

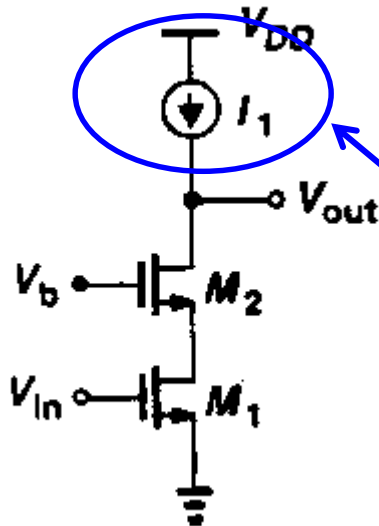


Lecture – 11

Date: 17.09.2015

- Non-idealities in Current Mirror
- Cascode Current Mirror
- Current Mirror Configurations
- Examples

Cascode Amplifier with Current Source Load



We know the gain of Cascode stage is given by:

$$\Rightarrow A_v \approx -\frac{g_{m1}(g_{m2} + g_{mb2})r_{o1}r_{o2}}{[R_D + r_{o1} + r_{o2} + (g_{m2} + g_{mb2})r_{o1}r_{o2}]}R_D$$

A constant current source possesses **very high output impedance** ($R_D \rightarrow \infty$), therefore the gain equation changes to:

$$A_v \approx -g_{m1}(g_{m2} + g_{mb2})r_{o1}r_{o2} = -(g_{m1}r_{o1}) \cdot (g_{m2} + g_{mb2})r_{o2}$$

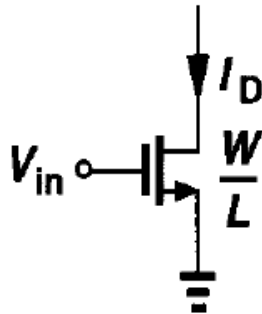
It is apparent that the maximum small-signal voltage gain is the multiplication of gains from CS and CG stages \rightarrow definitely a big plus!

$$R_{out} \approx (g_{m2} + g_{mb2})r_{o2}r_{o1}$$

Cascode – Amplifier (contd.)

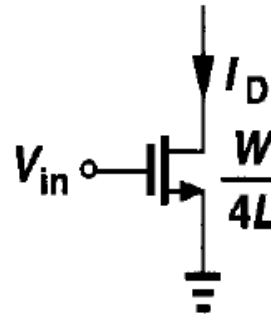
Discussion with respect to alteration in dimension

What happens if **Length (L)** of the main device is quadrupled while the **Width (W)** remains same?



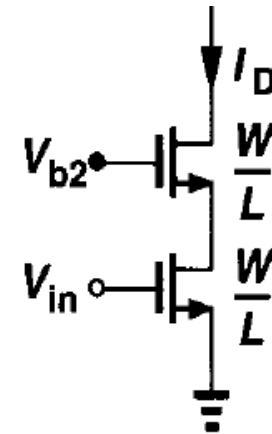
$$A_v \approx -g_m r_o = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \frac{1}{\lambda I_D}$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$



$$A_{v1} = 2A_v$$

$$g_{m1} = \frac{g_m}{2}$$



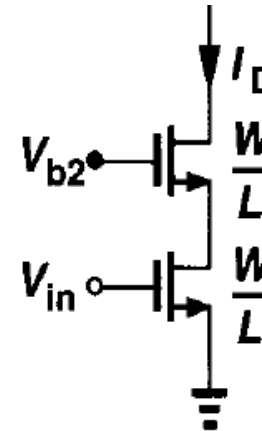
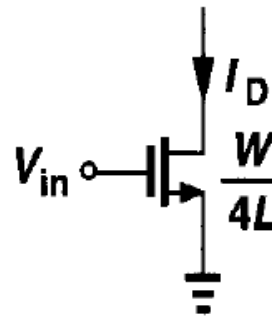
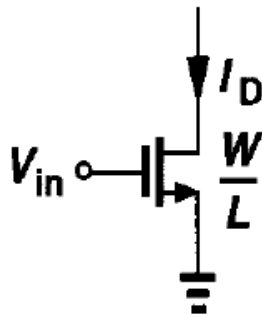
$$A_{v2} = (A_v)^2$$

$$g_{m2} = g_m$$

Cascode more suited for noise applications

Cascode – Amplifier (contd.)

Discussion with respect to alteration in dimension



$$(V_{GS} - V_{TH})^2 = \frac{2I_D}{\mu_n C_{ox} \left(\frac{W}{L}\right)}$$

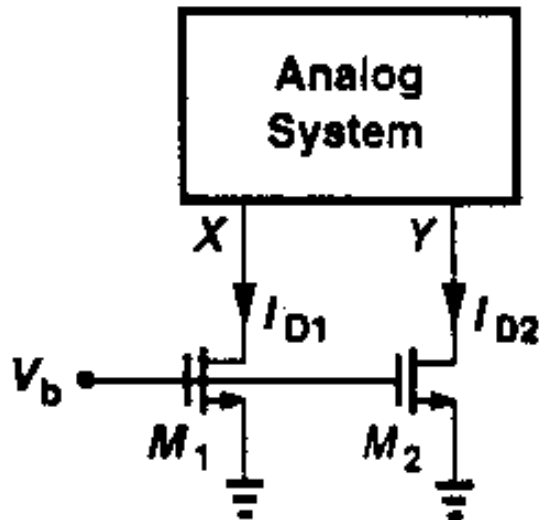
Quadruppling of Length (L) while keeping the Width (W) results in doubling of overdrive voltage

For **identical devices**, cascode also exhibits doubling of overdrive voltage

Shielding Property of Cascode

Cascode amplifier and Cascode current source → could be used in applications where the output varies drastically due to any reason!
 → This variation doesn't affect the subsequent sections greatly →
 shielding property of Cascode

Example: Two identical NFETs are used to generate **constant current sources**. However, due to internal circuitry of the system, V_X is higher than V_Y by ΔV .



Q: determine the resulting difference between I_{D1} and I_{D2} if $\lambda \neq 0$

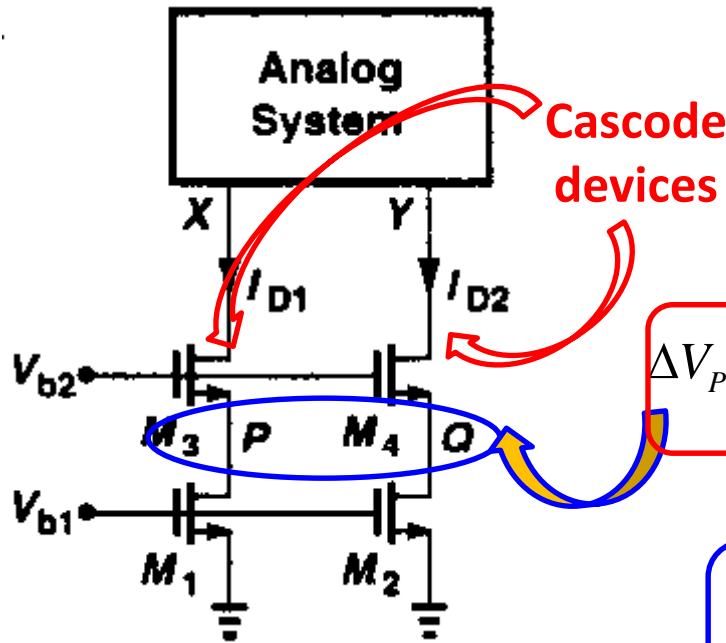
$$I_{D1} = \frac{1}{2} \mu_n C_{ox} (V_b - V_T)^2 (1 + \lambda V_X) \quad \leftarrow V_{DS1}$$

$$I_{D2} = \frac{1}{2} \mu_n C_{ox} (V_b - V_T)^2 (1 + \lambda V_Y) \quad \leftarrow V_{DS2}$$

$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} (V_b - V_T)^2 (\lambda \Delta V)$$

Shielding Property of Cascode (contd.)

Q: Add Cascode devices to M1 and M2 and then check the difference between I_{D1} and I_{D2} if $\lambda \neq 0$



$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} (V_b - V_T)^2 (\lambda \Delta V_{PQ})$$

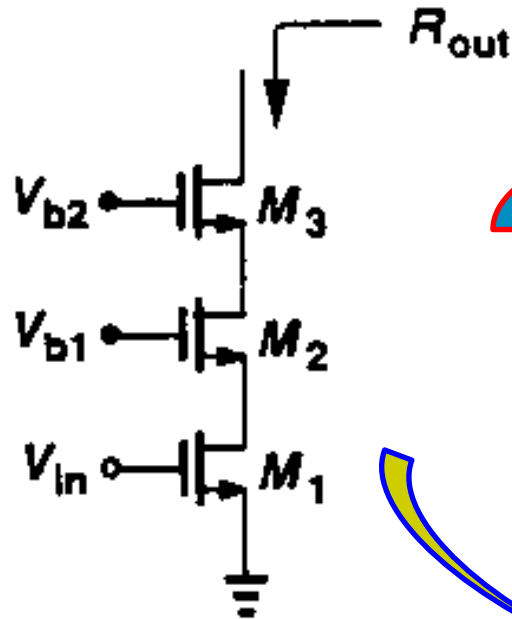
$$\Delta V_{PQ} = \Delta V \frac{r_{o1}}{[1 + (g_{m3} + g_{mb3})r_{o3}]r_{o1} + r_{o3}} \approx \frac{\Delta V}{(g_{m3} + g_{mb3})r_{o3}}$$

$$\therefore I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} (V_b - V_T)^2 \left(\frac{\lambda \Delta V}{(g_{m3} + g_{mb3})r_{o3}} \right)$$

This is a large value and thus the Cascode structure gives smaller variation → perfect example of Shielding property!!!

Triple Cascode

- Cascoding can be extended to **three or more devices** to achieve higher output impedance



It limits the voltage swing

Here the minimum output voltage equals the sum of three overdrive voltages

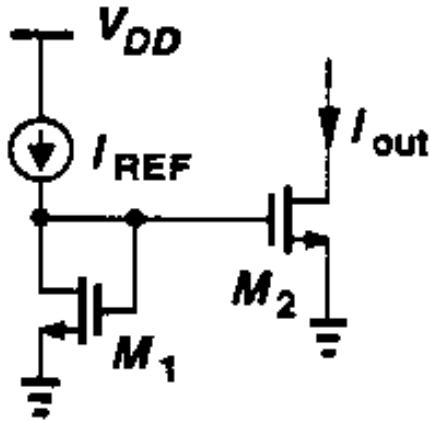


Folded Cascode

[Self Study](#)

Current Mirror

- Now let us take the generic equations for the following current mirror:



$$I_{REF} = \frac{1}{2} \mu_{n1} C_{ox1} \left(\frac{W}{L} \right)_1 (V_{GS1} - V_{T1})^2$$

$$I_{out} = \frac{1}{2} \mu_{n2} C_{ox2} \left(\frac{W}{L} \right)_2 (V_{GS2} - V_{T2})^2$$

Even if these transistors are identical and have been fabricated on the same chip [thus practically possessing similar parameters such as V_T , μ_n , C_{ox}], there are three effects that causes current mirror to be different from ideal situation

These effects are: (a) Channel Length Modulation, (b) V_T offset between the two transistors, (c) Imperfect Geometrical Matching

Current Mirror (contd.)

Channel Length Modulation Effect

- Assuming all other aspects of the transistor are ideal and the ratio of aspect ratios of both the transistors are unity then:

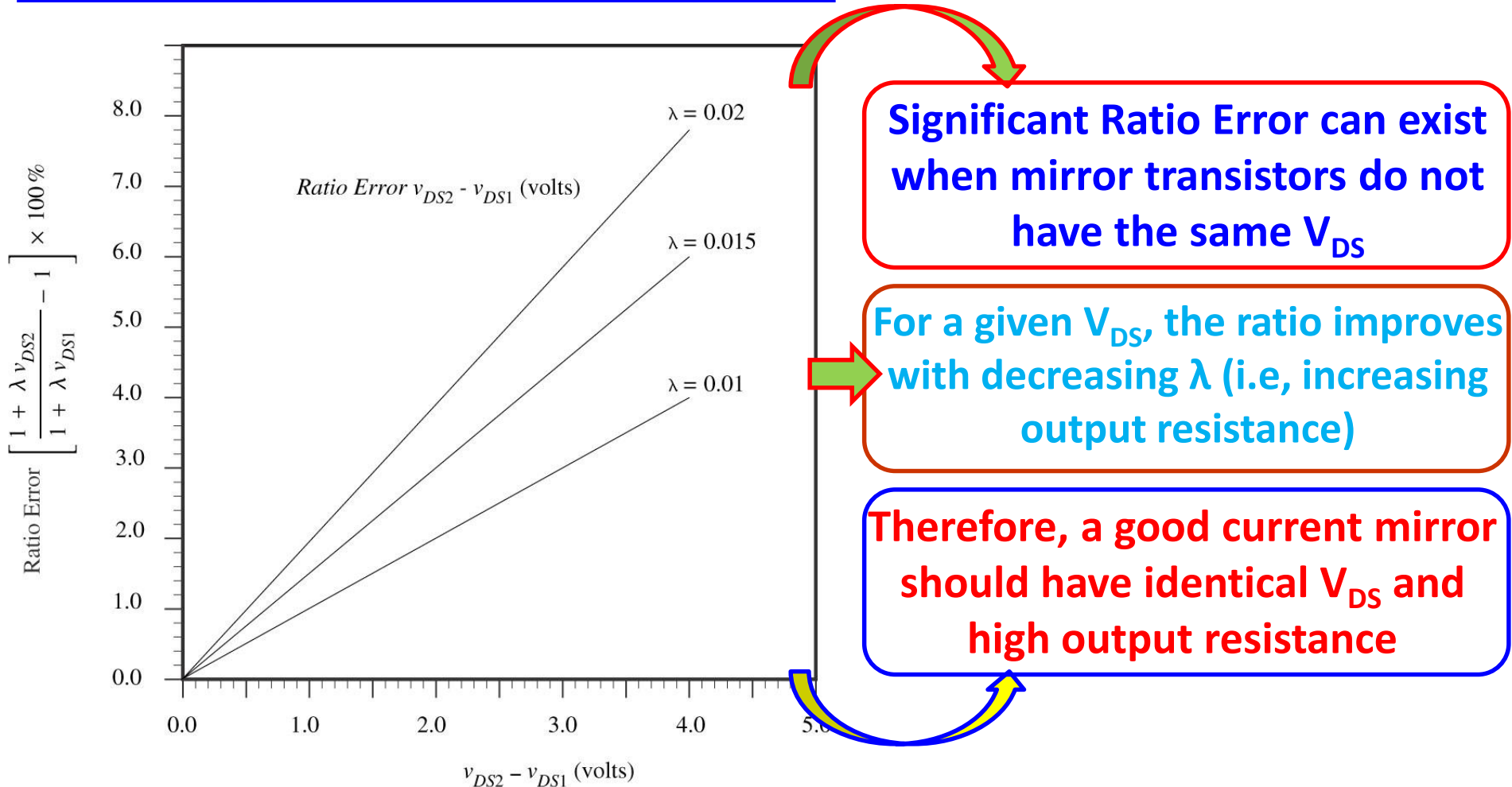
$$\frac{I_{out}}{I_{REF}} = \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}}$$

It is assumed that λ is the same for both M_1 and M_2

It is apparent that the difference in V_{DS} causes deviation from the ideal unity current gain or current mirroring

Current Mirror (contd.)

Channel Length Modulation Effect (contd.)



Current Ratio Error vs ($V_{DS2} - V_{DS1}$) for different values of λ

Current Mirror (contd.)

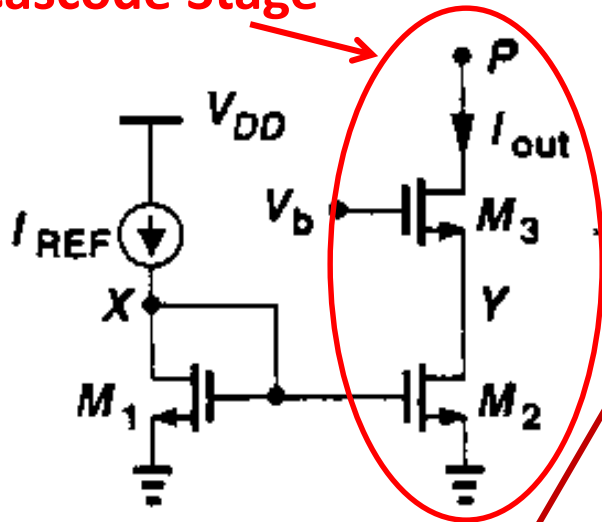
Channel Length Modulation Effect (contd.)

- **Therefore, the apparent solution seems the use of long channel device**
 - **however this also requires increase in width** → results in problems for area and power constraint designs
 - **furthermore, increase in width also increases the output capacitance** → high frequency performance suffers
 - **short channel devices are commonplace and therefore this solution is not appropriate**

Cascode Current Mirror

- In order to overcome the error due to channel length modulation, instead of a simple two transistor “current mirror” it is recommended to use “cascode current mirror”

Cascode Stage



- In this architecture, **small changes in potential at node-P** does not have any effect on the potential at node-Y → **shielding property of Cascode**
- Through some technique make $V_Y = V_X$ → then the effect of channel length modulation is insignificant as the mirror equation becomes:

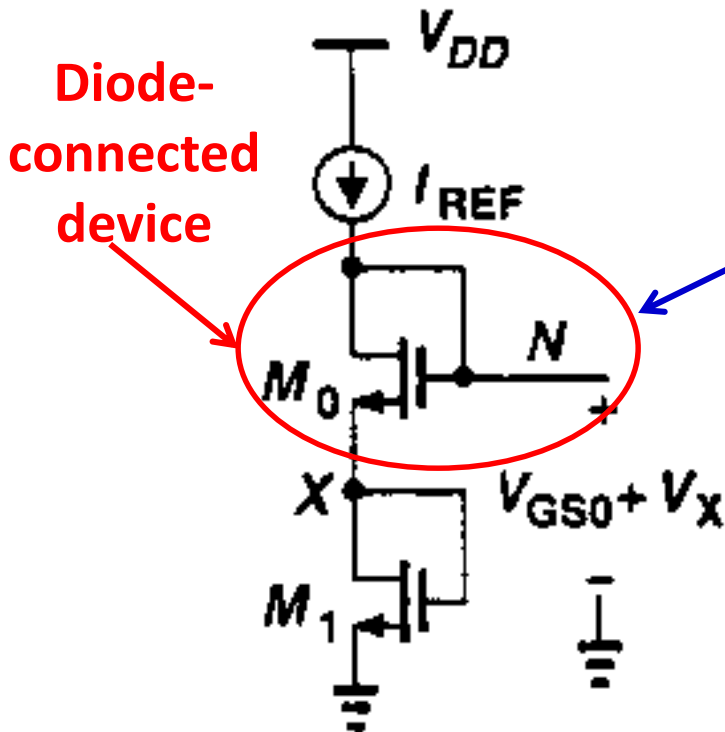
$$I_{out} = I_{REF} \frac{(W/L)_2}{(W/L)_1}$$

Once again only dependent on the scaling of devices

$$\Delta V_Y = \Delta V_P / [(g_{m3} + g_{mb3})r_{o3}]$$

Cascode Current Mirror (contd.)

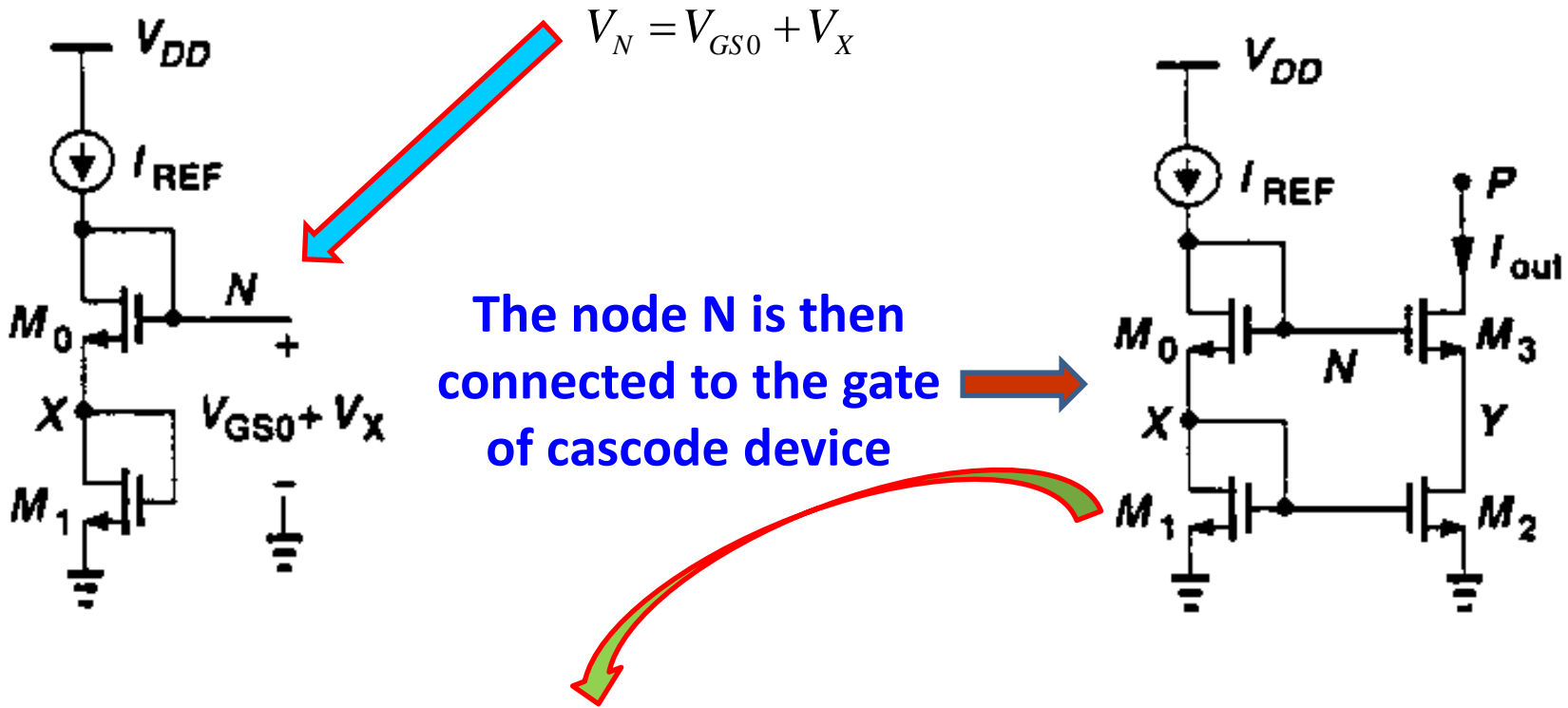
- How do we generate the condition: $V_Y = V_X$
- For this to happen, we must guarantee $V_b - V_{GS3} = V_X \rightarrow$ It means one gate-source voltage should be added to achieve this \rightarrow this is easily achieved by placing a diode-connected device M_0 in series with M_1



Addition of this device will ensure fulfillment of:
 $V_b - V_{GS3} = V_X$

Then proper choice of dimensions of M_0 and M_3 yields $V_{GS0} = V_{GS3}$

Cascode Current Mirror (contd.)



$$V_{GS0} + V_X + V_{GS1} = V_{GS3} + V_Y + V_{GS2}$$

$$\therefore V_{GS0} + V_X = V_{GS3} + V_Y$$

proper choice of dimensions of M_0 and M_3 yields $V_{GS0} = V_{GS3}$

Cascode Current Mirror (contd.)

- Thus appropriate choices of dimensions of M_0 and M_3 gives: $V_{GS0} = V_{GS3}$

- For this to happen:

$$\frac{(W/L)_3}{(W/L)_0} = \frac{(W/L)_2}{(W/L)_1}$$

- Once $V_{GS0} = V_{GS3}$, we get: $V_X = V_Y$

- $V_X = V_Y$ leads to the condition: $V_{DS1} = V_{DS2} \rightarrow$ transforms the mirror equation:

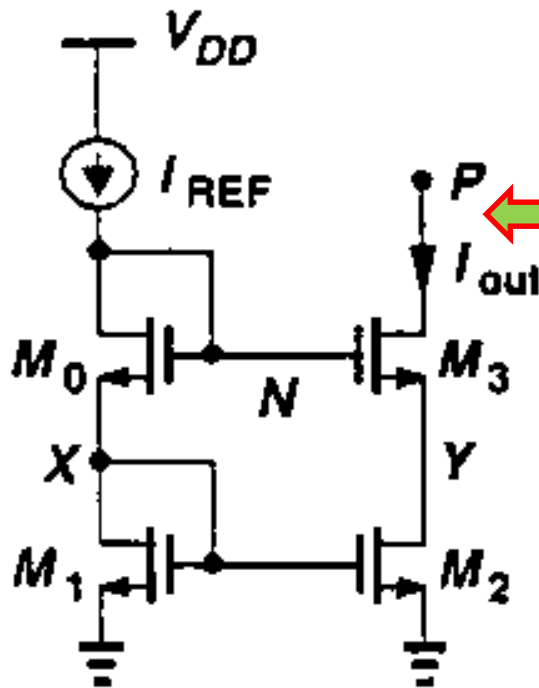
$$I_{out} = I_{REF} \frac{(W/L)_2}{(W/L)_1}$$

Valid even when there exist body effect in transistors M_0 and M_3

- Cascode configuration improves the accuracy of current copying capability \rightarrow but what is the major drawback?

Cascode Current Mirror (contd.)

- As the cascode current mirror provides a constant current source, it should also possess very high output impedance
- Consider once again the following configuration:

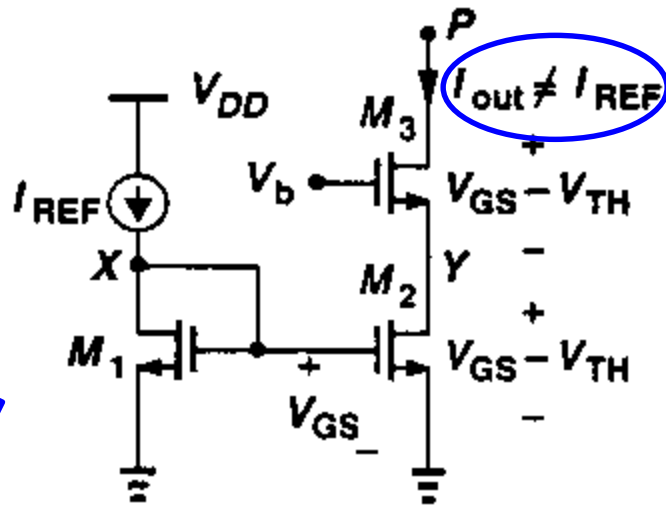


What is R_{out} here?

$$R_{out} \approx g_{m3} r_{o3} r_{o2}$$

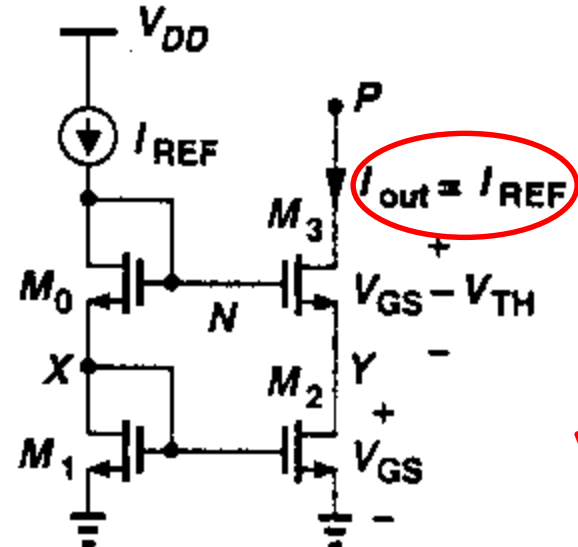
Cascode Current Mirror (contd.)

Accuracy and Voltage Swing Trade-off



Standard Cascode
Current Mirror

- V_b is chosen to allow minimum V_p
- **Problem:** $V_x \neq V_y$
- $I_{out} \neq I_{ref}$



Modified Cascode
Current Mirror

- V_b is chosen to allow $V_x = V_y$
- V_p is not minimum
- **However,** $I_{out} = I_{ref}$

Current Mirror (contd.)

Threshold Offset Effect

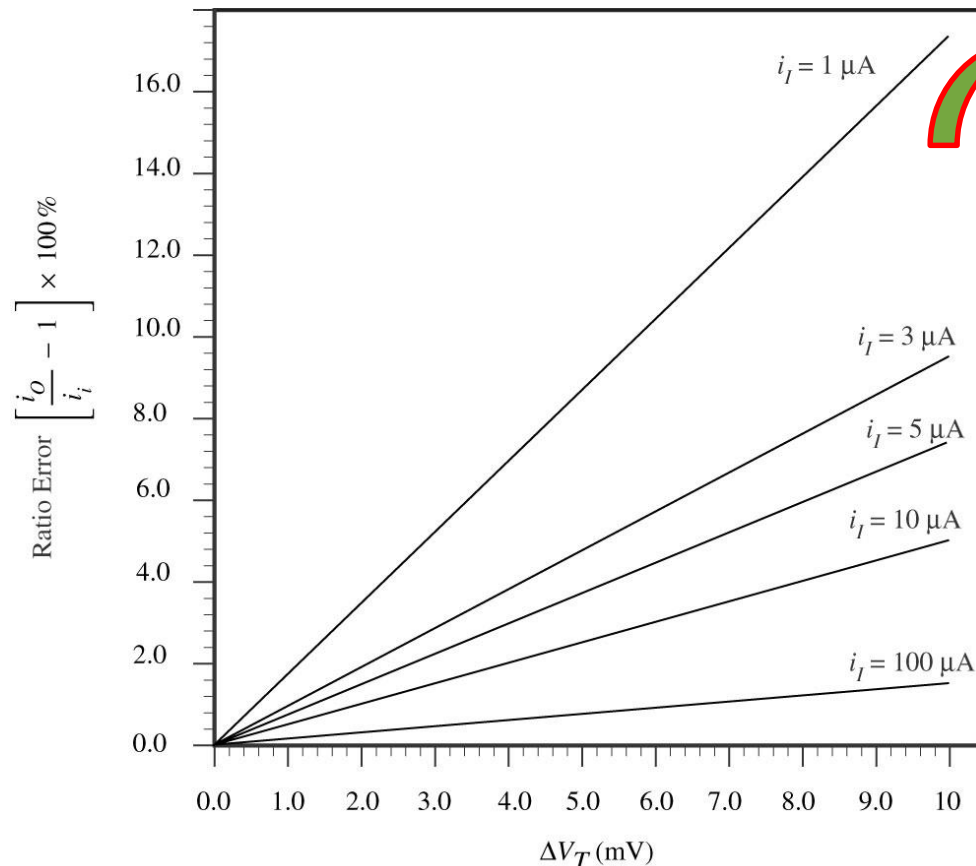
- The offset between the threshold voltage **of two transistors** also causes problems in the optimal operation of current mirror
- The threshold offset is typically less than 10mV for identical transistors
→ even this small offset causes substantial error!!!
- Let us now consider a current mirror configuration **where both have the same V_{DS}** and all other aspects of the transistors are equal **except V_T** .
The expression simplifies to:

$$\frac{I_{out}}{I_{REF}} = \left(\frac{V_{GS} - V_{T2}}{V_{GS} - V_{T1}} \right)^2$$

Current Mirror (contd.)

Threshold Offset Effect (contd.)

- The plot of ratio error **between ideal and imperfect current mirroring** as a function of $\Delta V_T = V_{T1} - V_{T2}$ results into:



Clearly identifies that better current mirroring is obtained for higher currents

Its due to the fact that V_{GS} is higher for higher currents \rightarrow ensures that ΔV_T is a smaller percentage of V_{GS}

Current Mirror (contd.)

Threshold Offset Effect (contd.)

- Sometimes it may happen that the factor $\mu_n C_{ox}$ (let us call it K') is also mismatched alongwith the offset in the threshold.
- The current mirror equation then transforms to:

$$\frac{I_{out}}{I_{REF}} = \frac{K'_2 (V_{GS} - V_{T2})^2}{K'_1 (V_{GS} - V_{T1})^2}$$

In this case its assumed that the aspect ratio is identical (considering that its designer driven!).

- Let us define:

$$\Delta K' = K'_2 - K'_1 \qquad K' = \frac{1}{2}(K'_2 + K'_1) \qquad V_T = \frac{1}{2}(V_{T1} + V_{T2})$$

- Then:

$$K'_1 = K' - 0.5\Delta K' \qquad K'_2 = K' + 0.5\Delta K' \qquad V_{T1} = V_T - 0.5\Delta V_T \qquad V_{T2} = V_T + 0.5\Delta V_T$$

Current Mirror (contd.)

Threshold Offset Effect (contd.)

- Let us substitute the mirror equation using these assumed parameters:

$$\frac{I_{out}}{I_{REF}} = \frac{(K' + 0.5\Delta K')(V_{GS} - V_T - 0.5\Delta V_T)^2}{(K' - 0.5\Delta K')(V_{GS} - V_T + 0.5\Delta V_T)^2}$$

- Factor out K' and $(V_{GS} - V_T)$ to get:

$$\frac{I_{out}}{I_{REF}} = \frac{\left(1 + \frac{\Delta K'}{2K'}\right) \left(1 - \frac{\Delta V_T}{2(V_{GS} - V_T)}\right)^2}{\left(1 - \frac{\Delta K'}{2K'}\right) \left(1 + \frac{\Delta V_T}{2(V_{GS} - V_T)}\right)^2}$$

- Assuming these quantities to be small, we get:

$$\frac{I_{out}}{I_{REF}} = \left(1 + \frac{\Delta K'}{2K'}\right) \left(1 + \frac{\Delta K'}{2K'}\right) \left(1 - \frac{\Delta V_T}{2(V_{GS} - V_T)}\right)^2 \left(1 - \frac{\Delta V_T}{2(V_{GS} - V_T)}\right)^2$$

Current Mirror (contd.)

Threshold Offset Effect (contd.)

- Retaining only the first order products gives:

$$\frac{I_{out}}{I_{REF}} \cong 1 + \frac{\Delta K'}{K'} - \frac{2\Delta V_T}{V_{GS} - V_T}$$

If the percentage change of K' and V_T are known a priori, then this expression can predict the worst-case error in the current mirroring capability of the current mirror

For example, if : $\frac{\Delta K'}{K'} = \pm 5\%$

and: $\frac{\Delta V_T}{V_{GS} - V_T} = \pm 10\%$

then: $\frac{I_{out}}{I_{REF}} \cong 1 \pm 0.05 \pm (-0.2) = 1 \pm (-0.15)$

In this example, the maximum error amounts to 15% provided the tolerances in K' and V_T are correlated

Current Mirror (contd.)

Mismatch in Aspect Ratio

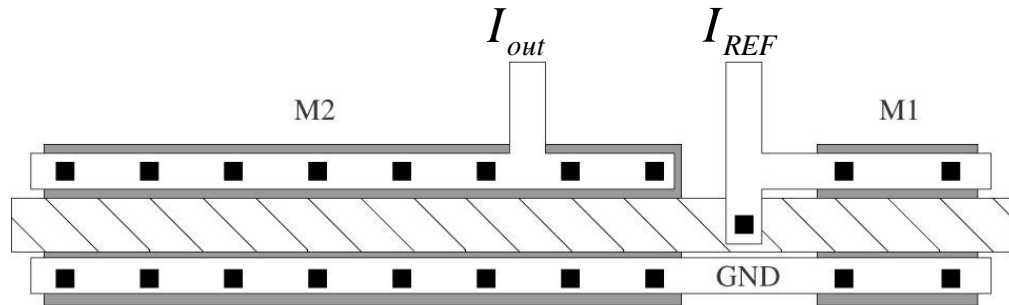
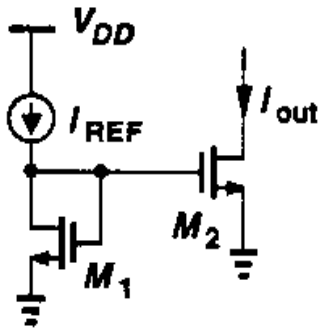
- mismatches are commonly present even in identical transistors on the same die \leftarrow W and L are often mismatched due to mask, photolithography, and diffusion variations \rightarrow this can be significant even for two transistors placed side by side
- One way to overcome these effects is to make transistors much larger than these variations \rightarrow e.g., for transistors of identical size with W and L greater than $10\mu m$, the errors due to the mismatched aspect ratio will be insignificant \leftarrow when compared to errors contributed by offset V_T and Channel Length Modulation
- However, many applications (for high current gain applications!) require aspect ratio of transistor (M_2) to be much larger than the aspect ratio of the reference transistor (M_1) \leftarrow necessitates creativity in layout techniques !!!

Current Mirror (contd.)

Mismatch in Aspect Ratio (contd.)

- Example: we see layout of one-to-four current amplifier below. Its assumed that the lengths are identical ($L_1 = L_2$). Find the ratio error if:

$$W_1 = 5 \pm 0.1 \mu m \quad W_2 = 20 \pm 0.1 \mu m$$



$$\frac{I_{out}}{I_{REF}} = \frac{W_2}{W_1} = \frac{20 \pm 0.1}{5 \pm 0.1} = 4 \left(\frac{1 \pm (0.1/20)}{1 \pm (0.1/5)} \right) \approx 4 \left(1 \pm \frac{0.1}{20} \right) \left(1 - \frac{\pm 0.1}{5} \right)$$

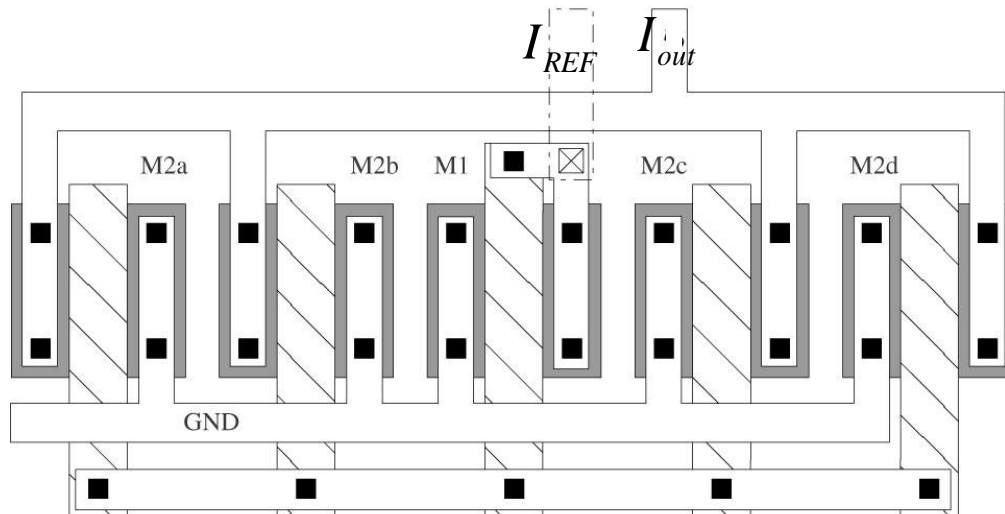
$$\Rightarrow \frac{I_{out}}{I_{REF}} = 4 \left(1 \pm \frac{0.1}{20} - \frac{\pm 0.4}{20} \right) \approx 4(1 - (\pm 0.06))$$

It is assumed that variations would have the same sign. In this case it is apparent that the ratio error is 1.5% of the desired current ratio

Current Mirror (contd.)

Mismatch in Aspect Ratio (contd.)

- For large W , it's a good strategy to have W not much larger than L and to put equal transistors in parallel.
- A solution to this problem is to use appropriate layout technique. For example, use four duplicates of transistor M_1 to achieve one-to-four ratio. This way the tolerance on W_2 is multiplied by the nominal current gain.

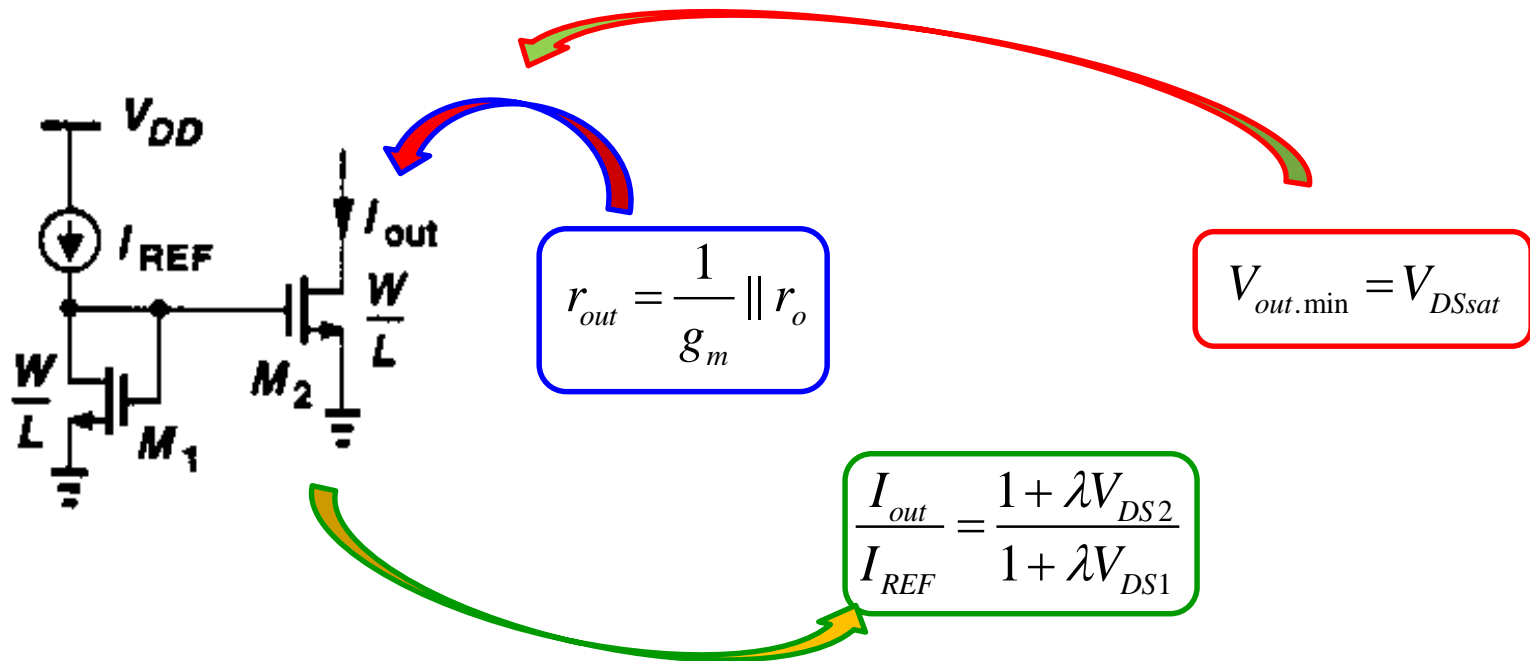


$$\frac{I_{out}}{I_{REF}} = \frac{W_2}{W_1} = \frac{4(5 \pm 0.1)}{5 \pm 0.1} = 4$$

Here its assumed that ΔW should be the same for all the transistors

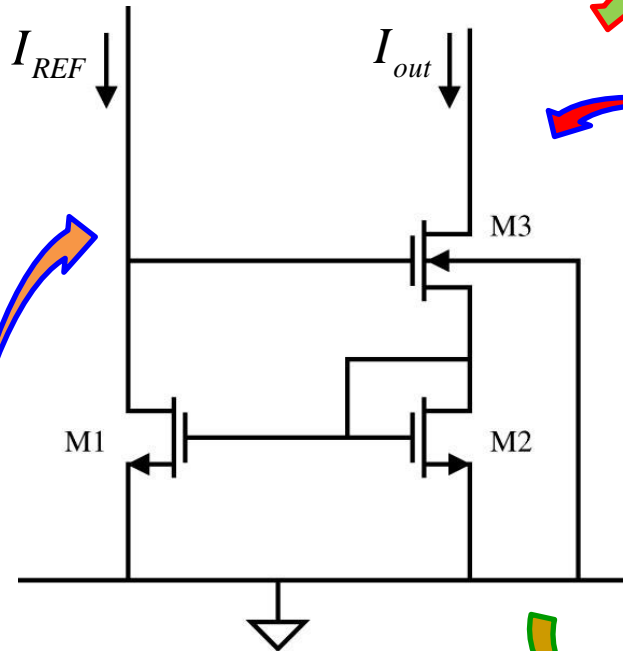
Current Mirror Configurations

- It is a common practice to design current mirror circuits for high output impedance [for achieving near ideal current source!]
- No less important is the voltage headroom [specially for low voltage applications!!!]



Current Mirror Configurations (contd.)

- Wilson Current Mirror



$$V_{out.min} = V_{DSsat,3} + V_{GS2} > V_T + 2V_{Dsat}$$

Headroom is smaller

$$r_{out} \approx r_{o3} \frac{g_{m3}}{g_{m2}} g_{m1} (R_L \parallel r_{o1})$$

High Value

$$\frac{I_{out}}{I_{REF}} = \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}}$$

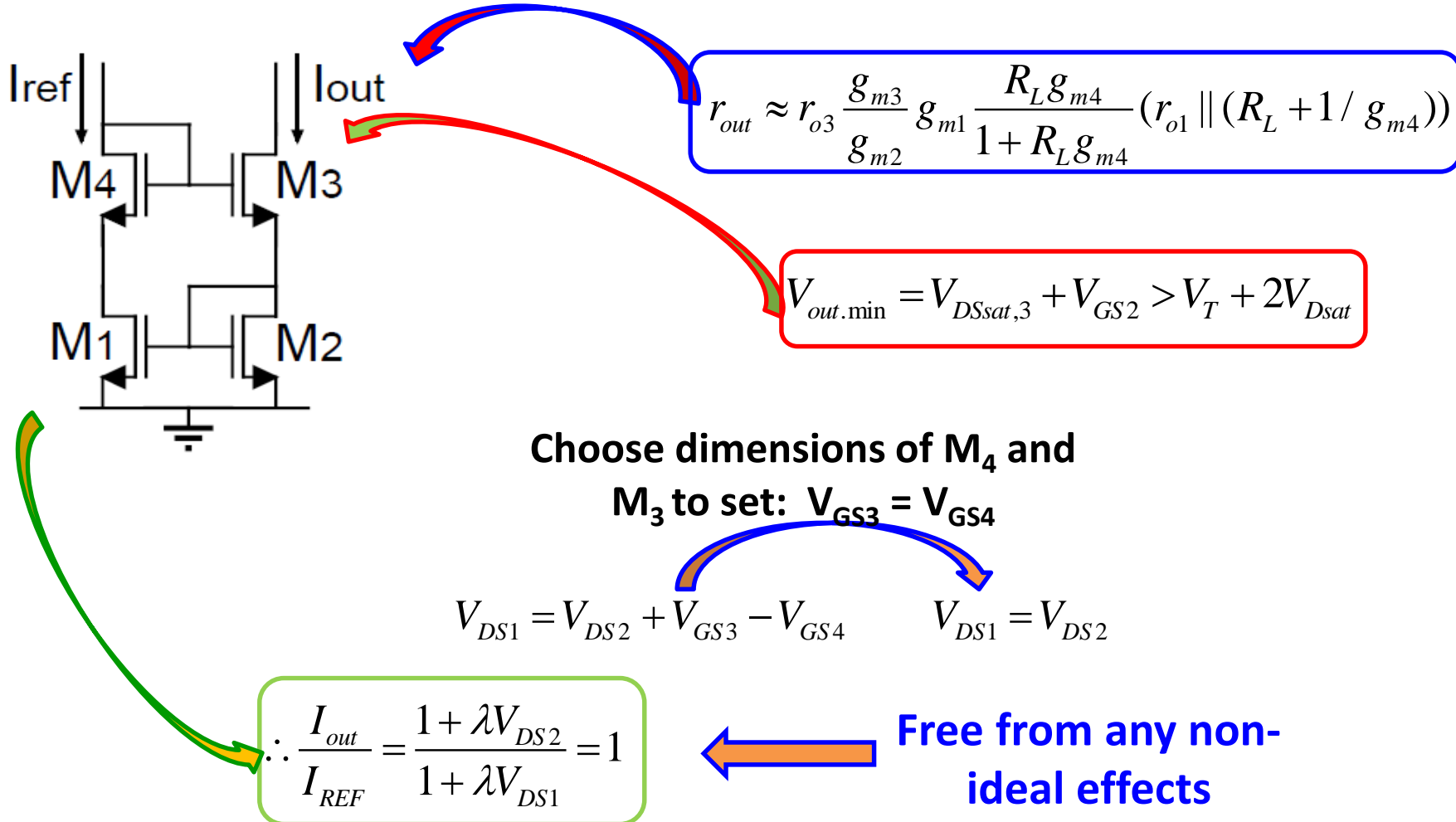
$$\frac{I_{out}}{I_{REF}} = \frac{1 + \lambda V_{DS2}}{1 + \lambda (V_{DS2} + V_{GS3})}$$

R_L is reference current generator impedance

The dependence on V_{DS2}

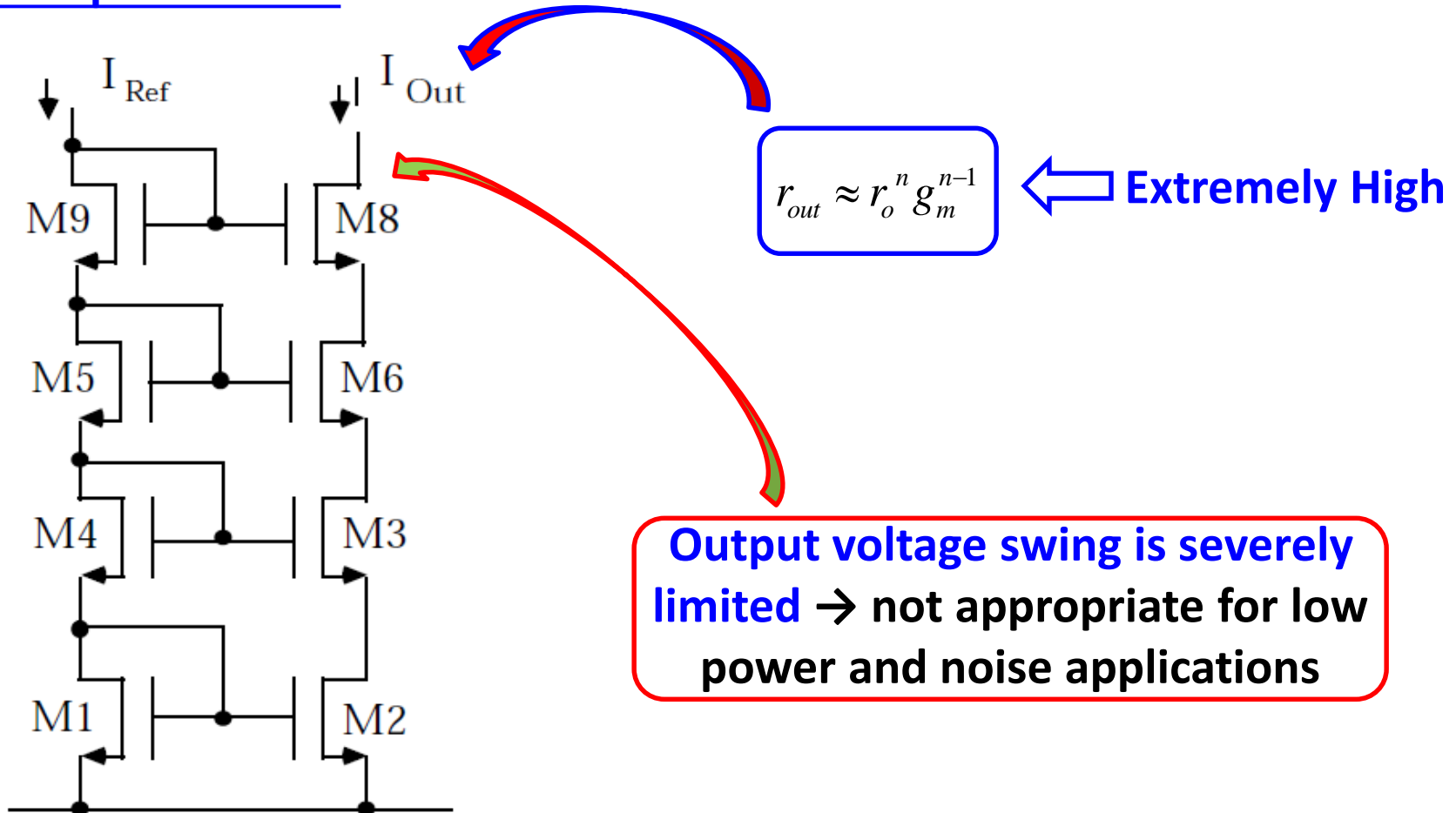
Current Mirror Configurations (contd.)

- Improved Wilson Current Mirror



Current Mirror Configurations (contd.)

- Multiple Cascode

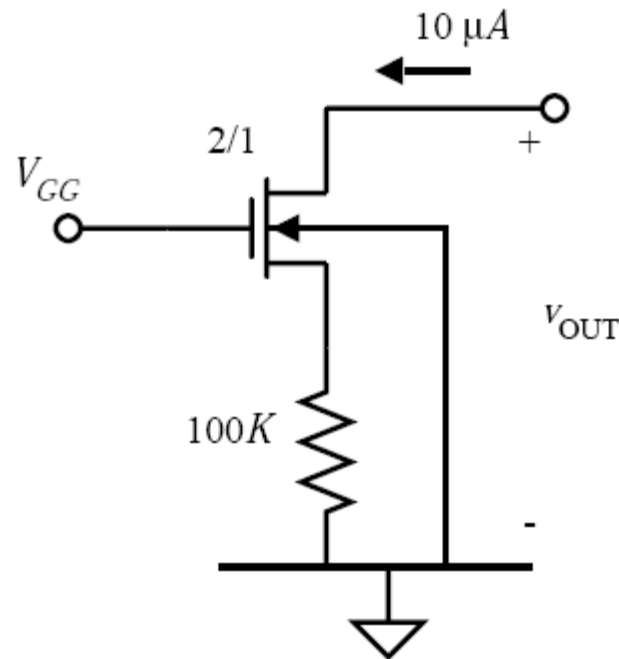


Current Mirror Configurations (contd.)

Configurations	Current Ratio	Output Swing	Output Impedance
Simple	$\frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}}$	V_{DSsat}	$\frac{1}{g_m} \parallel r_o$
Cascode	1	$2V_{DSsat} + V_T$	$r_o^2 \cdot g_m$
Triple Cascode	1	$3V_{DSsat} + 2V_T$	$r_o^3 \cdot g_m^2$
Wilson	$\frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}}$	$2V_{DSsat} + V_T$	$r_o^2 \cdot g_m$
Improved Wilson	1	$2V_{DSsat} + V_T$	$r_o^2 \cdot g_m$

Example-1

Following figure illustrates a source-degenerated current source. Calculate the output resistance at the given bias current by using the following model parameter: $\mu_n C_{ox} = 110 \mu\text{A}/\text{V}^2$, $\lambda = 0.04$ ($L=1 \mu\text{m}$) or 0.01 ($L=2 \mu\text{m}$) /V, $2|\phi_F|=0.7$, $\gamma=0.4 \text{ V}^{1/2}$



Example-1 (contd.)

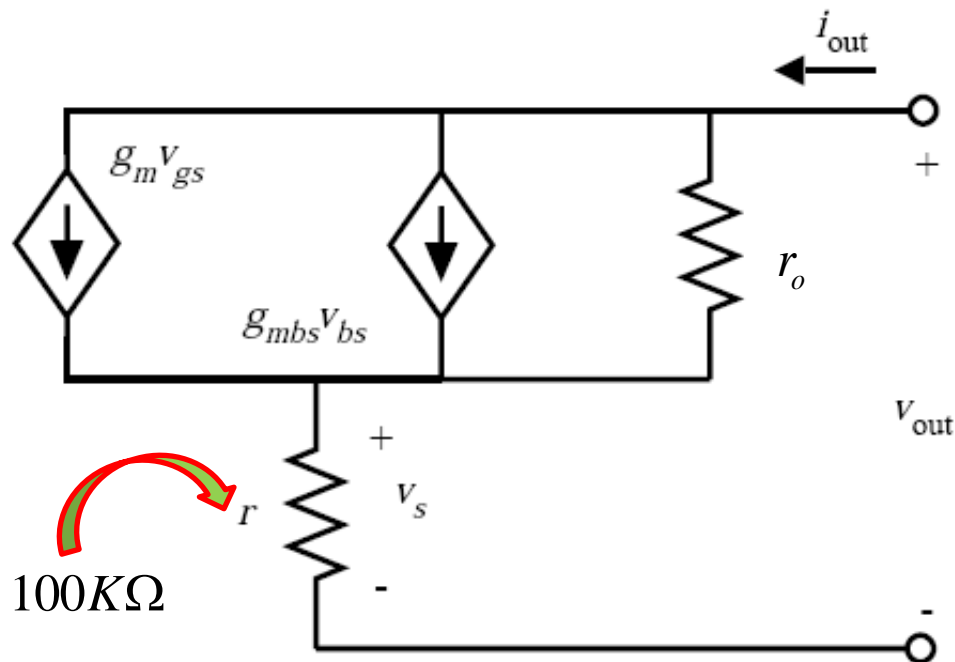
The dc terminal conditions are:

$$I_D = 10\mu A$$

$$V_S = I_D * R = 10 * 10^{-6} \times 100 * 10^3 = 1V$$

$$V_{SB} = V_S$$

Now the small signal model of the circuit is:



Simplification gives:

$$r_{out} = \frac{V_{out}}{i_{out}} = r + r_{out} + [(g_m + g_{mbs})r_o]r$$

Can be approximated to:

$$r_{out} = g_m r_o r$$

Example-1 (contd.)

The device parameters can be computed as:

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \quad \longrightarrow \quad g_m = \sqrt{2 \times 110 \times 10^{-6} \times \frac{2}{2} \times 10 \times 10^{-6}} \quad \longrightarrow \quad \therefore g_m = 66.3 \times 10^{-6}$$

$$g_{mbs} = g \frac{\gamma}{2(2|\phi_F| + V_{SB})^{1/2}} \quad \longrightarrow \quad g_{mbs} = 66.3 \times 10^{-6} \frac{0.4}{2(0.7 + 1)^{1/2}} \quad \longrightarrow \quad \therefore g_{mbs} = 10.17 \times 10^{-6}$$

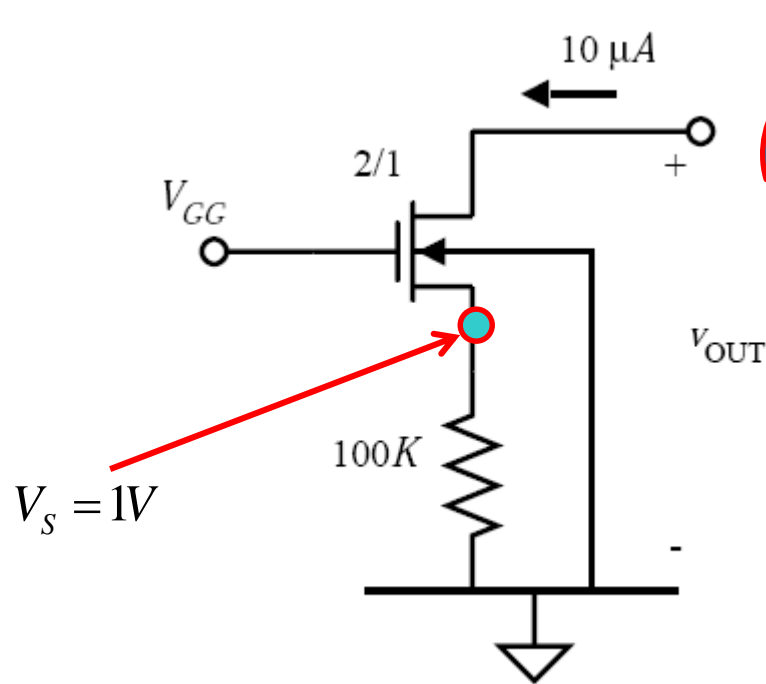
$$r_o = \frac{1}{\lambda I_D} \quad \longrightarrow \quad r_o = \frac{1}{0.04 \times 10 \times 10^{-6}} \quad \longrightarrow \quad \therefore r_o = 2.5 \times 10^6 \Omega$$

Thus: $r_{out} = 100 \times 10^3 + 2.5 \times 10^6 + \left[(66.6 \times 10^{-6} + 10.17 \times 10^{-6}) 2.5 \times 10^6 \right] 100 \times 10^3 = 21.7 \times 10^6 \Omega$

The approximated: $r_{out} = 66.6 \times 10^{-6} \times 2.5 \times 10^6 \times 100 \times 10^3 = 16.65 \times 10^6 \Omega$

Example-2

Calculate the minimum output voltage required to keep device in saturation in example-1. The model parameters: $\mu_n C_{ox} = 110 \mu\text{A}/\text{V}^2$, $\lambda = 0.04$ ($L=1 \mu\text{m}$) or 0.01 ($L=2 \mu\text{m}$) /V, $2|\phi_F|=0.7$, $\gamma=0.4 \text{ V}^{1/2}$



$$I_D = \frac{1}{2} (\mu_n C_{ox}) \left(\frac{W}{L} \right) (V_{GS} - V_T)^2$$

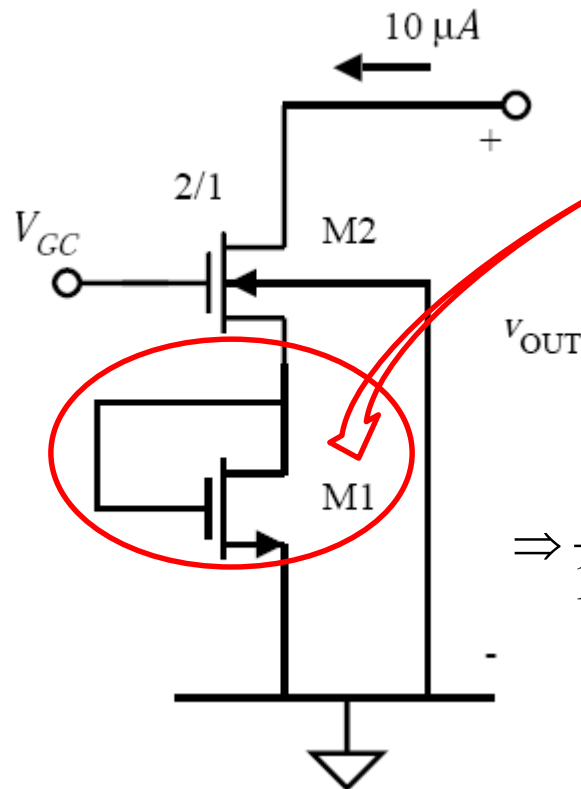
$$\Rightarrow V_{GS} - V_T = \sqrt{\frac{2I_D}{(\mu_n C_{ox}) \left(\frac{W}{L} \right)}}$$

$$\therefore V_{GS} - V_T = 0.302\text{V}$$

$$V_D(\text{min}) = V_S + (V_{GS} - V_T)(\text{min}) = 1 + 0.302 = 1.302\text{V}$$

Example-3

Using the Cascode circuit shown below, design the W/L of M1 to achieve the same output resistance as the circuit in example-1. Ignore body effect.



This device should provide 100K impedance to maintain overall same output resistance

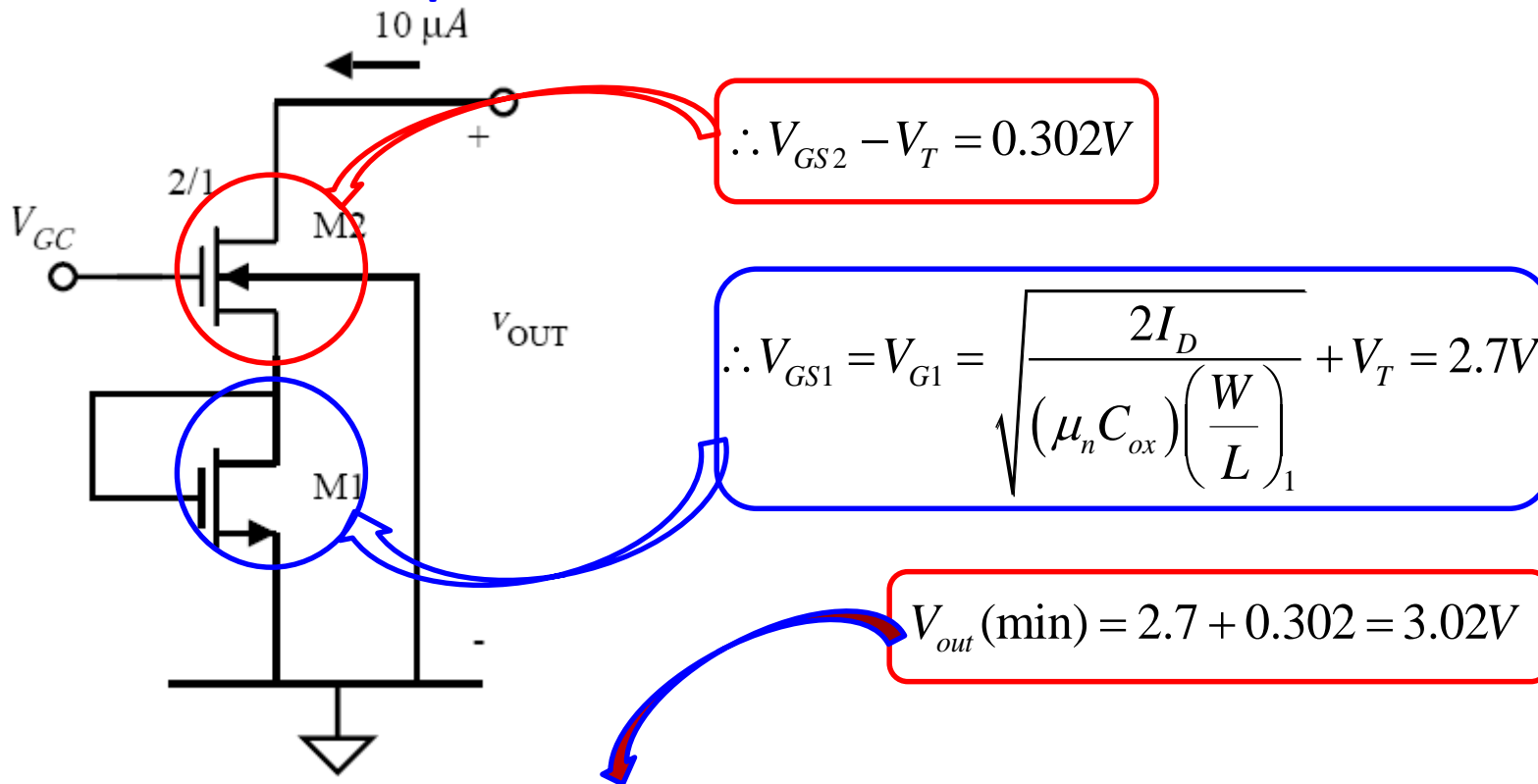
$$r_{o1} = 100 * 10^3 = \frac{1}{g_{m1}}$$

$$\Rightarrow \frac{1}{100 * 10^3} = g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_D} \rightarrow \left(\frac{W}{L}\right)_1 = \frac{1}{22}$$

Note that the terminal conditions of M2 must change to support the large gate voltage required for M1

Example-4

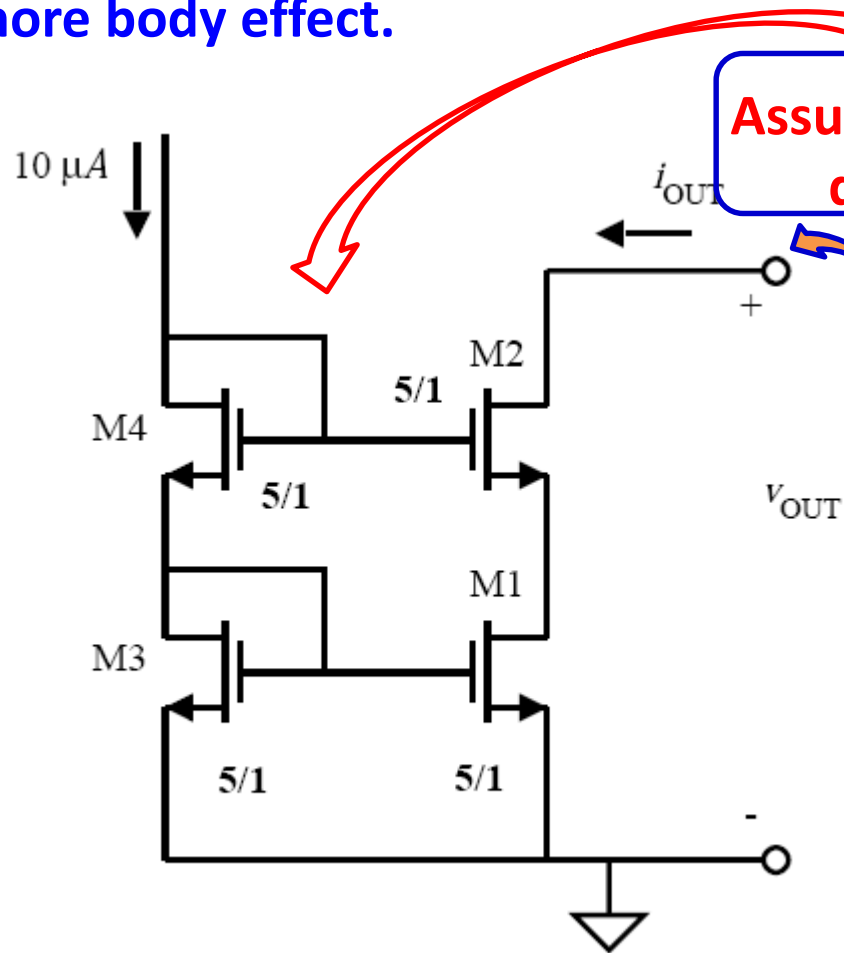
Now calculate the minimum output voltage required to keep the devices in saturation in example-3.



The minimum output voltage for circuit in example-1 is lower than the minimum output voltage for circuit in example-3, therefore is a better choice for low voltage applications

Example-5

Calculate the output resistance, while maintaining all the devices in saturation, for the circuit given below. Assume that I_{out} is actually $10\mu A$. Ignore body effect.



Assume a near perfect current mirror (all devices have $10\mu A$ drain current)

$$r_{out} = \frac{v_{out}}{i_{out}} = r_{o1} + r_{o2} + [g_{m2}r_{o2}]r_{o1}$$

Example-5 (contd.)

$$r_{out} = \frac{v_{out}}{i_{out}} = r_{o1} + r_{o2} + [g_{m2}r_{o2}]r_{o1}$$

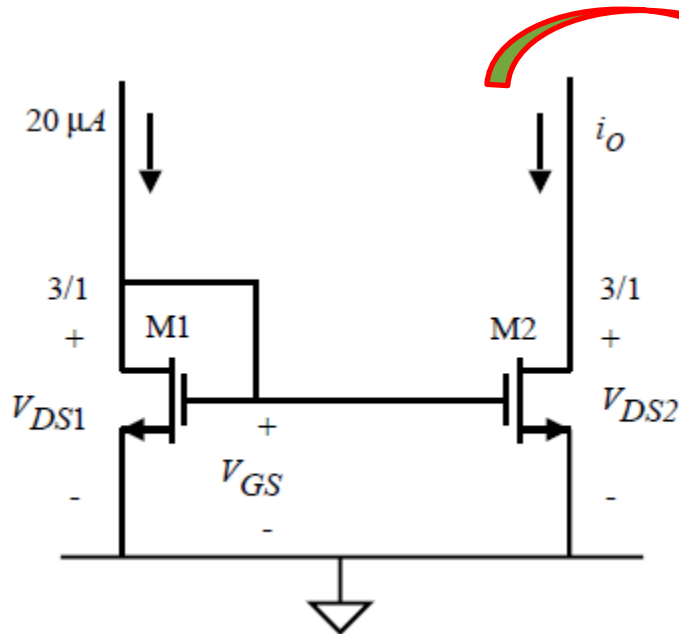
$$r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = 2.5 * 10^6 \Omega$$

$$g_{m2} = \sqrt{2(\mu_n C_{ox}) \left(\frac{W}{L}\right)_2 I_D} = 104.9 * 10^{-6}$$

$$\therefore r_{out} = 2.5 * 10^6 + 2.5 * 10^6 + [104.9 * 10^{-6} \times 2.5 * 10^6] 2.5 * 10^6 \approx 661 * 10^6 \Omega$$

Example-6

Consider the simple current mirror given below.



Over process, the absolute variations of physical parameters are as:

Width Variation $\pm 5\%$

Length Variation $\pm 5\%$

V_T variation $\pm 5\%$

$\mu_n C_{ox}$ Variation $\pm 5\%$

Assuming that the drain voltages are identical, what is the minimum and maximum output current measured over the process variations given above. The model parameters: $\mu_n C_{ox} = 110 \mu\text{A}/\text{V}^2$, $\lambda = 0.04$ ($L=1 \mu\text{m}$) or 0.01 ($L=2 \mu\text{m}$) / V , $2|\phi_F|=0.7$, $\gamma=0.4 \text{V}^{1/2}$

Example-6 (contd.)

We know:

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2$$

K

$$\Rightarrow V_{GS} = \sqrt{\frac{2I_D}{K \left(\frac{W}{L}\right)}} + V_T$$

- Assuming equal V_{GS} for both the transistors, we can express the output current as:

$$i_o = \frac{1}{2} K_2 \left(\frac{W}{L}\right)_2 \left(\sqrt{\frac{2 \times I_{Ref}}{K_1 \left(\frac{W}{L}\right)_1}} + V_{T1} - V_{T2} \right)^2$$

- We can deduce from this equation that the minimum and maximum of output current will happen under respective following conditions.

	K_1	K_2	$(W/L)_1$	$(W/L)_2$	V_{T1}	V_{T2}
i_o (min)	Max	Min	Max	Min	Min	Max
i_o (max)	Min	Max	Min	Max	Max	Min