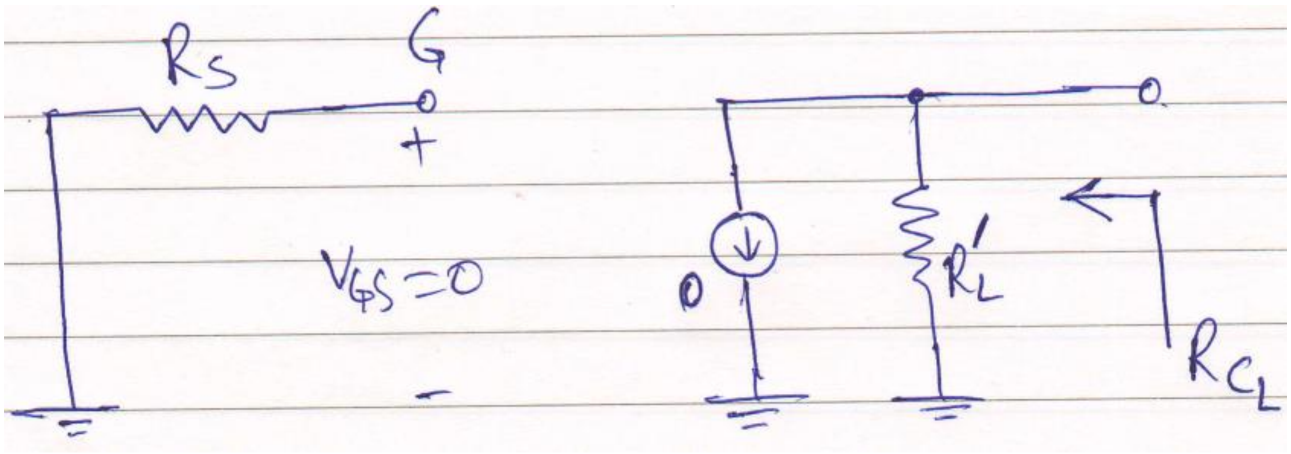
$$\frac{V_{GS}}{R_S} + I_X = 0$$

$$I_X = g_m V_{GS} + \frac{V_{GS} + V_X}{R_L'}$$

$$\Rightarrow R_{GD} = \frac{V_X}{I_X} = R_S + (1 + g_m R_S) R_L'$$

Common Source Amplifier (contd.)

- Considering only C_L → open C_{GS} and C_{GD}



$$R_{C_L} = R_L^{'}$$

Thus, the effective time constant: $\tau_H = C_{GS}R_{GS} + C_{GD}R_{GD} + C_L R_{C_L}$

Therefore the 3-dB roll-off frequency is: $f_H = \dfrac{1}{2\pi\tau_H}$

Provides a better estimate than Miller's approximation