

===== Part A =====

1. Let L be a set of strings over $\{0, 1\}$ which
 * do not contain the substring "00" in the start or in the middle (it may appear at the end
 — so, "00" is itself in L , but 000 is not in L)
 * do not end with "01"
 Show that L is regular.

2*. Construct an NFA that accepts all strings over the alphabet $\{a,b\}$ that contains at most one occurrence of aa . The string aaa has two occurrences of aa . Prove correctness (write only the level-1 claims).T

3*. Prove that this language is regular: $\{0^k u 0^k : k \geq 1 \text{ and } u \text{ belongs to } \{0,1\}^*\}$.

===== Part B =====

4. Define $\text{Switch1}(L) = \{ \text{flip every odd bit of } w : w \text{ in } L \}$. Show that regular languages are closed under Switch1 . *Hint: Since L is regular, there is a DFA D accepting L . Start with that D . If 110010 is in L , then 011000 is in $\text{Switch1}(L)$.*

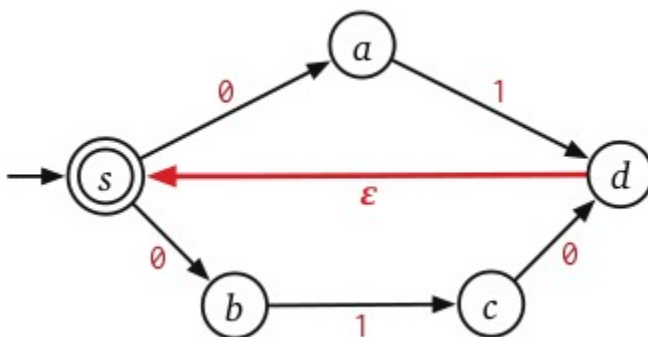
5. Define $\text{Switch2}(L) = \{ w \text{ such that } L \text{ contains } w' \text{ where } w' \text{ is a version of } w \text{ in which every alternate 1 is switched to zero, starting from the first 1} \}$. For example, if 10101010 is in L , then 00100010 is in $\text{Switch2}(L)$. Show that regular languages are closed under Switch2 .

6!. Define $\text{Switch3}(L) = \{ w \text{ such that } L \text{ contains } w' \text{ and } w \text{ is a version of } w' \text{ in which every alternate 1 is switched to zero, starting from the first 1} \}$. Show that regular languages are closed under Switch3 .

7*. $\text{Zerofy}(L) = \{0^{|w|} : w \text{ is in } L\}$ which are unary languages over $\{0\}$. Prove that the regular languages are closed under Zerofy .

===== Part C =====

8* (easy). Construct an equivalent DFA for this NFA. *Hint: Create the states of the DFA as you need (by following the transitions), instead of first creating one for each subset of states.*



A table like this will be helpful. You need not show the transitions going to the $\{\}$ (empty set) state, as long as you mention that “transitions that go to the stop state $\{\}$ are not shown”. Verify that the DFA is correct by taking some examples.

q : A state of DFA / A subset of NFA states	Is this a final state of DFA?	d(q,0)	d(q,1)