Lecture-12 Date: 01.10.2015

- Differential Amplifier

Differential Amplifier

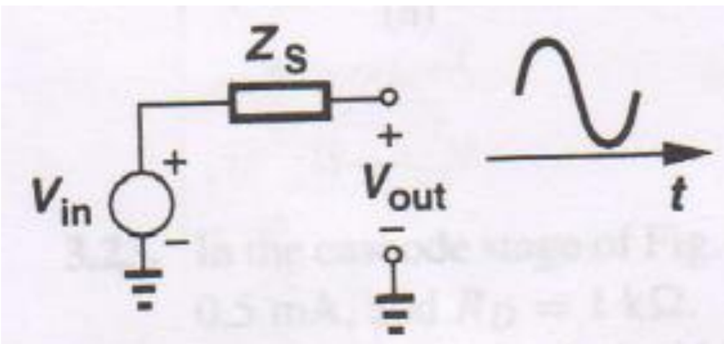
- Why differential?
- What do we want from an amplifier?
  - robust operation → free from external effects such as noise
  - High output voltage swing → optimal headroom / legroom
  - High gain → such as cascode configuration
  - Linear Performance
- Differential amplifiers exhibit/provide following features:
  - robust operation → noise do not affect its performance
  - Higher output voltage swing
  - Higher gain in comparison to single-stage amplifiers
  - Linear Performance and simpler biasing

Major Drawback: more area on a chip (however not always true!)

Differential Amplifier (contd.)

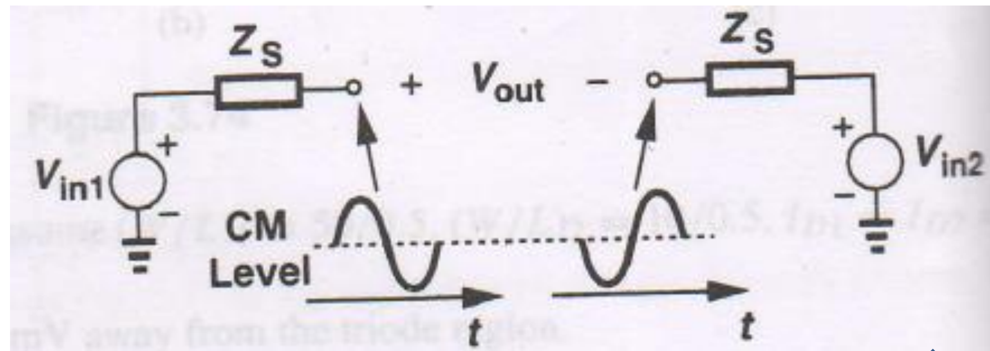
Differential amplifier deals with differential signals

Single-ended signals



Defined with respect to a fixed potential (usually ground)

Differential signals

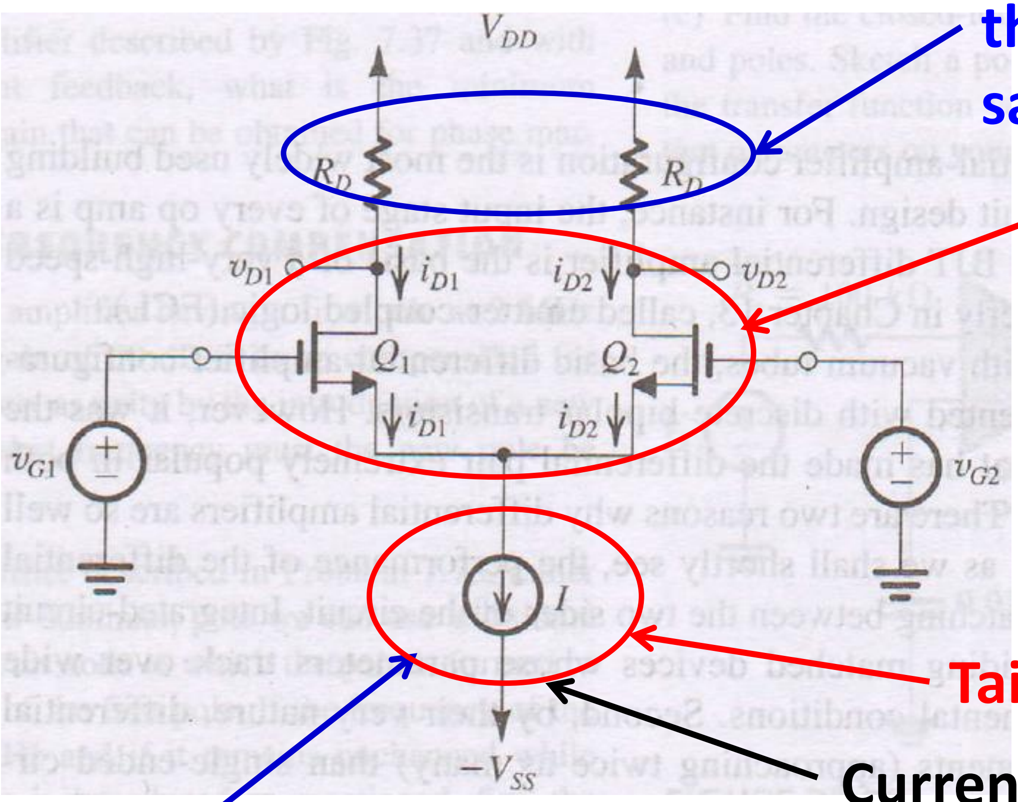


Defined between two nodes that have equal and opposite signal excursions around a fixed potential

The fixed potential in differential signal is called "common-mode" CM level

Differential Amplifier (contd.)

MOS Differential Pair



Loads (active!) → must ensure that the transistors remain in saturation

These transistors are perfectly matched (at least in theory!)

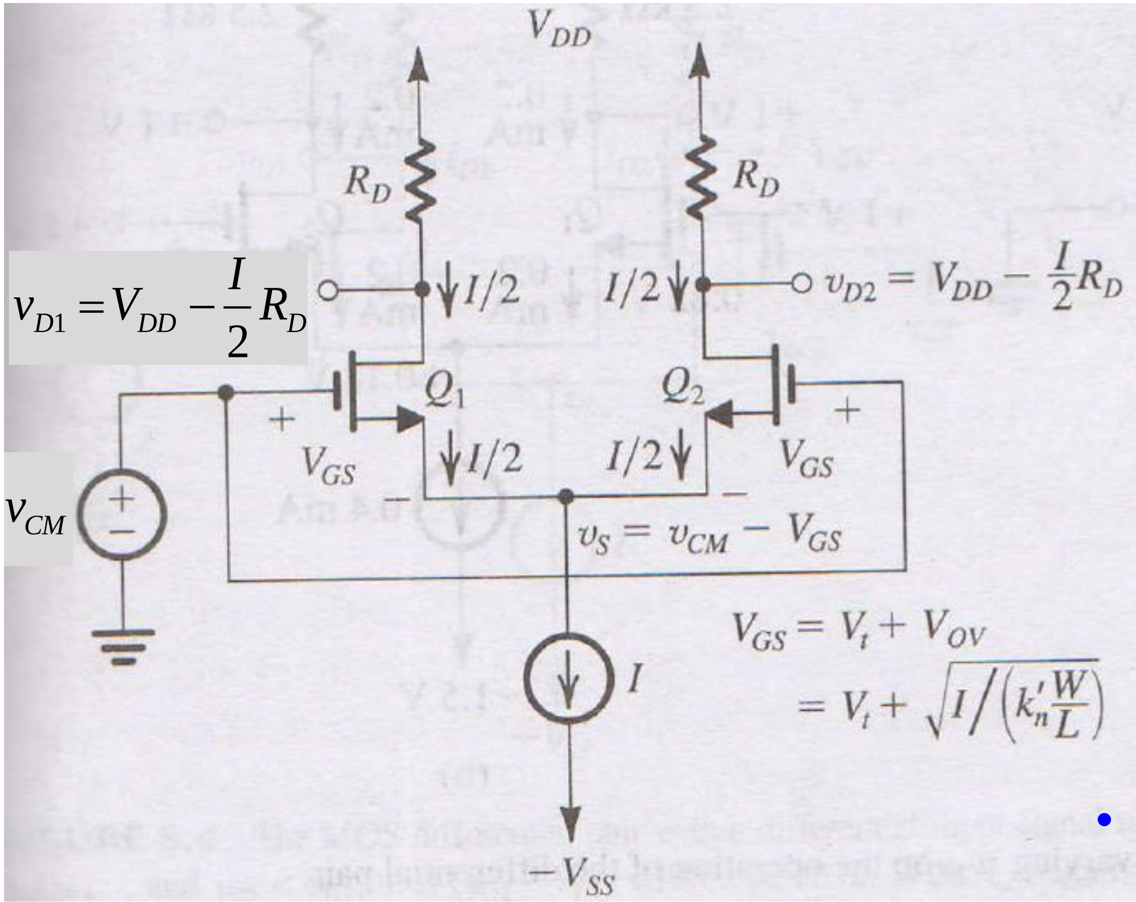
Tail Current

Current source is ideal: constant current, infinite output impedance

What is the need?

-to overcome the issues emanating from non-ideal CM level

Operation with a Common-Mode Input Voltage



$$v_{G1} = v_{G2} = v_{CM}$$

- **Since Q1 and Q2 are matched, the symmetry gives:**

$$i_{D1} = i_{D2} = \frac{I}{2}$$

- **Source Voltage is:**

$$v_s = v_{CM} - V_{GS}$$

- **Where, V_{GS} is obtained from:**

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right) (V_{GS} - V_T)^2$$

- **Voltage at each drain is:** $v_{D1} = v_{D2} = V_{DD} - \frac{I}{2} R_D$

Operation with a Common-Mode Input Voltage (contd.)

- **Drain Voltage Difference:** $v_o = v_{D1} - v_{D2} = 0$
- **Voltage difference is zero → what does it suggest?**
  - **The differential pair doesn't respond to CM input signals**
- **Let us look further: suppose v_{CM} is varied in such a way that Q_1 and Q_2 remain in saturation → "*I*" will divide equally between Q_1 and Q_2 → the drain voltages (v_{D1} and v_{D2}) will not change → difference in drain voltages will remain zero**
  - **This clearly supports the hypothesis that the differential pair doesn't respond to CM input signals → alternatively, differential pair rejects variations in CM input signals → provides protection against any abrupt changes in CM level originating from any factors such as noise etc.**
- **How about the common-mode input range? → The range of v_{CM} over which Q_1 and Q_2 operate properly**

Operation with a Common-Mode Input Voltage (contd.)

- **For Q_1 and Q_2 to remain in saturation:**

$$V_{GS} - V_T \le V_{DS} \qquad \Rightarrow (v_{CM} - v_S) - V_T \le V_{DD} - \frac{I}{2}R_D - v_S$$

$$\Rightarrow v_{CM} \le V_T + V_{DD} - \frac{I}{2}R_D \qquad \therefore (v_{CM})_{\max} = V_T + V_{DD} - \frac{I}{2}R_D$$

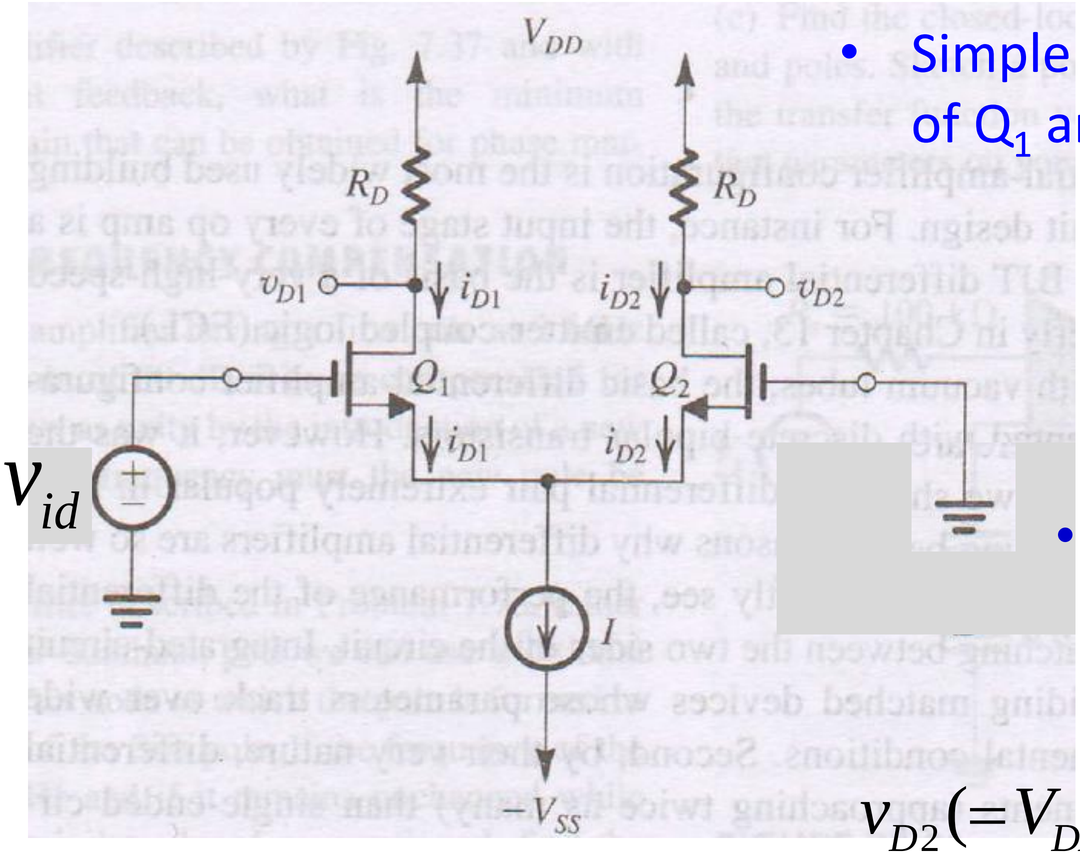
- **The lowest value of v_{CM} is determined by the need to keep the constant current source operational:**

$$v_{CM} - V_{GS} - (-V_{SS}) \ge V_{CS} \qquad \Rightarrow v_{CM} - V_{GS} + V_T - V_T \ge -V_{SS} + V_{CS}$$

$$\Rightarrow v_{CM} - V_{OV} - V_T \ge -V_{SS} + V_{CS} \qquad \Rightarrow v_{CM} \ge -V_{SS} + V_{CS} + V_{OV} + V_T$$

$$\therefore (v_{CM})_{\min} = -V_{SS} + V_{CS} + V_{OV} + V_T$$

Operation with a Differential Input Voltage



- Simple differential input: v_{id} to the gate of Q_1 and grounding of gate of Q_2:

$$v_{GS1} = v_{id} - v_s$$

$$v_{GS2} = -v_s$$

$$v_{id} = v_{GS1} - v_{GS2}$$

- For +ve v_{id} → v_{GS1} is greater → i_{D1} will be greater than i_{D2}

$$v_{D2}(= V_{DD} - i_{D2}R_D) > v_{D1}(= V_{DD} - i_{D1}R_D)$$

- Differential output voltage: $v_{D2} - v_{D1} > 0$

Operation with a Differential Input Voltage (contd.)

- For -ve v_{id} → v_{GS2} is greater → i_{D2} will be greater than i_{D1}

$$v_{D2}(=V_{DD}-i_{D2}R_D) < v_{D1}(=V_{DD}-i_{D1}R_D)$$

Differential output voltage: $v_{D2}-v_{D1}<0$

It is thus apparent that the differential pair respond to differential-mode signals → by providing differential output signal between the two drains

Operation with a Differential Input Voltage (contd.)

- **Differential pair as an amplifier**

- **Consider the scenario: (for small v_{id})**
  - **For +ve v_{id}, a small increase of v_{id} will bring an increase of ΔI in the current "I" through Q_1 and a decrease by same amount in current "I" through Q_2.**

Therefore:

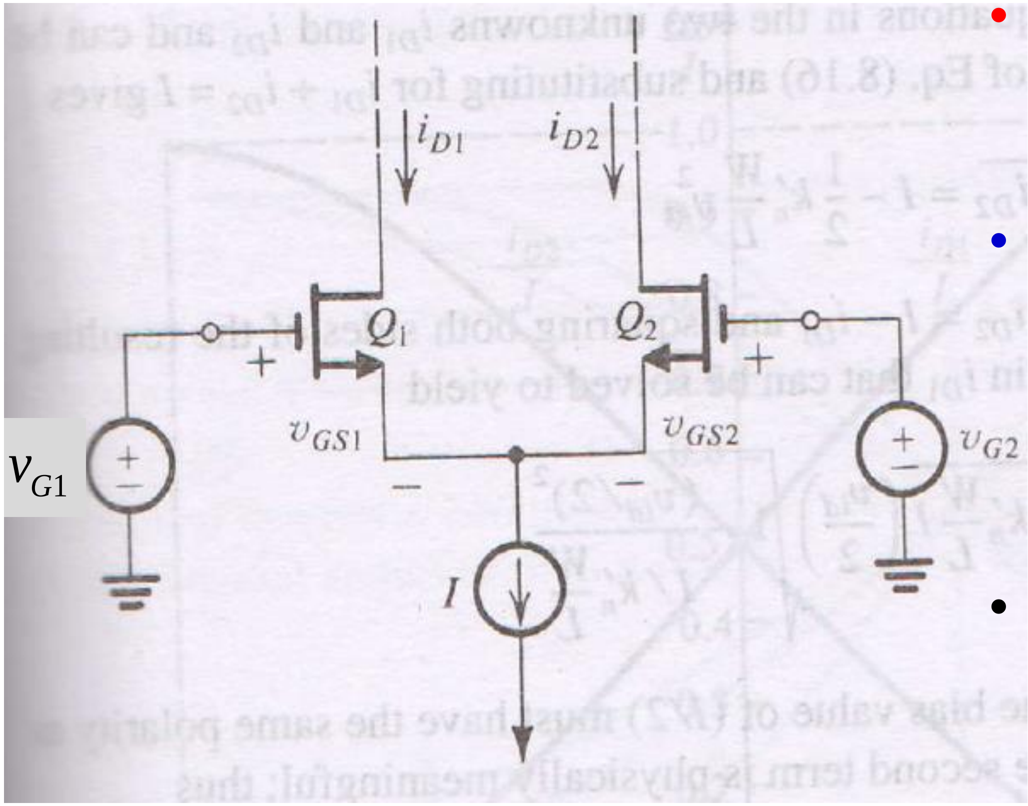
$-\Delta I R_D$: develops at the drain of Q_1

$\Delta I R_D$: develops at the drain of Q_2

Thus, additional differential output: $\Delta v_o = 2\Delta I R_D \propto v_{id}$: develops

Amplifier Action

Differential Pair – Large-Signal Operation



- The idea is to define i_{D1} and i_{D2} in terms of input differential signal $v_{id} = v_{G1} - v_{G2}$
- The circuit doesn't include connection details considering that these drain current equations do not depend on the external circuitries
- **Assumptions:** Q_1 and Q_2 are always in saturation; differential pair is perfectly matched; channel length modulation is not present

$$i_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS1} - v_T)^2$$

$$i_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS2} - v_T)^2$$

Differential Pair – Large-Signal Operation (contd.)

$$i_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS1} - v_T)^2$$

$$i_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS2} - v_T)^2$$

$$\sqrt{i_{D1}} = \sqrt{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}(v_{GS1} - v_T)$$

$$\sqrt{i_{D2}} = \sqrt{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}(v_{GS2} - v_T)$$

$$\sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}(v_{GS1} - v_{GS2})$$

$$\sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}.v_{id}$$

$$v_{GS1} - v_{GS2} = v_{G1} - v_{G2} = v_{id}$$

Differential Pair – Large-Signal Operation (contd.)

Now: $\sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}.v_{id}$ **and:** $i_{D1} + i_{D2} = I$

$$i_{D1} = \frac{I}{2} \pm \sqrt{\mu_n C_{ox}\frac{W}{L}I}\left(\frac{v_{id}}{2}\right)\sqrt{1 - \frac{(v_{id}/2)^2}{I/(\mu_n C_{ox}\frac{W}{L})}}$$

i_{D1} has same polarity as v_{id} → here +ve

$$\therefore i_{D1} = \frac{I}{2} + \sqrt{\mu_n C_{ox}\frac{W}{L}I}\left(\frac{v_{id}}{2}\right)\sqrt{1 - \frac{(v_{id}/2)^2}{I/(\mu_n C_{ox}\frac{W}{L})}}$$

$$\Rightarrow i_{D2} = I - i_{D1} = \frac{I}{2} - \sqrt{\mu_n C_{ox}\frac{W}{L}I}\left(\frac{v_{id}}{2}\right)\sqrt{1 - \frac{(v_{id}/2)^2}{I/(\mu_n C_{ox}\frac{W}{L})}}$$

Differential Pair – Large-Signal Operation (contd.)

- **At bias point:** $v_{id} = 0 \qquad \Rightarrow i_{D1} = i_{D2} = \dfrac{I}{2}$

$$v_{GS1} = v_{GS2} = V_{GS} \qquad \Rightarrow \frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{OV})^2$$

- **Therefore:**

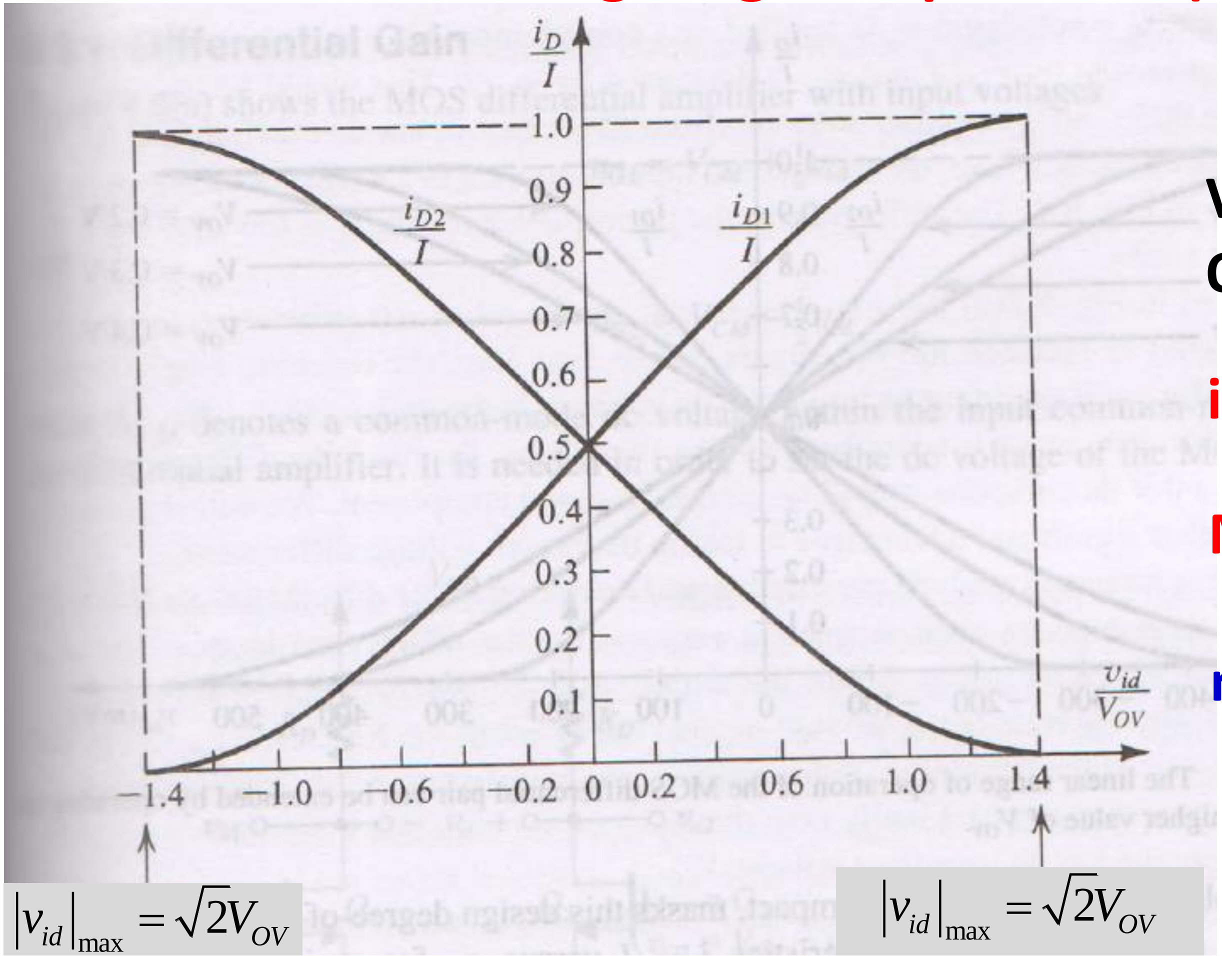
$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right)\sqrt{1 - \frac{(v_{id}/2)^2}{V_{OV}}}$$

$$i_{D2} = \frac{I}{2} - \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right)\sqrt{1 - \frac{(v_{id}/2)^2}{V_{OV}}}$$

Drain currents in terms of v_{id}

Basically describes the effect of differential input on drain currents

Differential Pair – Large-Signal Operation (contd.)



V_{OV}: overdrive voltage for Q_1 and Q_2 for $I/2$.

$i_{D1} = i_{D2} = I/2$: for $v_{id} = 0$

Making v_{id} +ve causes i_{D1} to increase while making v_{id} –ve causes i_{D2} to increase

$v_{id} = \sqrt{2}V_{OV}$: the current I is steered through Q_1

$v_{id} = -\sqrt{2}V_{OV}$: the current I is steered through Q_2

Differential Pair – Large-Signal Operation (contd.)

Observations:

- **The currents are nonlinear due to presence of square term**
- **Our interest: linear amplification ↔ how to achieve that?**
- **By keeping (v_{id}/2) much smaller than V_{OV}**

$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right)\sqrt{1 - \frac{(v_{id}/2)^2}{V_{OV}}} \cong \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right) = \frac{I}{2} + i_d$$

$$i_{D2} = \frac{I}{2} - \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right)\sqrt{1 - \frac{(v_{id}/2)^2}{V_{OV}}} \cong \frac{I}{2} - \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right) = \frac{I}{2} - i_d$$

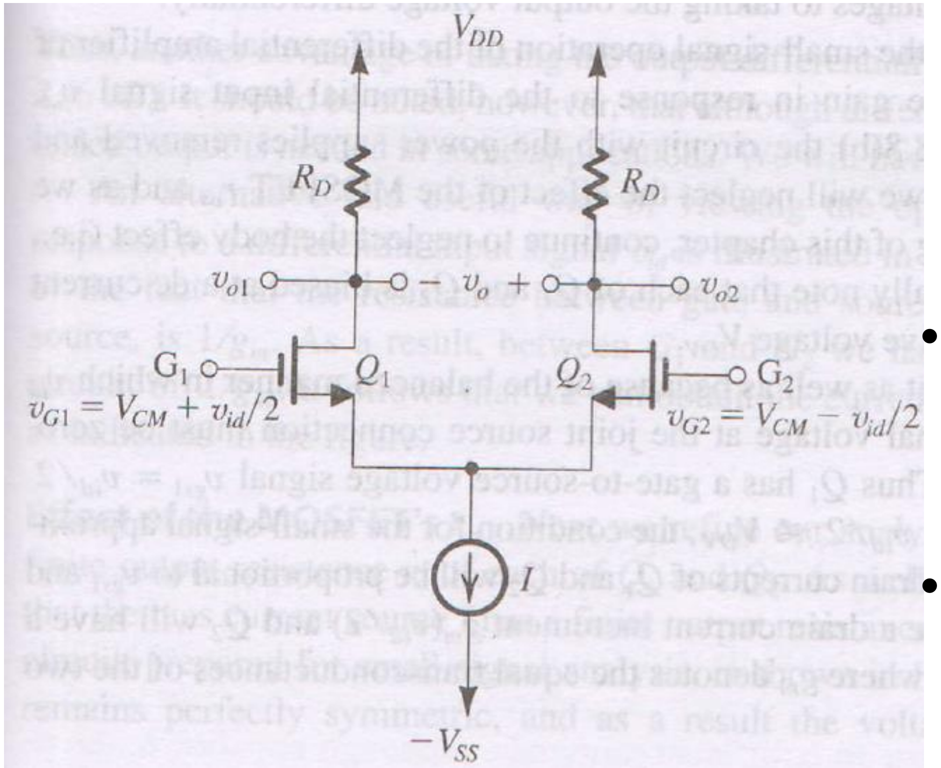
Where: $$i_d = \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right)$$

Differential Pair – Large-Signal Operation (contd.)

Observations:

- **However, reduction in v_{id} leads to reduction in g_m → consequently reduction in gain**
- **How can we increase g_m?**
  - **By using smaller (W/L) ← for a constant bias current**
- **Linearity can also be improved by increasing V_{OV} → however this results into reduced g_m ↔ g_m can be increased by increasing bias current but at the expense of higher power consumption**

Differential Pair – Small-Signal Operation



Input Voltages

$$v_{G1} = v_{CM} + \frac{1}{2} v_{id} \qquad v_{G2} = v_{CM} - \frac{1}{2} v_{id}$$

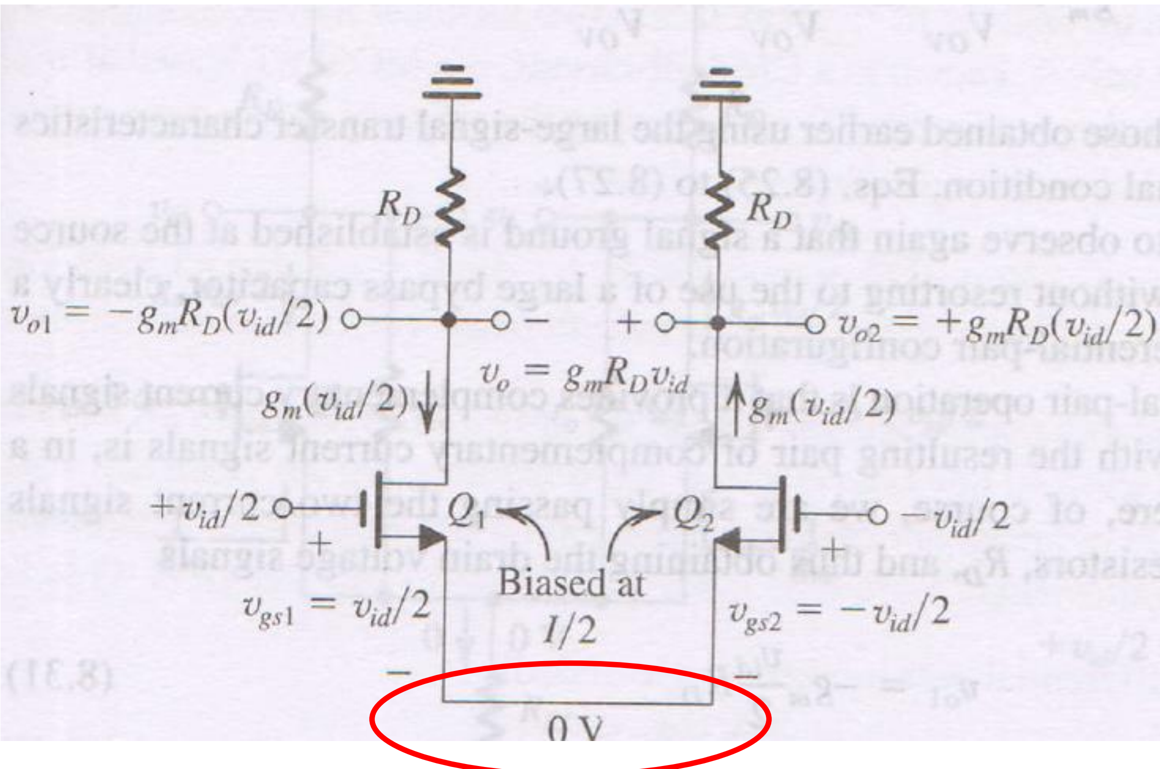
- **v_{CM} is common-mode dc voltage → is needed to set the dc voltages (biasing point) of the NFETs**
- **In general, v_{CM} is at the middle of the power supply → therefore, for dual-rail supply it is typically zero**

- **The differential signal v_{id} is applied in a balanced manner → v_{G1} increases by $v_{id}/2$ and v_{G2} decreases by $v_{id}/2$**

Q: Do you expect any difference in performance while applying the differential signal v_{id} in a balanced manner or in an unbalanced manner?

Differential Pair – Small-Signal Operation (contd.)

What is the small signal voltage gain?



Virtual Ground! → Why?

Discards the need of a bypass capacitor for ac grounding

From large-signal analysis:

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

For CS mode Q_1:

$$v_{o1} = -g_m \frac{v_{id}}{2} R_D$$

For CS mode Q_2:

$$v_{o2} = g_m \frac{v_{id}}{2} R_D$$

Differential Pair – Small-Signal Operation (contd.)

For single-ended operation:

$$A_v = \frac{v_{o1}}{v_{id}} = -\frac{1}{2} g_m R_D$$

OR

$$A_v = \frac{v_{o2}}{v_{id}} = \frac{1}{2} g_m R_D$$

For differential operation:

$$A_d = \frac{v_{o2} - v_{o1}}{v_{id}} = g_m R_D$$

Twice the single-ended gain

Is it always advantageous?

Differential Pair – Small-Signal Operation (contd.)

Alternative Approach:

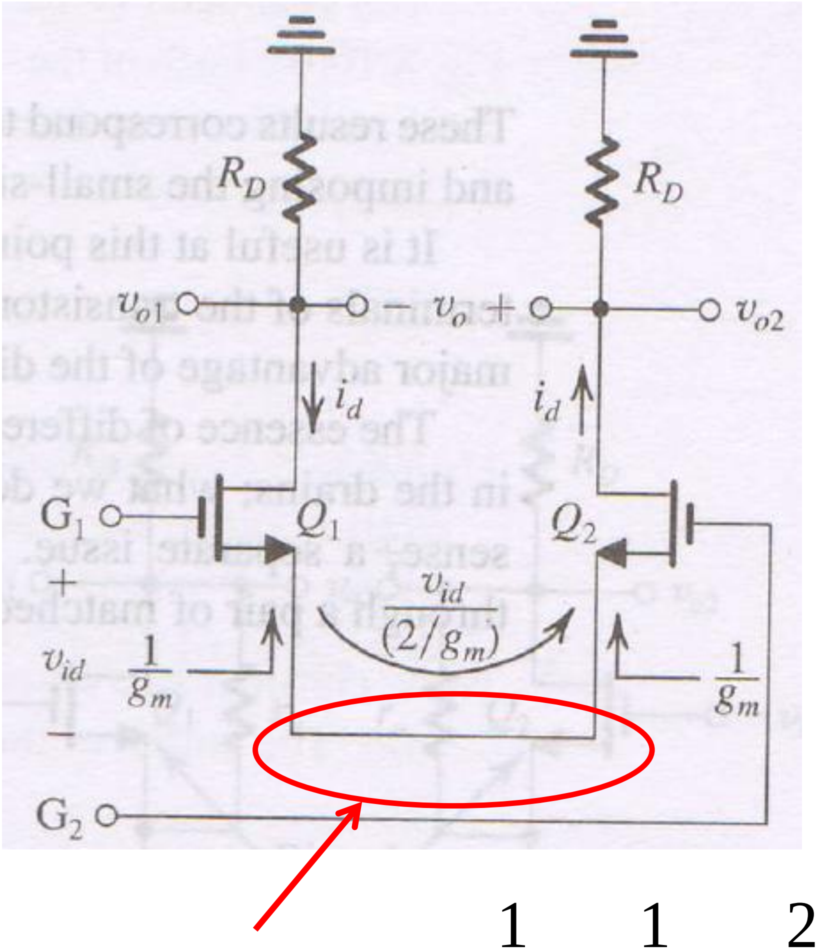
Drain current is given by:

$$i_{d1} = i_{d2} = \frac{v_{id}}{2/g_m}$$

Therefore,

$$v_{o1} = -g_m \frac{v_{id}}{2} R_D \qquad v_{o2} = g_m \frac{v_{id}}{2} R_D$$

$$\therefore A_d = \frac{v_{o2} - v_{o1}}{v_{id}} = g_m R_D$$



Total Impedance= $\frac{1}{g_m} + \frac{1}{g_m} = \frac{2}{g_m}$