Lecture – 17

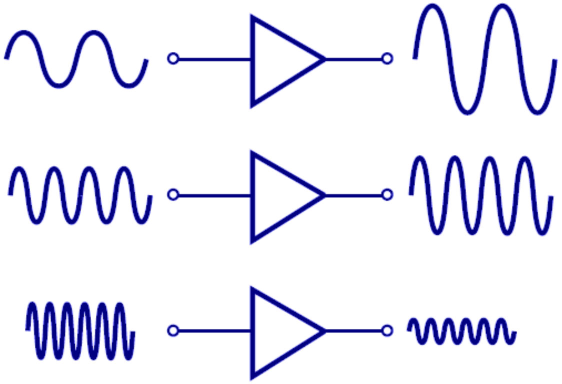
Date: 26.10.2015

- Introduction to Frequency Response
- 3-dB Frequency
- Bode’s Rules
- Association of Poles with Nodes
- Miller’s Theorem
- General Frequency Response
- High Frequency MOSFET Model
- Transit Frequency
- Determination of 3-dB Frequency

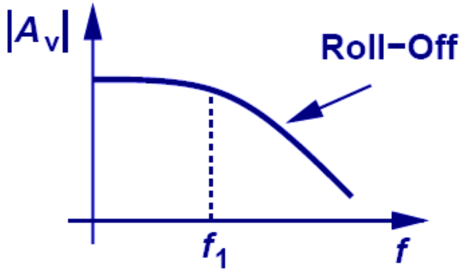
Intro to Frequency Response

What is meant by frequency response of an amplifier?

- The idea is to apply varying frequency signal to the amplifier and then observe the behavior → the gain may present three different scenario → falling, rising or constant with frequency



As frequency of operation increases, the gain of amplifier decreases

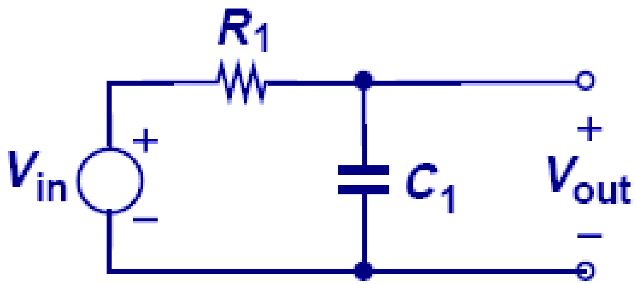


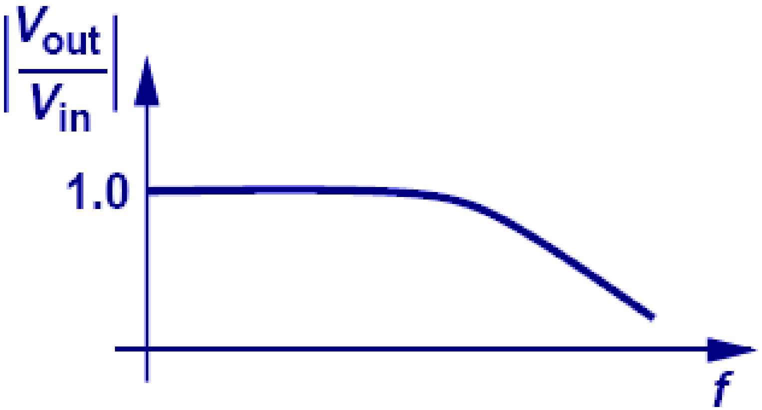
In this example, the gain rolls-off with the increase in frequency → the frequency *'f_1'* is termed as the useful bandwidth of the amplifier

Intro to Frequency Response (contd.)

- **Gain Roll-Off**

Simple Low-Pass Filter





In this simple example, as frequency increases the impedance of C_1 decreases and therefore the voltage divider consisting of C_1 and R_1 attenuates V_{in} to a greater extent at the output

Intro to Frequency Response (contd.)

Why is it important?

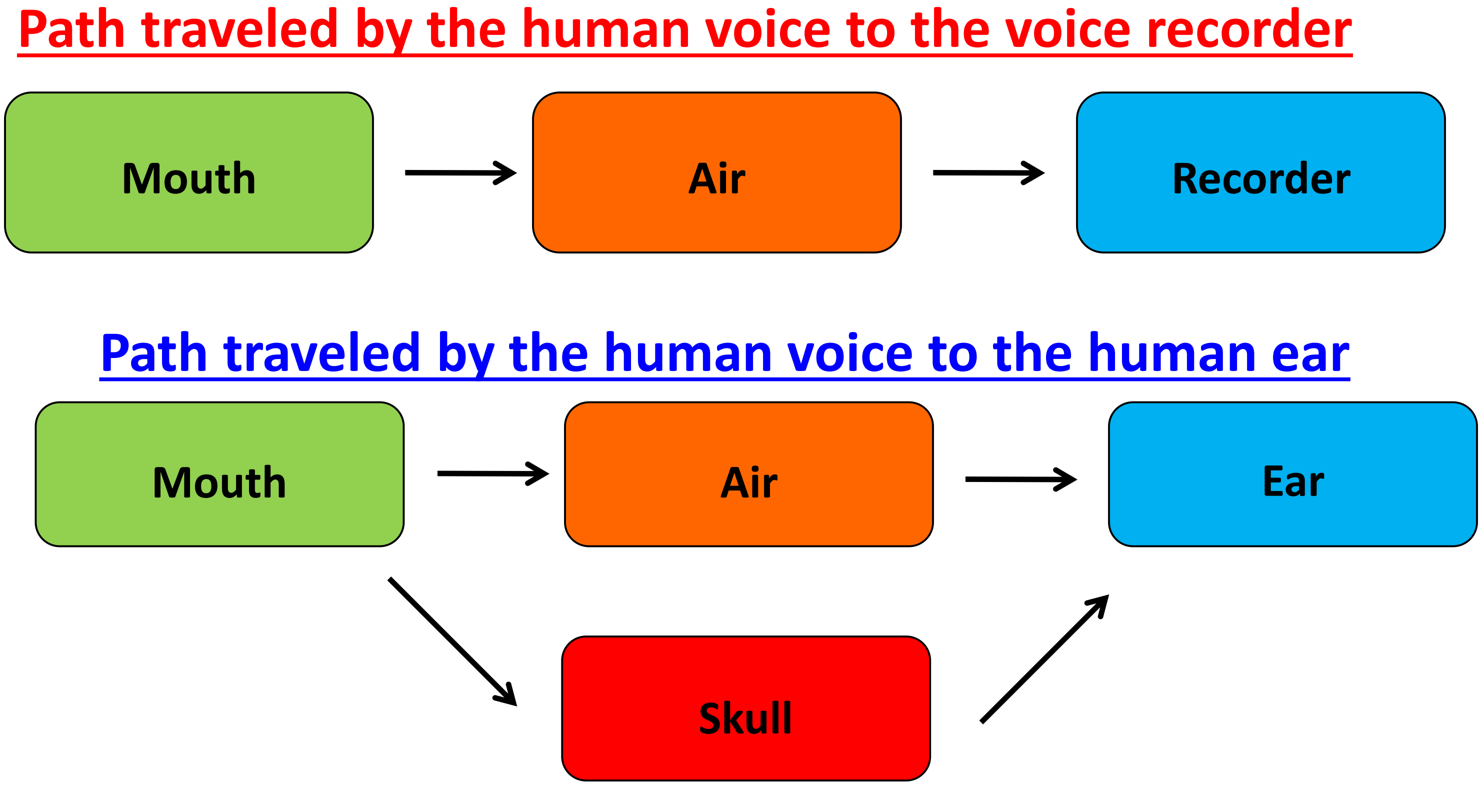
What happens if a CDMA signal has to pass through a GSM standard transceiver?

What happens if your DVB-H enabled hand phone possess circuit components capable of transmitting only CW signals?

What happens if the bandwidth of video system in your computer is insufficient to process video signals?

Intro to Frequency Response (contd.)

- When you record your voice and listen to it, it sounds little different from the way you directly hear. Why?

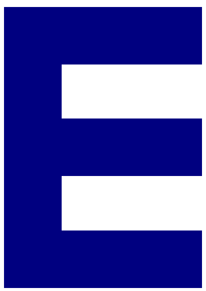


Since the paths are different, the results will also be different

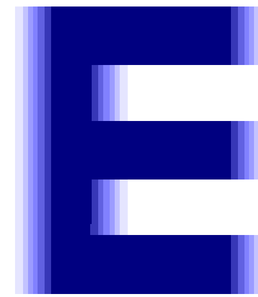
Actually, skull passes some frequencies more easily than air

Intro to Frequency Response (contd.)

- **Example:** The graphics card delivering the video signal to the display must provide at least 5MHz bandwidth. What happens if the bandwidth is not sufficient?



High Bandwidth



Low Bandwidth

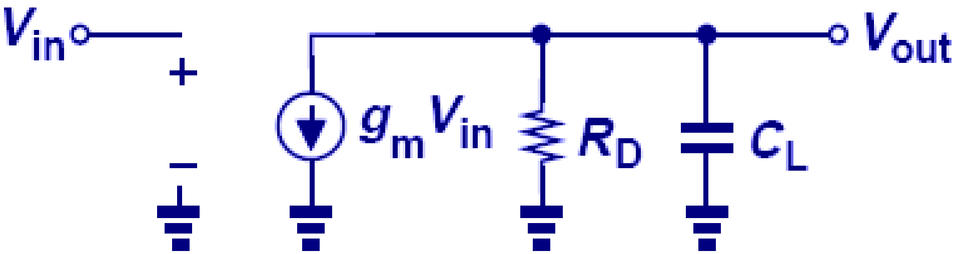
Insufficient bandwidth leads to soft edges
→ yields fuzzy picture

This is due to the fact that the circuit driving the display is not fast enough to abruptly change the contrast from e.g., complete dark to complete white from one pixel to the next

Frequency Response (contd.)

What causes the roll-off in frequency response?

Impedance, current, voltage?



$$V_{out}(s) = -g_m V_{in}(s)\left(R_D \,||\, \frac{1}{X_{C_L}(s)}\right)$$

At low frequencies

The signal current produced by FET flows through R_D → reason is the almost open condition provided by C_L

At high frequencies

The signal current produced by FET flows through the parallel combination of R_D and the impedance contributed by C_L

Reduces with increase in frequency

Results in gain reduction with increase in frequency

The capacitive load, C_L, is the culprit for gain roll-off since at high frequency, it will “steal” away some signal current and shunt it to ground

Frequency Response (contd.)

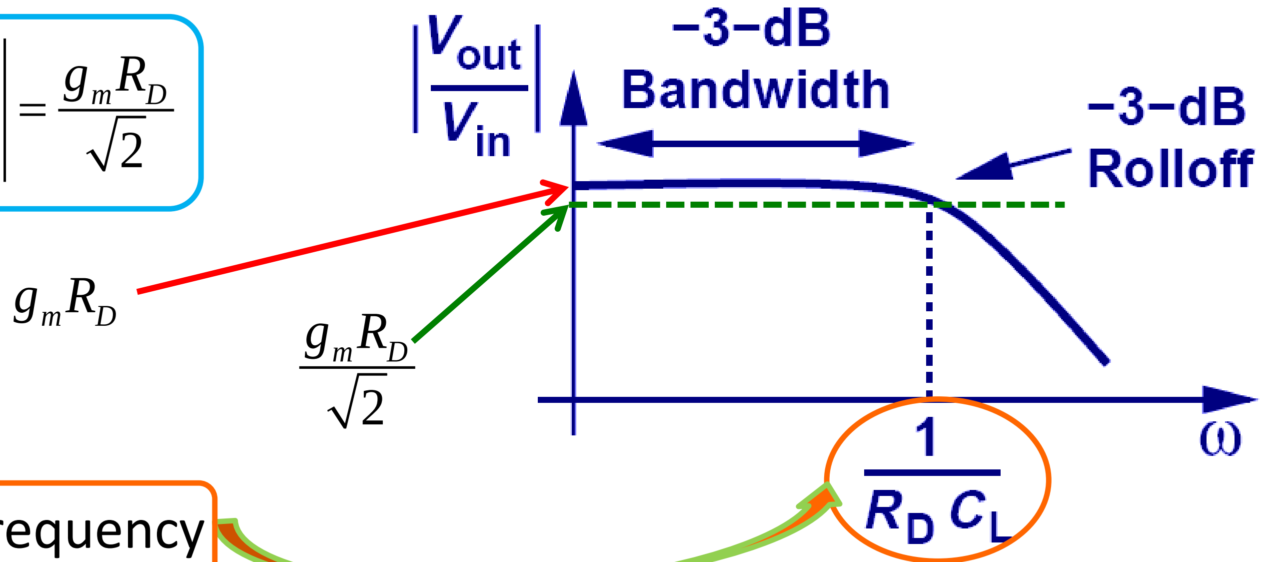
$$\Rightarrow\left|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = g_m\left(R_D \,||\, \frac{1}{X_{C_L}(j\omega)}\right) = \frac{g_m R_D}{\sqrt{R_D^2 X_{C_L}^2 \omega^2 + 1}}$$

At low frequency, the capacitor is effectively open and the gain is flat ($g_m R_D$). As frequency increases, the capacitor tends to a short and the gain starts to decrease.

- A special frequency is $\omega = 1/(R_D C_L)$, where the gain drops by 3dB.

$$\Rightarrow\left|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = \frac{g_m R_D}{\sqrt{2}}$$

3dB – Gain



3-dB frequency

Frequency Response (contd.)

Bode's Rules: The task of obtaining |H(jω)| from H(s) is often tedious. In such cases, it is easier to approximate the response by looking at H(s) → Bode's rules help in approximation

- As ω passes each pole frequency, the slope of |H(jω)| decreases by 20dB/dec. (A slope of 20dB/dec simply means a ten fold decrease in H for a ten fold increase in frequency)
- As ω passes each zero frequency, the slope of |H(jω)| increases by 20dB/dec.

$$H(s) = A_v \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

- When we hit a zero, ω_{zj}, the Bode magnitude rises with a slope of +20dB/dec.
- When we hit a pole, ω_{pj}, the Bode magnitude falls with a slope of -20dB/dec

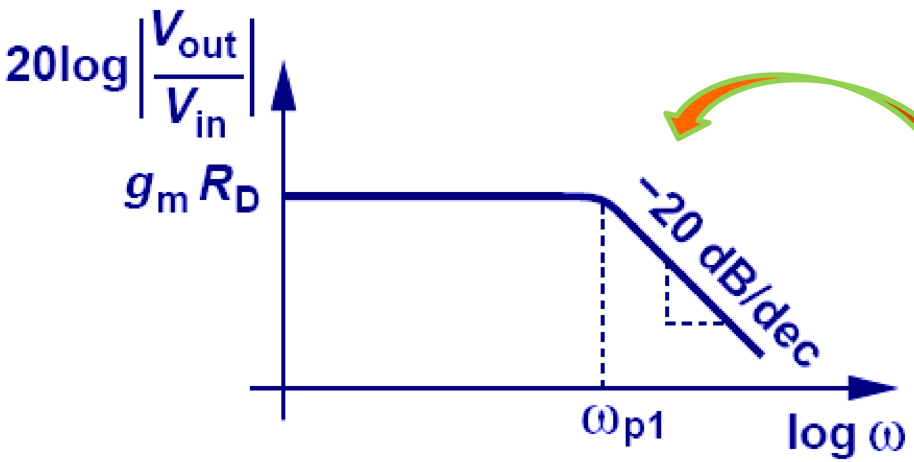
Bode's Rules (contd.)

$$\Rightarrow \left|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = \frac{g_m R_D}{\sqrt{R_D^2 X_{C_L}^2 \omega^2 + 1}}$$

At this frequency the slope changes from 0 to –20 dB/dec

- In this a pole frequency is:

$$|\omega_{p1}| = \frac{1}{R_D C_L}$$



Bode's rule ignores the 3dB roll-off effect at the pole frequency

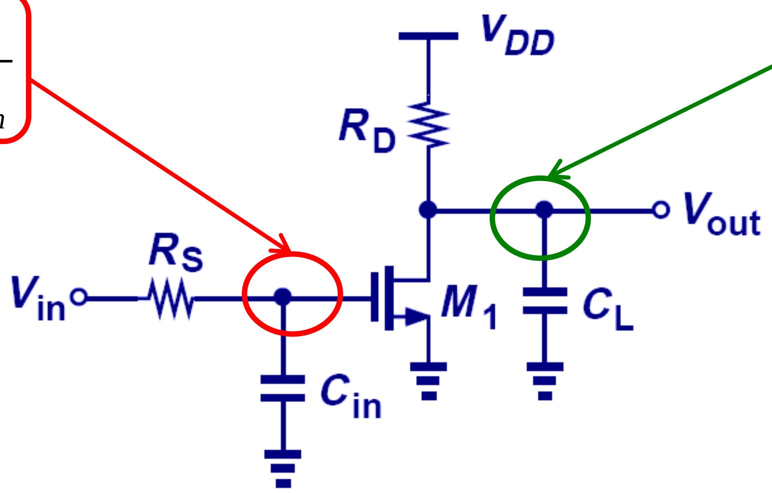
The circuit only has one pole (no zero) at $1/(R_D C_L)$, so the slope drops from 0 to -20dB/dec as we pass ω_{p1}

Frequency Response (contd.)

Association of poles with nodes: poles of a circuit transfer function is key in the frequency response → it aids in determining the speed of various parts of the circuit

$$|\omega_{p1}| = |\omega_{in}| = \frac{1}{R_S C_{in}}$$

$$|\omega_{p2}| = |\omega_{out}| = \frac{1}{R_D C_L}$$



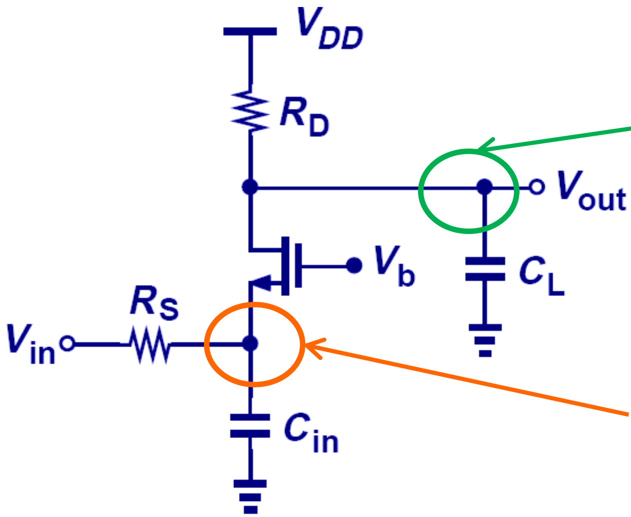
Low Frequency Small Signal Gain

$$\Rightarrow \left|\frac{V_{out}}{V_{in}}(j\omega)\right| = A_v \frac{1}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)}} \cdot \frac{1}{\sqrt{\left(1+\omega^2/\omega_{p2}^2\right)}}$$

$$\therefore \left|\frac{V_{out}}{V_{in}}\right| = \frac{g_{m1}R_D}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}}$$

Frequency Response (contd.)

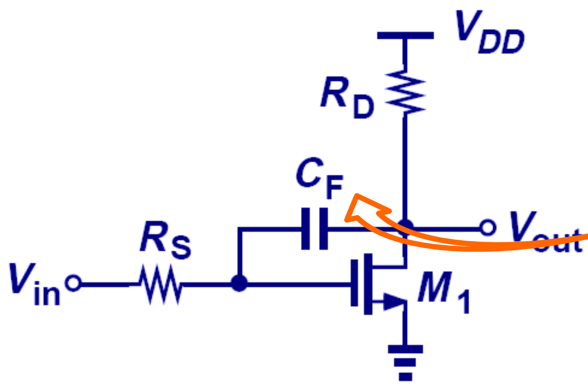
Example:



$$\left|\omega_{p2}\right| = \left|\omega_{out}\right| = \frac{1}{R_D C_L}$$

$$\left|\omega_{p1}\right| = \left|\omega_{in}\right| = \frac{1}{\left(R_S \,\|\, 1/g_{m1}\right) C_{in}}$$

Example: circuit with floating capacitor

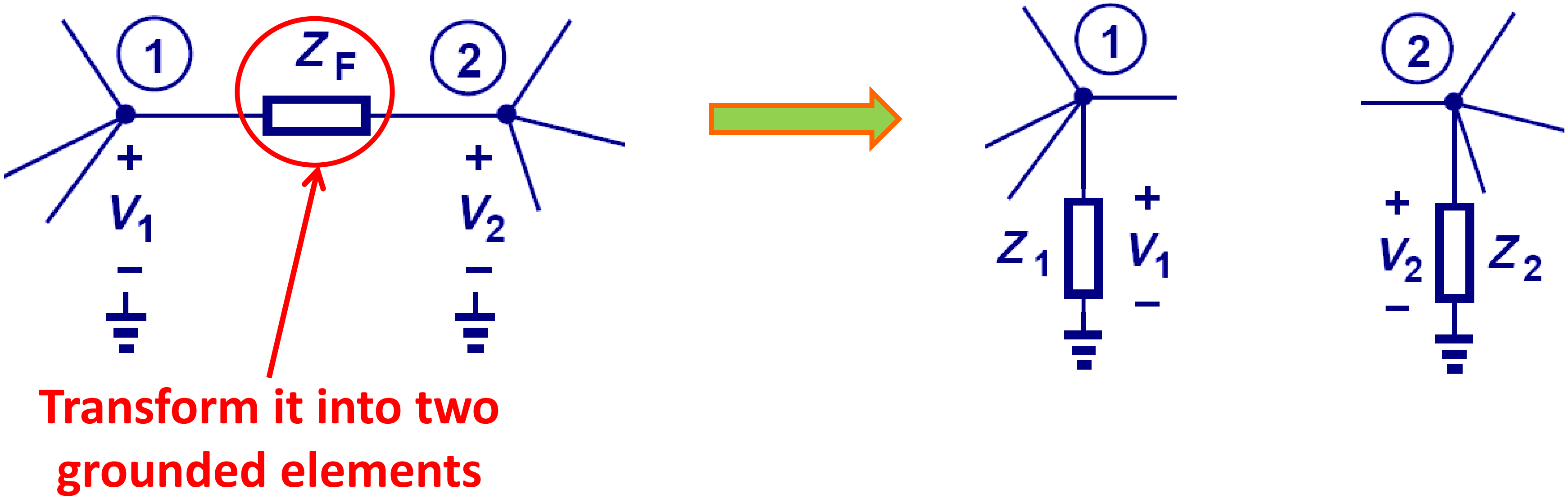


Now the capacitor C_F isn't connected between just one node and ground. What do we do?

Miller's Theorem Provides the Solution

Miller's Theorem

- It coverts floating impedance element into two grounded elements



- The current drawn by Z_F from node 1 must be equal to that drawn by Z_1

$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1}$$

- Current injected to node 2 must be equal to that injected to node 2 in both situations:

$$\frac{V_1 - V_2}{Z_F} = -\frac{V_2}{Z_2}$$

Miller's Theorem (contd.)

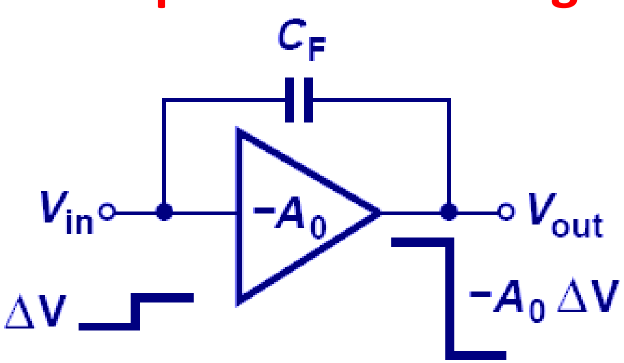
$$Z_1 = Z_F \frac{V_1}{V_1 - V_2} = \frac{Z_F}{1 - A_v}$$

$$Z_2 = Z_F \frac{V_1}{V_1 - V_2} = \frac{Z_F}{1 - \frac{1}{A_v}}$$

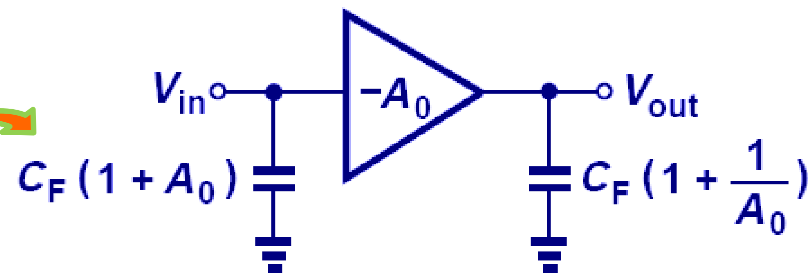
A_v = low frequency small-signal gain

If A_v is the gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2.

- If Z_F is capacitive and amplifier is inverting then:



Miller's Theorem



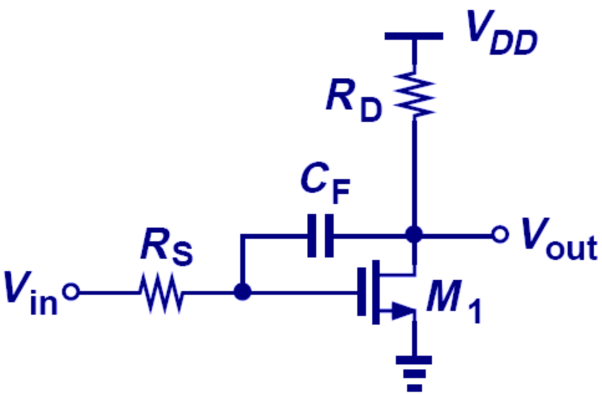
$$Z_1 = \frac{1}{(1 + A_0) C_F s}$$

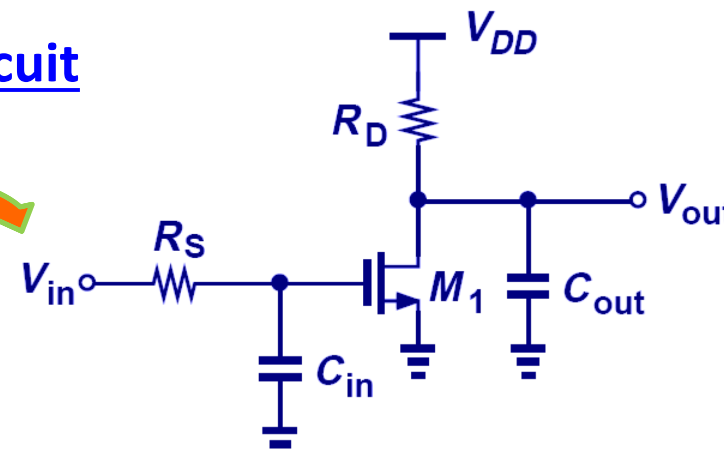
$$Z_2 = \frac{1}{\left(1 + \frac{1}{A_0}\right) C_F s}$$

With Miller's theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.

Miller's Theorem (contd.)

Example: determine poles of the following circuit





$$C_{in} = (1 + A_0)C_F = (1 + g_m R_D)C_F$$

$$\therefore \omega_{in} = \frac{1}{R_S C_{in}} = \frac{1}{R_S(1 + g_m R_D)C_F}$$

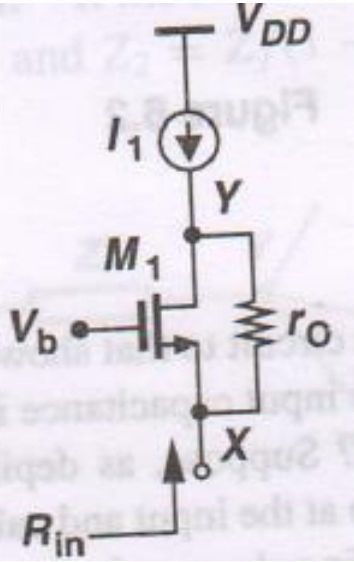
$$C_{out} = \left(1 + \frac{1}{A_0}\right)C_F = \left(1 + \frac{1}{g_m R_D}\right)C_F$$

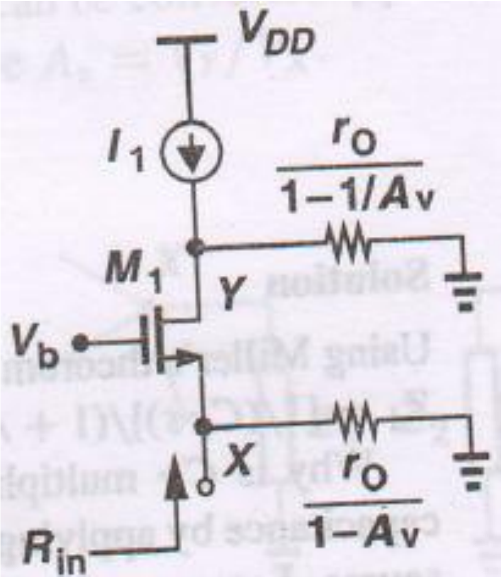
$$\therefore \omega_{out} = \frac{1}{R_D C_{out}} = \frac{1}{R_D\left(1 + \frac{1}{g_m R_D}\right)C_F}$$

Miller's Theorem requires that the floating impedance and voltage gain be computed at the same frequency. However, apparently we always use low-frequency gain even at high frequencies. It is done for simplifying the analysis, otherwise the use of Miller Theorem will be no simpler. Therefore it is often called Miller's Approximation

Miller's Theorem (contd.)

Q: Calculate the input resistance of the following:





Here,

$$A_v = 1 + (g_m + g_{mb})r_o$$

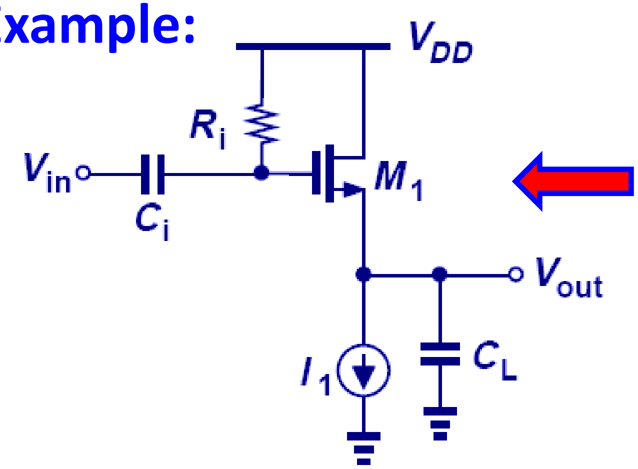
$$R_{in} = \frac{r_o}{1 - A_v} \,||\, \frac{1}{g_m + g_{mb}}$$

$$R_{in} = \frac{r_o}{1 - [1 + (g_m + g_{mb})r_o]} \,||\, \frac{1}{g_m + g_{mb}}$$

$$\therefore R_{in} \approx \infty$$

General Frequency Response

Example:



High quality audio amplifier: R_i establishes a gate bias voltage equal to V_{DD} for M_1, and I_1 defines the drain bias current. Assume $\lambda=0$, $g_m=1/(200\Omega)$, and $R_i=100k\Omega$. Determine the minimum required value of C_i and the maximum tolerable value of C_L

- The input network consisting of R_i and C_i attenuates the signal at low frequencies. The roll-off frequency for audio signal is given as:

$$2\pi*(20Hz)=\frac{1}{R_iC_i}=\frac{1}{100*10^3*C_i}$$

$$\therefore C_i = 79.6nF$$ **Min. Value**

- The load capacitance creates a pole at the output node, lowering the gain at the high frequencies. Let us suppose pole frequency at 20kHz (upper end of audio):

$$\omega_{p,out}=\frac{g_m}{C_L}=2\pi*20*10^3$$

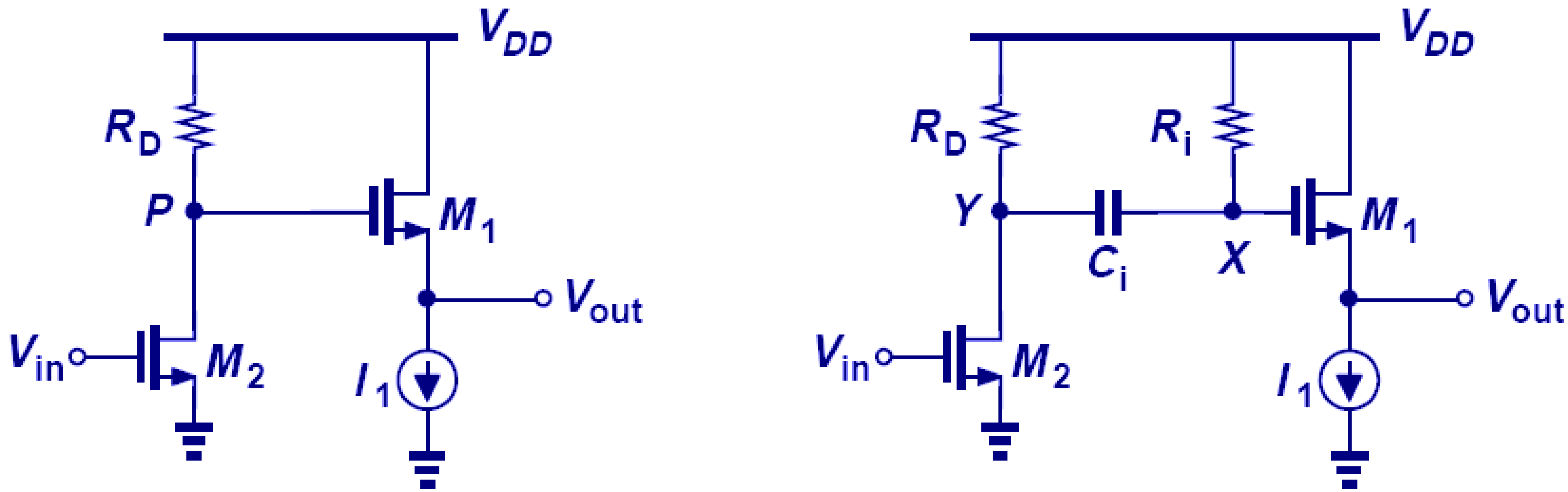
Max. Value

$$\therefore C_L = 39.8nF$$

General Frequency Response (contd.)

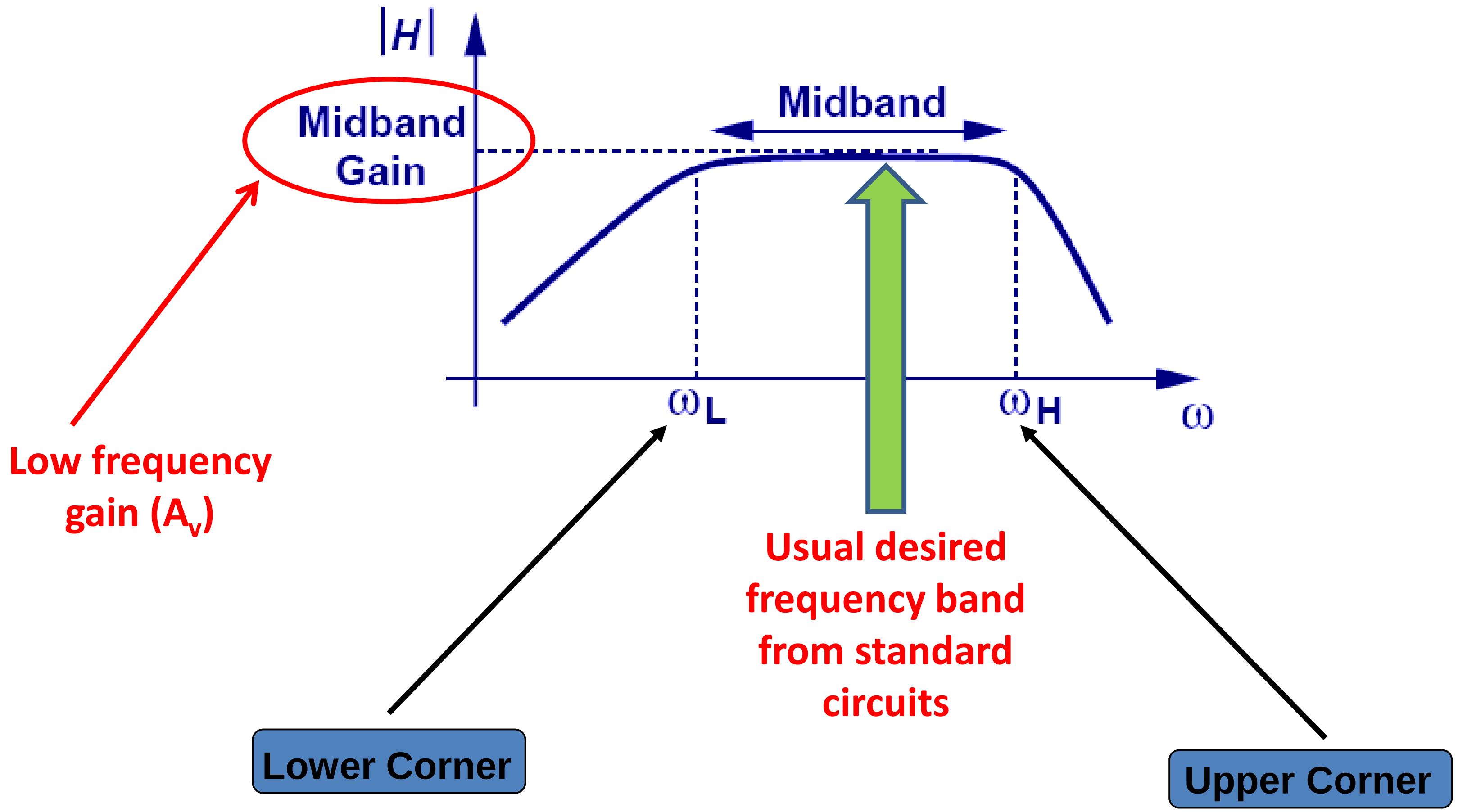
Why do we need capacitor C_i at the input in the previous example?

The absence of C_i could be blessing as it will not affect the performance at low frequencies → we would be saved from computing C_i as well

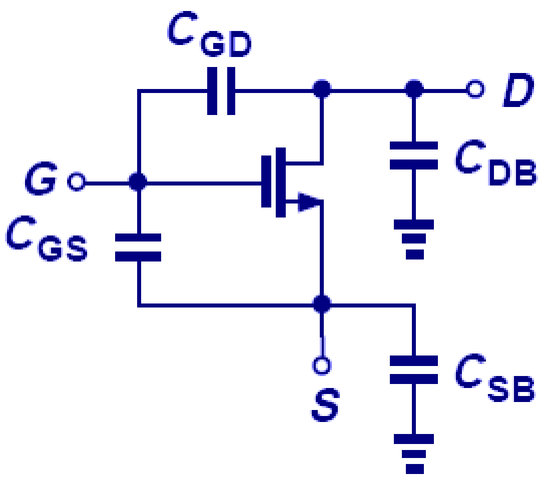


- **Capacitive coupling, also known as AC coupling, passes AC signals from Y to X while blocking DC contents.**
- **This technique allows independent bias conditions between stages. Direct coupling does not.**

General Frequency Response (contd.)



High Frequency MOSFET Model



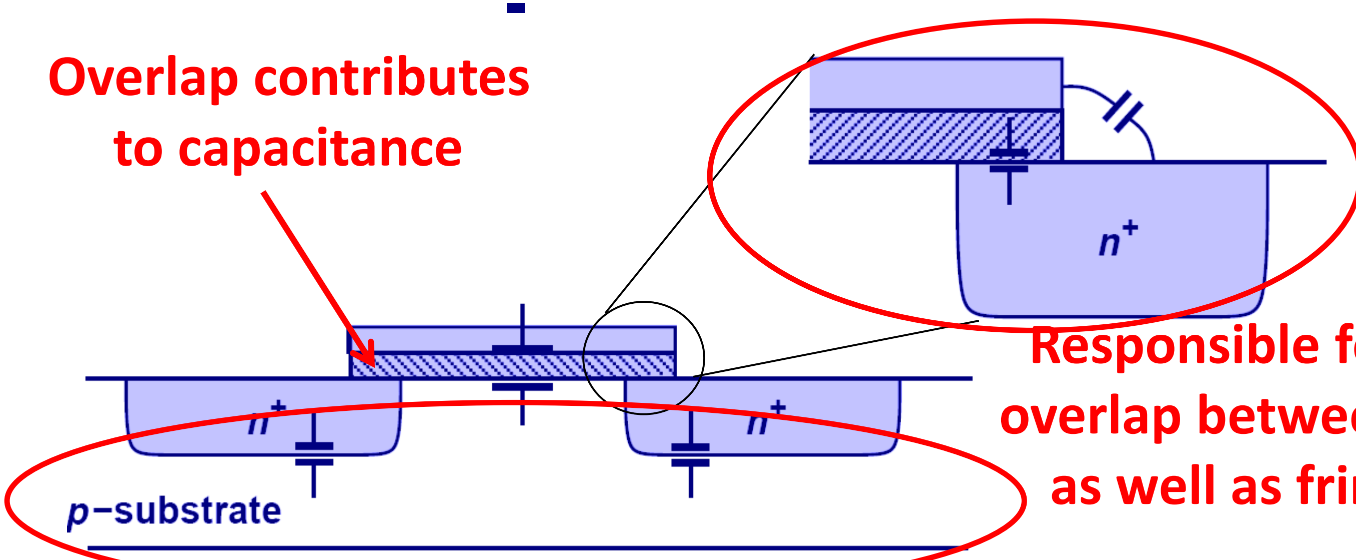
C_{GD}: gate-drain capacitance

C_{GS}: gate-source capacitance

C_{SB}: source-bulk capacitance

C_{DB}: drain-bulk capacitance

Overlap contributes to capacitance

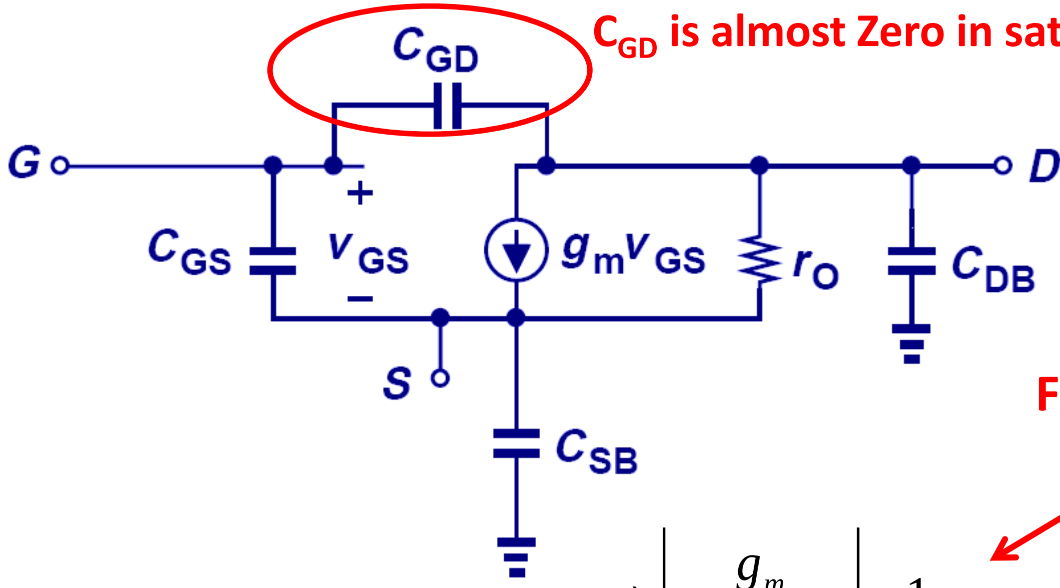


Responsible for C_{GD} and C_{GS}: overlap between gate and S/D as well as fringe field lines

Junction capacitance contributes to C_{SB} and C_{DB}

Cut-off Frequency or Transit Frequency

- So many capacitances in the MOSFET reduces the performance of amplifiers → cut-off or transit frequency, f_T, regulates the speed of MOSFET
- It is the frequency at which the small-signal current gain falls to unity



C_{GD} is almost Zero in saturation

Input Current $= j\omega C_{GS} V_{GS}$

Output Current $= g_m V_{GS}$

For f_T: $\left| \dfrac{g_m V_{GS}}{\omega_T C_{GS} V_{GS}} \right| = 1$

$$\Rightarrow \left| \frac{g_m}{2\pi f_T C_{GS}} \right| = 1$$

$$\therefore f_T = \left| \frac{g_m}{2\pi C_{GS}} \right|$$

The source-bulk and drain-bulk capacitance doesn't affect the speed of transistor

Determination of 3-dB frequency (f_H)

- As a designer it is important to understand the implications of various capacitive effects (present in the circuit) on the overall performance of the circuit
- In order to understand such implications there are three different techniques to determine f_H (a key parameter in high frequency performance estimation)
- **Miller's Approximation Technique:** It is useful for certain cases when the input resistance is relatively large and output capacitance (C_L) is relatively small → in such a case the high-frequency response is dominated by the pole formed at the input node
- **OCTC Method:** Its useful for circuits when its not easy to determine the poles and zeros by hand analysis → is an approximate method
- **Exact Analysis:** Involves full analysis of the circuit to find the transfer function

Determination of 3-dB frequency (f_H) – contd.

Physical Significance of Poles and Zeros in a Transfer Function:

- Think of Poles and Zeros as INFINITY's and ZEROs.
- At Zeros: the system produces ZERO output
- At Poles: the system produces INFINITE output
- Obviously, you cannot produce infinite voltage with any electronics

→ So, it means that, the output will be unbounded (in theory) and saturated at the highest possible value (in practice).

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Now, let's talk about a specific case: The TRANSFER FUNCTION can be the IMPEDANCE of a filter, it will be zero (short circuit) at zeros, and INFINITY (open circuit) at poles

Miller Approximation Technique

- **High Frequency Gain function of an amplifier can be given as:**

$$A(s) = A_M . F_H(s)$$

Mid-band gain → small-signal gain

Transfer function of amp

- **$F_H(s)$ can be represented in terms of poles and zeros as:**

$$F_H(s) = \frac{(1+s/\omega_{z1})(1+s/\omega_{z2}).....(1+s/\omega_{zn})}{(1+s/\omega_{p1})(1+s/\omega_{p2}).....(1+s/\omega_{pn})}$$

- **If a dominant pole (ω_{p1}) exists then:**

$$F_H(s) \cong \frac{1}{(1+s/\omega_{p1})}$$

Assuming that zeros are usually either at infinity or possess very high value

Miller Approximation Technique (contd.)

- Thus presence of a dominant pole provides 3-dB roll-off frequency as:

$$\omega_H \cong \omega_{p1}$$

- Condition for the existence of dominant pole: the lowest-frequency pole is at least two octave away from the nearest pole or zero.

- If a dominant pole doesn't exist then:

$$F_H(s) = \frac{\left(1 + s/\omega_{z1}\right)\left(1 + s/\omega_{z2}\right)}{\left(1 + s/\omega_{p1}\right)\left(1 + s/\omega_{p2}\right)}$$

For 2-pole and 2-zero network

$$\Rightarrow F_H(j\omega) = \frac{\left(1 + j\omega/\omega_{z1}\right)\left(1 + j\omega/\omega_{z2}\right)}{\left(1 + j\omega/\omega_{p1}\right)\left(1 + j\omega/\omega_{p2}\right)}$$

Miller Approximation Technique (contd.)

$$\Rightarrow |F_H(j\omega)|^2 = \frac{\left(1+\omega^2/\omega_{z1}^2\right)\left(1+\omega^2/\omega_{z2}^2\right)}{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}$$

- For $\omega=\omega_H \rightarrow |F_H|^2 = 1/2$ and therefore:

$$\Rightarrow \frac{1}{2} = \frac{\left(1+\omega_H^2/\omega_{z1}^2\right)\left(1+\omega_H^2/\omega_{z2}^2\right)}{\left(1+\omega_H^2/\omega_{p1}^2\right)\left(1+\omega_H^2/\omega_{p2}^2\right)}$$

- ω_H is smaller than all other poles and zeros and as a consequence terms with ω_H^4 could be neglected. Therefore simplification gives:

$$\omega_H \cong \frac{1}{\sqrt{\left(\frac{1}{\omega_{p1}^2}+\frac{1}{\omega_{p2}^2}\right)-2\left(\frac{1}{\omega_{z1}^2}+\frac{1}{\omega_{z2}^2}\right)}}$$

Open Circuit Time Constant (OCTC) Method

- Its not always straightforward to apply Miller technique and determine the poles and zeros
- In such cases OCTC method prove handy
- Alternate form of $F_H(s)$ for n-zero and n-pole network is:

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \ldots\ldots.. + a_n s_n}{1 + b_1 s + b_2 s^2 + \ldots\ldots.. + b_n s_n}$$

Where, **a** and **b** are related to zeros and poles respectively. For example, $\mathbf{b_1}$ is given by:

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \ldots\ldots + \frac{1}{\omega_{pn}}$$

Ref: Paul E. Gray and Campbell L. Searle, Electronic Principles: Physics, Models, and Circuits (1969), John Wiley & Sons Inc., New York

Open Circuit Time Constant (OCTC) Method (contd.)

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \ldots\ldots + \frac{1}{\omega_{pn}}$$

- b_1 can be determined by considering various capacitances in the network one at a time while reducing all other capacitors to zero i.e, replacing them with open circuits
- Determine C_iR_i for each capacitors and then compute:

$$b_1 = \sum_{i=1}^{n} C_i R_i$$

- If one of the poles is dominant (say P1) then:

$$b_1 \cong \frac{1}{\omega_{p1}} \qquad \Rightarrow \omega_H \simeq \frac{1}{b_1} = \frac{1}{\sum_i C_i R_i}$$

Open Circuit Time Constant (OCTC) Method (contd.)

Advantage of OCTC method:

- It tells the circuit designer which of the various capacitances is significant in determining the network (amplifier) frequency response
- The relative contribution of the various capacitances to the effective time constant b_1 is immediately obvious
- For example, if in any amplifier the contribution of $C_{GD}R_{GD}$ in the overall time constant is maximum → then C_{GD} is dominant capacitor in determining f_H → to increase f_H, either use MOSFET with smaller C_{GD} or for a given MOSFET reduce R_{GD} by either reducing the load impedance or by employing smaller source impedance → furthermore, if source impedance is also fixed then the only way to increase f_H (and hence the bandwidth) is by reducing the load impedance
- Reduction in load impedance → leads to reduction in A_M